TEACHING FOR MATHEMATICAL LITERACY IN SECONDARY AND HIGH SCHOOLS IN LESOTHO: A DIDACTIC PERSPECTIVE

By

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TO WHOM IT MAY CONCERN

This is to certify that I have in my personal capacity, edited the Ph.D. thesis of Mrs. Fungai Shava and can to be best of my knowledge declare it free from grammatical errors.

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DECLARATION

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Fungai Munashe Mavugara-Shava

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DEDICATION

This work is dedicated to my nieces and nephews

and

to all my students, past and present.

ABSTRACT

The main purpose of this study is to inquire, from a didactical perspective, into the question of teaching mathematics for mathematical literacy in secondary and high schools in the district of Maseru, Lesotho. In the study, mathematical literacy and didactical practices relating to mathematics are viewed as related variables that directly impact upon each other. In order to appropriately place the concept of didactical practices in school mathematics education, the study engages support from literature to explore a range of related areas in mathematics education and in mathematical literacy. These areas include, amongst other factors, aspects such as: the position of mathematics in education, the role, meaning and neighbours of mathematics education, and the psychological theories and philosophies that influence trends in didactical practices related to mathematics.

In the study, mathematical literacy itself is defined from different perspectives. In the light of these definitions, the study views mathematical literacy as the individual's aggregate of mathematical skills and knowledge that empowers the individual to participate meaningfully and make well-founded mathematical judgements in a society that is imbued with technology.

Didactical practices and the nature of mathematics that are purported to inculcate mathematical literacy in learners are discussed, in the study, to serve as a premise on which the teaching of mathematics, for mathematical literacy in secondary and high schools in the district of Maseru, is investigated.

The investigation itself seeks to establish the current didactical practices relating to mathematics, which are employed in secondary and high schools in the district of Maseru, Lesotho, and to determine the extent to which these didactical practices correspond to and correlate with indicators of teaching mathematics for mathematical literacy. The study further examines whether the nature (content, objectives, and recommended didactical practices relating to mathematics) of the mathematics curriculum offered in the district of Maseru, concurs with that recommended in literature on teaching mathematics for mathematical literacy.

In conclusion, the investigations of the study culminate in assessing which didactical practices relating to mathematics still need to be improved, embraced, or redefined.

Recommendations based on the findings of the study include: the use of open-ended problem solving techniques, real-life problem investigations, and the use of projects as a didactical approach. Other recommendations are: themes across the school curriculum should be unified, real-life data should be used in statistics and probability, and mathematics problems should encompass actual, real-life problems rather than contrived problems related to real life situations.

OPSOMMING

Die hoofdoel met hierdie navorsing is om vanuit 'n didaktiese perspektief die vraag na die onderrig van wiskunde vir wiskundige geletterdheid in Lesotho se sekondêre en hoër skole na te vors. Wiskundige geletterdheid en wiskundig didaktiese praktyke word in hierdie navorsing as verwante veranderlikes beskou wat direk op mekaar inspeel. Ten einde didaktiese praktyke in wiskundeonderrig in skole toepaslik te

plaas, verkry hierdie navorsing ondersteuning uit die literatuur om ’n reeks verwante gebiede in wiskundeonderrig en wiskundige geletterdheid te ondersoek. Hierdie areas sluit onder andere aspekte soos die volgende in: wiskunde se posisie in onderrig, die rol, betekenis en genote van wiskundeonderrig, die psigologiese teorieë en filosofieë wat tendense in wiskundige didaktiese praktyke beïnvloed.

Wiskundige geletterdheid self word in hierdie navorsing vanuit verskillende perspektiewe gedefinieer. Die navorsing beskou wiskundige geletterdheid in die lig van hierdie definisies as die individu se totale wiskundige vaardighede en kennis wat hom/haar bemagtig om betekenisvol deel te neem en goedgefundeerde wiskundige oordele aan die dag te lê in ’n samelewing wat van tegnologie deurdrenk is.

Die navorsing bespreek didaktiese praktyke en die aard van wiskunde wat na bewering wiskundige geletterdheid by leerders inskerp. Die bedoeling is dat dit dien as ’n vertrekpunt van waar die onderrig van wiskunde vir wiskundige geletterdheid in Lesotho se sekondêre en hoërskole ondersoek kan word.

Die ondersoek self probeer vasstel wat die huidige wiskundig didaktiese praktyke is wat in sekondêre en hoërskole in Lesotho in gebruik is. Dit probeer ook vasstel in watter mate hierdie didaktiese praktyke met indikatore om wiskunde vir wiskundige geletterdheid te onderrig, ooreenstem en korreleer. Die navorsing ondersoek verder of die aard (inhoud, doelwitte en aanbevole wiskundig didaktiese praktyke) van die wiskunde-kurrikulum wat in Lesotho se sekondêre en hoërskole aangebied word ooreenstem met dit wat in die literatuur oor die onderrig van wiskunde vir wiskundige geletterdheid aanbeveel word, ooreenstem.

Ten slotte loop die navorsingsondersoek uit op die assessering van watter wiskundig didaktiese praktyke nog verbeter, aanvaar of geherdefinieer moet word. Aanbevelings, wat op die bevindings van die navorsing gebaseer is, sluit in: die gebuik van oop probleemoplossingstegnieke, ondersoeke na probleemstellings in die werklike lewe en die gebruik van projekte as ’n didaktiese benadering. Ander aanbevelings is: temas in die skool-kurrikulum behoor verenig te word, data uit die werklike lewe behoort in statistiek en waarskynlikheid gebruik te word en wiskunde

probleme behels werklike probleme uit die werklike lewe eerder as versinde probleme wat met situasies uit die werklike lewe verband hou.

TABLE OF CONTENTS

CHAPTER 2

SCHOOL MATHEMATICS AND MATHEMATICAL EDUCATION

CHAPTER 3

MATHEMATICAL LITERACY AND DIDACTICAL PRACTICES

CHAPTER 4

RESEARCH METHODOLOGY AND INSTRUMENTS

CHAPTER 5

DATA ANALYSIS: RESULTS AND THEIR QUALITY

CHAPTER 6

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

TABLES

FIGURES

CHAPTER 1

PROBLEM STATEMENT AND EXPOSITION OF STUDY

1.1 INTRODUCTION

Whether it is conceded or not, history has ceaselessly shown that mathematics permeates the whole of our world, society, and human activities. In fact, authors such as Becker and Shimada (1997:4), Begle (1970:10), Bell (1978:6), Bochner (1966:v), Dowling (1998:1-23), Howson and Kahane (1990:20), Hoyles, Morgan, and Woodhouse (1999:48-74), ICMI (1979:234), Murtly, Page, and Rodin (1990:xiii, 3), and Siegel (1988:75) all affirm this assertion. In particular, mathematics is an integral part of people's cultural, social, economic, and technological environment (Dowling 1998:xiii-xv, Tymoczko 1998:xiii). To this effect, Kline (1985:v) writes:

> Major phenomena of our physical world are not perceived at all by the senses …, realities of our physical world are known through the medium of mathematics … mathematics reveals … major phenomena of our world.

Holt and Majoram (1973:v) emphatically point out that no person worth his or her salt "dares to be innumerate". In a similar perspective, Restivo, Bendegen, and Fischer (1993:13, 113) describe mathematics education as "a collective effort to study and shape the relationship between human beings and mathematics".

1.2 ORIENTATION AND BACKGROUND TO THE STUDY

1.2.1 Orientation

The focus of this study is on the didactical practices relating to mathematics that enhance mathematics literacy in secondary and high school students in the district of Maseru, Lesotho. Most common didactical practices relating to mathematics mainly involve presenting information to the class by chalkboard and overhead projector and giving assignments to individual students or the whole class. These didactical practices leave much to be desired. In particular, with the inevitable technological advancement and globalisation, societies place an irresistible pressure on mathematics education to turn out mathematically literate citizens. These citizens should be confident in mathematics and able to competently use mathematics in real contextual situations. Didactical practices relating to mathematics that place emphasis on the acquisition of facts, axioms, theorems, skills, procedures, and processes in some way removes mathematics from the contexts in which mathematics arises and thrives. Such practices are increasingly becoming obsolete, unproductive, and inappropriate in a world that is imbued with technology (Avital 1983:276, Cangelosi 1994:1-4, Hirsch 1992:v, National Council of Teachers of Mathematics (NCTM) 1991:1-3, Neyland 1994:3, Orton and Wain 1994:212, Siemon 1983:250, Sitia 1983:274).

Borasi (1992:1-3), Hoyles, Morgan, and Woodhouse (1999:6-7), Kline (1985:v), Tanner and Jones (2000:104-108), and Zeitz (1999:ix-xi) all purport that this pressure may be attributed to the fact that mathematics gives one knowledge and mastery of major areas of our physical world and of quantitative aspects in our daily social life. In the face of these influences, didactical practices relating to mathematics in Maseru, Lesotho are studied in this research in order to find out whether the practices meet the pressing need of society (both locally and globally) that requires mathematics education to turn out mathematically literate citizens.

The requirement placed on mathematics education is based on both its development as a body of knowledge and on its utilitarian value. Siegel (1988:75) concurs with Kline (1985:v) on the utility of mathematics. Actually, Siegel takes the notion further and posits that mathematics is in essence “a service subject” although, to most mathematicians, its attraction has frequently been the sheer beauty of the subject without regard for its applications. However, the utilitarian value of mathematics itself is well documented. For instance, Grobler (1998:1) lists the various uses of mathematics in different fields of knowledge. The continual change in technology, the increased plethora of areas in which to apply mathematics, and the growth of mathematics itself as a body of knowledge elicit changes in didactical practices. For instance, literature posits that the teaching of mathematics at secondary and high schools should afford an individual the acquisition of mathematics for intelligent citizenship since it teaches one to think and to use mathematics in various real life problems (Bondi 1991:1, Howson 1988:33-34). It is, therefore, maintained that mathematics offers one the ability to reason from given data, to deal with probabilistic situations, to think algorithmically and discretely, and, in general, to participate in decisions involving quantitative matters in an informed and intelligent way.

Factors pointed out here, together with other expectations in modern society, urge that instruction in mathematics must focus on training people to be mathematically literate: people whose mathematics is meaningfully integrated into real-life contexts (Bondi 1991:I, Woodbury 1998:303). In fact, Fraser (in Neyland 1994:173) affirms this by further pointing out that students need to be instructed in mathematics so as to "own" mathematics. Fraser maintains that students need to view themselves as competent in the use of mathematics, and that they need to appreciate its power as a form of communication, truly to regard it as a human activity. Then they need to go on to explore its integral role across their school curriculum. Fraser, here, does not deprive mathematics of its valued importance in and by itself. She is merely indicating that mathematics is virtually impossible to divorce from other subjects and, hence, from other areas of one's knowledge. Fraser is, in a way, urging for didactical practices in mathematics that produce mathematically literate people and individuals whose mathematics knowledge is integrated with realities in life and in other school subjects.

In similar collocations, the same aspect is pointed out by many scholars, such as: Bell (1983:252), Biehler (1983:291-293), Burkhardt (1983:284), Miwa (1983:294), Niss (1983:247), Wheeler (1983:290), and Yeluda (1993:89). Further, Mokoena (in AMESA 1998:33-45) also gives a succinct description of how concept mapping within mathematics itself can enhance meaning and understanding in learners that, by implication, renders one to be mathematically literate.

Due to previously stipulated causes, many countries initiated a major mathematics curriculum review with the objective of shaping the instruction of mathematics to the extent that learners may meaningfully "own" mathematics. Booss and Niss (1979), House and Coxford (1995), and other authorities advance arguments along the same lines. In the USA in particular, the National Council of Teachers of Mathematics (NCTM) (1989:v) maintains that instruction of mathematics "must be significantly revised." They hoped that this revision would result in mathematics education that is capable of producing people who are "mathematically literate both in a world that relies on … computers and in a world where mathematics is rapidly … applied in diverse fields" (NCTM 1989:1).

To this end, there is ample documentation of work from other countries to affirm what the next section portrays.

1.2.2 Background to the study: A synopsis of mathematics education in some countries.

The quest for meaningful mathematics education is of central concern in many countries today (Burkhardt (1981), House and Coxford (1995), Mohyla (1984), the USA National Council of Teachers of Mathematics ((a)1989, (c)1992, (d)1993), and Zweng Green, Kilpatric, Pollak, and Suydam (1983). The type of mathematics education required is that which produces mathematically competent and knowledgeable, creative, critical learners who are able to lead productive and self-fulfilled lives (South Africa Department of Education Government Document (2002:4, 9). As a result, many countries are, currently rising to the challenge of shaping classroom instruction in mathematics in schools in order to produce students who have a meaningful knowledge of mathematics and who are mathematically literate (Yahoo, OECD, PISA countries, 2001 home page). Among these countries are the following: the USA, Canada, Vietnam, South Africa, Hungary, and the United Kingdom.

As has been mentioned earlier, in the USA, the National Council of Teachers of Mathematics (NCTM, 1989:1) maintains that instruction of mathematics in schools must be significantly revised. The revision is intended to shape mathematics education so that it can produce people who are mathematically literate in a world that is embedded in and embellished with technology. To this end, the NCTM has curriculum standards for mathematics (for mathematical content, for teaching, for assessment) set for different levels of school mathematics. For instance, in secondary schools, standards for mathematics content such as number sense, symbolism and algebra, geometry, functions, discrete mathematics, probability, and statistics are given in terms of mathematical competences that students are intended to acquire as they interact with the learning environment (Hirsch 1992:28-63, NCTM Crossroads in Mathematics 2001:6-8, NCTM 1989:123-184). This change in the emphasis of mathematical content has naturally necessitated a reshaping of the whole pedagogy of mathematics (Hirsch 1992:vi). Thus, reforms in mathematical content also triggered reforms in didactical practices (Hirsch 1992:6-16, NCTM 1991:104-160, NCTM 1989:189-244).

In Canada, Geoffrey Roulet (1998:2) points out that the Ontario Mathematics Coordinators Association (OMCA) "… call on teachers to develop mathematics curricula in which pupils actively construct their own personal mathematical understanding through investigating, conjecturing, testing hypothesis and the sharing and discussing of ideas."

The proposition, here, sounds, in many ways similar to that encapsulated in the USA NCTM curriculum reform in Standards for School Mathematics. Thus literature, here, reveals that mathematics education is being shaped in American countries so as to meet the demands from changes in today's society.

On the teaching and learning of mathematics in Vietnam, Dat Do (2001:3) acknowledges that curriculum reforms are also taking place in that country. The mathematics curriculum has undergone a number of adjustments and has been made "more progressive". In particular, didactical practices by and large:

- … concentrate on learners, … emphasise active learning, … develop pupils' initiative and creativity,

- provide applicable knowledge and skills necessary to their life in the community and future, and

- encourage thinking and individual learning, group work, cooperative learning, … problem solving, constructivism, educational games, investigations, … improvement of learning environment, … meaning in learning, pupils to study more actively, confidently and creatively (Dat Do 2001:5-6).

Dat Do's report concurs with most reforms in mathematics curricula in other parts of the world. For instance, the International Baccalaureate Middle Years Programme (IBMYP) that is offered in many international schools of the world, provides mathematics programmes that set out "to give students an appreciation of the usefulness, power and beauty" of mathematics by considering it as a means of "modelling the real world" and other real contextual physical situations (IBMYP 1995:5). In fact, the IBMYP for mathematics places emphasis on "understanding" in a context of interest and stresses "interrelationship of knowledge, skills and attitude" in learning mathematics. Different didactical approaches are encouraged and adopted, viz. portfolios, projects, games, investigations, open and closed problem solving, and computer and calculator work. At the same time, students are encouraged to "investigate mathematics independently, to explore relationships within the subject and to recognise and exploit the interaction between mathematics and other subjects" (IBMYP 1995:55).

In South Africa as in most parts of the world, mathematics education has also undergone reform. As a subject, it is seen as the "construction of knowledge that deals with qualitative and quantitative relationships of space and time" and has both utilitarian and intrinsic value (S A Government document on Mathematical Literacy, Mathematics and Mathematical Sciences 1997 (a):1). Here mathematics is viewed as "a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves" (Department of Education, South Africa 2000:21).

In this context, didactical practices relating to mathematics are envisaged as a means of effecting specific competences in mathematics and, at the same time, instruction in mathematics is seen as a means of yielding specific mathematical outcomes in learners using the school mathematics learning environment. The uppermost desired outcomes of mathematics education are thus seen to be a demonstration of understanding of mathematical concepts, procedures, and other related skills as well as the development of critical thinking and analysis of relationships (S.A. Pilot Model for Standardisation in the Senior Phase, Section Clearing Area Pack – Mathematics, Mathematical Literacy, Mathematical Sciences 2000:12).

Tibor Szalontai (2001:1-5) of the Institute of Mathematics and Informatics in the College of Nyiregyhaza, Hungary, outlines the "good mathematics teaching", "methodology", and "practice of lessons" used in Hungary. Szalontai points out that mathematics education in Hungary has attracted international interest, mainly because the practice has been successful since it is rooted in the reputed work of Hungarians such as George Polya, Zoltan Dienes, Tamas Varga, and Istvan Lakatos. Szalontai maintains that some of the main features of the reputed practice include, amongst others, the following factors:

> Whole class activity and individual work which is followed by whole class discussion: report, reasoning, arguing, debate, feedback, agreement, feedback, self-correction, praising, evaluation, teachers' extra comments or extension, spoken and written abilities, clear mathematical language, frequent mental calculations, … questioning, investigations, … realistic problems, models internalisation, … conceptual thinking, … associational, … reflectional, … problem oriented theories of learning (Szalontai 2001:1-2).

Szalontai, here, portrays a realistic mathematics classroom situation (though in Hungary) that could be observed in most parts of the world. In fact, Szalontai (2001:1) indicates that, in the United Kingdom, Professor David Burghes of the Centre for Innovation in Mathematics Teaching (CIMT) at the University of Exeter has built the "Experimental Mathematics Enhancement Programme" (MEP) for secondary stages on Hungarian didactical approaches to mathematics.

It is necessary to discuss mathematics education as is envisaged in the UK and, in particular, by the University of Cambridge Local Examination Syndicate (UCLES), which, in actual fact, is the basis on which Lesotho and other Commonwealth countries build their mathematics education. In fact, for its overseas candidates, UCLES has offered the Cambridge Overseas School Certificate (COSC) at Ordinary Level and Advanced Level for many years until, more than a decade ago, it phased out the COSC to embrace the International General Certificate of Secondary Education (IGCSE). According to the explanatory booklet on the University of Cambridge's International General Certificate of Secondary Education (IGCSE 2001:1), UCLES has "provided international examinations of the highest quality based on contemporary curriculum and assessment" since 1863. In fact, UCLES points out that the Syndicate "remains at the forefront of research at a time of social, educational and technological change" and always seeks to incorporate "the latest developments in education" by improving the "quality of education and its suitability for each and every student" (Ibid). In essence, the aim of UCLES with the IGCSE is to:

- support modern curriculum development,
- promote international understanding,
- encourage good teaching practice, and
- set widely recognised standards (IGCSE Syllabus 2001:2).

In didactical approaches, UCLES, amongst other things, encourages:

- the development of oral and practical skills,
- an investigative approach,
- the initiative needed to solve problems,
- the application of skills, knowledge, and understanding, and
- the ability to undertake individual projects and to work as a part of a team (IGCSE 2001:4).

However, all IGCSE syllabuses follow the same pattern, with most subjects divided into core and supplement syllabuses (extended syllabus). The core is aimed at candidates in the lower range of ability whereas the core and supplement together comprise the extended option, which is intended for candidates of higher ability.

Due to differing needs in the countries served by UCLES and due to the varied competence of teachers to undertake the school-based assessment that is required in Coursework, the IGCSE offers two courses in mathematics: mathematics syllabus without coursework and mathematics syllabus with coursework. Both syllabuses have a core and an extended option to cater for candidates of different abilities.

It should be pointed out that the components in coursework help candidates to develop competence in using mathematics in context and in a practical way, and sometimes across the curriculum. They thus enable candidates to solve real-world problems independently. In this light, projects, modelling, and investigations naturally form part of the IGCSE mathematics with coursework (Cambridge International Examinations, IGCSE Mathematics Syllabus for examinations in coursework 2003:2). Hence, these elements meet the requirements for teaching mathematics for mathematical literacy that are discussed in chapter 3 of this study.

At this juncture, it is apt to point out that Lesotho is no exception in the pursuit of relevant and meaningful mathematics education. Hence, the concerns of this study focus on the problem of didactical practices relating to mathematics in secondary and high schools particularly in the district of Maseru, Lesotho. It also focuses on whether the mathematics education provided produces mathematically literate people in a society where all local educational issues need to rise to global expectations.

1.3 STATEMENT OF PROBLEM

The problem of school mathematics education where learners actively construct their own personal understanding through investigations, conjectures, testing hypotheses, and other relevant interactions with mathematically imbued situations is crucial in a world where technology has permeated most areas of life (Amit, Hillman, and Hillman 1999:17, Goldstein, Mnisi, and Rodwell 1999:83-85, Tanner and Jones 2000:71-73). Nevertheless, as Roulet (1998:2) points out, though leaders of the teaching profession may call "for change in mathematics curricula and pedagogy and government policies (may reflect) this thinking", still school teachers, for one reason or another, do not always endorse the recommended didactical practices that go with the change. However, research in mathematics education may help to keep account of what actually goes on in relevant points of enquiry.

Thus, the focus of this study is on didactical perspectives in the teaching of mathematics for mathematical literacy in secondary and high schools in the district of Maseru, Lesotho. As part of the quest to clarify the dimensions of the problem of the research study, we need to consider factors that steer and determine the didactical practices as well as that of mathematics education in Lesotho. These factors include, amongst others, the following: political decrees and policies for education in Lesotho, the needs of the society, and developments in the teaching of mathematics itself as a subject world-wide. We can deduce this from policies that govern mathematics education in Lesotho. Currently, broad goals and policies for the educational system in Lesotho, which, in turn, govern and direct the didactical practices of mathematics itself touch upon factors such as the following:

- Everyone should be provided with the opportunity to develop competencies necessary for personal growth (Education Sector Development Plan 1992:3).

- Individuals should be provided with appropriate … skills to ensure the country's socio-economic development (Education Sector Development Plan 1992:4).

- Education should provide opportunities for literacy and numeracy (Education Sector Development Plan 1992:5).

- Educational programmes should incorporate cultural values (Education Sector Development Plan 1992:5).

- Emphasis should be places on education for life and education should offer relevant knowledge, skills, and attitudes that foster, among other things, education for the production and development of creative faculties (Lesotho Educational Policy and Localisation 1995:30).

- Secondary Education should equip students with knowledge, attitudes, and skills that enable them to adapt to changing situations (Ibid).

However, according to Shava (1999:9), Lesotho is part of the British Commonwealth countries and, as such, from the very nascency of her education, she inherited the British system of education. Therefore, secondary and high school mathematics in Lesotho have been fashioned upon and tailored to those of the Cambridge Overseas School Certificate examination (COSC). Furthermore, this set-up also means that the syllabuses that have hitherto been followed were basically foreign and left little room for adaptation to local conditions (Lesotho paper at the Nairobi Eastern and Southern Africa Regional Consultation on Education for all 1989:12). Issues regarding the irrelevance of education due to social realities and expectations from parents and society, i.e. that education and literacy should be put to effective use, are forcing Lesotho to examine carefully the education provided to learners (Gay, Gill, and Hall 1995:69, 72, Lesotho Ministry of Economic Planning 1997:169, 171). This is further encouraged by the fact that, for most people, primary, secondary, or high school education is the only education they will receive. Hence, education that is relevant to the needs of the Basotho is considered necessary and long over due. As a consequence, this elicited the current localisation of the Cambridge Overseas School (COSC) examinations. The following are some of the reasons and justification for the decision to localise COSC examinations:

- Localised examinations may lead to coherent and relevant education programmes.

- Programmes must be made to reflect and respond to the needs and circumstances of the country.

- Changes in the UK itself to a new system of curricula and examinations (to meet their own needs) led to a reconsideration of Lesotho's own educational needs (Pule 1995:9).

Currently (in Lesotho), curriculum planners and mathematics subject advisors are translating these educational expectations and policies into instructional practices that will produce the kind of citizen Lesotho expects to be produced at secondary and high school. At the same time, the curriculum planners and subject advisors are expected to take the following into consideration when they prepare instructional curricula materials for different school levels:

- whatever is proposed in the form of syllabus content and methodology, as well as examination procedure, should as far as possible be comparable to and compatible with what obtains in the region and to the rest of the developed world …,

- place emphasis on the teaching of mathematics to meet the needs of the country …,

- give depth of subject content and leave students competent enough to be self reliant,

- reflect the Sesotho context …, and

- adapt the content and style to the local situation (Khati, 1995).

In the light of this background, mathematics education in Lesotho has taken on board new perspectives. In fact, the current mission statement and aims for mathematics education in Lesotho are to:

- provide students with knowledge and skills by enhancing their abilities to think logically and analytically, and

- … promote positive attitudes towards the subject as mathematics provides an investigative environment that stimulates curiosity to investigate and solve problems (Secondary School Mathematics Syllabus 2000).

In this regard, the main themes in teaching mathematics are classified under the following headings:

- knowledge and skills,
- applications and problem solving, and
- appreciation of the environment (Ibid).

Taking all the above factors into consideration, one could summarise and suggest that, by implication, Lesotho seeks to embrace mathematics education that gives students meaningful mathematical knowledge, skills, attitudes, and values. This mathematics education is, amongst other things, intended to be of use in different contextual applications and in problem solving as well as in the appreciation of the environment. Thus, the mathematics education propounded here aims at producing a student who is in every way mathematically competent and literate. In the light of this, it is the task of the researcher to explore didactical practices in the ordinary classroom where the actual didactical scenario of mathematics education in secondary and high schools in Lesotho is located.

As indicated before, the quest for meaningful mathematics education is of central concern in many countries today. Literature shows that many countries are currently rising up to the challenge of shaping the instruction of mathematics in schools in order to produce students who acquire meaningful mathematical skill and are thus mathematically literate in every way (OECD 2001).

Again, as pointed out in one of the previous sections in this chapter, Lesotho is no exception to this change in emphasis in the instruction of mathematics. In fact, the discussion in Section 1.2 implies that the issue of appropriate, meaningful, and relevant mathematics education is pertinent for Lesotho as it is for all other countries. In a way, Lesotho’s mission statement and educational policies cited earlier expect mathematics education to produce mathematically literate citizens who have become adept at meaningful mathematics, which they can competently use in real contextual situations. Furthermore, the references cited also point out that the teaching of mathematics in secondary and high schools requires mathematics teachers to construct and manage learning environments where students develop meaningful mathematical knowledge, skills, attitudes, and values. In the light of these expectations, mathematics education and the didactical practices going with it are of pertinent enquiry in this study. Particular areas that are explored are discussed in the following section of this chapter.

1.4 PURPOSE OF RESEARCH AND OBJECTIVES

In this research study, the major aim is to explore, from a didactical perspective, the question of teaching mathematics for mathematical literacy in secondary and high schools in the district of Maseru, Lesotho. Thus, in the study, mathematical literacy and didactical practices relating to mathematics are viewed as related variables. Mathematical literacy in learners is viewed as a variable dependent on didactical practices (the independent variable) that are used in the classroom. Literature itself (Borg and Gall 1974:364, Caulcutt 1991:169, Gillespie and Glisson 1992:167-176, Ostle and Mensing 1975:165) posits that the dependent variable has a functional relationship with the independent variable. In fact, in the functional relation, a change is effected by the independent variable on the dependent variable.

One can ask many particular, relevant, and pertinent questions in order to explore such a relationship. Nonetheless, in this study, there are four specific questions to be explored, and these are:

1. What are the present didactical practices relating to mathematics in secondary and high schools in the district of Maseru?

2. To what extent do the present didactical practices and mathematics curriculum in Maseru district offer students mathematics education that is necessary for mathematical literacy?

3. Does (content, objectives, and didactical practices) the mathematics curriculum offered in secondary and high schools in Maseru concur with that suggested in literature on mathematical literacy?

4. What didactical practices relating to mathematics (if any) still need to be improved/ embraced/redefined in order to achieve mathematical literacy in students?

From these questions, the following specific objectives are generated in order to explore and meet the general aim of investigation of the study:

1. to determine the actual current didactical practices relating to mathematics in secondary and high schools in Maseru district,

2. to establish the extent to which current didactical practices followed in secondary and high schools in Maseru correspond to and correlate with indicators of teaching mathematics for mathematical literacy as reflected in literature,

3. to examine and assess whether the nature (content, objectives, and mode of assessment) of the mathematics curriculum offered in Maseru's secondary and high schools concurs with that suggested in literature on mathematics education for mathematical literacy, and

4. to assess what didactical practices relating to mathematics in Maseru (if any) still need to be improved/embraced/redefined in order to achieve mathematical literacy in students.

However, in order to carry out and build up a meaningful, valid, and reliable study, appropriate methods and instruments for gathering the relevant data for each question need to be followed. The following section outlines the general methods of research and relevant investigation that the researcher will pursue.

1.5 RESEARCH METHODS

According to literature, research can be done in one of two main approaches: qualitative and quantitative (Bell 1989:4, Best and Kahn 1993:184, Bliss, Monk, and Ogbon 1983, Gillespie and Glisson 1992, Hitchcock and Hughes 1989:24, McMillan and Schumacher 1989:384). The researcher will use both quantitative and qualitative approaches to collect the data required to investigate research questions in this study. Questionnaires, interviews, and documentary analysis are the instruments that will be used.

According to Cohen and Manion (1992:41), procedures and operations that will be followed in carrying out the investigations in the study are "methods and methodologies" of the research. Furthermore, Bell (1992:50) posits that these research methods need to be selected on the basis of whether the methods will be used to collect data that is required to produce a complete picture of reliable and valid research.

1.5.1 Validity and reliability

Operations and procedures that are carried out in order to generate data for the purposes of this research study are important since they influence both the worthiness and dependability of the findings of the research. The worthiness and dependability of findings depend on the validity and reliability of the instruments used to obtain the resulting data and of the findings of the research study.

With regards to reliability, Bell (1992:50-52), Cohen and Manion (1992:272), Frith and MacIntosh (1992:21), Hopkins (1989:81), Nunnally (1964:79), Openheim (1992:144), Popham (1981:58), Singleton, Straits, and McAllister (1988:111), and Wiersma and Jurs (1985:65) all concur that reliable instruments give measures that are consistent, replicable, dependable, precise, and stable. On the other hand, literature actually indicates that validity is a concept that researchers need to take into consideration in the whole process of research. Thus, the method of research, the construction of the research instruments, the recording of the data, and even the analysis stage need to yield valid data (Cohen and Manion 1992:116, 199-203, 253, 278, 317-319, Frith and MacIntosh 1991:19, Hammersley 1987, Hammersley 1986:201, Henerson, Morris and Fitz-Gibbon 1987:132-133, Hopkins 1989:78-79, Lloyd-Jones and Bray 1986:35, Pidgeon and Yates 1974:61-63, Oppenheim 1992:147-148, 160-163, Singleton *et al*, 1988:110-111). As Henerson *et al*, (1987:133) point out, in essence, validity actually indicates how "Worth while a measure is likely to be in a given situation for telling you what you need to know. Validity boils down to whether the instrument (also whole method used) is giving you the true story or at least something approximating the truth."

Therefore, in the following paragraphs, procedures that will be used to collect relevant data that will assist in answering the pertinent questions in this study are discussed.

1.5.2 Target group

There are exactly 225 registered secondary and high schools in all the ten districts (see map in Appendix A) of Lesotho (Lesotho Ministry of Education and Development Plan 1996 (c), Lesotho Ministry of Education List of Schools by District, 2002). Of these, 16 schools are in Botha-Bothe, 50 in Leribe, 25 in Berea, 51 in Maseru, 26 in Mafeteng, 17 in Mohale's Hoek, 12 in Quthing, 11 in Qacha's Nek, eight in Mokhotlong, and nine in Thaba-Tseka. To include the whole population in this study is difficult due to limitations of cost and time of the research, distances between schools, and accessibility due to the mountainous terrain of the country under study. The study targets the secondary and high schools in Maseru. Hence, as is justified and discussed in Chapter 4 of this study, the researcher shall use a representative sample group of five secondary and high schools in and around the city of Maseru (the capital city of Lesotho). The schools are selected by the purposive cluster sampling technique. From each of these five schools, 25 secondary-school students, 25 high-school students and two mathematics teachers as well as the respective mathematics supervisors will be taken into the sample group. Furthermore, Lesotho's two mathematics curriculum developers, one member of the inspectorate for mathematics, and the mathematics subject resource person and advisor will be part of the sample group.

The idea of using a representative group of the population is justified since this fulfils the desired relationship categories between parent population and the representative sample group that is pointed out by Borg and Gall (1974:114-115), Henerson, Morris and Fitz-Gibbon (1987:104), Oppenheim (1992:8, 38, 39-49), Ostle, and Mensing (1975:49-51). These include, amongst others, the existence of a relationship between the research subjects and parent population, the existence of a random choice of sample subjects (though not totally arbitrary), and the use of a cluster selection method for ease of control. Sample and sampling techniques are discussed in Chapter 4.

1.5.3 Instruments

The instruments used to gather data for each research question are discussed at length in Chapter 4. Triangulation will be employed in collecting data for this study. Specifically, the instruments used will include the following: interval scale Likert type questionnaires, interviews, ordinal scale questionnaires (placing given items in rank order on an ordinal scale), and documentary analysis. On the whole, three questionnaires will be administered: the first to students, the second to teachers, and the third to mathematics curriculum planners, the inspectorate, and the mathematics resource person and advisor. Similarly, three sets of interviews will be carried out: the first with students, the second with teachers, and the third with mathematics curriculum planners, the inspectorate, and the mathematics resource person and advisor.

The process of triangulation shall be followed because research findings may easily become artefacts of particular methods of collecting research data. Hence, to avoid this, triangulation shall reduce the probability that "any consistent findings are attributed to similarities of methods" (Cohen and Manion 1992:270). To build up content validity in these questionnaires, preliminary fact-finding, informal interviews and open-ended questionnaires will be conducted on groups of ten students and two teachers from a school different from the five schools in the sample group. Information from these fact-finding questionnaires and interviews, together with information from the literature review in chapters two and three, will be used to construct a questionnaire used to gather data for the study.

Every questionnaire to each of the three sample groups (students, teachers, and administrators) is divided into three sections. Section C seeks to collect data that addresses the first objective: "to determine the actual current didactical practices relating to mathematics in Maseru's secondary and high schools". In fact, Section C of each of the three questionnaires is an ordinal scale where respondents are asked to put didactical practice items in rank order.

Section B of the questionnaire consists of questions based on didactical practices that literature purports to entrench mathematical literacy in learners. Section A has didactical practice items that reflect current didactical practices relating to mathematics in Lesotho's secondary and high schools.

Both Section A and Section B of the questionnaire are of the Likert agreement five-point interval scale type where the responses "Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D), and Strongly Disagree (SD)" are expected from the research subjects. Correlating scores of respondents on Section A and Section B will address the second objective:" to establish the extent to which current didactical practices followed in Maseru's secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematical literacy as reflected in literature".

For each group of respondents, the reliability of findings will be tested by using Pearson's product moment correlation formula (using split-half scores) followed by Spearman-Brown's formula to calculate the reliability of the whole instrument (Cohen and Manion 1992:274-275, Terreblanche and Durrheim 1999:89, Tuckman 1988:173-174, Wiersma and Jurs 1985:74). Furthermore, 27 interviews will be conducted: 10 with students (two students from each school), 10 with teachers (two from each school), five with subject supervisors (one from each school), and two with curriculum developers. These interviews will be aimed at qualitatively verifying and supplying in-depth information and facts that were gathered from the questionnaires. Interview responses will also be used to check the reliability and validity of responses to questionnaires by triangulation between methods and, thus, to avoid findings that are method bound (Babbie 1994:105-106, Cohen and Manion 1992:269, 270, 272, Oppenheim 1992:158). Further triangulation will be achieved by comparing findings from questionnaires and those from in-depth interviews.

On the other hand, qualitative document analysis will be employed to examine whether the content, goals and objectives, and assessment procedures of the mathematics curriculum offered concur with those suggested in literature on mathematics education for mathematical literacy. The nature of the curriculum may confine, prescribe, and limit didactical practices. Hence, it is necessary in this study to take an in-depth look into the prescribed mathematics curriculum for Lesotho's secondary and high schools and compare it with mathematics educational practices for mathematical literacy. In fact, coupled with this exercise, a qualitative analysis of mathematics textbooks used in the sample schools will also be conducted to assess how fit they are for giving instruction that promotes mathematical literacy in students.

The last objective (to assess what didactical practices relating to mathematics in Lesotho, if any, still need to be improved/embraced/redefined in order to effect mathematical literacy in students) of the study shall be explored by a qualitative analysis of the findings from the exploration of the first three objectives.

1.6 DEFINITION OF TERMS

Terms connected to this research need to be defined in order to set the arguments in the right perspective. For instance, the researcher sets out to investigate didactical perspectives of teaching mathematics for mathematical literacy in secondary and high schools in Lesotho. As such, major terms in the research are: **"didactical perspectives**", "**mathematical literacy**" and "**secondary and high schools in Lesotho**". In the following subsections, the respective terms are discussed, defined, and explained.

1.6.1 Didactical perspectives

First, "didactical" is an adjective, which, according to Webster's Third New International Dictionary of the English Language, means:

- fitted for or intended to teach, and
- concerned with or functioning in the conveyance of instruction as in teaching some lesson,

- or intended to convey instruction and information, (hence) …overburdened with instructive or factual matter to the exclusion of graceful and pleasing detail, and

- involving lecture and textbook instruction.

The Encyclopaedia Americana International Edition (1992) also underlines "didactical" with the same import as the colloquial definition above and points to the same meaning and major purpose of "instruction or guidance" that exist mainly to communicate some fact or idea. More scientifically, Restivo, Bendegen, and Fischer (1993:13, 113) specifically view the didactics of mathematics as "a collective effort to study mathematics and to shape the relationship between human beings and mathematics". However, it is Griessel (1988:6) who clearly maps it out in a purely academic and wide educational context when he writes:

> Didactic education is a particular … perspective on instruction and learning in the education situation and can also be called the science of educative teaching.… The contents of life (including learning content skills and techniques) have to be mastered by the child as a means used by the adult to accompany him to his own adulthood. Teaching (Greek: didaskein) may not be separated from education.… If it is, it may result in a situation where a child gains outstanding scholastic achievements and can always show a brilliant report to prove it, but he never become really adult in the manner that he can fulfil the criterion of responsible self-determination.

Van der Stoep and Louw (1992:28-29) concur with Griessel. They also view didactics as "the theory of teaching or the scientific analysis of the teaching activity". Therefore, didactics is, here, viewed as the scientific study of the teaching activity. Specifically derived from the Greek word "didaskein", the term "didactics" implies "to teach", "to offer content", or "to impart knowledge" while the Greek word "didaskalia" means the profession of teaching and "didache" implies the content to be taught (Van der Stoep and Louw 1992:29). Hence, in the light of these definitions, "didactical perspectives" in this study means the fitting, appropriate, and recommended practices in the teaching and learning of mathematics in Maseru, Lesotho.

1.6.2 Mathematical literacy

The second pivotal term in the research is “mathematical literacy”. Currently, “literacy” has become a fashionable word. For instance, Bhola (1994:8) points out that “all reading in all settings is called “literacy”. Thus, there is talk of “cultural literacy, scientific literacy, political literacy, computer literacy”, and other forms of literacy. Nonetheless, according to Curry, Schmitt, and Waldron (1996:2), the import of the term “mathematical literacy” is captured in the following words:… mathematical literacy and numeracy are used interchangeably ... Both terms should be viewed as loosely referring to the aggregate of skills, knowledge, beliefs, patterns of thinking, and problem-solving processes that individuals need to effectively interpret and handle real world quantitative situations, problems, and tasks.

On the other hand, The Organisation for Economic Co-operation and Development (Yahoo, OECD Home page) defines “mathematical literacy” as: “… an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well founded mathematical judgements and to engage mathematics in ways that meet the needs of that individual’s current and future life as a constructive concerned and reflective citizen.”

The OECD itself is a unique international organisation, which currently has 32 member countries (mainly developed) working together to produce a method for assessing students across countries. In fact, the OECD sees mathematical literacy in three dimensions viz.:

- the content of mathematics,
- the process of mathematics as defined by general mathematical competencies, and

- the situation in which mathematics is used, ranging from private contexts to wider scientific and public issues.

However, some literature on mathematical literacy, for instance, Elliot and Kenney (1996: 1-19), Pimm (1987), and Restivo, van Bendegen, and Fisher (1993:117) point out that some elements of the ability to express oneself and communicate both in speech and writing are expected in mathematical literacy. Further, Schifter and Fosnot (1992:8) indicate that the understanding of mathematics was derived from a process of concept construction and active interpretation as opposed to absorption and accumulation of items of information. This provides individuals with aspects of a map of reality. Moreover, Borasi (1996:201) and Restivo, van Bendegen, and Fisher (1993:113) also include attitudes, beliefs, cultural values, values of mathematical meaning, and utilisation as integral parts of meaningful mathematics education.

From the above definitions, one can deduce that some of the salient indicators of mathematical literacy are an individual's meaningful and functional mathematical knowledge, competencies in certain skills, and beliefs and patterns of thinking rooted in real world contexts and cultural settings. In a way, it is a form of interactive dialogue between the individual's aggregate mathematical knowledge indicators and the contextualised real world. However, the relationship between mathematics and reality is not new. In this regard, Strauss (Regional Conference in Mathematics Education UOFS 1998:11) alludes to the historical fact that "The Pythagoreans, for instance, believed that everything in reality is number". However, the definitions of mathematical literacy that are cited in the preceding paragraphs, reveal that mathematical literacy has everything to do with meaningful physical and numerical reality and also has indicators that have significant cultural bearings. This becomes clear in the fact that mathematical literacy touches utilitarian and intrinsic values that have meaning in the context of the immediate physical phenomena and, further, in taking on board the culture in which the mathematics arises and is practised.

Further, according to Casey (2000:2), a much deeper understanding of mathematics is gained by considering how mathematics arises and is used in various cultural settings. It is alleged, here, that cultural settings, with their diverse human activities, embody functional mathematical literacy for people operating in those societies. Hence, it is instructive to note that successful mathematical literacy is shaped when we take on board the understanding, reasoning, and mathematical thinking patterns prevalent in the society from which learners come. This allegation is affirmed by Getz (1999:434) when he maintains that there are possibilities for embedding mathematics in a familiar cultural background via the introduction of multicultural material into the curriculum. Furthermore, Gayfer, Hall, Kidd, and Shrivastava (1979:7) concur with this when the y point out that:

> Literacy is functional when it arouses in the individual a critical awareness of social reality, enabling him or her to understand, master and transform the physical reality … (Thus) to be effective, functional literacy should deal with … cultural and social aspects…. so that teaching should relate subject matter to local conditions … (Consequently) teaching methods that cater for the learner's felt needs are most effective.

1.6.3 Secondary and high schools in Maseru, Lesotho

Lesotho is one of the African Commonwealth states. It is a sovereign country that is completely land locked and bordered by South Africa. The eastern and northern parts of Lesotho share borders with Kwa-Zulu Natal, which is a province of South Africa. The Southern and Western areas of Lesotho are bordered by South Africa's Eastern Cape and the Free State provinces respectively.

After primary education, in Lesotho, there are five years of secondary and high school before one goes for tertiary education. In fact, secondary and high schooling in Lesotho is a five-year, post-primary, formal education programme that culminates in the student obtaining the COSC Ordinary Level (commonly referred to as O'Level). The first three years of this programme is generally referred to as secondary school and lead to the Lesotho Junior Certificate, which prepares students for the COSC. The last two years comprise high school and this prepares students for the COSC examination.

Having defined the terms that govern the research study, it is only appropriate to indicate how the research fits into the structure of education as a discipline.

1.7 DEMARCATING THE RESEARCH AREA

The research falls in the discipline of didactics, in the specific area of the teaching and learning mathematics in Maseru, Lesotho. As an enquiry in didactics, the study, hence, focuses on the instruction and learning of mathematics. However, there cannot be any didactical situation in school mathematics without a formally set curriculum. According to Howson (1991:4-5), curricula in mathematics is mainly concerned with the learning of mathematics and can be considered to be comprised of the "aims and objectives, content, teaching methods, evaluation, and assessment". Actually, Howson maintains that the most important component of the mathematics curriculum is the creation of a powerful learning environment, using appropriate "teaching methods" since this is what gives a lasting impact on the learner as the learner interacts with the curriculum in the rich learning atmosphere. For this reason, the major focus of this study is on teaching mathematics. In particular, the study focuses on the teaching activities that effect mathematical literacy in students in Lesotho's secondary and high schools. This pursuit is justified since, according to Begle and Gibb (Shumway 1980:3), researchers in mathematics education and teachers of mathematics both have a common goal. This goal is to jointly improve the teaching and learning of mathematics.

In fact, Jaworski (1994:xi) and Paul Ernest (Hoyles, Morgan, and Woodhouse 1999:x) concur that "mathematics education" is now established world wide as a major area of study where instructional matters are an ongoing field of research and review. Therefore, a research study focusing on didactical practices relating to mathematics in secondary and high school students in Maseru (generalised for the whole of Lesotho) neatly falls in line with all the other research works in mathematics education in other parts of the world whose ultimate goal is to improve the teaching and learning of mathematics.

Actually, Biehler, Scholtz, Straber, and Winkelmann (1994:1, 2) maintain that the didactics of mathematics does, in fact, exist as a discipline although it is fairly young when compared to other sciences such as mathematics itself, or educational psychology. They point out that neighbouring disciplines to the didactics of mathematics are mathematics itself, general education, educational psychology, and cognitive sciences. As such, it is influenced, draws ideas, and gains benefits from its various neighbours. As the didactics of mathematics taps its modus operandi from these many disciplines and, hence, is influenced by many factors, the teaching of mathematics itself can be complex, intriguing, challenging, exciting, or frustrating (Bell 1978:2, Wilson 1993:3). Therefore, in exploring this study, consideration should be given to these influential neighbours to the instruction of mathematics in Lesotho.

We need to take cognisance of the fact that the researcher explores these intricate didactical practices relating to mathematics in Maseru, Lesotho where it is probable to find some people who regard mathematics as the acquisition of algorithmic skills while others view it as the understanding of mathematical concepts and relationships that define the structure of mathematics itself (Kokome 1991:9, 39, 41). Kokome, in fact, indicates that other people take it as the development of problem-solving skills (Ibid). However, Wilson (1993:6) maintains that as we move into the 21st century, we should move away from these stereotypes and rather adapt our teaching of mathematics to the present needs of students. To do this, the instruction of school mathematics must provide experiences that encourage and enable students "to value mathematics, gain confidence in their own ability, become mathematical problem solvers, communicate mathematically and reason mathematically" (Ibid).

Besides what we have indicated in the preceding paragraphs of this section, students must also be taught to appreciate the purpose, power, and relevance of mathematics to local and global contextual situations. Students need to be equipped with mathematical literacy in order to function intelligently in both local and global contextual situations (Booss and Niss 1979:1). Hence, in this study, investigation is conducted to find out the extent to which the teaching of mathematics in Lesotho actually inculcates mathematical literacy in secondary and high school students. The overall student population in secondary and high schools in Lesotho is approximately 7 000 and the teacher/ student ratio in these schools is 1:25 (Lesotho Fifth year development Plan 1996 (c):53). As pointed out earlier on in this section of the chapter, 250 students will be in the representative research sample group. However, the actual research outline and exposition of the study is given in the following section.

1.8 THE EXPOSITION OF THE STUDY: RESEARCH OUTLINE

There are five subsequent chapters that follow the present one. A literature review is given in two chapters: Chapter 2 and Chapter 3. In Chapter 2, school mathematics and mathematics education are discussed, reviewing literature in order to seek the import and purposes of mathematics education. The chapter will also juxtapose mathematics education with its neighbours such as cognitive learning theories. Common didactical practices are discussed in the light of these aspects. Didactical practices in Lesotho as well as educational assessment procedures and the use of technology in school mathematics education are also reviewed in this chapter. Chapter 3 addresses the issue of mathematical literacy and didactical practices that enhance mathematical literacy. The need for mathematical literacy, its meaning and indicators, and related issues such as mathematical literacy and communication and the integration of mathematics with the real physical world and with other bodies of knowledge and school subjects are treated in this chapter.

Chapter 4 deals with the methodology and instruments of research. Detailed operations and procedures that are to be carried out to generate the data required to answer the questions posed in this study are provided in this chapter. The methods of investigation and the justification of these methods are fully discussed and explained. Furthermore, the chapter will deal with the construction of instruments needed for use in the collection of appropriate data.

Chapter 5 involves the actual collection of data required in the answering of the research questions. The analysis of data and the interpretation of findings are also done in this chapter.

In Chapter 6, conclusions and a summary of results and findings for each question of investigation are indicated. Recommendations, if any, are suggested. The chapter also discusses the importance of the study and gives suggestions for further research.

1.9 CONCLUSION

This chapter has discussed the background to the study and outlined the central problem of the study as the quest for didactical practices relating to mathematics that entrench mathematical literacy in students in secondary and high schools in Lesotho. Specific aims and research methodology are also outlined. Major terms in the study such as didactical practice and mathematical literacy are defined.

According to this exposition of the study, the next two chapters deal with the literature review. It is in these two chapters that the issues involved in mathematics education, didactical practices relating to mathematics, and didactical practices that entrench mathematical literacy are fully addressed. Actually, Chapter 2 explores, among other issues, the meaning of mathematics education together with learning theories in mathematics education and common didactical practices in mathematics education.

CHAPTER 2

SCHOOL MATHEMATICS AND MATHEMATICS EDUCATION

2.1 INTRODUCTION

Although this study focuses on didactical practices of mathematics education in the district of Maseru in Lesotho, a developing country, background literature related to this study, in essence, pertains to global views and universal didactical practices relating to mathematics. This position agrees with Khati's (1995:v, 29-31) overview of education in Lesotho. Khati maintains that, in practice, school curricula in Lesotho should as far as possible be both comparable and compatible with standards in the region (of which Lesotho is part) and with the rest of the world.

In fact, as far as mathematics education is concerned, literature points out that, in view of the universal nature of mathematical content and the ongoing global research in classroom instruction, the practice in mathematics education is similar in all countries (Kaiser, Luna and Huntley 1999:25). Further, literature also purports that the similarities and running threads and trends in mathematics education stem mainly from the history, the nature, and the development of mathematics as a subject and from incumbent learning theories that are harnessed in didactical practices relating to mathematics (Howson 1991:3-6, Christiansen, Howson, and Otte 1986:8, Kress 1997:8, Nelson, Joseph, and Williams 1993:7, Roulet 1998:1). At the same time, that similarity does not imply congruency. It is implied that factors such as the goals and purposes for teaching the subject in the different countries, societies, and cultural settings contribute to breaking the congruency properties.

In this chapter, a literature review of mathematics education and common didactical practices relating to mathematics is presented. Literature related to such a heading is wide ranging. For purposes of brevity, the literature review in the chapter focuses on:

- a systematic presentation of mathematics within education and a discussion of its neighbours and sub-disciplines,

- psychological and philosophical theories that influence trends in didactical practices relating to mathematics, and

- common learning difficulties encountered by learners due to inappropriate didactical practices.

The purpose of mathematics education in general is portrayed and literature on didactical practices relating to mathematics in secondary and high schools in Lesotho are also reviewed. Further, a review of assessment and the place of technology in mathematics education that is recorded in literature are discussed.

2.2 MATHEMATICS EDUCATION, ITS NEIGHBOURS, AND SUB-DISCIPLINES

According to Jaworski (1994:xi), "mathematics education is now established world wide as a major area of study". In fact, Biehler *et al*, (1994:1-2) maintain that the didactics of mathematics does exist as a discipline although it is fairly young when compared to other sciences such as mathematics itself, or educational psychology. They point out that neighbouring disciplines to the didactics of mathematics are mathematics itself, general education, educational psychology, and cognitive sciences. As such, it is influenced and draws ideas from its various neighbours. As the didactics of mathematics taps the particular way in which it operates (its modus operandi) from these many disciplines, it, therefore, has many influencing factors, which make the teaching of mathematics complex, intriguing, challenging, exciting, or frustrating (Bell 1978:2, Wilson 1993:3).

2.2.1 Mathematics education: Its meaning and neighbours

According to Greer and Mulhern(1990:xiii), "mathematics education is concerned with the learning and teaching of mathematics". In the same light, Davies, Maher, and Nodding (1990:ix) concur with Greer and Mulhern when they point out that:

> At the 1985 meeting of the North American, Section of the International Group of the Psychology of Mathematics (in Columbus, Ohio), a group of … mathematics educators concerned about the current status of mathematics education met to discuss the need to address important issues regarding research on teaching and learning mathematics.

From this indicative quotation, it can thus be deduced that mathematics education is indeed concerned with the teaching and learning of mathematics. In fact, Davies, Maher, and Noddings go on to allude to the fact that "views about the nature of mathematical activity have direct bearing on the ways in which mathematics education can be approached". Therefore, Davies, Maher, and Noddings collocate mathematical education with the teaching and learning of the subject. This refers to some of the mathematical activities in mathematical education apart from research in this area. Hence, regarding a definition, Niss (1999:5) views mathematics education as the didactics of mathematics. Niss perceives mathematics education as a scientific and scholarly field of both research and development that aims, amongst other things, to identify, characterise, and understand "phenomena and processes actually or potentially involved in" the didactics of mathematics at any educational level.

Similarly, Shumway (1980:14) points out how research in mathematics education has afforded bricks for building up cognitive development, concepts, and principles in learning skills of mathematics. Servais and Varga (1971:30) informatively mention how the striking difference between mathematics (a subject which is a prototype of exact science) and education (as a social subject area) has narrowed over the years. Over the time, the two have merged to such an extent that mathematics education has been accepted as a discipline in its own right and, at the same time, has become an important area of research (Ibid).

Further, Niss (1999:1) instructively points out that, although mathematics education has become established as an academic discipline on the international scene, one has to be aware of the slightly different names given to it in different quarters. Niss points out that the field is sometimes called "mathematics education research" or "the science of mathematics education". Nonetheless, "mathematics education" remains predominant in everyday usage. However, in some parts of Europe, preference for using "the didactics of mathematics" is prevalent (Ibid). Even in some quarters of South Africa, this is the case (Van der Stoep and Louw 1992:28, Van Tonder 1998:96).

2.2.2 Role players in mathematics education

Greer and Mulhern (1990:xiii) maintain that mathematicians, teachers of mathematics, and psychologists mutually work in and involve themselves with mathematics education. In fact, Greer and Mulhern passionately point out that "those who come to the subject from or through mathematics have a healthy respect and veneration for mathematics" (Ibid). From this proposition, it can thus be deduced that some of the neighbours and sub-disciplines of mathematics education include: the subject mathematics itself and the didactics of mathematics. It further suggests that mathematics has a distinct place in education.

Paul Ernest (in Kaiser, Luna, and Huntley 1999:vii, Jaworski, Wood, and Dawson 1999:v), posits that even though mathematics education was originally rooted in mathematics and psychology, new disciplines have pervaded it. Paul Ernest indicates that fresh perspectives have been injected into didactical practices in mathematics education from fields and disciplines such as philosophy, logic, sociology, anthropology, history, women's studies, cognitive science, semiotics, hermeneutics, post-structuralism, and post-modernism (Ibid).

2.2.3 The position of mathematics within education

According to Begle (1970), Freudenthal (1983), Kroeze (1992), Orton and Wain (1994), Simpson (2001), Walsh (1993), Woodbury (1998), and others, mathematics holds a key position within education in general. In fact, Greer and Mulhern (1990:xiii) perceive mathematics as "part of the universal education … for all …" Kroeze (1992:3) points out that mathematics is, actually, one of the important subjects taught at school and it, thus, forms a vital part of the education one acquires at school. However, Walsh (1993:6) distinguishes between two aspects of education. First, Walsh sees education in the widest sense where much of it is "elusive … highly personal and … serviced by a whole array of agencies". Second, Walsh (1993:6-7) perceives education in the context of what a school actually provides and what parents think the school should provide. On the one hand, Kroeze (1992:3) maintains that school education, together with other types of education at home, at church, and elsewhere, actually form the total education transmitted to an individual. On the other hand, taking a different perspective, Butler *et al*, (1970:40) purport that total education has two patterns: special education, which provides one with specific qualifications needed for excellent performance in some specific areas and general education concerned with the preparation of individuals for life as a productive responsible citizen. In essence, both Butler *et al*, and Kroeze are holding the same views about the dual facets of education: the specific contributing to the general.

However, the various kinds and aspects of education should not be compartmentalised. In fact, although education should allows some degree of specialisation, it must also form a coherent whole in which the distinct forms of knowledge specific to (different) subjects can be critically evaluated, compared, contrasted and, to an appropriate degree, integrated To this effect, South Africa (Discussion Document Curriculum 2005 1997:17) maintains that "a mechanical and permanent grouping" of learning outcomes is indeed not desirable in an educational setting. For the same reason, the European International Baccalaureate Organisation's Middle Year Programme (1995:6) for 11 to 16 year-old students offers an approach to teaching and learning that "shuns the fragmentation of knowledge" but rather "accentuates the inter-relatedness" of various educational disciplines and "advances a holistic view of knowledge" (Ibid). In the light of all these views, all educational aspects form a harmonious and continuous unit. Mathematics education forms part of this continuous harmonious unit.

In fact, literature argues that the actual inclusion of mathematics in one's general education is regarded as essential because, amongst other factors, it helps in the development of common values, attitudes, knowledge, and skills needed by all for common democratic citizenship (Butler *et al,* 1970:30-31, Costello 1991:1-122, Orton and Wain 1994:33, and Woodbury 1998). Further, one of the factors that opens a place for mathematics in any intelligent citizen's functional knowledge and education is the stark reality that mathematics, in nature, manifests itself in multi-variant, multifaceted dimensions that are limitless in extent and depth (Bochner 1966:v, Grobler 1998:1, Holt and Marjoram 1973:v, Howson 1988:33, Page and Rodin 1990:xiii, 3, and Siegel 1988:75). In this regard, Kline (1979:1) points out that not even any formal education can encompass all that the subject can offer. Kline passionately argues that, even at secondary and high school level, mathematics education severely limits the presentation of many mathematical values because educators are too preoccupied in meeting set objectives to prepare students for examinations. However, Johnson-Wilder, Johnson-Wilder, Pimm, and Westwell (1999:9-18) and Wilson (1993:324) clearly establish how mathematical education is intricately intertwined with both the nature of mathematics as a subject and with cognitive learning theories.

2.2.4 The place of theories of learning in mathematics education

Orton and Wain (1991:1-6) discuss the place of learning theories in supporting and enlightening the process of learning mathematics. Orton and Wain (1991:139) point out that due to the "complexity of the nature of human abilities … general theories of learning … cannot be ignored". In essence, Orton and Wain (1991:2) posit that, amongst other things, learning theories enable educators to "explain what we see in school and take appropriate action ... In this sense our theory explains, and could even predict, phenomena …, our theory might present a systematic view of phenomena whilst at the same time remaining relatively simple to grasp."

From what Orton and Wain are propounding, it can be deduced that speculations in learning seem to have a fertile yield when they are informed by a sound framework of learning theories.

Greer and Mulhern (1990:24-27) also discuss at length the forged partnership that exists between mathematics education and cognitive psychology. In fact, Begle (1970:23), Biehler *et al*, (1994:1-4), Borasi (1996:19), Mellin-Olsen (1987:18-29), Niss (1999:5), and Shumway (1980:14) indicate that mathematics education and psychology are neither strange nor only passing bedfellows. Their affair has a long history where mathematics instruction has been responding to psychological theories. Daniels and Anghileri (1995:3-57) extensively explore the inter-relatedness and the mutual influence of psychology, pedagogy, and mathematics on mathematical education, and, thus, by implication, also spell out some of the neighbours to mathematical education (as is pointed out in Section 2.2.1). It, therefore, seems that mathematics education is a complex problem field in which theoretical constructs from various scientific areas mutually interact.

Burton (1999:3-89) also discusses the politics of mathematics education from a bedrock of psychology taking both psychological and sociological learning theories ranging from Piaget, Bruner, Skinner, and Gagne to Dienes, and Vygotsky. In a way, this further indicates the relationship between mathematics education and psychological and sociological factors (Burton, 1999: 93.100). Again, Nelson, Joseph, and Williams (1993) explore a multicultural approach to the teaching of mathematics in their book, thus pointing to the relationship between mathematics and culture, which, itself, is a sociological aspect.

Nevertheless, the challenges in mathematics education are many. Apart from building up and organising appropriate mathematical content, there still remains the daunting didactical task of carrying out instructional activities that equip students with mathematical skills, attitudes, values, and knowledge relevant to the needs of the contemporary society. However, taking cognisance of effective instruction is deeply embedded in learning theories. These theories are impacted by philosophical, sociological, and psychological factors.

2.3 SOME PHILOSOPHICAL AND SOCIOLOGICAL ISSUES AND PSYCHOLOGICAL THEORIES AND PHILOSOPHIES THAT INFLUENCE TRENDS IN DIDACTICAL PRACTICES OF MATHEMATICS EDUCATION

The purpose of this subsection is to map out some of the contributing philosophical, sociological, cultural, and psychological factors that have influenced trends in the didactical practices of mathematics education.

2.3.1 Philosophical issues

In order to view philosophical issues in mathematics education, it is instructive to first seek and perceive philosophical issues in education in general. Actually, according to Hamm (1989 1), for one to understand the philosophy of education, it is necessary to understand what philosophy is since the philosophy of education is but a branch of philosophy. However, Hamm (1989:2-3) strongly points out that the philosophy of education is not synonymous with education theory. Again, it is not about building theories of education, neither is it a history of education thought nor a matter of "drawing conclusions … and eliciting implications from bodies of systematic … thought" (Ibid). Rather, according to Hamm, the philosophy of education is all about how philosophers of education think and function.

In fact, Hamm (1989:5-8) posits that, in their thinking, philosophers of education constantly ask key questions such as: What? How? Why? Hamm's basic argument is that philosophical issues in education, as embodied in the above questions, concern the search for meaning, definitions, and explanations of concepts of education. However, the same questions also govern the philosophy of mathematics since mathematics itself is a paradigm of meaning, precision, rigor, and certainty. That is why it "stands at the pinnacle of rational thought" (Tymoczko 1998:xiii). Hence, according to Hart (1997:1), philosophical issues and mathematics have been attracted to each other since the time of Plato and other great philosophers of his time.

Actually, Goodman (in Tymoczko 1998:xiii) posits that "The philosophy of mathematics begins when we ask for a general account of mathematics, a synoptic vision of the discipline that reveals its essential features and explains just how it is that human beings are able to do mathematics.

Further, Goodman (in Tymoczko 1998:97) points out that rigorous reasoning is "the special property of mathematics and logic". In the same vein, Goodman asserts, that "the philosophy of mathematics … consists in the explanation of demonstrative reasoning (Ibid). Actually, Goodman (in Tymoczko 1998:97) captures essential but relational phenomena in the philosophy of mathematics in the statements that:

- Mathematic practice provides important material for a philosophical understanding of mathematics,
- The questions of mathematical discovery and development are essential to a philosophy of mathematics,
- There is a fundamental similarity between the practices of mathematics and the practices of science,
- Pedagogy is an important topic in the philosophy of mathematics,
- Constructivism is a vehicle through which mathematical concepts are understood and internalised.

However, mathematics practice itself has changed and evolved over time, and it can now be viewed in a new way "as a social or cultural practice" Kitcher (in Tymoczko 1998:215). Hence, apart from the philosophical issues, sociological issues also influence the practice of mathematical education.

2.3.2 Sociological issues

Dowling (1998:xiii) asserts that, sociology does not only concern individuals, groups, and their patterns of interrelationship. It also goes further and "weaves knowledge and social practice into a complex whole". For example, in mathematics education, learners weave mathematical knowledge into social practice to allocate meaning to their real world. To this end, Paul Ernest (in Biehler *et al*, 1994:135) views the philosophy of mathematics as "giving an account of mathematics acknowledging the centrality of mathematical practice and social processes".

To this end, Silver (in Schoenfeld 1987:33) convincingly shows that mathematics education places emphasis on learners' ability to use and apply mathematical knowledge for problem solving within and outside the school setting. Thus, problems of mathematics in the real world, particularly problems that are relevant to the lives and the social environment of learners, inculcate meaningful knowledge in learners. In fact, Silver (in Schoenfeld 1987:54) posits that one way of facilitating this aspect within the classroom set-up is "to use prototypical situations for introducing and developing instruction on mathematical concepts and skills". Pea (in Schoenfeld 1987:104-105) actually perceives social environments as fertile grounds for discussing, reflecting upon, and establishing mathematical thinking. Pea further posits that recognising and encouraging mathematics as a social activity in the world "would not only be beneficial and more realistic, but would also make mathematics more enjoyable, sharable rather than sufferable".

However, in propounding a theory that connects constructive learning and social activity, Dowling (1998:127-128) points out that social activity can link with Piaget's egocentric thought and sociocentric thought that portray "the development of rationality as the interaction between three systems: the sensory-motor, the operational, and the symbolic". To this end, Paul Ernest (in Biehler *et al*, 1994:343) accords with Pozzi, Noss, and Hoyles (1998:105-122) in purporting that one of the crucial foundations of understanding mathematics emerges from the consequences of philosophical issues of mathematics that flow from the linguistic basis of social constructivism. Therefore, this line of thinking leads to cognitive and psychological issues that also influence thought relating to mathematics.

2.3.3 Psychological issues

As alluded to in one of the preceding sections, teaching, particularly teaching mathematics, is a complex activity (Glaister and Glaister 2000, Niss 1999, Reynolds and Muijs 1999, Romberg (in Campbell and Gristein 1988:23), and Stiff, Johnson, and Johnson (in Wilson 1993:3)). Factors that contribute to the complexity of teaching mathematics are many, but interrelated. Romberg (in Campbell and Grinstein 1988:23) indicates that one of these contributing factors is what teachers are actually faced with when they undertake the task of teaching. For instance, when teaching any mathematics topic, teachers make implicit assumptions about how students learn. Then, based on these assumptions, they decide how the instruction should take place.

However, in all cases, teachers consider the nature of the mathematics topic that they deal with as well as the time available to cover the topic. At the same time, the number of students in the class and their predisposition and ability to understand the lesson are factors that teachers need to address (Ernest 1989:21-55, Romberg (in Campbell and Grinstein 1988:23), and Vorster 1997:23-25).

De Corte and Weinert (1996:33-43) identify some underlying meaningful learning traits and processes that research recommends as building blocks for an operative educational learning theory. In fact, De Corte and Weinert concur with Du Toit *et al*, (2001:17) that the repertoire of characteristics of the definition of learning processes, in literature, includes the fact that learning is constructive, cumulative, self-regulated, goal-oriented, situated, co-operative, individually different, learning to learn, and that it entails cognitive apprenticeship. By positing that learning is constructive, it is implied that learners actively construct their knowledge and skills through organising their already acquired mental structures. This factor is connected to the fact that learning is cumulative since learners can select and actively process information that they have already encountered to build new meaningful skills.

However, theories of learning are many. This is because how people learn is extremely complex as is indicated in the definition on learning that has been cited. At the same time, Romberg (in Campbell and Grinstein 1988:23-24) maintains that there exists no agreement on the actual details of how learning takes place and, also, there is no consensus about what categorically spells out evidence that learning has taken place. Furthermore, on the various kinds of learning, psychologists make different philosophical assumptions about the real nature of the learning process. For instance, some of the conflicting and contrasting assumptions involve the questions of whether learning occurs by passive reflection or by active constructivism, whether limits and confining restrictions of learning are biologically or environmentally determined, and whether learning is a rational or an irrational process (Ibid).

A close look at some of these learning theories will throw more light on the arguments purported here and, at the same, time display the various theories on which teachers can effectively fashion their didactical approaches to promote the meaningful learning of mathematics (Niss 1999:12). As Bell (1978:2) points out, each teacher can select and apply elements of each theory in his or her own class.

According to the literature, Jean Piaget, Jerome Bruner, BF Skinner, Robert Gagne, Zoltan Dienes, Richard Skemp, and Lev Vygotsky are some of the people who have contributed a great deal to theories of learning that have had an extensive and intensive impact on the teaching of mathematics (Borasi 1996:20-43, Daniels and Anghileri 1995:39-57, Fosnot 1996:73-89, Stiff and Curcio 1999:46, 82, and Wilson 1993:3-13). It is instructive to briefly capture the essence of each of the theories of these important contributors.

2.3.3.1 Piaget

One of most fundamental and influential theories relating to the teaching of mathematics in the last century is that of Piaget. In the 1960s, Piaget determined four main developmental stages through which people progress in their intellectual growth from birth to adulthood viz. the sensory-motor stage (from birth until about two years), the pre-operational stage (three to about six years old), the concrete operational stage (from age seven to age twelve, thirteen or even later), and the formal operational stage (Daniels and Anghileri 1995:39-40, Bell 1978). From these stages, we see that learners in the secondary and high schools fall in the latter two, but mainly in the last one. Literature (Wilson 1993:6) informs us that, in the concrete stage, students learn to cope with the immediate environment by building things and manipulating objects. Hence, children in the concrete stage have difficulty in applying formal intellectual processes to verbal symbols and abstract ideas. The formal stage itself is characterised by abstract thought operations. It is here that the formal operational thinker is able to formulate theories, generate hypotheses, and test various hypotheses.

What is important, however, is the fact that Piaget considers that the order in which the developmental stages manifest themselves is not fixed and exclusive. Students may move between a concrete operational stage and a formal operational stage when confronted with new concepts, skills, and principles. Furthermore, it is noteworthy to perceive that Piaget's developmental stages indicate that knowledge of mathematics is social and physical as well as logical.

Thus, in essence, Piaget's theory of intellectual development maintains that learners construct their own knowledge through interaction with the environment (yielding social as well as physical knowledge) in a cognitive structured way that predominantly involves processes of assimilation and accommodation (yielding logical knowledge) (Daniels and Anghileri 1995:40, Wilson 1993:6). In the process of assimilation, learners incorporate new experiences into their existing cognitive structure, whereas in the accommodation process, the restructuring and modification of the existing cognitive structure occur in order to accommodate new a cognitive schema (Ibid).

According to Daniels and Anghileri (1995:40-41), Piaget introduced a version of the constructivist approach where the teacher's role changes from transferring knowledge to constructing with the student a conceptual framework for understanding. Constructivism itself is a developmental philosophy for learning (Fosnot 1996:ix, Campbell and Grinstein 1988:26). According to Borasi (1996:19-23), this implies that knowledge is constructed by the learner by rationalising new experiences and incorporating acquired knowledge into a personal framework of understanding. In this regard, all knowledge is thus perceived as a product of one's own cognitive acts as one interacts with the learning environment. Hence, Piaget can be termed a developmental constructivist (Schoenfeld 1987:20).

2.3.3.2 Bruner

Bruner and Dienes are two of the developmental psychologists who improved on Piaget's work. Bruner proposed a theory based on three mental modes viz. enactive, iconic, and symbolic. Learners progress from the enactive mode (where they actively manipulate concrete materials) to the iconic (where pictorial representations involve mental images) to symbolic representation with competent use of both language and mathematical symbols (Daniels and Anghileri 1995:41). Unlike Piaget, Bruner did not support the argument that learners need to be ready before proceeding to the next mode. Rather, Bruner suggested that anything can be presented to the learner provided it is simple enough for the learner to understand as is the case in a spiral curriculum model of learning (Ibid). Thus, Bruner opened the ground for the structuralist approach to the instruction of mathematics where experiences with concrete materials and pictorial representations are commonly used to support computational practice in the learning of mathematics. According to Wilson (1993:7), Brunner supports the constructivist view that learners should "be given the opportunity to construct their own representation of mathematical concepts, rules and relationships."

2.3.3.3 Dienes

Dienes worked closely with Bruner, but Dienes stressed the much-used didactical practice that stresses progression from the concrete to the abstract in mathematical teaching/learning activities (Schoenfeld 1987:107). In fact, Dienes perceived three types of mathematical concepts: pure mathematical concepts, notational concepts, and applied concepts, and he regarded the learning of these concepts as a creative art. However, according to Daniels and Anghileri (1995:42), Dienes advocated the fact that students acquire structural aspects of mathematics as they engage in activities that involve "materials that embody the concepts which are the objects of instruction" irrespective of the concept under study. Dienes' structural approach stemmed mainly from the realisation of the fact that learning a specific procedure or skill without reference to its broader structural context is often unproductive.

Commenting on Dienes' work, Bell (1990:213) maintains that Dienes' proposition of mathematics as basically the study of structures made mathematics more interesting and easier to learn. Regarding mathematics as the study of structures, the classification of structures, the differentiation of relationships within structures, and the grouping of relationships within structures actually reduced the once difficult and tricky subject to an exciting and less difficult one. Nevertheless, according to Bell, for students to be able to study mathematical structures, they must be able to:

- Analyse the structures and identify logical relationships between them,
- See common properties (in the abstract) from a number of different structures or events and be able to classify the structures as belonging together.
- Generalise groups of structures learnt before by expanding them to broader classes that have similar properties to those found in smaller classes,
- Employ abstractions learned before to build up more complex and higher orders of abstractions (Ibid).

2.3.3.4 Skemp

In the pursuit to build understanding in students when they learn mathematics, one can also turn to the contribution by the psychologist Skemp. In his psychology of learning mathematics, Skemp stresses the importance of connecting new ideas to what has already been grasped (Daniels and Anghileri 1995:43; Kroeze 1992:1). Nevertheless, in his view of structuralism in mathematics, Skemp makes a distinction "between relational and instrumental understanding" (Daniels and Anghileri 1995:13). In relational understanding, new ideas are incorporated into existing cognitive schemata so that the student knows what to do and w hy a certain procedure is taken, but in instrumental understanding, the student learns to obtain correct answers by using rules without reasons (Ibid).

2.3.3.5 Gagne

Another theory of cognition and intellectual development is that of Robert Gagne (Stiff, Johnson, and Johnson in Wilson 1993:4-5). Gagne suggests that desired learning can be successfully achieved in students when the correct experiences of concepts and ideas are appropriately sequenced and sufficiently practised (Schoenfeld 1987:5). Gagne further proposes that even higher order tasks can be fully internalised, provided they have first been broken down into small units that make up the whole (Stiff, Johnson, and Johnson in Wilson 1993:5). As a consequence of Gagne's view point, there is the notion that students cannot manage more advanced mathematical skills, concepts, and procedures until they are proficient in basics and less advanced mathematics (Ibid).

2.3.3.6 Vygotsky

The role of social interaction and verbal communication is central in Vygotsky's blend of constructivism for learning mathematics (Daniels and Anghileri 1995:45, Romberg in Campbell and Grinstein 1988:26). This is inferred from the fact that students' mathematics learning is mediated by their interaction with parents, family, peers, as well as teachers. Coupled with this mediation, the student's interaction with the rest of his/her social environment plays a part in enabling the student to construct his/her own mathematical meaning. Thus, learning becomes a social product where the type and amount of informal knowledge the student brings into school affects his/her construction of mathematical meaning.

According to Dowling (1998:44-45), Vygotsky propounds a form of constructivism that leans on the linguistic comprehension of knowledge of mathematics. In this linguistic model of cognitive development, speech provides the thought structure. In fact, "development occurs as thought grasps new linguistic tools and operates with them in its own restructuring" (Dowling 1998:45). This state of development occurs within a "zone of proximal development" (ZPD), which is actually a region or zone of uncertainty charged with a "disequilibrating tension" that urges the learner forward (Ibid).

The (ZPD) is “the gap between what the learner can achieve on his own, and what he can achieve with help from a more knowledgeable adult or peer” (Goulding in Johnston *et al*, 1999:44). This is in accord with Jaworski’s (1994:26) verbatim echo of Vygotsky’s own words that the ZPD is: “The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers”.

From these definitions, it is, therefore, a false notion that students only learn when the task given is exactly at the level of their preparation by previous background knowledge and experience. Students actually learn when the task comes within range of their ability. Based on Vygotsky’s ZPD, Tanner and Jones (2000:80-81) describe three possible levels of difficulty for students. The first level is one at which students can work without help using only knowledge that is well grasped and known. At this level, it is highly possible to achieve the consolidation and practice of previous material, but there is no new learning for the students. The second level is one where the work is so far above the student’s current level that the result is that the student is in off-task behaviour and no learning takes place. However, the third level is between the two extremes and is actually the learning zone (the ZPD) in which students can operate only with some form of help. Tanner and Jones (2000:80) posit that, within the ZPD, “working noise can be heard, teachers are kept busy, and maximum learning occurs”.

However, apart from the psychological, sociological, and philosophical theories discussed in this section of the chapter, there are other cognitive factors that throw light on didactical issues of mathematics. For instance, it is alleged that learning does not automatically take place because the teacher has singly used this cognitive theory or another (Tanner and Jones 2000:70-85). It still remains a complex activity where all that is purported to help in the learning should be harnessed and utilised. The teacher, therefore, constantly faces the challenge of the quest to find a combination of strategies that may enable all his/her students to reach their full potential. In this area, the teacher again faces a particular challenge of cognitive differences in ability, inert personal traits, and tendencies (Greer and Malhern 1990:x-xi).

2.4 COMMON LEARNING DIFFICULTIES DUE TO INAPPROPRIATE DIDACTICAL PRACTICES

Davis, Maher, and Noddings (1990:51) point out that mathematics instruction needs to be tailored in such a way that it “meshes with” students’ cognitive thinking, otherwise, difficulties in learning arise. For instance, students do not merely absorb new material, they assimilate it. In fact, Davis, Maher, and Noddings mention the following possible learning difficulties in mathematics due to inappropriate didactical practices:

- A gap between formal instruction and a (student’s) existing knowledge prevents assimilation. It can make … taught skills and concepts seem foreign and difficult (to students),

- A lack of readiness to learn a mathematical concept or skill may prevent the assimilation of new information. This may be due to gaps in students’ mathematical knowledge or due to the fact that the prerequisite knowledge is not thoroughly grasped,

- When learning is conducted in an abstract and lockstep manner, (students) are forced to memorise mathematics by rote. Some … fail to memorise what is meaningless (and may) construct beliefs that interfere with further learning and problem-solving efforts.

- Mechanical learning and thinking may result in students’ failure to see how their mathematical knowledge applies to new situations (Price, 1996: 604-608).

Nevertheless, the consequences of difficulties in learning mathematics are many. One of the problems the teacher of mathematics is faced with is student phobia to mathematics. Often one of the causes of phobia is repeated failure to perform well in the subject (due to any of the factors mentioned in the previous paragraph). However, the student who performs well in the subject will experience intrinsic motivation that spurs him/her on to achieve well (Grobler 1998:9, Larcombe 1985:1-24, Winter in Nickson and Lerman 1992:87).

Therefore, in mathematics education, the affective responses of students are important (Mcleod and Ortega in Wilson 1993:21). Anxiety, frustration, confidence, joyfulness, satisfaction, beliefs, enthusiasm, apathy, and interest are feelings and moods that teachers often use to describe students' responses to mathematical tasks (Costello 1991:22-129, Land 1963:83). Therefore, in planning instruction, teachers need to consider the cognitive factor, viz. the ability or inability to achieve well in mathematics. This factor seems to give rise to all positive or negative attitudes to the object of the lesson.

However, in conclusion, it must be pointed out that, for successful didactical practices relating to mathematics, insights into cognitive theories of the learning processes are enlightening and instructive (Greer and Malhern 1990:18-28, Noss, Heally and Hoyles 1997: 203-231) Johnson-Wilder *et al*, 1999:42-48). From the discussion in the previous section and this section, it is evident that learning theories have the potential to illuminate the grey social, personal, and psychological areas that may surround learners. I can also equip teachers with strategies to organise their instruction in ways that may best benefit their students. Nevertheless, learning theories also seems insufficient to understand theories about how people learn. The ability to select the appropriate theory and to apply it is equally important, as each of these theories suggest merely a method of organising an informed and effective teaching environment. However, before looking into how teachers often carry out didactical practices relating to mathematics in the ordinary classroom, we need to expound why mathematics education is necessary. What are the aims and purposes expected to be realised in offering mathematics education.

2.5 THE PURPOSE OF MATHEMATICS EDUCATION

According to Christiansen, Howson, and Otte (1986:9, 11), the aims of mathematics education vary with respect to community expectations concerning the type of mathematics education envisaged for that group of people. However, as Kroeze (1992:6-7) maintains, the aims of teaching mathematics can also be derived from the nature of the subject. These aims may be immediate (enabling attainment of specific skills or enhance certain abilities), intermediate (enabling intellectual development or acquisition of knowledge), or long term. In this section, long-term, general, and universally renowned purposes of mathematics education are considered, since short-term and specific ones vary from country to country. According to Christiansen, Howson and Otte (1986:9-11) the long term, general, and universally held purpose of mathematics education emanate mainly from the inherent nature of mathematics as a subject. This renders it useful and intellectually engaging. Hence, mathematics education is offered for purposes of both its utilitarian and aesthetic value.

As pointed out in Chapter 1, whether one agrees or not, history has throughout all ages ceaselessly shown that, by nature, mathematics permeates the whole of our world, society, and human activities, irrespective of location, colour, or creed. In fact, Kline (1985:213) points out that, "there is in nature a hidden harmony that reflects itself in our minds in the form of simple mathematical laws". This compelling nature of mathematics reveals order and law in situations where mere observation probably would show only chaos.

In particular, today, mathematics seems to have become an integral part of people's cultural, social, economic, and technological environment. In many parts of the world, it seems almost impossible to live a normal life without making use of some kind of mathematics. Hence, Hoyles, Morgan, and Woodhouse (1999:2) strongly argue that the theme of teaching mathematics as a tool in various contextual applications should be the central concern of the technological age.

This in itself gives enough ground and purpose for teaching mathematics at school. By nature, mathematics provides a means of communication that is powerful, concise, and unambiguous (Morgan in Johnson-Wilder *et al*, 1999:132-135, NCTM 1993:5). It can also represent information in many ways: tables, diagrams, graphs, geometrical shapes, symbols, formulas, and otherwise (Hoyles, Morgan, and Woodhouse 1999:64, IB Middle Years Programme Mathematics 1995:5-7). Furthermore, mathematics can frequently be used to study other subjects and often opens doors to sought-after careers. To this effect, even thirty years ago, Holt and Marjoram (1973:v) emphatically pointed out that no person worth his/her salt "dares to be innumerate". However, the need is even more pressing in today's society where we have continual change in technology and a respective increased plethora of areas regarding the application of mathematics (Bondi 1991:1, Neyland 1994:173, Restivo, van Bendegen and Fischer 1993:185).

Historically, the standing utility, value, and purpose of mathematics education in society has enabled us to address questions about our physical world and to understand quantitative operations of nature along with certain human activities (Cockcroft 1982:2, Howson and Kahane 1986:2, Johnson-Wilder, Johnson-Wilder, Pimm, and Westwell 1999:13, King 1992:95, Kline 1979:1, Restivo, van Bendegen and Fischer 1993:115-116, Travers, Pikaart, Suydam, and Runion 1977:2). Yet, it is deplorable, as Campbell and Grinstein (1988:263) point out, that mathematics instruction is often given as "disjoint topics and drill-and-practice activities", which is said to be of use in applications of mathematics in the real world. If, however, the utility of mathematics is pervasive and ubiquitous, there is a need to provide students with real situations in which to apply mathematics.

Nevertheless, for some people, the utility purpose of mathematics education is of little interest in comparison to the aesthetic considerations and fascination sought and found in mathematical pursuits per se (Borasi 1996:23, International Baccalaureate Mathematics Higher Level 1998:4). On the one hand, the beauty of mathematical abstraction, its logical order and ageless purity act as a spur to wit. On the other hand, the richness of mathematical ideas and multiplicity of its aspects affords it the diversity and ubiquity of applications (Kline 1985:39-49, Kurt 2001). Closely related to this purpose is the aspect of teaching mathematics for the development of critical thinking, problem-solving skills, and mathematical reasoning (Niss 1993:157-165, Wilson 1993:40).

O'Daffer and Thornquist (in Wilson 1993:40) perceive critical thinking as a process of effectively using thinking skills to help the student to make, evaluate, and apply decisions of mathematical incline to solve situational problems. Thus, the process of critical thinking essentially involves the following: understanding the situation, addressing the evidence, data, or assumptions concerned, going beyond the evidence/data/assumption and drawing the desired conclusion/decision/solution in order to finally apply the result (Ibid). O'Daffer and Thornquist further point out that specific skills needed in mathematical critical thinking are: visualising, comparing, contrasting, relating, evaluating, formulating ideas, conjecturing, deducing, inducing, generalising, exploring, sequencing, ordering, classifying, validating, analysing, and predicting (Ibid). The list is endless, but what is noteworthy is that these outcomes support higher abilities of cognitive levels of learning in the renowned Bloom's Taxonomy of Educational Objectives.

However, as pointed out earlier, today, mathematics education is operating in a socially, economically, environmentally, and technologically changing world. These changes effect other changes in the nature of mathematics as a subject, the emphasis on mathematical knowledge, as well as changes in the perception of didactical practices of mathematics education (Hirsch 1992:1, National Council of Teachers of Mathematics 1989:3). In the face of these changes, there is a corresponding shift in the function of mathematical knowledge in society and hence in the purpose of mathematics education in schools.

A utilitarian purpose of mathematics education that is deemed to be concomitant with today's changing world concerns "the development of mathematical power for all students" (National Council of Teachers of Mathematics (NCTM) 1991:1). Mathematical power here is seen as taking on board the inculcation of self–confidence and the development of the inclination/tendency to seek, evaluate, and appropriately utilise "quantitative and spatial information in solving problems and in making decisions" (Ibid). Ideally, in order to attain mathematical power, students need to be versatile in their use of mathematical knowledge. They also need to develop "perseverance, interest, curiosity, and inventiveness" in pursuit of mathematical power (NCTM 1991:1). This mathematical empowerment goes hand in hand with goals of the teaching and learning of mathematics.

In achieving these goals, it is expected, amongst other things, that students will:

- appreciate the role played by mathematics in both contemporary and past societies and explore the relationships between mathematics and other bodies of knowledge, be aware of how theoretical mathematics has become concrete even in a technologically oriented society,

- be capable of employing their mathematical power with informed confidence,

- work with perseverance for hours, days, or even weeks to solve problems that maybe closed or open-ended,

- use mathematical language to read, write and discuss learnt ideas during which process they will learn to refine, clarify, and advance their mathematical knowledge and thinking, and

- develop authentic mathematical reasoning through use of appropriate mathematical factual evidence to support the mathematical argument at hand (NCTM, 1989:6).

It is evident that the kind of mathematical empowerment described here requires that students should acquire meaningful mathematical knowledge and skills in order to bring together personal, technologically oriented, and individual thinking skills in their application of mathematics in situations that may arise.

However, there may be other and different purposes to mathematics education depending on the needs of specific societies. As has been discussed in this section, mathematics is taught in schools today for its utilitarian purposes, for its fascinating aesthetic beauty, for the development of a logical and critical mind, and, recently, for mathematical empowerment of the student in a technologically fast-growing world. From this, we deduce that the goals for school mathematics bent on merely acquiring specific skills and techniques are left out. In fact, Borasi (1996:17) points out that goals such as these are irrelevant in today's society. Rather, in this age, students need to become efficient problem solvers and thoroughly proficient as critical thinkers who are, at the same time, confident in their mathematical ability to employ their mathematical knowledge in familiar and novel situations (NCTM 1989:5-7).

In essence and by implication, the purposes of mathematics education themselves foreshadow the respective general didactical practices of mathematics education since the question of why mathematics is to be taught naturally goes together with how it is taught. The next section of this chapter deals with common didactical practices in mathematics education today.

2.6 COMMON DIDACTICAL PRACTICES IN MATHEMATICS EDUCATION

Cooney, Davies and Henderson (1975:3-10) view didactical practices as referring to what consciously or unconsciously goes on in the (mathematics) classroom. From this point of view, didactical practices may thus include activities of which teachers are in control and things they plan to do with students for a variety of reasons. In fact, Cherry Ward (2001:94-96) maintains that didactical practices have come and gone "with only minor variations from the traditional instructional" practices. This point of view resonates with that held by many scholars such as Borasi (1992:2), Bressenden (1980:1), Goldman (1997:19), and others. The objective of this subsection of the present chapter is to explore the different didactical practices in mathematics education.

Of necessity, effective didactical practices relating to mathematics need to create powerful learning environments, since the situation pertaining to mathematics in the classroom, with the habitual interaction that takes place there, dictates to students the mathematics learning that can possibly occur (Cangelosi 1996:79-303, Hoyles, Morgan, and Woodhouse 1999:105). In fact, Hatch (in Hoyles, Morgan, and Woodhouse 1999:104-116) refers to this as "maximizing energy in the learning of mathematics". Hatch maintains that, by providing high-energy classrooms for students, the didactical practices relating to mathematics keep all students in a powerful learning environment. In this environment and with previous learning in a high-energy classroom, Hatch posits that students can confront any mathematical problem given to them or invented for them.

Actually, Hatch (in Hoyles, Morgan, and Woodhouse 1999:106-108) indicates that there are factors that contribute to creating a high-energy state in students, such as, "lesson pace", "know-how", "investigating, conjecturing and proving", and "struggle". Hatch maintains that creating a high-energy classroom involves keeping the lesson pace, which has nothing to do with the speed of movement through the curriculum, but has to do with the "sort of tautness of expectation where all pupils' energy is bent towards the learning of the task". Hence, pace is not getting through more sums per lesson, but rather, it focuses on "the energy expended in understanding the meaning behind the sums and reflecting on the mathematics". In doing this, the teacher also needs to ensure that students fully acquire the "know-how" of the mathematical proceedings involved in the lesson rather than acquiring the knowledge through memorisation.

Hatch further propounds that the process of investigation, where the student "confirms, proceeds at a definitely non-linear and unpredictable rate" (with work related and integrated into the curriculum), rather than being pressed to complete set tasks within a certain time span is central to the didactical practice that creates a powerful learning environment. Investigational work advocated by Hatch has to have mathematical significance that is tested through proven conjectures. The procedure calls for the process of internalising, articulating, and reflecting upon the mathematical concepts involved. This involves encountering difficulties, excitement, and expending lots of energy and time, but through the struggle emerges real and long lasting "know-how".

Nevertheless, creating a powerful learning environment does not divorce itself from other approaches to the practice of didactics. For instance, Goldstein, Mnisi, and Rodwell (1999:84-85) report that modelling, reflective practices, collaborative work, and other didactical approaches assisted in creating a powerful classroom-based approach for teachers in South Africa.

According to Borasi (1996:24), one of the practices of didactics used in mathematics education is the interactive approach. Borasi, here, promotes the argument that classrooms, within which powerful learning environments exist, are, of necessity, environments in which students play an active role in the learning activities. Furthermore, students should engage in solving mathematical problems that transcend the mere performance of routine operations and processes. In the same light, Hirsch (1992:iv) alludes to and supports this view by pointing out that the teacher's role in the learning environment should be that of a "catalyst and facilitator of learning" as opposed to being an "authority figure and dispenser of information". Specifically, the role and strengths of teachers in a powerful learning environment are spelt out as being good in:

- selecting mathematical tasks to engage students' interests and intellects,

- providing opportunities to deepen their understanding of the mathematics being studied, and its application,

- orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas,

- using and helping students to use technology and other tools to pursue mathematical investigation,

- seeking and helping students to connect to previous and developing knowledge, and

- guiding individuals, small–group, and whole -class work (NCTM 1991:1).

Similar principles of teaching are, also, indicated by Borasi (1992:2-3), Cooney, Davis, and Henderson (1975:10-11), Hoyles, Morgan, and Woodhouse (1999:196-197), and Wray and Lewis (1997:21). Yet, teaching itself remains a complex practice. In fact, Bell (1978:2) points out that the complexity of teaching and learning, coupled with the difference among teachers and students, is a good recipe for making the didactical task a highly "individualised and personalised activity".

Therefore, because of the individualised, personalised, and different teaching activities that take place in various classrooms, there are widely practised and accepted practices pertaining to the didactics of mathematics. These practices and approaches of didactics assist students in acquiring mathematics education by experiencing, among other things, the following:

- direct instructional lessons that lead them to acquire knowledge of conventional facts as well as to develop and polish algorithmic skills and to retain knowledge,

- inquiry lessons that lead them to reason inductively and deductively to devise solutions to real life problems and to be creative with mathematics, and

- comprehension lessons that help them to communicate with and about mathematics, to eagerly construct concepts for themselves, and to discover relationships (Cangelosi 1996:vii).

However, there are a number of didactical approaches that can be used in lessons described here, viz.:

- exposition that focuses on explanations of concepts and skills and promotes students' understanding of material (Johnson-Wilder *et al,* 1999:57-58, 61-64, 70, 96, 99, 135, Simmons 1993:2-3),

- discussion of mathematical material that promotes communication among students and between teacher and students, and entrenches further understanding of concepts (Elliott and Kenney 1996:21, Johnson-Wilder *et al,* 1999:64, Pimm 1987:22-74),

- problem solving that includes the application of mathematics to everyday situations (Avital in Zweng *et al,* 1983:279, Burkhart in Zweng *et al,* 1983:283, Isaacs in Zweng *et al,* 1983:282, Jarworski in Pimm 1988:3, Neyland 1994:19),

- project and practical work (Jarworski in Pimm 1988:3),

- investigational work (Jarworski in Pimm 1988:3),

- mathematical modelling of situations from the real world (Crossroads 2001:2, NCTM, Ormell in Zweng *et al,* 1983:295, Treilibs in Zweng *et al,* 1983:297),

- individual work (Johnston *et al,* 1999:69-70),

- group work (cooperative learning) (Burton 1999:128, Davidson 1990:1-20, Johnston *et al,* 1999:68-69),

- whole-class work (Pimm and Johnson-Wilder in Johnson-Wilder, Johnson-Wilder, Pimm, and Westwell 1999:64),

- textbook approach (Johnson-Wilder *et al,* 1999:70),

- making mathematical links and connections that affords contextualised instruction (House and Coxford 1995:3, Henderson and Landesman 1995, Noss, Heally, and Holyles 1979),

- open-ended approach (Becker and Shimada 1997:1-9), and

- constructivism approach (Fosnot 1996:ix, 73-89).

However, didactical practices discussed in this section are those which mainly appear in literature and allegedly fit for practice in any country. Nevertheless, it is instructive in this study to explore pertinent didactical practices in Lesotho's mathematical education.

2.6.1 Didactical practices in Lesotho

Lesotho's mathematics teachers have thirteen guidelines (from the government's National Secondary and High School Curriculum Panel) for planning learning experiences. These guidelines include the following:

- Check (and if necessary, first develop) the pre-requisite knowledge that the pupils need for a new topic or concept,

- Do not lecture much,

- Do not start with definitions and formal language,

- Allow pupils to explore freely the limits of the concept,

- Create structured experiences where pupils can use known concepts to develop new concepts,

- Approach new concepts from different directions,

- Initially, use everyday informal language before introducing the “official” terminology,

- Give pupils a chance to use concrete materials, such as cubes, graphs, real data, chart paper, etc,

- Use visual aids,

- Give pupils the time to really understand the new concepts so that they do not rely on rote learning,

- Give pupils a sufficient number of examples and exercises to allow the new concepts to be firmly established,

- Give pupils sufficient time to allow new ideas to take root before going on to further developing the topic,

- Give pupils the opportunity to enjoy mathematics (Lesotho Secondary Mathematics Teachers’ Handbook, 1990).

The suggested teaching guidelines above indicate that Lesotho’s mathematics teachers are expected to incorporate Dienes, Skemp, and Bruner’s theories of learning mathematics: concrete to abstract, known to unknown, spiral approach, creating understanding rather than promoting rote learning, structured learning, intrinsic motivation, and other related positive learning factors.

Furthermore, Ntsekhe-Mokhehle (2003:1-4) indicates that, in teaching the forms D and E mathematics syllabus, some of the approaches recommended are guided discovery, emphasis on real-life applications, induction, deduction, explanation of concepts, and exposition. This concurs with what Ntsekhe-Mokhehle (2000:1) purported in addressing teachers of mathematics at a workshop on the teaching and learning of mathematics in the year 2000.

2.6.2 Practices of didactics relating to mathematics: An international perspective

Roulet (1998:1) maintains that instruction practices relating to mathematics do, in fact, not differ widely throughout the world. By citing practices in several countries, Roulet argues that "the nature of classroom teaching is quite similar in all countries and is stable over time" (ibid). For instance, in three nation-wide American studies, it was found that "a pattern in which extensive teacher-directed explanations and questions … followed by students seatwork on pencil-and-paper assignments" was the order of the day in mathematics instruction practices. Further, Roulet (1998:2) states that, in 1980, data from the United States of America "confirm(ed) that students at the 12th grade level" have a great part of their class time spent on "listening to teacher presentation, … doing seatwork and taking tests accounted for other blocks of time". This concurs with what Dat Do (2001:4) says about the teaching of mathematics in Vietnam schools. Dat Do points out that almost all teachers present concepts first. This is then followed by "lots of practice but with little context". Dat Do specifically says that the teacher does "99% of the talking in highly structured lessons in order to complete daily objectives". In fact, Do says that in any mathematics lesson, the teacher takes about 90% of class time explaining and illustrating to the whole class, then 8% is given to individual work, and 2% to group work.

Roulet (1998:1-6) cites three international survey reports that point out similar practices pertaining to the didactics of mathematics internationally. Firstly, teachers surveyed from 22 countries that were at the 1980 Second International Mathematics Study (SIMS) conference reported that most "of their (class) time was used in whole class instruction" (ibid). At the same time, data on the attitudes of students sampled in the SIMS survey indicated the prevalence of a style of instruction dominated by teachers wherein students perceived mathematics "as a set of rules rather than a discipline involving creativity, speculation and conjuncture" (Roulet 1998:2).

Secondly, in 1991, in an International Assessment of Educational Progress (IAEP) study, data collected from eight classes in 20 countries indicated that " students in many countries regularly spent their instructional time listening to mathematics lessons" lectured by teachers and "that another common classroom activity is to require students to work mathematics exercises on their own" (Roulet 1998:2). Thirdly, Roulet cites another international research reported in The 3rd International Mathematics and Science Conference. It was found that "the most frequent approaches used across countries involved students working individually with assistance from teachers and working as a class with the teacher leading" (Roulet 1998:3).

From the literature discussed here, it seems that, although professional leaders in the teaching of mathematics are inviting teachers of mathematics to practise instructional strategies concomitant with prevailing educational demands from society, there still is no real change in didactical practices. Little has been made in moving classroom instruction to one that promotes active learning. A further look at literature reveals that only slight variations on available didactical strategies are unfortunately in use.

As far back as twenty years ago and even fourty years ago, authors such as Christensen, Howson, and Otte (1986:245) and Land (1963:86) maintained that in any ordinary mathematics class, prominent features were as follows:

- Learners do one or more exercises, usually after the teacher's explanations/demonstrations of procedures that are directly linked to the given exercise,

- Students learn from their work either individually or in groups, but the learning activity is predominantly limited to drill and practice of previously described concepts and procedures,

- Diagnostic results are controlled by the teacher and maybe discussed with the whole class,

- Should the teacher find feedback from previous work negative, he/she repeats the standard procedure of further explanation and further drill. Should the teacher evaluate the feedback as positive, the standard procedure is applied to the next topic, concept, procedure, or exercise (ibid).

However, it needs to be pointed out that in instructional practices discussed above, textbooks are still extensively used. Textbooks contain most of the work both teachers and students use. In fact, the drill problems and, sometimes, the accompanying answers are from textbooks (Howson 1974, Smith 1996, Zweng *et al*, 1986).

Tibor Szalontai (2001:1-4) admits that, although mathematics teaching–learning approaches in Hungary are rooted in more than a 100 years of tradition, there are still elements of whole-class interactive activities. Classes are kept together as far as possible and individual work is always followed by whole-class discussion. The class discussion involves reports, reasoning together, arguing/debate, feedback, self-correction, evaluation, and agreement, and, finally, the teacher gives extra comments and extends the discussion. Here, the spoken and written abilities are a means of developing clear mathematical language and communication. However, Szalontai (2001:2) further points out that in Hungarian mathematics instruction, the method of mathematical investigation is also employed. The development of mathematical thinking through manipulative and demonstrational models, set in realistic problem contexts, is nurtured. We can define mathematical investigation itself as an enquiry into a particular area of mathematics leading to a general formula that may have been known to the student.

John Costello (1991:12) sums up these reported and commonly used practices of didactics when he says that "mathematics teaching at all levels should include" the following: exposition by the teacher, discussion between teacher and students and between student and student, appropriate practical work, consolidation and practice of fundamental skills, procedures, techniques or any routine work, problem solving including the application of mathematics to everyday situations, and investigational work.

In all of the above cases, instructional activities will hopefully furnish students with required mathematical skills and provide them with some understanding of basic mathematical rules, procedures, or techniques that students may use in mathematical settings of application. In reality, as literature points out, the prevalence of practices of didactics dominated by the teacher yield only passive learners who practise rote learning and this stifles critical thinking (Howson 1991:128, Marjoram 1974:4, Watson 1995:3, Zweng *et al,* 1983).

However, effective instruction of mathematics is more than a simple transfer of facts. Among other requirements, the instruction needs to be effective. Reynolds and Muijs (1999) report of an effective programme for teaching mathematics through what they call "the active teaching method". This method was studied and tried between 1970 and 1983 on British primary school students, but with modification, the method was equally successful with secondary school students, even in American schools. In essence, the method requires lessons to be structured as follows:

- Daily review (10 minutes): This requires the teacher to review concepts and skills associated with the previous day's work. The teacher collects and deals with previously given homework and he/she can ask students several questions on mental exercises by means of re-capping previously taught concepts,

- Development (20 minutes): The teacher briefly focuses on prerequisite skills, procedures, and concepts by lively explanations and demonstrations (yet focusing on meaning so as to promote understanding in students). Then he/she assesses students' competence by using the active interaction of process and product questioning, followed by controlled practice. If necessary, the teacher repeats and elaborates on meaning,

- Individual work (15 minutes): Here the teacher is required to give a period of uninterrupted drill practice, let students know that the work will be checked, and actually check the work,
- Homework: This is to be assigned on a regular basis (Reynolds and Muijs 1999).

A second British study on the effective teaching of mathematics was Mortimore's Junior School Project (JSP) in 1988. According to Reynolds and Muijs (1999), this significant study showed that effective teaching is achieved when teachers prepare structured lessons in which frequent high order questioning and statements are used. In this method, teachers need to restrict teaching sessions to a single area of work and involve students in the lesson. Notably, a proportion of time spent in communicating with the whole class yielded better results than when teachers spent a high proportion of time with individual students.

On the whole, in 1996, it was finally reported that there was agreement between American and British based research that using whole-class interactive instruction is a productive way of teaching. As is described above, in this interactive teaching, the teacher spends a high proportion of time in high order questioning or communicating with the whole class and making sure that pupils are actively involved (Reynolds and Muijs 1999).

However, literature itself discusses a plethora of various modes of mathematical instruction that are summarised here, but, for the sake of brevity, are not fully propounded. The most popularly recorded and practised are:

- lecture/telling/expository instructional method (Daniels and Anghileri 1995:37, Johnson-Wilder, Johnson-Wilder, Pimm, and Westwell 1999:57, Kroeze 1992:11, Price 1996, Simmons 1993:3-4, Smith 1996, Wilson 1995:3, Wilson 1993:218),

- discovery learning (Borasi 1996:6, Daniels and Anghileri 1996, Orton and Wain 1994:36-148, Wilson 1993:218),

- discussion (Borasi 1996, Simmos 1993:2-4),

- problem solving (Costello 1991:12, Du Toit, Kotze, and Du Plooy 1995, Neyland 1994:77, Simmon 1993:5, Thompson 2001),

- investigation (Costello 1991:12, Gardiner 1987, Simmons 1993:108),
- open-ended approach (Becker and Shimada 1997),
- computer-aided instruction (Berry, Graham, and Watkins 1996, Costello 1991:101), and
- mathematical modelling and cross-curricular instruction (Booss and Niss 1979, Cundy and Rollet 1989, House and Coxford 1995, Zweng *et al*, 1983).

Furthermore, another force to reckon with in didactical practices relating to mathematics education throughout the world is the development and acceptance of technological devices such as calculators and computers. This element is explored in the following section.

2.7 THE USE OF TECHNOLOGY IN SECONDARY AND HIGH SCHOOL MATHEMATICS EDUCATION

According to Heid, Sheets, and Matras (in Cooney and Hirsch 1990:194), unlike in earlier decades, a "vast array of powerful computing technology" can today be harnessed in didactical practices relating to mathematics. A plethora of literature supports this view: Barron and Hynes (in Elliott and Kenney 1996:126-135), Boge in Booss and Niss 1979:45-52, Clayton in Hoyles, Morgan, and Woodhouse 1999:22-28), Greer and Mulhern 1990:73-81, Higgo 1994:49, Hirsch 1992:9, Howson 1991:16, Johnson-Wilder and Pimm in Johnson-Wilder *et al*, 1999:144-166, Little 1995:36, Tuma and Reif 1980:ix, Sheets and Heid in Davidson 1990:265-292, Tanner and Jones 2000:164-178). Literature indicates that scientific calculators, graphic calculators, and computers are some of the technological devices that are currently being used in mathematics education to enhance the instruction and learning of mathematics.

According to literature, the development and acceptance of technology in didactical practices relating to mathematics have been urged to take root because of factors such as the following:

- It has profound implications and offers great opportunities for mathematics education that can be translated into classroom realities (Barrett and Goebel in Cooney and Hiesch 1990:205; Barron and Hynes in Elliott and Kenney 1996:127, Higgo 1994:49, Howson 1991:16, Little 1995:36, Sheets and Heid in Davidson 1990:265-292,),

- By using technology, students can learn existing curricular content, skills, and concepts more readily, enjoyably, with better understanding, and with the enhanced ability to apply mathematics (Barron and Hynes in Elliott and Kenney 1996:128, Higgo 1994:49, Little 1995:37, Tanner and Jones 2000:170-178),

- Mathematics, as practised in industry, commerce, and educational institutions, has itself changed in response to the availability of technological devices (Clayton in Hoyles, Morgan, and Woodhouse 1999:22-28, Higgo 1994:49),

- Concepts hitherto regarded as difficult become accessible to learners much earlier when technological devices are appropriately used (Heid, Sheets, and Matras in Cooney and Hirsch 1990:195, Higgo 1994:49, Tanner and Jones 2000:170-178),

- Using technological devices takes away the need for repeated drudgery and extensive computation/processing and allows greater concentration on understanding than on practising techniques (Higgo 1994:49),

- It encourages learners to play an active role in their own learning and, thus, changes the task of the teacher from giver of information to mentor. The mentor is the person who guides learners towards methods and approaches that deal with the problem under consideration and helps learners through appropriate discussion to forge links between related concepts and topics. In this way, the mentor is helping to establish sound mathematical knowledge and development (Heid, Sheets, and Matras in Cooney and Hirsch 1990:195, Higgo 1994:49; Sheets and Heid in Davidson 1990:289-291).

Although a diversity of computing technological devices has gained profound impact on didactical practices in mathematical education, Barrett and Goebel (in Cooney and Hirsch 1990:205-206) point out some disadvantages and limitations of the use of computing technology in the teaching and learning of mathematics. For instance, they note that "many schools still do not have a computer in each mathematics classroom". Further, they allege that, in spite of the wide use of technological devices in the classroom, "mathematics instruction has changed little". Sometimes, "teachers who have a computer to use in front of their classes have had trouble defining its role in the classroom" (Ibid). Here, by implication, Barrett and Goebel suggest that there are inherent challenges that come with computing technological devices in the classroom. Teachers need to be well informed of technological devices that they can use in their teaching.

The International Baccalaureate Organization (IBO), in their Curriculum Review Report on mathematics and technology, raised issues pertinent to possible difficulties encountered in using graphical calculators/computers in teaching mathematics that most educational systems, including Lesotho, can relate to. These issues included inter alia:

- difficulties associated with the cost and availability of calculators to students/schools,

- the possible need for teachers themselves to learn the skills associated with the effective use of calculators,

- the recognition that the time taken for students to learn the skills associated with using a calculator is substantial, but could be integrated into the curriculum,

- the widespread introduction of technological devices such as computers or calculators would produce the need for mathematics curricula to be written in a way that incorporates appropriate usage of calculators, this can lead to the possible de-emphasis of some areas and the enhancement of others ...,

- the advance notice required by schools to prepare for changes in regulations about the use of calculators,

- the fact that the technology is already moving on, and

- the danger of being left behind if one fails to implement some use of technology (International Baccalaureate Organization Group 5 Mathematics and Computer Science Curriculum Review 2002:7).

In fact, Heid, Sheets, and Matras (in Cooney and Hirsch 1990:194) point out two major challenges for both teachers and students in the use of computing technological devices in the instruction of mathematics. (What they allude to, in a way, summarises the considerations of the International Baccalaureate that are referred to in the previous paragraph). The first challenge is "the creation and adoption of a curriculum that openly builds on available computing power". Then come challenges that "deal with the ways in which the roles of teachers and students will change in the implementation of such a curricula" (Ibid). One of these challenges lies in assessment procedures that may be adopted if computers/calculators are used as didactical tools. For instance, one paper could be calculator free, testing basic skills and techniques, while other papers may require the use of a calculator and coursework included in portfolio assignments or projects that make use of relevant computer programmes.

2.8 ASSESSMENT PROCEDURES

According to Lloyd-Jones *et al,* (1992:1), educational assessment "lies at the core of learning". In spelling out the meaning of "assessment", Lloyd-Jones *et al,* distinguish "between evaluation, assessment, testing and examinations" (Ibid). They maintain that evaluation is the broadest of these terms and that it involves the identification of the actual educational outcomes, comparing these with the anticipated outcomes. They then proceed to the process of judging the nature, value, and desirability of the educational outcomes. Actually, Lloyd-Jones *et al,* (1992:1) see "assessment" as "… an all-embracing term (that) covers any situation in which some aspects of a pupil's education is, in some sense, measured, whether this measurement is by a teacher, an examiner or indeed the pupil himself or herself."

In fact, Lloyd-Jones *et al,* view assessment as focused on judging how well a student is performing in some educational task, while evaluation is concerned with judging whether it was worth carrying out the assessment in the first place (Ibid). Hence, evaluation cannot take place without assessment. At the same time, assessment that is completely divorced from evaluation is a half-measure (Satterly 1989:4, Frith and Macintosh 1991:9). However, Lloyd-Jones *et al,* do not equate assessment to tests and examinations. Rather they conceive a test as "… a particular situation set up for the purpose of making an assessment, while an examination is just a large-scale test, or a combination of several tests and other assessment procedures" (Lloyd-Jones *et al*, 1992:1).

Niss (1993:3) points out that, although assessment and evaluation are often used interchangeably, they may be regarded as slightly different. In particular, Niss views assessment in mathematics education as concerned with "the judging of the mathematical capability, performance and achievement (all three notions taken in their broadest sense) of students whether as individuals or in groups" (Ibid). This concurs with Webb (in Webb and Coxford 1993:1) who defines "assessment for the mathematics classroom" as "the comprehensive accounting of a student's or a group of students' knowledge". Webb and Coxford (1993:1) actually maintain that assessment involves "measuring students' understanding and use of content, obtaining instructional feed back, grading and monitoring growth in mathematical achievement."

In a way, assessment thus deals with the outcome of mathematical teaching at student level. Evaluation can be viewed as "the judging of educational or instructional systems in its entirety or in parts" such as the curricula, programs, teachers, and other specific segments of the educational system (Niss 1993:3).

Johnson-Wilder (in Johnson-Wilder *et al*, 1999:103) maintains that, in a didactical scenario, teachers need to make various judgements about students' attainment for two major reasons:" to evaluate the effectiveness of their teaching and to inform their planning". He further points out that, in mathematics education, one specifically needs to assess what students already know since "mathematical understanding is hierarchical" and misconceptions can easily lead to students' inability to "comprehend new topics" (Ibid). However, literature agrees that, in didactical practices, assessment is carried out:

- to inform teachers in their planning,
- to help teachers to diagnose the source of students' difficulties,
- to provide teachers with a basis for reporting progress to both students and their parents, and
- to meet the requirements of external examinations (Cangelosi 1996:307-336, Hoyles, Morgan, and Woodhouse 1999:231-233, Johnson-Wilder *et al*, 1999:103, Stenhouse 1975:57-59, 64-66, 72-74, 79, 82-83, 86, 94-95, 101, and Tanner and Jones 2000:68).

Furthermore, literature posits that the main modes of educational assessment are formative and summative assessment (Cangelosi 1996:307-308, Frith and MacIntosh 1991:17, Johnson-Wilder *et al*, 1999:104, Lloyd-Jones *et al*, 1992:2-4, Niss 1993:7-8, Satterly 1989:6-7). According to Lloyd *et al*, (1992:2), formative assessment is an integral part of the didactical situation. It is the process through which the teacher and the student receive "feedback information about whether the learning objectives are being reached" (Ibid). Thus, formative assessment provides information about a student pertaining to his or her areas of weakness, strength, and potential. It may be viewed as diagnostic and may also be utilised as an important ingredient of motivation (Lloyd *et al*, 1992:2). On the other hand, summative assessment is the end product, an assessment that gives the final summing up of one's knowledge as opposed to the assessment of one's immediate learning stage that affords formative feedback. Actually, summative assessment is usually entrenched in educational systems that offer external and public examinations (Lloyd *et al*, 1992:2-3, Satterly 1989:6-7).

The performance of students in such examinations may be compared with that of other members in their class, their year group, other students in their whole country, or in other countries according to the set norm for that examination. Hence, such assessment is known as norm-referenced assessment (Lloyd *et al*, 1993:3). However, when assessment is providing a comparison between students' performance and given standards set (criteria) for the achievement of particular learning objectives, the assessment is called "criterion–referenced" (Ibid).

There are various modes of educational assessment depending on the purpose and on the use of the assessment. Friths and MacIntosh (1991:17) discuss formal versus informal, formative versus summative, continuous versus terminal, coursework versus examination, process versus product, internal versus external, convergent versus divergent, and idiographic versus nomothetic. Formal assessment is for the "for public use" while informal assessment is "for private use" (Ibid). Frith and MacIntosh maintain that continuous assessment should not be identified with formative assessment, nor should terminal assessment be associated with only summative assessment. Actually, continuous assessment may be used formatively at the time it is taking place, but may also subsequently contribute to summative assessment as in the case of continuous coursework contributing to the final result in a summative examination (Ibid). For instance, Niss (1993b:21) points out that continuous assessment can actually take place concurrently with and is often integrated into the didactical situation. However, convergent assessment refers to tasks such as objective tests, while divergent assessment is exemplified by open-ended tasks. Idiographic assessment, however, "aims to find out about the uniqueness of individuals, what they do, what they know and what they are". In contrast, nomothetic assessment "collects data about individuals with a view to comparing one with another and using generalisations made from those assessed for the assessment of others" (Frith and MacIntosh 1991:18). Hence, formative assessment can be idiographic and can also, to an extent, be nomothetic.

However, Swan (in Niss 1993b:196) points out that, in recent years, there has been a movement away from norm-referenced towards criterion-referenced assessment with the intention of crediting students for their positive achievements. Thus, instead of ranking students, teachers prefer a search for evidence of particular proficiencies and these are reported in a form of profile of achievement. This approach is believed to be essential if the results are to be used formatively. The criteria used are usually based on hierarchically ordered lists of behavioural objectives that are developed from research. In order to be meaningful, the criteria need to be specific and detailed.

However, in recent years, mathematical education has claimed new territories where traditional assessment modes such as examinations and tests administered from the outside have, in many cases, hindered curriculum reform and were in need of reform to reflect suitable assessment practices (Niss 1993a:4). According to Niss, the current modes of assessment allow one to assess the knowledge, insights, and abilities that are related to the understanding and mastering of mathematics.

The assessment modes also provide assistance to individual learners in monitoring and improving their acquisition of mathematical insights and power. At the same time, current assessment modes assist teachers in monitoring and improving their teaching, guidance, supervision, and counselling (Ibid). Further, as Niss (1993a:4) points out, new mathematical applications, modelling, and explorations aided by computers and graphic calculators have enabled assessment to be in the form of extended investigations of both pure and applied mathematics, project work, scientific enquiry, out-of-classroom activities, experimentation, and group work.

2.9 CONCLUSION

From the discussion in this chapter, it is evident that, today more than ever, mathematics education requires that learners must be competent in contextually employing their mathematical knowledge instead of merely memorising rules, concepts, and formulae, followed by drill in respective procedures, processes, and techniques. This need for a change in didactical practices is urged by current changes in our society where there is need to empower citizens in the field of mathematics so that they can meaningfully participate in the technological world of which they are part. Hence, the question of how to organise didactical practices in order to develop a mathematically literate student becomes pertinent in such a society. Fortunately, in America, Britain, Australia, and other parts of the world, didactical practices relating to mathematics are being reshaped. The next chapter addresses the issue of mathematical literacy, its indices, and the fitting didactical practices.

CHAPTER 3

MATHEMATICAL LITERACY AND DIDACTICAL PRACTICES

3.1 INTRODUCTION

The previous chapter explored mathematics education in schools, its influential neighbours, and philosophical, sociological, and psychological issues that influence learning theories of mathematics together with details of common didactical practices relating to mathematics. The present chapter explores issues that are relevant to mathematical literacy and didactical practices that enhance mathematical literacy in students. Thus, the chapter deals with current conceptions of what it means to be mathematically literate and the kind of mathematics that builds up mathematical literacy in learners. Furthermore, the need to justify mathematical literacy is discussed in view of pressures from the changing society, the changing nature and growth of mathematics as a subject, the changing needs at work places, and the ubiquity and utility of mathematics in real life situations. Didactical practices that are recommended in literature to entrench mathematical literacy are also explored in detail.

3.2 MATHEMATICAL LITERACY: ITS MEANING, INDICATORS AND INDICES

In essence, what is mathematical literacy and which didactical practices can achieve that kind of literacy? What are the observable and recognisable aspects of mathematical literacy that are spelt out in literature as indicators and indices of mathematical literacy? In the sections that follow, first, mathematical literacy itself, its indicators, and its indices are explored. In subsequent sections, the need for mathematical literacy is spelt out. Thereafter, a discussion of didactical practices and the kind of mathematics that are purported, according to literature, to achieve mathematical literacy are addressed.

3.2.1 The concept of mathematical literacy

In mathematics education today, much emphasis is placed on mathematical literacy (Bhola 1994:8, Cooper 1997:1, De Turck 2000:1, Getz 1999:434, Restivo 1993:117, Hirsch 1992:3, and The NCTM Standards 1989:3). There is, thus, considerable talk about teaching students so that they are mathematically literate in a world that needs a mathematically literate workforce. Yet, according to Cooper (2000:1), there seems to be no simple consensus on the meaning and definition of the term "mathematical literacy". Nevertheless, a description of what mathematical literacy entails has been propounded in literature often enough for us to envisage the import of the term. It needs to be pointed out that the different definitions, which are independently given in literature, are inter-related, and, hence, may be seen to enlarge on and complement each other in order to portray a clear picture of the meaning of the term.

Hoyles, Morgan, and Woodhouse (1999:18) see a mathematically educated person as a mathematically literate person. In fact, Hoyles, Morgan and Woodhouse view the mathematically literate person as one who has knowledge of mathematics as a discipline in its own right and has knowledge of mathematics, which he/she employs as a necessary tool for coping with and understanding the complex world in which we live. On the other hand, Cooper (2000:1-3) envisages mathematical literacy as referring first and foremost to education in mathematics itself. He then also sees it as referring to "familiarity with mathematical language" and to "acquaintance with the fundamentals of mathematics". In other words, Cooper sees mathematical literacy as the attainment of education in mathematics, which, in turn, enables one to be knowledgeable and conversant with its fundamental skills, concepts, and language. This, in essence, concurs with Hoyles, Morgan, and Woodhouse (1999:18). However, when mathematics becomes part and parcel of one's functional knowledge and language as Cooper propounds, one has become mathematically literate. Further, there is also The Organisation for Economic Co-operation Development (OECD) that views mathematical literacy in three dimensions: "the content of mathematics, the process of mathematics and the situation in which mathematics is used" (OECD 2000:1). This view takes functional mathematics education as a means of equipping one with mathematical literacy.

Nevertheless, the views of Hoyles, Morgan, and Woodhouse, Cooper, and the OECD about the meaning of mathematical literacy are not mutually exclusive. When the three descriptions are explored, major similarities emerge. For instance, the OECD talks about "content of mathematics", "process of mathematics", and "the situation in which mathematics is used. This can be parallel to Cooper's (2000:2-3) and Hoyles, Morgan, and Woodhouse's (1999:18) description of mathematics education that leads to mathematical literacy. In fact, on the one hand, Cooper (2000:3) maintains that for one to be considered mathematically literate, his/her mathematical proficiency needs to be beyond mere arithmetic. On the other hand, Cooper (2000:1-3) simultaneously envisages mathematics education as a means of equipping the mathematically literate person with the ability to engage in problem-solving techniques pertaining to real life. This essentially concurs with the quote from Hoyles, Morgan, and Woodhouse (1999:18) at the beginning of this paragraph. Hence, Cooper (2000:2-3), and Hoyles, Morgan, and Woodhouse (1999:18) independently agree with the NCTM Standards (1989:23, 137), which propound that, through mathematics education, students should, amongst other things, be able to:

- formulate problems from mathematical and from everyday life situations,

- develop methods and apply strategies to solve a wide variety of problems, apply integrated, mathematical problem-solving strategies to solve problems from within and outside mathematics itself,

- recognise and formulate problems from within and outside mathematics as a subject, and

- apply the process of mathematical modelling to real-world situations.

At the same time, these views agree with those of the OCED (2000:1) when they maintain that the real life situations in which mathematics arises and is used actually range from individual and private contexts to those situations that relate to wider scientific and public issues. However, what Cooper (2000:4) proposes is that mathematics education, which seeks to inculcate mathematical literacy in students, should develop students' abilities to reason, think independently, and solve problems in mathematics both inside and outside of mathematics per se. The reason for this is that students acquire the knowledge of computation and other specific mathematical contents.

Indeed, as Hirsch (1992:v-vi) points out, mathematical education for mathematical literacy needs to demarcate contemporary, appropriate, and relevant mathematical content at particular levels of education. However, it is the OECD (2000:1) that points out that, with the acquired mathematical content, learners are introduced to general processes of mathematics that develop their competencies and skills. The latter should, amongst others, consist of:

- the normal, familiar, conventional definitions of mathematical terminology and simple computations,

- making connections within mathematical content and cross-curricular connections, as well as connections with real-world situations in order to solve problems, and

- mathematical thinking, conjectures, generalisations, and insight that require students to engage in analysis, identify the mathematical elements in a situation, and develop the ability to pose their own problems.

Further, it should also be noted that the mathematical competencies to which the OECD (2000:1) refers also include skills in the use of mathematical language, investigation, and modelling of problem solving. The same point is raised by Cooper (2000:4) when he indicates that familiarity with mathematical language, coupled with a thorough grasp of mathematical content, builds up mathematical literacy in learners. In fact, the NCTM New Standards (1997:58) maintain that, among other things, mathematical literacy itself empowers learners with the ability to:

- use mathematical language and representations with appropriate accuracy, including the use of numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams,

- organise work, explain facets of a solution orally and in writing, label drawings, and employ other relevant techniques to make the mathematical situation clear to the audience, and

- use mathematical language in order to make complex situations easier to understand.

It is instructive at this point to further examine other views on the meaning of mathematical literacy in order to build a wide spectrum of ideas from which we can draw indicators and indices. For instance, another view about the meaning of mathematical literacy is that of Curry, Schmitt, and Waldron (1996:1). In fact, Curry, Schmitt, and Waldron maintain that mathematical literacy and numeracy can be used interchangeably. This point resonates with that of Cooper (2000:2) where the terms "mathematical literacy", "numeracy", and "quantitative literacy" are used either synonymously or as complementing one another depending on the user. However, mathematical literacy has a broader import than mere "numeracy" and "quantitative literacy". The latter are but components of the former. For example, adequate number sense, familiarity with basic ideas of everyday used mathematics, and notions of simple ideas of probability can be associated with being numerically knowledgeable, and it leads to being mathematically literate. Numeracy and quantitative literacy pertain to numeric and computational abilities, which are but tools for a mathematically literate individual.

Hence, while viewing mathematical literacy and numeracy as terms that can be used interchangeably, Curry, Schmitt, and Waldron (1996:3) maintain that "…

both terms should be viewed as loosely referring to the aggregate of skills, knowledge, beliefs, patterns of thinking and problem solving processes which individuals need to effectively interpret and handle real world quantitative situations"

Actually, similar factors are pointed out by the OECD (2000:1) when they describe mathematical literacy as "…an individual's capacity to identify and understand the role that mathematics plays in the world, to make well founded mathematical judgements and to engage mathematics in ways that meet the needs of that individual's current and future life as a constructive concerned and reflective citizen."

However, as pointed out earlier, mathematical literacy is an aggregate of mathematical abilities that is built up from various facets and components, one of which is the ability to communicate mathematically.

3.2.2 Mathematical literacy and communication

Some literature on mathematical literacy, points out that elements of the ability to communicate both in speech and in writing are expected in spelling out mathematical literacy (Dorfler in Hoyles, Morgan, and Woodhouse 1999:63, Hoyles, Morgan, and Woodhouse 1999:17, Jenner in Pimm 1988:78-79, Restivo *et al,,* 1993:117, Lindquist and Elliott in Elliott and Kenney 1996:ix, 1-19, Pimm 1987:x-xi, xv, 1-49). The same idea is captured in South Africa's critical outcomes for "Learning Area: Mathematics" where students are expected, amongst other things, to:

- collect, analyse, organise, and critically evaluate information …, and

- communicate effectively using visual, mathematical, and/or language skills in the modes of oral and/or written presentation… (South Africa National Department of Education Document 2000:5).

However, the fact that mathematics is thought of as a language is a well-accepted theorem in mathematics circles. In fact, in the NCTM Standards (1989:26), mathematics is regarded as a meaningful language, which assists students in communicating mathematically and in applying mathematics productively. For instance, through mathematical symbolism, students form links and connections between the concrete and the abstract in pictorial, graphic, symbolic, verbal, and equation form. Hence, the NCTM Standards recommend that, in order to develop mathematical literacy in students, opportunities for mathematical communication should be provided so that students can, amongst other things, be able to:

- relate physical materials, pictures, and diagrams to mathematical ideas,

- reflect on and clarify their thinking about mathematical ideas and situations,

- relate their everyday language to mathematical language and symbols, and

- realise that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics (NCTM Standards 1989:26).

Therefore, in order to entrench mathematical literacy through communication, students should be encouraged to "talk mathematics". Actually, the NCTM Standards (1989:26) maintain that, in interacting with classmates, learners clarify their own thinking, get to construct knowledge, and acquire new ways of looking at given ideas. Furthermore, writing mathematics helps students to clarify their thinking and to develop a deeper understanding of mathematical concepts. At the same time, exploring, investigating, describing, and explaining mathematical ideas, procedures, and strategies to solve given mathematical problems actually promote communication (Ibid). As discussed in 2.3.3.6 of Chapter 2, this perspective is, indeed, in line with Vygotsky's theory of learning where speech is deemed to represent the thought structure.

3.2.3 Mathematical literacy and the integration of mathematics within itself, with the real physical world, and with other school subjects

Another facet of mathematical literacy is the formation of meaningful mathematical connections. These include mathematical connections within mathematics itself, with other subjects in the school curriculum, connections with realities in the physical world, or connections with other contexts arising from real-life situations. In this respect, Bell (in Zweng *et al,*, 1993:252), Bondi (1991:1), Burkhardt (in Zweng *et al,* 1993:284), Fraser (in Neyland 1994:173), Mokoena (in AMESA 1998:33-45), Wheeler(in Zweng *et al,* 1993:290), Woodbury (1998:303), and Yelueda (in Zweng *et al*, 1993:247) all concur with the assertion that mathematically literate people are those whose mathematics is meaningfully connected and integrated into real-life contexts. In fact, Elliott and Kenney (1996:ix) emphatically point out that students must, of necessity, be able to "make sense of and communicate in the quantitative world of which they are part".

According to Coxford (1995:3), connections in mathematics for the purposes of fostering mathematical thinking in other subject areas and "to contextualize mathematics" for the effect of enabling students to "see mathematics as a means to help make sense of their world" are not new. Mathematics has been known for its utilitarian value for time immemorial. In fact, one of the long-serving purposes of education in mathematics has been to equip students to solve mathematically inclined problems with the aid of mathematical concepts and methods. Hence, mathematical connections easily lend themselves to connections across the school curriculum.

In the light of this point of view, some of South Africa's learning programme guidelines for Curriculum 2005 are to include "integration within and across learning areas … relationships between learning outcomes …" (South Africa Department of Education Document 2000:16). Further, advancing the same point of view, the International Baccalaureate Middle Years Programme (IBMYP) for the ages 11 to 16 years, offers didactical practices that "embrace", but at the same time, "transcends the focus on traditional school subjects" and "accentuates the inter-relatedness of them" so as to "advance a holistic view of knowledge". The

IBMYP, thus, encourages to "shun the fragmentation of knowledge that so often results when students move from biology to history to mathematics to technology as if the classes had nothing to do with each other" (International Baccalaureate Middle Years' Programme Fundamental Concepts 2000:2).

However, being mathematically literate also entails making connections within mathematics itself. According to the NCTM Standards (1989:32), making mathematical connections within mathematics itself enables students to:

- link conceptual and procedural knowledge,
- relate various representations of concepts, formulas, or procedures to one another, and
- recognise relationships among different topics in mathematics.

In a way, the mathematical connections that the NCTM Standards are recommending here are supposed to foster in students the ability to see how mathematical ideas are integrated and, hence, entrench meaningful understanding of the concepts in students. Such mathematical conceptual connections, which link topics and concepts, are often referred to in literature as "concept mapping". As pointed out in section 1.2.1 of Chapter 1, Mokoena (in AMESA 1998:33-34) also gives a description of how concept mapping can, within mathematics itself, enhance meaning and understanding in learners that, by implication, yield mathematical literacy in students. Further, Coxford (in House and Coxford 1995:4-12) independently gives an exposition and illustration of how mathematical connections are facilitated because of the pervasiveness of mathematical conceptual themes and processes.

However, apart from integrating concepts within mathematics, connecting mathematical concepts with other subject areas (interdisciplinary connection) is an important mark in mathematical literacy. For example, Begle (1970:10), Berlin and White (1995:22-33), and Bochner (1966:v) also allude to the integration of school mathematics and science. Actually, Berlin and White (1995:22-23) point out several ways in which to view the connection between school mathematics and science. One way is to view the connection "from the perspective of the learner and how scientific and mathematical concepts, processes, skills … are processed and organised in the cognitive structure of the learner" (Berlin and White 1995:23). For example, in both science and mathematics, knowledge:

- is built in a spiral approach on previous knowledge,
- is organised and used as bricks to build upon or around big ideas, concepts, or themes,
- involves the interrelationship of concepts, formulas, and processes,
- is situation- or concept specific,
- is propagated and advanced through social discourse or contextually related to the real and physical world, and
- is socially constructed over time (Ibid).

From these factors, it can be deduced that constructivist principles of learning (discussed in Chapter 2.2.4 and 2.3) are employed in the study of both mathematics and science. Hence, Berlin and White (1995:24-25) maintain that ways of knowing mathematics are similar to those used to advance knowledge in science. In science, nature is explored by observation and the manipulation of phenomena, with patterns and generalisations captured in mathematical symbols, expressions, and equations. Thus both mathematics and science meet at the same natural/physical phenomena to assist the learner to attain holistic and meaningful understanding of the phenomena. Here, mathematical modelling is used to capture physical reality, patterns or relationships. Mathematics can facilitate scientific studies in other various ways such as the use of mathematical objects in graphic, symbolic, numeric, geometric, or functional forms (Berlin and White 1995:24).

It is, thus, possible to connect mathematics to other school subjects. Actually, Reimer (1995:104–115) shows the connections between mathematics and its history and, thus, establishes a powerful connection that can be forged between school mathematics and history. Whitin (1995:134-140) also shows a connection between literature and mathematics, whilst Lambdin and Lambdin (1995:147-151) reveal the connection between school mathematics and physical education through spatial awareness.

However, the connection between literature (the written word in text books) and mathematics is interesting. Whitin (1995:134-140) explores how, through the use of literature, students are enabled to see that mathematics is only a common, human, and, therefore, social activity that is employed by people in different contexts for different purposes. Further, Whitin (1995:134-140) draws attention to the fact that using textbooks in school mathematics is an important influence on how students develop the understanding of mathematics itself. In fact, in many classroom situations, the textbook is often the only resource, apart from the teacher, to which the students have access. By using textbooks, students gain a fuller and deeper understanding of the way mathematics is connected, first, to other mathematical concepts and, then, to the real world. At the same time, Whitin (1995:134-140) points out that mathematical material of any sort as well as its terminology and symbolic notations do not make easy and fast reading even to the best of learners. Hence, it can be deduced that the way a book is written often tells whether or not it promotes the understanding of the mathematical material it discusses (Ibid).

In fact, Coxford (1995:4) summarises mathematical connections (recommended by the NCTM Standards) that students should experience in order to achieve mathematical literacy. The summary includes enabling students to:

- see mathematics as an integrated whole,

- apply mathematical thinking and modelling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business,

- use and value the connections among mathematical topics, and

- recognise equivalent representations of the same topic (Ibid).

Actually, Berlin (1995:22-28), Coxford (1995:7-12), Day (1995:54-64), Hodgson (1995:13-20), and Rubenstein (1995:65-78) all argue that aspects of mathematical connections can actually be achieved through mathematical processes such as representations, applications, problem solving, and reasoning. This can be done by using "mathematical connectors", which include functions, matrices, algorithmic procedures, graphs, variables, ratios, and transformations.

However, on the whole, the discussion in this entire section describes mathematical literacy as:

- the individual's confidence in, overall command of (that comes from meaningful understanding), and effective use of mathematical knowledge (that includes mathematical structure, skills, concepts, principles, and procedures),

- mathematical reasoning and problem solving skills in competently identifying, predicting, interpreting, and handling real-world quantitative situations and a variety of other mathematically and technologically oriented contexts, and

- the ability to explore and reason logically, to solve routine and non-routine problems, to communicate about and through mathematics, and to connect ideas within and outside mathematics.

3.2.4 Indices and indicators of mathematical literacy

A break down of the whole concept "mathematical literacy" (as discussed in sections 3.2 and 3.2.1-3.2.3 of this chapter) provides variables that can be used in developing indicators of the mathematically literate person. For the purpose of this research, a group of similar indicators will be called an index of the concept "mathematical literacy". Hence, from the discussions in this chapter, four major indices of mathematically literate individuals can be identified, namely:

- the appreciation of the language and beauty of mathematics that comes with the meaningful understanding and command of mathematical knowledge and skills,
- the possession of creative, logical, and critical thinking in mathematics,
- having appropriate problem-solving skills in mathematics, and
- having the ability to appreciate and apply mathematics in appropriate real-world situations and other environmental and social contexts.

From the discussion in the aforementioned sections of this chapter, indicators of each of these indices, though inter-related, can be extracted, arranged, and grouped in tabular form as in Table 3.2.1 below.

Table 3.2.1 Indices and indicators of mathematical literacy

INDEX	**INDICATORS**
1. The appreciation of the beauty of mathematics that comes with meaningful understanding and command of mathematical knowledge and skills	• mastery of knowledge and skills, understanding and appreciation of mathematical structure, concepts, procedures, and principles, • ability to enjoy and engage in mathematical pursuits, • appreciation of the beauty, power, and utility of mathematics, • represent, discuss, read write, and listen to mathematics as a vital part of learning and understanding mathematical concepts, • understand mathematics through review of their historical origins and concept

	mapping, • appreciate the power of mathematical abstraction through algebra and generalisations, • ability to organise mathematical facts and develop strategies to solve a wide variety of problems, • describe, analyse, extend, and create a wide variety of mathematical patterns and relationships, • define, verbalise, and represent concepts in many different forms,
2. Possession of creative, logical, and critical thinking skills in mathematics	• thinking patterns that involve mathematics to make ideas more meaningful and concrete, • make mathematically informed generalisations, conjectures, and judgements, • mathematical insight that requires one to meaningfully engage in analysis to identify mathematical elements in a situation and even pose mathematically meaningful problems, • use inductive and deductive reasoning to verify arguments and conclusions, • judge the validity of arguments and construct logical arguments, • analyse situations and determine common properties and structures, • develop plausible mathematical statements, • appreciate the axiomatic nature of mathematics.
3. Having appropriate problem solving skills	• understand and are able to formulate mathematical problems, • apply a variety of methods to solve problems, • verify, interpret, and generalise results, • patient and persistent in investigating

	solutions to problems, • able to solve problems with number and language, and knowing when answer is reasonable, • know enough mathematical structure to be able to use what they know, to be able to work out what they do not know, • discuss and communicate clearly and fluently in the process of problem solving, • apply integrated problem-solving techniques to solve problems within and outside mathematics.
4. Having the ability to appreciate and to apply mathematics in appropriate real-world situations and other environmental contexts	• use mathematical language to make complex situations easier to understand, • use mathematical representations, e.g. tables, equations, charts, graphs, and diagrams to give appropriate accuracy to real life contexts and situations, • form problems from physical and every day mathematical situations, • apply the process of mathematical modelling to real world situations, • make mathematical connections across the curriculum, • relate everyday language to mathematical language and symbols, • relate and understand the physical world through mathematics, • can carry out research and mathematical investigations using mathematical literature and/or experimental resources, • cope with the demands of everyday life and know how to choose efficient processes in any situation – which lead to reliable answer. •

It needs to be pointed out that neither the indices nor the respective indicators portrayed in Table 3.2.1 are by any means exclusive. Nevertheless, they represent factors about mathematical literacy that can be deduced from the discussions ensuing from the literature review in this research study. However, having explored the meaning, indices, and indicators of mathematical literacy, the next issue is to examine and justify the need for mathematical literacy to offer mathematics education.

3.3 THE NEED FOR MATHEMATICAL LITERACY

In this section, the increased call for mathematics education that entrenches mathematical literacy is viewed from five interrelated angles, *viz.* change in didactical perspective, the technologically changing society, widening areas of applicability of mathematics, the change in the nature of mathematics due to its growth, and the pressure from the changing needs at work places.

3.3.1 Limiting didactical practices

Didactical practices that place emphasis on the acquisition of facts, axioms, theorems, skills, processes, and procedures in a way that divorces mathematics from the contexts in which it arises and thrives are becoming increasingly obsolete, unproductive, and inappropriate in a world that is imbued with technology (Bondi 1991:1, NCTM 1989:v, Neyland 1994:173, Woodbury 1998:303). This argument is also well documented by Avital (in Zweng, Green, Kilpatrick, Pollak, and Suydam 1983:276), Cangelosi (1994:1-4), Hirsch (1992:v), Marjoram (1974:4), NCTM (1991:1-3), Neyland (1994:3), Orton and Wain (1994:212), Smith (1996:390-391), Siemon (in Zweng *et al*, 1983:250), and Sitia (in Zweng *et al*, 1983:274). However, this citation is by no means exclusive. What is noteworthy, are the limitations of such didactical practices, which, at the same time, justify the need to offer mathematics education that cultivates in learners mathematical literacy. In fact, there are numerous and varied factors that urge and pave adequate ground for the need for didactical practices that entrench mathematical literacy in learners.

For instance, Fraser (1994:173) points out that the instruction of mathematics through drill portrays mathematics to learners "as a difficult, mechanical and as a linear set of concepts". Fraser goes on to say that even drilling learners to master mathematical concepts through graded problems in a spiral and hierarchical manner tends to put learners at a disadvantage in three inter-related ways:

- the learner gets stuck on the teacher's instruction at once,
- the learner desperately tries to make sense of the problem and failing this, rote learns the acceptable way to solve the given problem, and
- the learner fails to see the relevance of the learnt mathematics to his/her own life (Fraser 1994:173).

Cangelosi (1994:1-4) also observes that learners who acquire their mathematical education in this way perceive mathematics as a "boring string of terms, symbols, facts and algorithms understood only by rare geniuses". Consequently, such learners often fail to extend their mathematics beyond what is memorised, let al,one being able to discover or invent mathematical concepts or creatively apply the learnt mathematics outside the classroom. Mathematics becomes a life-long terror and a useless subject. Yet, according to Devi (2001:1, 9) mathematics is not boring, tedious, and dull, but thrilling, beautiful, full of sheer ecstasy, and offers a world of adventure where calculations are thrilling and applications are a plethora. Didactical practices need to shift. They need to address this shortfall and seek to produce learners who are mathematically literate and who have a lifelong functional knowledge of mathematics.

3.3.2 Pressure from the technologically changing society

It needs to be pointed out here that the unfortunate end product of the kind of mathematics education described in Section 3.3.1 is probably a mathematically illiterate citizen. Inevitably, this citizen is inculcated with a deep mathematics phobia, which becomes a hidden curriculum passed on to others (Borasi 1992:3, Borasi 1996:18, Blake 1994:30-31, Cangelosi 1996:vii, Johnson-Wilder, Johnson-Wilder, Pimm, and Westwell 1999:176, 181, 186). Worse still is the often observed fact that this kind of mathematically illiterate person becomes unfit and unproductive in an increasingly growing technological world (Borasi 1996:18, Curry, Schmitt, and Waldron 2000:2, Higgo 1994:49, Holt and Marjoram 1973:v).

Social and economic changes have caused low-cost calculators, computers, and other related technology to be part of the real, day-to-day tools and innovations that are used in technology, which continue to increase (NCTM 1989:3). Hence, in order to meet the demands of continual change in technology and other related issues, we have Pollak (in Howson 1988:34) advocating that didactical practices of mathematics itself "must change in the light of the changes in technology and applications of mathematics".

3.3.3 Pressure from the widening scope of the applicability of mathematics in real-life situations

The need for mathematically literate people is even more acute in today's society, which requires people with a functional body of mathematics that can readily be harnessed for application at every turn in life (Gayfer, Budd, Kidd, and Shrivastava 1979:7). As pointed out in Chapter 1, Section 1.1, history has continuously demonstrated that mathematics pervade the whole of our world, society, and human activities irrespective of location, colour, or creed. In particular, mathematics seems to have become an integral part of people's cultural, social, economic, and technological environment today. Hence, Holt and Marjoram (1973:v) emphatically point out that no person worth their salt "dares to be innumerate" when they write:

> "Politicians … lawyers and industrial magnets must all at least be able to read a graph in their daily decision - making … farmers interpret the growth curves of their live stock, ethnographers create mathematical models of primitive societies. All need some, if not much mathematics ... The intelligent layman, if he is to make rational decisions and vote prudently, will need at least an intuitive grasp of economies and social

> political principles involved and these … are becoming increasingly more mathematical.… It is as though the earth shrinks to a global village while knowledge explodes to uncontrollable proportion …, as though the only way knowledge can be handled, comprehended and communicated is by encapsulating it in formulae and symbol" (Ibid).

The same necessity and importance of mathematically competent and literate citizens is also captured in a stark but humorous manner by De Turck (2001:1-2) in his essay "Talk about Teaching Mathematics Literacy":

> "There is a story about the great eighteenth century Swiss mathematician, Leonhard Euler. He was summoned to court to debate with an esteemed but nameless philosopher about the existence of God. The philosopher offered a long, eloquently worded argument to refute the existence of a deity. Then, Euler stepped up to a blackboard and wrote some complicated mathematical equation…, stepped back and intoned "Therefore God exists". The philosopher was speechless in the face of the mathematics because he was not mathematically literate to recognise its irrelevance".

Even though the citations from Holt and Marjoram and from De Turck clearly indicate the expediency of mathematical literacy, we still have Morris (1981:161) alluding to the same notions and going on to demarcate areas permeated by the power and utilitarian value of mathematics when he posits that "… mathematics pervades the whole environment and every individual encounters the use of mathematics in three broad contexts:

- in the context of his private life,

- in the context of his working life, and

- in the wider context of the social, economic and political life of the country of which he is a citizen (Morris 1981:161).

All arguments we have here call attention to the need for and significant worth of mathematical literacy as mathematics itself pervades and forms an integral part of certain human activities and physical situations (Booss and Niss 1979:2, Durham 2000:1, Glaister and Glaister 2000:1, Tanner and Jones 2000:1). Yet, over time, mathematics seems to have increasingly appeared in spoken or written language, in other areas of knowledge, and even in the humdrum day to day social and life transactions between people (Bondi 1991:viii, ix, Courant and Robbins 1981:v, Howson and Kahane 1988:1-9, Neyland 1994:173). These real life situations, therefore, elicit the fact that individuals need to be equipped with functional mathematical knowledge and skills in order that that the individuals may operate in daily situations in an intelligent, competent, and productive manner. Furthermore, it places a compelling demand on educators. This demand justifies the allegation that no longer is it adequate for teachers of mathematics to solely cultivate in students skills to perform mathematical operations and procedures that are divorced from the contexts in which they appear in the social and practical contexts that are familiar to the learner. In fact, literature posits that, in order to afford students the acquisition of functional meaning and understanding of mathematics, mathematical skills and concepts need to be embedded, grounded, and explained in and through contextual real-life problems (Educational Studies in Mathematics Vol. 38, 1999:197, House and Coxford 1995:vii, Marjoram 1974:4, Neyland 1994:3, Steen 1981:31).

The arguments advanced in the preceding paragraphs lead to the assertion that the teaching of mathematics without reference to its enigmatic power to succinctly capture and assist explanations and solve contextual mathematical real problems is to rob students of one of the most vital qualities of mathematics. Such didactical practices may produce mathematically illiterate students since, in essence, learners do not interact with mathematics in a real and meaningful way.

3.3.4 Pressure from the changing nature of mathematics due to its growth

Another factor that justifies the need to instruct mathematics for mathematical literacy is the changing nature of the discipline of mathematics itself. Actually, Beagle (1979) points out that, since 1960, there have been major changes and developments in mathematics education at school. There has been significant change in both content and methods of instruction. In the article "Crossroads" on their internet home page, the NCTM (2001) clearly explains this factor. They show how mathematics as a growing discipline has developed algorithms needed for computer-based processes. Other areas of mathematical growth include the use of probability in understanding the status quo of chance and the process of randomisation, applications of non-Euclidean geometry in other fields, and the growing and extensive use of vectors in geometry and matrices in areas once thought to be none-mathematical (for example, information technology) (NCTM 2001:1).

This view concurs with that of Restivo, Bendegem, and Fischer (1993:185) as they maintain that today's world is full of mathematisations that were not here last year or ten years ago. The same notion is pointed out in the NCTM Standards (1989:1) where they say that "mathematics is rapidly growing and is extensively being applied to diverse fields". Probably, this change in the nature and dimensions of mathematics is due to the fact that today's society imbued with information has created a dual need within mathematics education. This dual need involves both the aspects of mathematics that must be transmitted to students and the concepts and procedures that must be mastered if students are to be self-fulfilled productive citizens (Ibid).

3.3.5 Pressure from the change in needs at workplaces

Change in society itself dictates particular requirements in the type of mathematics education appropriate to people at a certain point in their course of social development. For instance, Hirsch (1992:3) points out that, in yesterday's industrialised society, a lack of mathematical expertise was neither much of a disadvantage nor an impediment of consequence to success at the work place. This is because, at most work places, tasks were largely arithmetic. If more complex mathematics was needed at all, the attention of specialists such as engineers, accountants, statisticians, or quality control analysts was called for. Furthermore, understanding social and political issues was equally not complicated by sophisticated mathematics, since for personal needs the use of arithmetic and measurement sufficed. In fact, high school mathematics normally served to teach students how to reason or to prepare them for post high school studies. On the whole, mathematics had a minute resemblance as to how it was applied in the real world situations (Ibid).

However, the society today requires competence in mathematics that goes beyond arithmetic. Basically, for full and meaningful participation in society, all areas of life require a higher standard and quality of competence in mathematics (Hirsch 1992:3, Holt and Marjoram 1974:v, NCTM 1989:3). There is, thus, an increased need for mathematical literacy in our societies today, such that even workers in industry are expected to possess skills in mathematical reasoning in interpersonal relations (Hirsch 1992:4). To this end, Restivo, Bendegen, and Fischer (1993:174) also point out that there is a need to make a distinction between mathematical competence and competence in mathematical applications. The distinction involves recognising the meaning of mathematical knowledge itself, of technical knowledge that includes the ability to use mathematics to advantage, and of reflective knowledge that, essentially, involves a dialectical process. Perhaps, what society needs today is a combination of all these forms of knowledge. Actually, the prerequisite for this combination of knowledge helps us to understand our world and society, which have changed remarkably.

Further, Hirsch (1992:4) maintains that, today, the success of industries depends on the availability of a skilled and adaptable work force who possesses not only mathematical content, but must also have reasoning skills, interpersonal relations, as well as communications skills and tolerant work ethics. Specifically, Hirsch goes on to say that, in order to maintain today's democratic values, we need citizens who are able to:

- sift through arguments, interpret quantitative information" and "make critical judgements …,

- reason … think and act independently…, and

- make sense of data, …, interpret technical material, …, manipulate formulas and symbols, …, distinguish arguments, …, appreciate and act on uncertainty (Ibid).

3.3.6 The change in the emphasis of didactical practices

Mortimore (1999:7) points out that much has been learnt about how students learn. Consequently, new didactical approaches have been developed. According to Hirsch (1992:6), these approaches assist students to widen their views of the nature and value of mathematics and to become more productive citizens. Furthermore, changes in didactical strategies also provide students with and engage them in valuable learning experiences that are intended to entrench, in students, the acquisition of mathematical literacy.

In summary, we can expect the factors that we discussed in this section to be pushing for the need to instruct mathematics for mathematical literacy in three inter-related broad categories, viz.:

- changes in the needs of society that emphasise the expediency that all individuals have command of mathematics at all levels of life (at work place, in daily lives, and at professional level),

- changes in mathematics itself as a body of knowledge, coupled with the very nature of mathematics, continue to extend its utility and ubiquity to diverse fields of knowledge, and

- changes in didactical practices of mathematics that emphasise, amongst other factors, active learning and cooperative learning, which, in turn, enhance the understanding of concepts and holistic integrated linkage in knowledge.

However, in essence, what are the didactical practices that entrench mathematical literacy in learners? This question is explored in the section that follows.

3.4 DIDACTICAL PRACTICES THAT ENTRENCH MATHEMATICAL LITERACY

For several years now, a need has existed to embrace didactical strategies that foster in students a genuine disposition for mathematics, such that knowledge of mathematics is integrated with other kinds of knowledge and skills (Fosnot, 1996:ix). This is embraced in the hope that students will be fully equipped to use mathematics in real-world contexts as opposed to using it only at school in solving artificially contrived mathematics problems that have an artificial relationship to the real world (Christiansen, Howson, and Otte 1986:245, King 1992:113, Verschaffel and De Corte 1997:578). As is established in the previous sections, mathematics embedded in real-world contexts readily entrenches mathematical literacy.

3.4.1 The nature of didactical practices relating to mathematics that entrench mathematical literacy

According to Borasi (1996:24), didactical practices that advance mathematical literacy in students need to embrace, amongst others, the following:

- a view of learning as a generative process of meaning making that requires both social interaction and personal construction of meaning that is informed by context, and

- a view of teaching that is stimulating, supporting of students' own inquiry, and establishing a learning environment conducive to such inquiry.

Borasi is, thus, promoting the argument that classrooms where didactical practices produce mathematically literate students must, of necessity, be powerful environments in which students have an active role in learning activities. At the same time, students are expected to consider concepts and problems in their real contexts and are thus led to realise that solutions to problems often draw insights from natural settings.

This didactical view is intended to be ideal in stimulating students to learn mathematics in a meaningful manner. Furthermore, it ties in well with theories of learning that advocate that students should construct their own meaning of the material that is taught (Davis, Maher, and Noddings 1990:2-3, Ediger 1999, Fosnot 1996:ix, Goldman, Hasselbring, and Technology Group at Vanderbilt 2001:1-4, Johnson-Wilder *et al*, 1999:42-45, Noddings 1990:7-29, Szalontai 2001:1-4, Wubbels, Korthagen, and Broekman 1997:1-28). Furthermore, students are afforded the opportunity to build up their own understanding of the subject, make their own mathematical judgments, engage in mathematics in a way that is meaningful to them, and understand the diverse utility functions of mathematics in collocations other than in those artificial problems that are divorced from reality. At the same time, the kind of teaching propounded here also affords students to acquire, internalise, and use mathematical competencies in critical, insightful, and creative thinking. Thus, this didactical practice has the same impact on students as the "critical and developmental outcomes" that are advanced in South Africa's National Department of Education Document for 2005.

What is more, the role and strengths of teachers in a didactical environment that promotes mathematical literacy is that of being, among other things, good in:

- selecting mathematical tasks that actively engage students' interest and intellect (Vygotsky's zone of proximal development (ZDP) discussed in Chapter 2, Section 2.3.6),

- providing opportunities to deepen students' understanding of the mathematics that is being studied and its applications,

- orchestrating a powerful classroom environment and discourse in ways that promote the investigation and growth of mathematical ideas, using and helping students use technology and other tools to pursue mathematical investigation,

- seeking and helping students to seek and find concepts that connect and thus lead students to develop holistic knowledge, and
- guiding individual, small-group, and whole-class work to develop and retain knowledge that they meaningfully understand (NCTM 1991:1).

Similar didactical practices are advocated by Borasi (1992:2-3), Hoyles, Morgan, and Woodhouse (1999:196-197), and Wray and Lewis (1997:21). Yet, as pointed out in Chapter 2, Section 2.6, the teaching and learning process itself remains a complex practice, largely because of the complexity of the didactical activities that is coupled with differences among teachers and students.

Hence, a discussion on didactical practices that, according to literature actually entrench mathematical literacy, follows in the next section of this chapter.

3.4.2 Recommended didactical practices that entrench mathematical literacy

However, there are approaches that literature purports as having the capacity to entrench, in learners, aspects of mathematical literacy. These are: project and practical work, investigational work, mathematical modelling, whole-class or group work, making mathematical links and connections, open-ended approaches, interactive approaches, and constructivism perspectives in didactical practices relating to mathematics (Becker and Shimada 1997:1- 9, Burton 1999:128, Cangelosi 1996:1, Davidson 1990:1-20, Johnson-Wilder *et al,* 1999:64, Neyland 1994:19, Pimm 1988:3).

Project work is one of the didactical tools that, of late, literature points out to as gaining acceptance in many curriculum requirements (International Baccalaureate Group 5 Mathematical Studies Project Syllabus Requirement 2000:1, University of Cambridge International General Certificate of Secondary Education Mathematics (IGCSE) with Course Work Syllabus 2001:11, South Africa Department of Education Senior Phase Policy Document 1997:13). In fact, Simmons (1993:4) points out that appropriately designed mathematical projects can help to develop students' mathematical thinking, which, in turn, establishes a clearer understanding of concepts involved in the project itself. Nevertheless, it has been found and should herein be pointed out that the task of finding an appropriate mathematics project is normally not an easy one. For instance, Borasi (1992:106-123) gives a report of the independent project work of two students where, amongst other things, the students had to demonstrate evidence of careful and logical mathematical reasoning and creativity. However, she too admits that it was not easy for these students to come up with the desired project. Nevertheless, she advances advantages of project work that are instructive, such as:

- offering the opportunity to assign students to "real, open-ended tasks",
- providing students with the "opportunity to be inquisitive and creative",
- affording students the opportunity to transfer and apply acquired knowledge of mathematics,
- testing students on the ability to do without major guidance from the teacher,
- affording students the opportunity to take part in the formulation of the project problem,
- getting students to have an opportunity to have fun in mathematics,
- affording students the opportunity to explore the world around them through mathematics,
- affording students the opportunity to explore how the given context may change the meaning of mathematical definitions, and
- giving students the opportunity to make conjectures and pose further questions related to the problems (Borasi 1991:106-113).

The benefits of teaching students through project work, as indicated here, are by no means trivial. Project work enhances mathematical literacy in learners. That this advantage is enormous can be deduced from each of the advantages of teaching mathematics through project work. In fact, taken altogether, teaching mathematics through project work seems to forge a kind of hybrid view of mathematics instruction, which fosters a deep understanding of mathematical concepts and procedures through their application to real world and contextual situations. Hence, the University of Cambridge IGCSE Mathematics with Course Work (2001:10-11) states that each project "topic selected should be capable of extension, or development beyond routine solution". In fact, projects should be carried out in many mathematics areas, which include:

- statistical survey,
- inter-disciplinary projects (such as in geography, food science, business studies, physics, computer studies, music, and others),
- broadening mathematical knowledge (in branches such as topology, networks, loci and envelopes, spirals, curves of pursuit, exponential growth, and logarithms), and
- extending mathematical knowledge (in mechanics, calculus, further algebra and trigonometry, further statistics, and three dimensional geometry) (IGCSE Mathematics Syllabus 2001:11).

In all of these cases, the active involvement of students in harnessing their conceptual knowledge in solving real-world and contextualised problems clarifies mathematical meaning and widens and deepens appreciation by students of the power of mathematics in understanding the world around them. As students engage in project work, they may be involved in complex, open-ended, problem-solving techniques that call for mathematical connections to other subjects and to real-world contexts outside the classroom. Communicating mathematical results from meaningful contexts requires logical explanations. Sometimes, the making of conjectures and the formulation of new problems become a requisite. Hence, according to Goldman and Hasselbring (1997:2), it is found that project work sometimes engages students in problems that demand extended effort to solve.

Some projects are group projects that require students to use available technology and to engage in cooperative problem solving and discussion (Ibid).

As can be deduced from the discussion in the previous paragraphs, project work may involve mathematical investigation as well as mathematical modelling. The International Baccalaureate Organisation (2000:37) defines mathematical investigation as "an enquiry into a particular area of mathematics leading to a general result which was previously unknown to the" student. In particular, Bolt and Hobbs (1994:6) maintain that investigations in mathematics require independence of mind and keen initiative on the part of the individual who is taking part in it. The mathematics involved in investigation tasks may include finding and identifying relationships, recognition patterns, and making conjectures and generalisations (Morris, 1994:2).

In fact, according to Bolt and Hobbs (1994:6), investigation in mathematics involves:

- tasks in which various strategies and skills can be used,

- cases of situations, which can be investigated with opportunities for strategies such as trial and error and searching for patterns to be employed, and

- extended pieces of work which enable learners to investigate given topics or problems at length and with demanding concentration.

However, it should be pointed out that there may be no quick solutions or closed complete answers to problems explored through mathematical investigation (Becker and Shimada 1997:1, Bell, Brown, and Buckley 1990:4-5, Gardiner 1989:12, Gilblin and Porteous 1994:4). Investigation is, thus, meant to enable students to generate, discover, or develop salient characteristics of mathematical objects.

However, the International Baccalaureate Organisation (2000:37) defines mathematical modelling as "the solution of a practical problem set in a real world context in which the method of solution requires some … mathematical modelling skills". For instance, one could analyse the growth of a bacteria population by using an exponential model. In fact, Cundy and Rollett (1989:19, 76, 161, 208, 254) posit that models can be constructed in many areas such as: plane geometry, three dimensional polyhedra and other solid geometry shapes, mechanics, logic, and computing. The purpose of modelling is mainly to make abstract ideas to be "derived from, or illustrated by concrete examples" (Cundy and Rollett 1989:13). Modelling, thus, provides tangible means of connecting reality to the symbolic world of mathematics and vice versa.

Nevertheless, none of these didactical practices is normally used solely on its own. A combination of two or more of these instructional activities is the usual practice in most classroom situations (Do 2001:1-3, Johnson-Wilder *et al*, 1999:4, Szalonti 2001:1-4).

However, an instructional practice and approach that has stood the test of time is exposition. This approach was discussed at length in Chapter 2 of this study. What is noteworthy about the approach is that it is handy in individual, whole-class, cooperative, discussion, and any interactive didactical approach that is used in almost every other classroom, irrespective of the kind of mathematics in the didactical environment. As Simmons (1993:3) points out, if utilised properly and to advantage, exposition "can be a rich and rewarding approach" in all didactical situations.

To summarise didactical practices relating to mathematics for mathematical literacy, the researcher has drawn Table 3.4.1, which is an adaptation from the NCTM article on "The Crossroads in Mathematics".

Table 3.4.1 Recommended didactical practices relating to mathematics that entrench mathematical literacy

Increased Use	Decreased Use
Active involvement of students	Passive listening
Technology to aid in concept development	Paper-and-pencil drill
Problem solving and multi-step problems	One-step, single-answer problems
Mathematical reasoning	Memorisation of facts and procedures
Conceptual understanding	Rote manipulation
Realistic problems encountered by adults	Contrived exercises
An integrated curriculum with ideas developed in context	Isolated topics
Multiple approaches to problem solving	Requiring a particular method for solving a problem
Diverse and frequent assessment both in class and outside of class	Tests and final exam as the sole assessment
Open-ended problems	Problems with only one possible answer
Oral and written communication to explain solutions	Requiring only short, numerical answers or multiple-choice responses
A variety of teaching strategies	Lecturing

A literature-based exposition of the didactical practices that entrench mathematical literacy has been provided and summarised in Table 3.4.1. Yet, what does literature itself say about the kind of mathematics one needs to learn in order to be mathematically literate? Is it different in any way from the mathematics that is taught in most secondary and high schools? These questions are addressed in the subsection that follows.

3.5 THE KIND OF MATHEMATICS NEEDED TO ENTRENCH MATHEMATICAL LITERACY

Mathematical literacy and the type of mathematics expected to enhance it, are the crucial aspects of this section of the present chapter. However, from time immemorial, it has been traditionally and widely acknowledged that mathematics education focuses on mathematical skills and content knowledge (Freudenthal 1991:9-11, Greer and Mulhern 1990:108, NCTM Crossroads in Mathematics 2001:5).

In fact, literature posits that solving contextually meaningful problems through the in-depth study of specific mathematics topics that are presented in real-life contexts of application is what entrenches mathematical literacy (Anderson in Hoyles *et al*, 1999:8, Hee-Chan Lew in Hoyles *et al*, 1999:220, Hoyles *et al*, 1999:6, NCTM Crossroads in Mathematics 2001:5). Actually, in "Crossroads in Mathematics", the NCTM (2001:5) indicates that, with mathematical literacy, the focus is on providing learners with more engaging and valuable learning experiences in mathematics. The learning experience is intended, amongst others, for the intellectual development of students that nurtures desired outcomes in mathematical competence, which in turn, equips learners to function as productive workers and citizens.

From the discussion we are having, there seems to be a difference between traditional mathematics education and mathematics education for mathematical literacy. It seems that traditional mathematics education is content driven and seeks to enable learners to perform certain mathematical operations and procedures. While education for mathematical literacy, with the same mathematical content, seeks to empower learners with mathematical ways of thinking, analysing, organising, and structuring information and ideas in real-life contexts. Examples of how the same mathematical topic is treated in traditional mathematics education and in mathematics that entrenches mathematical literacy could be instructive at this point.

Number sense is a mathematical topic that has been treated in mathematics from time immemorial. In traditional mathematics education, number sense can be treated as in Table 3.5.1 (taken from Lesotho Mathematics Syllabus Form A and B 2001:8):

Table 3.5.1 Treatment of number sense in traditional mathematics education

CONTENT	**NOTES**
Types of numbers and their sequences	
Identification of sets of odd, even, prime, multiples, factors, square, and cube numbers in natural numbers	Identification includes: listing and describing
Expression of sets of natural numbers as product of their prime factors	Encourage students to have square numbers up to 144 and cube numbers up to 125 at their finger tips
Finding of common multiples and common factors (LCM and HCF)	
Identification and listing of directed numbers	The idea of directed numbers is introduced through use of practical topics, e.g. temperature readings followed by number lines (vertical and horizontal)
Finding the rule for a sequence and filling the missing number/s	The sequences include Fibonacci and Pascal. The topic can be introduced by pictorial representation

However, according to "Crossroads in Mathematics" (NCTM 2001:6):

> Number sense includes the ability to perform arithmetic operations, to estimate reliability, to judge the reasonableness of numerical results, to understand orders of magnitude, and to think proportionally. Suggested topics include pattern recognition, data representation and interpretation, estimation, proportionality and comparison…. (At the same time, with number sense) students (are expected to) perform arithmetic operations as well as reason and draw conclusions from numerical information.

From the two examples cited here, it can be deduced that both traditional mathematics and mathematics for mathematical literacy tap from the same mathematical content to effect different responses in a learner. With mathematics for mathematical literacy, students are equipped with an understanding of mathematical concepts as opposed to mechanically and "thoughtlessly grinding out answers" (NCTM 2001:8, 9). Again, in the first example in Table 3.5.1, mathematics is portrayed as a set of isolated concepts, rules, and procedures. The second example offers mathematics as an interrelated body of knowledge that empowers learners to use mathematics to think in contextual situations.

Specifically, the kind of mathematics that entrench mathematical literacy can be summarised (see Table 3.5.2 below) by citing the NCTM (2001:10-13) guidelines for content recommended for achieving mathematics standards.

Table 3.5.2 Recommended kind of mathematics that entrenches mathematical literacy

Increased Attention	**Decreased Attention**
Pattern recognition, drawing inferences	Rote application of formulas
Number sense, mental arithmetic, and estimation	Arithmetic drill exercises, routine operations with real numbers
Connection between mathematics and other disciplines	Presentation of mathematics as an abstract entity
Integration of topics throughout the curriculum	Algebra, trigonometry, analytical geometry, and so forth as separate courses
Discovery of geometrical relationships through the use of models, technology, and manipulatives	Establishing geometric relationships solely through formal proofs
Visual representations of concepts, for example, probability as area under a curve, timelines for annuity and interest, tables for logic and electrical circuits	Rote memorisation and use of formulas
Integration of the concept of function across topics within and among courses	Separate and unconnected units on linear, quadratic, polynomial, radical, exponential, and logarithmic functions
Analysis of the general behaviour of a variety of functions in order to check the reasonableness of graphs produced by graphing utilities	Paper-and-pencil evaluation of functions and hand-drawn graphs based on plotting points
Connection of functional behaviour (such as where a function increases, decreases, achieves a maximum and /or minimum or changes concavity) to the situation modelled by the function	Emphasis on the manipulation of complicated radical expressions, factoring, rational expressions, logarithms, and exponents
Modeling problems of change by constructing probability distribution or by actual experiment	Theoretical development of probability theorems
Collection of real data for analysis of both	Analysis of contrived data

descriptive and inferential statistical techniques	
Exploratory graphical analysis as part of inferential procedures.	"Cook book" approaches to applying statistical computation and tests, which fail to focus on the logic behind the processes
Use of curve fitting to model real data, including transformation of data when needed	Reliance on out-of-context functions that are overly simplistic
Connection amongst problem situations, its model as a function in symbolic form, and the graph of that function	"Cook-book" problem solving without connections
Discussion of the meaning of non-zero correlation and the independence of correlation from any implications of cause and effect	Blind acceptance of r (the correlation coefficient)
Use of statistical software and graphing calculators	Paper-and-pencil calculations and four-function calculators
Problems related to the ordinary lives of students, for example, financing items that students can afford and statistics related to sports participation by females as well as males	Problems unrelated to the daily lives of most students, for example, investments of large sums of money in savings or statistics related to sports only played by males
Matrices to organise and analyse information from a wide variety of settings	Requiring a system of equations to be solved by three methods
Graph theory and algorithms as a means of solving problems	Algebraically derived exact answers

From this table of guidelines of mathematical content, it can be deduced that the kind of mathematics that entrenches mathematical literacy is more focused on gaining general mathematical competence in a variety of contextually placed mathematical concepts. Williams, Wake, and Jervis (in Hoyles, Morgan, and Woodhouse 1999:95) actually posit that there exists a "relationship of general mathematical competence with (mathematical) content" and define general mathematical competence as "the ability to perform a … mathematical skill across a range of … applications with a specific, coherent body of mathematical knowledge, skills and models" (Williams, Wake and Jervis 1999:92).

However, general mathematical competence itself is an instructional approach (Williams Wake and Jervis 1999:97). Thus, the root of mathematics for mathematical literacy pertains more to a didactical approach than to the content of mathematics per se. The form of didactical approach becomes the vehicle and instrument through which mathematical literacy takes effect.

3.6 CONCLUSION

This chapter has discussed mathematics education that, according to different perspectives from literature, entrenches mathematical literacy. In the chapter, the need for mathematics education for mathematical literacy is argued from different perspectives. First, from a didactical point of view, mathematics education has traditionally failed to extend classroom mathematics to real and meaningful contextual situations. This has limited mathematics as a creative body of knowledge to a dry mechanical subject learned to be produced for examination purposes. Hence, in this respect, mathematical education has turned out unfit and unproductive citizens in a technologically growing world. In the light of this didactical shortcoming, didactical practices themselves must change.

Second, the need for mathematical literacy has been argued in the light of the requirements that are imperative in a changing society and culture that is driven by information and technology. This kind of society requires competence in mathematics that goes beyond arithmetic and mechanical mathematics and demands, amongst other things, citizens who reason, think, and act independently.

Third, the major changes in mathematics as a discipline have been considered. Changes in both mathematical content and didactical practices, where, among other factors, mathematisation and constructivism are encouraged as didactical approaches also formed the bases for arguing the need of mathematical literacy.

Furthermore, the chapter explored the meaning, indicators, and indices of mathematical literacy. The perspective of mathematical literacy as knowledge in mathematics in its own right and, at the same time, as a necessary tool in coping with and understanding contextual real life situations is discussed at length. Thus, the underlining factors in mathematical literacy are taken to be a meaningful understanding of and competence in mathematics that is set in real life contexts. Hence, mathematical literacy is taken as a tool in communication and as an integrating factor between mathematical content and both the real world and other school subjects. However, the main indicators of mathematical literacy are deemed as:

- the appreciation of the beauty of mathematics that comes with a meaningful understanding and command of mathematical knowledge,

- the possession of creative, logical, and critical thinking in mathematics,

- having appropriate problem-solving skills, and

- having the ability to appreciate real-world situations and other environmental contexts.

Related to the meaning of mathematical is the discussion of didactical practices that entrench mathematical literacy in learners. This discussion is further linked with the kind of mathematics that is needed to effect mathematical literacy. The kind of mathematics that effect mathematical literacy is deemed not to be the content driven kind. Rather, mathematics that entrench mathematical literacy uses the mathematical content to empower learners with mathematical ways of thinking, analysing, organising, and presenting and structuring information and ideas set in real-life contexts.

Having explored, in this and the previous chapter, literature related to the main focus of the thesis, the following chapter will discuss and justify methodology and instruments of research.

CHAPTER 4

RESEARCH METHODOLOGY AND INSTRUMENTS

4.1 INTRODUCTION

As indicated in Chapter 1 (Section 1.2), teaching mathematics in such a way that mathematical literacy is inculcated in learners is desirable in today's technologically dependent society (Becker and Shimanda 1997:1, Carpenter and Lehrer 1999:19, Elliott and Kenney 1996:ix, Gromov 2000:524-527, Hoyles, Morgan, and Woodhouse 1999:16-20, Tanner and Jones 2000:11-16). Lesotho is no exception to this need. In fact, this study seeks to examine and explore, according to generally accepted mathematical literacy indicators (stipulated in Chapter 3), to what extent the current didactical practices relating to mathematics, in the district of Maseru, Lesotho are purposefully set out to inculcate in students mathematical literacy within the Sotho context and in a wide world that is technologically inclined and imbued with mathematics. The specific objectives of the study are laid out in Chapter 1 (see Section 1.4) of this study. Chapters two and three reviewed related literature on mathematics education and didactical practices relating to mathematics, in general, and, in particular, those didactical methods that inculcate mathematical literacy in learners.

Actually, Chapter 2 discusses general mathematics education, theories that influence trends in didactical practices of mathematics, and common didactical practice relating to mathematics. In contrast, chapter 3 reviews the literature on didactical practices that purport to entrench mathematical literacy in learners. The task at hand in this chapter is to consider how to specifically collect required data relevant to this study and to outline how the research is going to be carried out. Therefore, the chapter looks into and justifies the aspects pertaining to the research approaches and instruments that are used to collect and analyse data for the study. The chapter also gives a description of how the research is to be carried out. In essence, the chapter addresses the question of methodology and the respective instruments of research.

According to Cohen and Manion (1992:41), methods and methodology in educational research are two seemingly different terms. Methods entail the range of approaches and techniques used to collect data relevant for the purposes of the research study (Ibid). In contrast, methodology seeks to "…describe and analyse these methods, throwing light on their limitations and resources, clarifying their presuppositions and consequences, relating their potentialities to the twilight zone at frontiers of knowledge" (Cohen and Manion 1992:42).

Thus, in essence, methodology justifies and throws light and understanding upon the approach and process of the research itself. However, in this study, one approach to the study does not preclude the use of other techniques of collecting research data. In fact, to this effect, the literature clearly purports that no research approach depends solely on one and only one method of collecting data (Bell 1999:7, Bell 1992:1, 4, 50, Borg 1987:155, Shumway 1980:33-35). Hence, in this study, the researcher is at liberty to use different research approaches simultaneously.

Although some research approaches lean heavily on one type of method to collect data, the overall guideline of action that the researcher will follow is as follows: whatever approaches are selected, they must provide the data required to put together a complete piece of the research study. This line of action in research is justified by Bell (1992:4). Furthermore, once the researcher has made decisions with regards to the approaches that best suit this research study, appropriate and relevant instruments with which to collect the data were designed to meet the purpose. In the present chapter, the researcher discusses the research population, sample, and sampling techniques that have been used for the study. Research approaches that will be followed, the instruments that will be used to collect the data, and issues of the validity and reliability of the instruments are also addressed. The chapter further discusses the type of data gathered using these instruments as well as giving a description of the procedure that is followed to accomplish the research.

4.2 POPULATION, SAMPLE, AND SAMPLING TECHNIQUES

4.2.1 The population

Literature posits that population for a research study is the target group that the researcher sets out to study and to generalise and apply the research findings (Borg 1981:75-76, Dyer 1979:149, Eichelberger 1989:165, Traver 1965:460, Tuckman 1988:239). In fact, according to Oppenheim (1992:38), population for a research study refers to "all those who fall into the category of concern". In this study, population refers to all secondary and high schools in the district of Maseru, in the Kingdom of Lesotho. However, it is not just the schools that are the focus of concern in this research study. Those characteristics of the population that are the concern of measurement in the study are the central focus of the research study. Literature posits that such traits, characteristics, or variables are referred to as "population characteristics" (Oppenheim 1992:38, Traver 1965:460). In this research study, the characteristics of the population that the researcher will focus on are didactical practices relating to mathematics in all these schools.

As indicated in Chapter 1 (see Section 1.6.3), after completing primary education, a student in Lesotho spends three years of secondary education at the end of which the student takes an examination to acquire the Lesotho Junior Certificate of Education. When the student obtains this certificate, he/she takes a further two years of high (called senior secondary in other parts of the world) school education. A school that gives only the first three years of education after completing primary education is a secondary school. While the school that offers the two years of education that culminate in obtaining the Cambridge Overseas School Certificate is referred to as a high school. A school may offer the full five school years after primary school education. Such a school is still referred to as a high school (Kokome 1991:2, Lesotho Education Sector Development Plan 1991/1992-1995/1996, 1992:8, Lesotho 1996(b) Official Yearbook 1996:136, The Kingdom of Lesotho Fifth Five-Year Development Plan 1991/1992-1995/1996, 1992:22, The Kingdom of Lesotho Sixth National Development Plan 1996(c) 1996/1997-1998/1999, 1997:174). Thus, after primary education, the first three years in a high school comprises the secondary school education.

As discussed in Chapter 1 (Section 1.5.2), Lesotho has 225 registered secondary and high schools in all of the ten districts of Lesotho (Lesotho Ministry of Education Development Plan, 1996(c), Lesotho Ministry of Education List of Schools by District, 2002). Of these, 16 schools are in Botha-Bothe, 50 in Leribe, 25 in Berea, 51 in Maseru, 26 in Mafeteng, 17 in Mohale's Hoek, 12 in Quthing, 11 in Qacha's Nek, 8 in Mokhotlong, and 9 in Thaba-Tseka (see map of the districts of Lesotho, Annexure 1 in Appendices). However, Cohen and Manion (1992:101) posit that, due to factors related to time, expenses involved, and accessibility, it is not always practically possible to collect data from the whole population under study. Hence, in this study the researcher will not study the complete population of 225 secondary and high schools in Lesotho. This is unnecessarily ambitious and difficult due to factors such as financial constraints, time limit of the research, distances between schools, and accessibility due to the mountainous terrain of the country under study. Therefore, the researcher will use a small representative sample group from the accessible population as similar characteristics exist between the complete population and any randomly selected sample as justified in the following paragraph.

Although the secondary and high schools in Lesotho are scattered throughout the country, from the figures given earlier on, 23% of the schools in Lesotho are in the Maseru district. Concerning similar didactical practices in secondary and high schools in Maseru district, it is noteworthy to point out that there are only two tertiary institutions that train teachers for the whole country. These are: the National University of Lesotho and the National Teachers' Training College. Both institutions are in the Maseru district. Given the same training experience for teachers, one may conclude that didactical practices are inherently similar. The assertion that teachers in Lesotho, by and large, share similar instructional practices is further accentuated and confirmed by the fact that all schools are, in fact, under the auspices of the same curriculum developers, the same inspectorate for mathematics teaching, and schools are also served by resource persons who often come together to share ideas and experiences (Kokome 1991:24).

Actually, curriculum developers hand out to teachers of mathematics syllabuses for each year group. These syllabuses already specify the content to be covered, the end objective in terms of what students are expected to be able to do with the content, instructional guiding notes for the teacher, and the expected number of periods required to cover that content (Kingdom of Lesotho Ministry of Education and Manpower Mathematics Syllabus 2001). Again, from time to time, curriculum developers hold seminars and workshops to disseminate curriculum innovations and recommended didactical practices with which to impart the curriculum. Apart from personal differences, didactical practices relating to mathematics in Lesotho are largely centrally controlled and, hence, they share similar traits that are characteristically the same (Kokome, 1991:38-68). Hence, wherever the sample may be taken within the district, similar didactical practices are expected, which affords the sample to share the same traits as those in the target population.

4.2.2 The sample

Literature (Borg and Gall 1974:115, Eichelberger 1989:165, Openheim 1992:8, Ostle and Mensing 1975:50, Travers 1965:304) posits that a sample is a representative group drawn from the accessible population of the target population. Therefore, in agreement with this citation, the researcher will take a sample from schools in the Maseru district. The sample includes some students and teachers from five schools and government administrators for mathematics in secondary and high schools. These schools are coded 1, 2, 3, 4, and 5 for purposes of the analysis of data. From each of these schools, three teachers (including the subject head) as well as 25 secondary and 25 high school students will be taken into the sample. A total of 125 students will, thus, be taken from each school along with three teachers. Furthermore, two developers of mathematics curricula for secondary and high schools from Lesotho's government, one member of the inspectorate for mathematics, and the mathematics resource person and advisor in Maseru are also part of the sample. All in all, the whole sample consists of 250 students, 15 teachers, and four administrators.

4.2.3 Sampling techniques

Although there are various sampling techniques in research, only those techniques that enable the researcher to collect appropriate data suited for the purposes of this study are selected (Borg 1981:73-75, Cohen and Manion 1992:101-104, Oppenheim 1992:39-42, Tuckman 1988:238-244). In the light of similarities in didactical practices in all of Lesotho's secondary and high schools (Kokome 1991:38), the researcher chooses to use cluster sampling where a specific number of adjacently positioned schools are randomly selected for ease of control since they are a good representation of the target population. Borg (1981:74) points out that cluster sampling uses "a naturally occurring group of individuals". In this study, the researcher uses a naturally clustered group of schools around the city of Maseru. Furthermore, Borg (1987:8, 89), Cohen and Manion (1992:102), and Oppenheim (1993:40) posit that the cluster sampling technique is justified when and where there are similar characteristics between the parent population and the research sample subjects. As pointed out at the beginning of this paragraph, similarities exist between the sample schools and the target population.

However, there are many secondary and high schools that are positioned adjacently in Maseru. Studying all these clustered schools could be expensive and also take a long time to complete. To select the five schools in the sample, purposive sampling was exercised. Cohen and Manion (1992:103) point out that "in purposive sampling, the researcher handpicks the cases to be included in his sample on the basis of his judgement of their typicality" so as to build up a satisfactory sample. From this description, the researcher purposively handpicked five easily accessible and fairly closely positioned secondary and high schools from the cluster of schools in Maseru (the city) as a sample that suits the study's specific needs.

From each of the sample schools, the researcher sampled 25 secondary school students and 25 high school students by simple random sampling from class lists provided by the administrators in the respective schools. For purposes of coding, the 25 simple randomly selected secondary students from School one are named S1, S2, S3, ... , S24, S25. A similar group of high school students from School one are named S26, S27, S28, ... , S49, S50. Similarly, S51, S52, S53, ... , S99, S100 are the corresponding 25 secondary school students from School 2 and the 25 high school students from the same school. Likewise, S101, S102, S103, ..., S149, S150 are the respective secondary and high school students from School three, S151, S152,..., S199, S200 are a similar group of students from School four and S201, S202, S203,..., S249, S250 students from School five.

Coding names are also given to teachers and administrators. The three teachers of mathematics from School one are named T1, T2, and T3. Teachers from School two are T4, T5, and T6. Those from School three are T7, T 8, and T9. T10, T 11, and T12 are teachers from School four while T13, T14, and T15 are teachers from School five. The two curriculum developers are given code names A1 and A2. The member of the mathematics inspectorate is named A3 and the mathematics resource person is A4.

4.3 RESEARCH APPROACHES: QUANTITATIVE AND QUALITATIVE APPROACHES

Literature indicates two main approaches to research: qualitative and quantitative (Bell 1989:4, Best and Kahn 1993:184, Bliss, Monk, and Ogbon 1983, Gillespie and Glisson 1992, Hitchcock and Hughes 1989:24, McMillan and Schumacher 1989:384). All research methods and styles fall under one or both of these two major research approaches.

Bell (1989:4) posits that quantitative research approaches are amenable to gathering quantified facts and measures from which one is enabled to study the relationship of one set of quantified facts to another in a statistical manner. According to Hitchcock and Hughes (1989:24), Best and Kahn (1993:184), and McMillan and Schumacher (1989:384), a qualitative research approach captures data as it occurs naturally. The approach affords an in-depth, detailed description of events, experiences, knowledge, views, and feelings that gives richness of data and allows a full understanding of what is being studied (Ibid). Although typical ethnographic, qualitative methods for collecting data will not be used, suitable qualitative in-depth interviews, open-ended questionnaires, direct observation, and documentary analysis will take place. Interviews will be a follow-up exercise that will address issues that emerge from the analysis of data from questionnaires in order to elucidate explanations of subjects' responses in questionnaires.

As indicated in Section 1.5 of Chapter 1, the researcher will use both quantitative and qualitative approaches where open-ended questionnaires, interviews, and documentary analysis only are used to collect data relevant to this study. Bell (1989:4) points out that these approaches do not preclude each other. Hence, all research tools with which to gather data used in this study will be amenable to collecting either qualitative or quantitative data.

The remaining part of this section deals with the method of collecting the information that is required to investigate the research questions of this study. The instruments that are used to collect the data, together with issues of their validity and reliability are discussed and structured. It is important to decide and justify the type of research instrument that is optimally capable of supplying the information necessary for answering the research questions, and, therefore, these will also be portrayed. Assuming that research instruments and approaches do not preclude or reject each other, various instruments and research approaches are simultaneously incorporated in this study in order to afford optimal insight into the research questions (Bell 1992:2, Bell 1999:7). At the same time, since the extent of the process to collect data is practically influenced by the amount of time normally prescribed for the completion of the research work, the number of data collection instruments that is used will be limited further (Ibid).

4.4 RESEARCH INSTRUMENTS

According to Borg (1987:107), instruments for data collection and procedures that are normally used in educational research are: paper-and-pencil tests, questionnaires, interviews, direct observations, and documentary analysis. However, in line with what Shumway (1980:41-42) posits, the selection of instruments for this research study is one of the most crucial components of the research design. In fact, Shumway maintains that careful thought should go into the selection of the instruments (Ibid). Among the various research instruments, the instruments selected for this research study are: questionnaires, interviews, and documentary analysis. The researcher chose these instruments because, by their nature, they will assist in gathering both qualitative and quantitative data needed to triangulate information for validity and reliability (Cohen and Manion 1989:269-280).

However, it is not sufficient just to make a decision about the type of instrument that will be used in the research. The instruments are to be defended as adequate and fit for the purpose of this research study. The adequacy of instruments is very important because, if the instruments opted for are not sensitive enough to gather the required data, they are not fit for the purpose of the study. There is a need to develop the kind of instrument that is appropriate for the research study. In fact, Shumway (1980:41-42) maintains that inadequate instruments of research limit research reports and findings. The data and findings of the study are rendered invalid and unreliable. In the light of this information and justification from literature, care was taken to construct instruments that will afford the researcher the opportunity to gather relevant data for the study.

To build up content validity in the instruments for this study, preliminary exploratory and fact-finding informal interviews as well as open-ended questionnaires were administered on groups of ten students and two mathematics teachers from each of the sample schools (see Appendices 1, 2, and 3). These students and teachers were not the same as those in the actual sample group of the research study. Information from these preliminary fact-finding questionnaires and interviews, together with information from the literature review in chapters two and three, were used to construct Section A, Section B, and Section C of the questionnaires to respondents (see Part 2 of Appendices 4, 5, and 6). Section A of Part 2 of the questionnaires is supposed to capture and reflect common didactical practices relating to mathematics, along with purported current didactical practices relating to mathematics in Lesotho as in Chapter 2 (see Sections 2.6, 2.6.1). Section B contains didactical practices relating to mathematics, which, according to the literature review in Chapter 3 of this study, posit to entrench mathematical literacy (see Section 3.4.2 of Chapter 3). Section C contains 15 didactical practices relating to mathematics extracted (and refined) from Table 3.5.2 in Chapter 3.

After the instruments were developed, a pilot study was performed. Oppenheim (1993:46) points out that the instruments that one may design do not always emerge fully fit and automatically appropriate for the purpose for which they are designed. They need to be adapted, fashioned, redeveloped, and refined to maturity through piloting. In piloting, instruments and research procedures are actually tried out on groups that are different from, but in every respect similar to the research sample. The results from piloting are then used to refine the instruments where necessary (Borg 1987:109, Oppenheim 1993:48-55).

The pilot study was carried out using ten secondary school and ten high school students at a high school just outside the city of Maseru. This school is not one of the five in the sample group. One typographical error was detected and then corrected for the final instruments. The questionnaires were again refined, tuned, fashioned, and, finally, designed (see Appendices 4, 5, and 6).

Bell (1992:59) maintains that there are different types of questionnaires. For example, some questionnaires allow the respondents to do any of the following:

- List: Here a list of items, any of which may be chosen by respondents, is given. The chosen item represents the subject's opinion on the research variable.

- Category: A set of categories is given and respondents choose the one they fit into.

- Ranking: This presupposes an ordinal scale. A collection of items is given and respondents place these in rank order.

- Scale: Careful handling is required here, since there are various scaling devices such as nominal, ordinal, interval, and ratio (Bell 1992:59, Cohen and Manion 1992:154-161, Coleman and Briggs 2002:235-236, Slavin 1984:160). An appropriate scale fit for the type of data required needs to be chosen.

For this study, questionnaires with nominal scales, ranking on an ordinal scale ,and responses to items on interval scales (Likert scale) are used to collect data that are linked to the exploration of the research objectives (see questionnaires in Appendices 4, 5, and 6). In line with Coleman and Briggs (2002:235-236), on the nominal scale, a set of categories are given to respondents to choose those they fit into (see Part 1 Appendices 4, 5, and 6 on Biographic Details of respondents). Actually, the categories are personal biographical details about respondents such as "gender", "age", or "teaching experience", where applicable. Biographic details of respondents only serve to give information about the respondents that helps to size up the nature and quality of subjects in the sample. Data directly amenable to the exploration of the research questions come from Part 2, Sections A, B, and C of Appendices 4, 5, and 6.

Apart from questionnaires, interviews will be carried out to obtain information that may help to illuminate some of the grey areas of the data collected by questionnaires. There are different types of interviews. For instance, structured interviews are intended to yield quantitative data. Here, the content, sequence, and wording of interview questions are predetermined in advance. According to Oppenheim (1992:91) for the structured type of interview:

> ... the researcher will find many questions to which she has to record the responses by putting a tick in boxes or by circling a response category. Sometimes these will be pre-coded answer categories which she has to read out to the respondent, but there will be others where she must immediately classify the response into one of a set of categories which the respondent will not usually see.

The researcher is not going to use structured interviews since this type of interview yields quantitative data. The researcher will use unstructured interviews to obtain in-depth information that helps to establish the truth and worthiness of data from the questionnaire responses.

Of the different types of interviews cited in literature (these include: formal/structured, informal/unstructured, non-directive and focused interviews), the researcher will use informal, non-directive interviews. Formal/structured interviews will not be used because a set of pre-organised questions and the answers that are recorded on a standardised schedule to yield quantitative data is not what the researcher intends to obtain (Cohen and Manion 1992:307-310).

The researcher requires qualitative data from the unstructured interviews where the interviewer is free to modify both the sequence and wording of the questions. In fact, the content, sequence, and wording of questions are entirely in the hands of the researcher. Furthermore, the completely informal interviews afford the researcher to fashion the interviews to be in conversational style instead of having a set of sequenced questions. The non-directive interviews also have minimal direction and control from the researcher and, as such, they will afford the respondents freedom to express opinions subjectively and spontaneously. The researcher's role here will be to encourage, probe, and request the respondent to elucidate doubtful points (Cohen and Manion 1992:309). In this way, the researcher will collect in-depth qualitative data. By permission of the interviewees, interviews will be put on a tape recorder for later analysis. Where permission is declined, the researcher will take field notes.

Besides questionnaires and interviews, documentary analysis will be carried out to gather qualitative data from set textbooks, syllabuses, and from teachers' schemes of work. The content in documents such as the prescribed mathematics curriculum for Lesotho's secondary and high schools, as well as mathematics textbooks that are used in the sample schools will be assessed in the light of indicators that literature purports to entrench mathematical indicators. The information here will also throw light on the actual didactical practices that are used in secondary and high schools in Lesotho.

From the report in this section of the chapter, it is evident that the three different types of instruments (questionnaires, interviews, and documentary analysis) that will be used in this study yield different types of data. The interval scale questionnaires (Likert type) will yield quantitative data. The ordinal scale will be used in Part 2, Section C of the questionnaire where respondents are asked to rank 15 mathematical didactical items into descriptive categories. Hence, this scale yields both quantitative and qualitative data. Interviews and documentary analysis will yield qualitative data.

The choice of different types of instruments will afford the researcher to triangulate the data both within method and between methods (Cohen and Manion 1992, 269-275). According to Cohen and Manion (1992:269), "Triangulation may be defined as the use of two or more methods of data collection in the study of some aspect ... by making use of both quantitative and qualitative data". There are different advantages gained in using triangulation. In this study, the process of triangulation will be followed because research findings may easily become artefacts of particular methods of collecting research data. Hence, to avoid the distortion of data, the process of triangulation will be used. Triangulation will, amongst other factors, increase the reliability of findings as well as reduce the probability that "any consistent findings are attributed to similarities of methods" of data collection (Cohen and Manion 1992:270).

Having selected the various instruments that will be used to collect the data, it is necessary to look into the structure of each of these tools for collecting data. The following section discusses specific details of the instruments that are employed in exploring this research study.

4.4.1 Specific description of instruments

On the whole, three questionnaires (see Appendices 4, 5, and 6) will be administered to the sample group: one to students (Appendix 4, Questionnaire for Students), the second to teachers (Appendix 5, Questionnaire for Teachers), and the third to the mathematics curriculum planners, the inspectorate, and the mathematics resource person and advisor (Appendix 6, Questionnaire for Administrators). Each questionnaire is divided into three sections: Section A, Section B, and Section C.

Didactical factors gathered from employing open-ended, information-finding questionnaires in Appendices 4, 5, and 6, together with common didactical practices relating to mathematics as discussed in Chapter 2 form the contents of items in Section A of the questionnaires. These didactical items are, therefore, supposed to be mostly content representations of current didactical practices relating to mathematics in Maseru, Lesotho. Each Section A of the three questionnaires consists of 30 items. There are 25 items in each Section B of the respective three questionnaires. The contents of items in Section B of the questionnaires are didactical practices relating to mathematics that are purported to entrench mathematical literacy as discussed in Section 3.4, Subsections 3.4.1 and 3.4.2 of Chapter 3 of this study.

In essence, Section A and Section B of each questionnaire collect quantitative data on an interval scale. They are the Likert agreement, 5-point interval scale type where the responses, **"strongly agree (SA), agree (A), Undecided (U), disagree (D), and strongly disagree (SD)"** to questionnaire items are assigned scores from five to one. All questionnaire items in Section A and Section B are positively framed, hence assigning scores from five to one is appropriate. In fact, Gay (1981:296), Henerson, Morris, and Fitz-Gibbon (1987:87), Oppenheim (1992:157), and Slavin (1984:160-166) concur that agreement interval scales permit scores to be treated as integers, which can be added subtracted, divided, and multiplied.

Hence, these scores can be analysed quantitatively through statistical techniques and measures of central tendency (such as mean, mode, and median) and measures of variability (such as range, quartiles, percentiles, variance, and standard deviation) can be calculated. Further, many types of correlation coefficients are possible to calculate (Gay 1981:296-305, Oppenheim 1992:157-158, 195-200, Slavin, 1984:198-202). For this study, total scores will be used to display findings in bar charts. To check aspects of correlation, scatter-grams will be used. Rigorous calculations of correlation coefficients will be done using Karl Pearson's product-moment correlation prediction formula (Naiman, Rosenfeld and Zirkel 1977:211-212):

$$r = \frac{n\sum(AB) - \sum A \sum B}{[(\sqrt{n\sum A^2 - (\sum A)^2})(\sqrt{n\sum B^2 - (\sum B)^2})]}$$

Upton and Cook (1997:551) refer to the formula above as "the population product moment, ... since population characteristics are simply sample characteristics taken to the extreme" (Ibid). Furthermore, each questionnaire includes a Section C an ordinal scale where respondents are asked to rank given didactical practice items. The section has 15 statements about didactical practices relating to mathematics to which respondents are asked to indicate how often each stipulated didactical practice is actually taking place within the teaching/learning environment that they experience. Respondents are asked to accordingly rank the didactical items as "**frequently practised, (F)", "rarely practised (R)",** or **"never practised (N)".** The frequency rated on the item will be used to rank the didactical items in positions 1, 2, 3, ..., 15. As in any ordinal scale, these numbers (1, 2, 3, ..., 15) do not reflect any numerically valued weighting (as in 15 being greater than 1 or 3, for instance). Rather, these numerals indicate the category in the ranking described earlier on. The frequency of respondents in ranking items will be considered to decide which item is ranked most "**frequently practised", "rarely practised", or "never practised".**

Apart from questionnaires, unstructured, non-formal interviews will be carried out after responses on the questionnaires are coded and analysed. The interviews seek in-depth explanations of the responses by the sample subjects in the questionnaire; hence, these interviews yield qualitative data. The interviews will consist of in-depth, unstructured and semi-structured questions that collect qualitative data. The interview questions will concentrate on acquiring information related to any aspect on the hitherto collected data of which the researcher feels the need to verify. For instance, justification and further explanation will be helpful to establish the rank of the 15 didactical statements relating to mathematics in Section C of the questionnaire that respondents rank as "**frequently practised, rarely practised, or never practised".** The 15 items concerning didactical practices relating to mathematics in Section C of the questionnaires, in a way, capture the essence of didactical approaches that literature purports to be practised in today's mathematics classes.

Further qualitative data will be obtained from unstructured interviews. This will involve respondents giving their own personal perspectives about the use of didactical items they will be asked about (Bell 1992:4). However, as Tuckman (1988:393) points out, "a reasonable representative picture of the phenomenon's occurrence and absence may soon emerge and thereby provide a basis for interpretation of the phenomenon."

Hence, depending on the saturation point, a maximum of 27 interviews will be carried out: ten with students (two students from each school), ten with teachers (two from each school), five with subject supervisors (one from each school), and two with curriculum developers. Interviews will be stopped as soon as satisfactory insight is gained into the occurrence of didactical practices. As pointed out in the previous section, tape-recorded (where interviewees oblige), unstructured, in-depth interviews will be carried out. In line with Oppenheim's (1993:91) point of view, the researcher will use interview questions, which will be backed up by probing and encouraging comments for respondents to be able to freely open up and supply the required information.

The interviews will focus on qualitatively verifying and supplying in-depth information and facts that were gathered from Section A, Section B, and Section C in the questionnaires. Interview responses will also be used to check the reliability and validity of responses to questionnaires by triangulation between methods. This will avoid findings that are method bound as pointed out in the previous section of this chapter (Babbie 1994:105-106, Cohen and Manion 1992:269, 270, 272, Oppenheim 1992:158).

However, apart from questionnaires and interviews, documentary analysis will be processed. The exercise will also afford the researcher to gather qualitative data. Again, findings from documentary analysis will further help in triangulating research findings and so assist in establishing results that are not method bound.

This section of the chapter has given a discussion of the instruments and the respective types of data that will be collected for the research study. The next section deals with how the questions of research will be answered using the data collected by administering the instruments.

4.4.2 Relating instruments to research objectives

As pointed out in the preceding section, each questionnaire for each of the three sample groups (students, teachers, and administrators) is divided into three sections: A, B, and C (see Appendices 4, 5, and 6). Of these sections, only Section C of each of the questionnaires, along with interviews and documentary analysis, actually seeks to collect data that addresses the first objective: "to determine the actual current didactical practices relating to mathematics in secondary and high schools in the district of Maseru". Since the interviews throw more light on didactical practices relating to mathematics that are used in secondary and high schools, actual interview questions only arise from shady factor components of didactical practices relating to mathematics that the researcher finds in analysing data gathered by other instruments. Best and Kahn (1993:201) purport that this type of interview set up increases comparability of responses.

On the whole, the information collected will assist the researcher to specifically find out and establish which didactical practices relating to mathematics are currently used in the sample schools. The procedure is in line with what Borg (1987:110) posits about having respondents in the sample who are "able to supply the information that the researcher wants". Actually, as pointed out earlier, respondents in the sample group of this study share similar didactical characteristics and traits with the rest of the population in Maseru's secondary and high schools. Therefore, according to Oppenheim (1992:39-42), findings that are obtained from the sample group will further be generalised as current didactical practices relating to mathematics in in the district of Maseru.

The second objective of this research study is: "to establish the extent to which current didactical practices followed in Maseru's secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematics literacy as reflected in literature".

To accomplish this objective, correlation between respondents' scores on two variables is required. Literature (Borg 1981:156-158, Dyer 1979:76-86, 198-199, Eichelberger 1989:115, Ostle and Mensing 1975:238-241, Tuckman 1988:191, 269, 273-275, Witte 1980:75) views correlation as the statistical indication of the relationship between two sets of scores (or the extent to which two sets of score are related). Specifically, Borg (1981:156) defines the correlation coefficient as "a statistical tool that can be used to compare measurements taken on two different variables in order to determine the degree of relationship between these variables". Therefore, in this study, respondents' scores on Section A of the questionnaire (didactical practices in Maseru) will be correlated with their scores on Section B of the questionnaire (indicators of teaching mathematics for mathematics literacy). Scores will be correlated on scatter-grams and through statistical calculations using Karl Pearson's product-moment formula (see 4.4.1).

According to Naiman, Rosenfeld, and Zirkel (1977:208, 212), the coefficient **r,** in the above formula, gives a measure of the relationship between scores in Section A and those in Section B. The coefficient **r** has a minimum possible value -1 when there is a perfect negative correlation. The maximum possible value of **r** is 1 when there is a perfect positive correlation (Naiman, Rosenfeld, and Zirkel 1977:212). The resulting correlation, captured in the calculated correlation coefficient will reflect the extent to which the two variables are related.

The third objective of the study is: “To examine and assess whether the nature (content, objectives, and recommended didactical practices) of the mathematics curriculum offered in secondary and high schools in Maseru concurs with that suggested in literature on teaching mathematics for mathematics literacy”.

To explore this objective, the researcher will use qualitative documentary analysis. The content of the prescribed mathematics curriculum for Lesotho’s secondary and high schools as well as the objectives and recommended and practised didactical methods are examined and assessed. In the light of the literature on mathematics education for mathematics literacy, it will be established whether secondary and high schools in Maseru offer education for mathematics literacy. However, this exercise goes together with the qualitative analysis of mathematics textbooks that are used in the sample schools.

The fourth and last objective of this research study is: “to assess what didactical practices relating to mathematics in Maseru, Lesotho (if any) still need to be improved, embraced, or redefined in order to achieve mathematics literacy in students”. This objective is to be explored qualitatively by analysing the findings from investigating objectives 1, 2, and 3.

In order to collect worthwhile and truthful data for this study, the procedure for collecting the data and the instruments used to gather the data need to be valid and reliable. Hence, the following section examines the issue of validity and reliability of the instruments that are used in this study so that the findings of the study themselves are made as valid and reliable as possible.

4.5 VALIDITY AND RELIABILITY OF INSTRUMENTS

4.5.1 Validity

The validity of an instrument has everything to do with whether the instrument performs its intended function well or not. In fact, literature purports that it is all about the capability of the instrument to measure truthfully what it is supposed to measure. It concerns the predictions and generalisations that is based on the data obtained to be dependable and meaningful (Bell 1992:51, Ebel 1979:468, Frith and MacIntosh 1991:19, Henerson, Morris, and Fitz-Gibbon 1987:134, Shava 1999:47, Singleton, Straits, Straits, and McAllister 1988:111).

Some of the most important qualities of a valid instrument are whether it has both content and construct validity. Content validity refers to the degree to which the instrument samples the content area that is measured and also refers to how well the items in the instrument represent the universe of all the items that might be embodied in the area under investigation (Abel 1979:447, Ary, Jacobs, and Razavieh 1972:191, Shava 1999:57, Wiersma and Jurs 1985:97). Construct validity refers to the extent to which the instrument items agree with the theoretical framework of the major construct that is being measured (Ary, Jacobs, and Razavieh 1972:197, Popham 1981:109).

In order to build content validity in the instruments that are used in this study, preliminary, fact-finding, informal interviews and open-ended questionnaires (see Appendices 1, 2, and 3) were administered on groups of ten students and two teachers from each sample school. These students and teachers are different from those in the actual study sample. Both the preliminary interviews and questionnaire are intended to determine the didactical practices relating to mathematics used in mathematics education in the sample schools. Again, the content of common didactical practices relating to mathematics was drawn from literature as portrayed in Chapter 2 of this study. Furthermore, indicators of teaching/learning mathematics for mathematics literacy were incorporated when constructing questionnaire items in Section B of the questionnaire. The same applies to the 15 items that are in Section C, and respondents are to put these in rank order. Section C pertains to what the literature in chapters 2 and 3 purports as didactical practices relating to mathematics that establish mathematics literacy. Hence, the researcher attempted to build content validity into constructing the instruments. However, to verify that the instruments collect valid data, the truthfulness will be sorted out by triangulation in interviews after data gathering and analysis.

Furthermore, didactical practices relating to mathematics in literature actually provide the theoretical framework of the construct that the study seeks to examine. Construct validity of didactical practices relating to mathematics is automatically incorporated in all the instruments as items in the instruments are drawn from what literature recommends for didactical practices relating to school mathematics. In fact, in defining construct validity, literature concurs that this type of validity refers to the extent to which items in the instrument agree with the theoretical framework of the major concept that is being measured (Ary, Jacobs, and Razavieh 1972:197, Henerson, Morris, and Fitz-Gibbon 1987:136, Popham 1981:113, Wiersma and Jurs 1985:107).

An important aspect of construct validity is a clear delineation of the concept that is being measured. According to Henerson, Morris, and Fitz-Gibbon (1987:137), the delineation itself needs to include clear distinctive features of the concept based on previous writing about the construct, including its sub-components and their relationships to one another. For this study, chapters two and three spell out the construct under study, hence, the assertion that didactical practices relating to mathematics in literature actually provide construct validation.

As pointed out in Section 4.4 of the current chapter, in this study, the validity of findings is checked through triangulation between methods of data collection (questionnaires, interviews, and documentary analysis) and through triangulation within the same method (Cohen and Manion 1992:269-275, 278). Triangulation will be achieved by comparing data gathered in the different methods and within the same method. According to Babbie (1994:105-106), triangulation is the use of several different techniques to collect research data in order to test the same aspect of research. In this regard, triangulation is actually a valuable research strategy since it helps to overcome the problem of method-boundness that may be created by the exclusive use of one method. Hence, in this study, data is of different types (quantitative and qualitative), drawn from different groups (students, teachers, curriculum developers, and the inspectorate), and obtained by using different instruments (interview questionnaires and documentary analysis) in the quest to establish a true and meaningful state of didactical practices relating to mathematics followed in the sample schools. As pointed out before, extrapolating these to the whole of Lesotho is appropriate since the sample schools share similar didactical practices with the rest of the country.

4.5.2 Reliability

Literature (Frith and MacIntosh 1992:21, Hopkins 1989:81, Lloyd-Jones and Bray 1992:39, Oppenheim 1992:144, Peil, *et al*, 1982:9) defines reliable research instruments as those that yield measures that are consistent, replicable, dependable, precise, and stable. From this definition, it can be deduced that, for a research instrument to be reliable, it needs to have stability and internal consistency. However, stability and consistency complement each other and, generally, imply the same traits and characteristics of reliability.

From the literature (Henerson, Morris, and Fitz-Gibbon 1987:147, Popham 1981:129), stability reliability is consistency of measurement that one would obtain from using the same instrument over a period of time. Such procedure is often referred to as test-retest and the central concern of the exercise is to demonstrate consistency of measures with the instrument using the same respondents across time. For the purpose of this study, test-retest reliability is not done. The procedure is time consuming and does not suit the time line for the completion of this study.

However, internal consistency of items in the instrument is also measured using two or more forms of equivalent/content parallel/alternate form tests. According to Wiersma and Jurs (1985:67), this type of reliability is established by checking consistency of measurement of the variable across two or more equivalent forms of the instrument. The instruments are said to have equivalent reliability. The researcher will use equivalent forms of tests by taking the split half of the items in each questionnaire. Each split half will consist of questions (questionnaire items) that measure the same trait. Oppenheim (1993:160) posits that, practically, items from the same instrument are divided into two halves at random to form two separate but parallel form tests. In this study, items are specifically separated according to the trait that they measure. Thus, after the whole instrument (see Appendices 4, 5, and 6) is administered to the sample group, the researcher will correlate the respondents' sub-scores (on the split halves) in order to establish the internal consistency of items in the instrument, where reliability on the whole test is given by the Spearman-Brown formula (**r**):

$$r = \frac{nr_1}{1 + (n-1)r_1}$$

where n = number of split parts (2)

r_1 = uncorrected correlation coefficient (given by Pearson's product moment formula).

To achieve the split halves meaningfully, for both Section A and Section B of the instrument, the researcher will split into two halves all corresponding items that measure the same trait. There are 30 items in Section A. These will be divided into two equivalent halves. In Section B there are 25 items. OF the items in this section of the Questionnaire, item 17 does not have an equivalent. Hence, there are only 24 items in section B that will be split into two characteristically identical halves. As discussed in the preceding paragraph, scores of respondents in each of the corresponding items in the respective split halves in each section are used to compute relevant correlation coefficients that reflect the extent to which respondents are consistently responding to the same didactical trait.

Having fully discussed the different research approaches that are employed in this research study, the population sample and sampling techniques, the instruments of research, and issues of their validity and reliability, the next section looks into the details of the research process of the study.

4.6 THE RESEARCH PROCESS

According to literature, the quest for discovering new knowledge and the need to come to grips with the truth and understanding of educational phenomena has led educationists to achieve this through relevant and insightful experience, sheer reasoning, and scientific research and enquiry (Borg 1993:5-6, Cohen and Manion 1992:1-5). However, the three categories are not independent of each other, neither are they mutually exclusive. Rather, they complement each other and usually overlap, culminating in producing a whole piece of research study. In fact, Cohen and Manion (1992:5) envisage research study as a combination of both experience and reasoning. Research must be taken as the most successful approach to discovering the truth. In itself, research is defined as a systematic, controlled, empirical, and critical investigation of a set of hypothetical propositions about presumed phenomena (Ibid). Hence, as such, subjective opinions from experience are checked against scientific objective reality that has been empirically established by following systematic procedural steps of gathering reliable and valid data.

4.6.1 Step 1: Exploratory enquiry

Exploratory enquiry was the first exercise in the quest to gather the required data to investigate the objectives of this research study. Open-ended questionnaires and interviews for fact finding were carried out on a group of respondents similar, but not the same, to that in the sample group (see Questionnaire 1). This was important for constructing content valid instruments (Cohen and Manion 1992:62, Oppenheim 1993:65-67, Shumway 1980:30, 40, 41). Therefore, spontaneity in free-style questionnaires and interviews is essentially the key factor in this stage of the research process. The objective of such an exercise in this study is to obtain unrestricted information about didactical practices relating to mathematics that are currently used in Lesotho's secondary and high schools.

4.6.2 Step 2: Piloting

The second exercise was piloting the instruments before the actual administration of the instruments on the sample group. Literature indicates that piloting is an exercise in collecting and correcting ideas. According to Cohen and Manion (1992:62), respondents in the pilot study must be as similar as possible to those in the main enquiry. Hence, in this study, the researcher used students and teachers at a high school just outside Maseru.

4.6.3 Step 3: Construction of final research instruments

After the piloting exercise, the final instruments were constructed. However, as indicated in Section 4.4 of this chapter, responses to both pilot questionnaires and exploratory questionnaires and interviews, together with didactical practices relating to mathematics recommended in literature (see Chapter 2 and three), are used to construct a set of questionnaires (see Appendices 4, 5, and 6) that are used to explore this research study. Therefore, after the initial exploratory pilot study, follows the construction of relevant questionnaires and interviews and the actual implementation of the data gathering plan, as described in Section 4.4 of this chapter.

4.6.4 Step 4: Administering instruments to respondents

Actually, data is collected almost simultaneously for each objective that the researcher sets out to investigate. The researcher will personally go to each school in the sample group according to agreed dates of appointment to administer the questionnaires or to conduct interviews. The same arrange ments will be made with appointments for administrators.

After the gathering of required data is accomplished, the data is analysed and a summary of results and findings for each question under investigation is discussed. Then conclusions and recommendations, if any, are suggested. At that juncture, the significance of the study and related areas for further research are indicated.

4.7 CONCLUSION

The present chapter explores the research methods and methodology and all the instruments that are used to complete this study. The population, sample, and sampling technique are covered in Sections 4.2.1, 4.2.2, and 4.2.3. The research will follow both qualitative and quantitative research approaches as discussed in Section 4.3.

Research instruments are discussed in Section 4.4. A description of the specific instruments that will be used in the study is given in Section 4.4.1. Section 4.4.2 describes how each instrument will gather data required to explore each research question. However, findings cannot be worthwhile and useful if the instruments fail to collect valid and reliable data. Hence, issues of validity and reliability of the research instruments are discussed in Section 4.5.1 and 4.5.2. All the instruments for the study appear in the Appendices Section, in order to avoid disrupting the flow of the main body of the research.

The steps that the research process will follow include: exploratory enquiry, piloting of draft instruments, the construction of the final instruments, and, finally, the administering of these instruments on the respective respondents.

The next chapter deals with the actual processing and analysis of the collected data. The chapter also addresses issues and a justification of the validity and reliability of the findings.

CHAPTER 5

DATA ANALYSIS: RESULTS AND THEIR QUALITY

5.1 INTRODUCTION

Chapter 4 describes the data collecting techniques that the researcher uses to gather information that specifically answers research questions and meets the objectives of the study. The questions of research and related objectives of the study are developed and described in Chapter 1, Section 1.4. The four objectives that the study seeks to meet are to:

- determine the didactical practices relating to mathematics that are currently being applied in secondary and high schools in Maseru district in Lesotho,

- establish the extent to which current didactical practices relating to mathematics followed in Maseru's secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematics literacy as reflected in literature,

- examine and assess whether the nature (content, objectives, and recommended didactical practices) of the mathematics curriculum offered in secondary and high schools, in Maseru, concurs with that suggested in literature on teaching mathematics for mathematics literacy, and

- assess what didactical practices relating to mathematics in Maseru, if any, still need to be improved, embraced, or refined in order to achieve mathematics literacy in students.

An inspection of the objectives reveals that both quantitative and qualitative data need to be collected in order to explore the questions pertinent to the study. In fact, the discussion in Sections 4.3, 4.4, 4.4.1, and in 4.4.2 purports that the nature of the data that was collected for this study need to be quantitative and qualitative in its own right for the purposes of triangulation, validity, and reliability. Furthermore, the instruments that were used to gather the data were fashioned to meet this requirement. These instruments are: questionnaires (see Appendices 4, 5, and 6), interviews, and documentary analysis. Sampling techniques for selecting participating schools and respondents for this study are discussed in chapter 4 (see Sections 4.2. 2 and 4.2.3). Five schools were purposively selected from clustered schools in Maseru for their accessibility and to reduce the cost and time of study. As described in Section 4.2.2 of Chapter 4, the five schools will be referred to in this study as School 1, School 2, School 3, School 4, and School 5, respectively. From each school, 25 secondary school students, 25 high school students, and three teachers of mathematics were chosen by simple random sampling as indicated in Section 4.2.3 of Chapter 4. Four administrators (two curriculum planners for mathematics, one member of the inspectorate for the teaching of mathematics, and one mathematics resource person and advisor) are also part of the respondents in the sample group.

From the description of the respondents, it is evident that there are three categories of respondents: students, teachers, and administrators. The different categories of respondents required the use of three distinct questionnaires for the collection of the data. The three separate questionnaires (see Appendices 4, 5, and 6) were administered to students, teachers, and administrators, respectively. The items in the separate questionnaires are the same in content, but different in wording in order to fashion the questionnaire to measure the same trait from the respective respondents in particular categories (students, teachers, and administrators, see Appendices 4, 5, and 6).

In this chapter, the researcher displays, describes, and analyses the collected raw data as it originally appears in the responses to the items in the instruments. From the analysis, research findings will crystallize out. Later in the chapter, the reliability and validity of the responses of members of the different categories are checked by triangulation. In particular, calculation of the split-half coefficient of correlation is used to assess the reliability of the findings.

5.2 DATA ON GENERAL AND BIOGRAPHIC DETAILS OF RESPONDENTS

Although general and biographic details of respondents are not directly pertinent to the objectives of the study, it is fitting and interesting to observe a few factors about the subjects in the sample group. To this effect, the first section in each questionnaire draws personal biographic details of respondents on a nominal scale (see Part 1 of Appendices 4, 5, and 6 of each respective questionnaire). The questionnaires were administered to respective subjects, collected back, and the researcher then analysed and coded responses of all subjects in this part of each questionnaire. The following subsections give a tabular display accompanied by a brief discussion of the raw data drawn from all the respondents in this section of each questionnaire.

5.2.1 Biographic information of students in all sample schools

Table 5.1.1 gives a summary of the gender of all students who responded to the questionnaire according to their class level (form 1, form 2, up to form 5). Numerals under forms 1 to 3 indicate the number of secondary school students who are in the indicated class level. Numerals under forms 4 and 5 refer to the number of students in high school.

Table 5.1.1 Gender of students in all sample schools per form

		Secondary School Students				**High School Students**		
	Form	**1**	**2**	**3**	**Total**	**4**	**5**	**Total**
Gender	**Female**	10	50	15	75	45	30	75
	Male	5	25	20	50	30	20	50

One evident and glaring aspect about gender of students is the female to male ratio in each category of the respondents. Of the 125 secondary school students who responded to the questionnaire, 75 were female and 50 male. The same ratio (female: male = 3:2) pertains to the 125 high school students. These numbers were not by design, since the researcher selected these subjects by a simple random sampling technique from each school by class list in alphabetical order. At the same time, this ratio, which gives female students a greater percentage, is not an indication that there are more females than males in the sample schools. However, the gender of students is not the direct concern of this research study neither is the age of students as reflected in Table 5.1.2 which follows.

Table 5.1.2 Age of students in all sample schools per form

	Age in years	Form 1	Form 2	Form 3	Total	Form 4	Form 5	Total
Female	11-12	3	1	0	4	0	0	0
Male	11-12	1	2	0	3			
Female	13-14	7	40	2	49	1	0	1
Male	13-14	4	15	6	25	9	2	11
Female	15-17	0	9	13	22	23	3	26
Male	15-17	0	8	14	22	21	1	22
Female	18-20	0	0	0	0	10	25	35
Male	18-20	0	0	0	0	5	17	22
Female	Over 20	0	0	0	0	1	2	3
Male	Over 20	0	0	0	0	3	2	5

An aspect to note in Table 5.1.2 is the age limit required to enter a particular class. A perfunctory scan over the recorded data reveals that age boundaries for class levels do not appear to be a major concern in schools in Maseru. Further, judging by the number of years spent at the same school, repeating classes seems to be permissible (see Table 5.1.3).

Table 5.1.3 Number of students who spent the indicated number of years at the same school

Number of years at same school	Years in Form 1	Years in Form 2	Years in Form 3	Total	Years in Form 4	Years in Form 5	Total
1 year	15	6	0	21	0	0	0
2 years	0	69	2	71	0	0	0
3 years			25	25	0	0	0
4 years			8	8	70	0	70

5 years					5	40	45
6 years						10	10

5.2.2 Biographic information of teachers in all sample schools

In the questionnaire (see Appendix 5 part 1) teachers were asked to give biographic details about themselves with regards to gender, age, educational qualifications, teaching experience and the classes which they teach. In this section a brief description of the information which was gathered is given.

Table 5.2.1 Gender of teachers in all sample schools

School	**1**	**2**	**3**	**4**	**5**	**Total**
Female	2	3	2	1	3	11
Male	1	0	1	2	0	4

Table 5.2.1 shows that, on the whole, there are more female mathematics teachers than males. In two schools, all three mathematics teachers in the sample are females. Looking at each school reveals that there are more female mathematics teachers than males except for one school where the ratio of male to female is 2:1. Again, this information is not deduced from a scientific enquiry since the issue of female versus male teachers is not a rigorous pursuit of this study.

Table 5.2.2 Age of teachers in all sample schools

Age in years:	**School 1**	**School 2**	**School 3**	**School 4**	**School 5**	**Total**
Less than 20 years	0	0	0	0	0	0
20 to 25 years	1	0	0	0	0	1
26 to 30 years	0	1	0	1	0	2
31 to 35 years	0	0	1	0	1	2
40 to 45 years	2	1	1	2	1	7
Over 45 years	0	1	1	0	1	3

Table 5.2.2 shows that the ages of mathematics teachers in the sample ranges between 20 and more than 40. This may be indicative of a balanced workforce with the young working together with the more experienced. Table 5.2.3 shows that most mathematics teachers teach form 1 up to and including form 5. This is in line with their qualifications as Table 5.2.4 indicates that the majority of the teachers in the sample group are aptly qualified.

Table 5.2.3 Forms currently taught by teacher per school

School	**1**	**2**	**3**	**4**	**5**	**Total**
Forms 1-3 only	1	0	1	0	0	2
Forms 1 to 5	2	3	2	3	3	13

Table 5.2.4 Teachers' educational qualifications

School	**1**	**2**	**3**	**4**	**5**	**Total**
Secondary Teacher's Diploma	1	0	1	0	0	2
B Sc in Education	2	2	2	2	3	11
Master's Degree	0	1	0	1	0	2

Looking specifically at Table 5.2.4, the data reveal that, except for two of the teachers, all teachers in the sample group have at least one degree relevant to teaching mathematics. The other two teachers hold a secondary teacher's diploma. Of the teachers who hold at least one degree, four have a Masters degree. The majority of these teachers have teaching experience ranging from 6 to over 20 years and, but for two, all of them work with all year groups (see Table 5.2.4 and Table 5.2.5)

Table 5.2.5 Teaching experience of teachers in years

School	1	2	3	4	5	Total
Number of teachers with 1-2 years of experience	1	0	0	0	0	1
Number of teachers with 3-5 years of experience	0	0	1	0	0	1
Number of teachers with 6-10 years of experience	0	1	1	1	1	4
Number of teachers with 11-15 years of experience	0	1	0	1	0	2
Number of teachers with 16-20 years of experience	1	0	1	0	2	4
Number of teachers with more than 20 years of experience	1	1	0	1	0	3

5.2.3 Biographic information of administrators

Table 5.3.1 reveals that the same feature, i.e. having a greater number of females compared to males, which is observed in students (see Table 5.1.1) and teachers (see Table 5.2.1), is also noted amongst the administrators who responded to the respective questionnaires.

Table 5.3.1 Gender of administrators

Gender	
Females	3
Males	1

Table 5.3.2 indicates that the ages of administrators range from 31 to over 45 years, which may be indicative of a mature and seasoned workforce. The factor of

academic qualifications of the administrators too, is impressive. But for one, all hold a Masters degree.

Table 5.3.2 Age of administrators

Age in Years	
31-35	1
36-40	1
41-45	1
Over 45	1

Table 5.3.3 Educational qualifications of administrators

B Sc in Education	**1**
Masters Degree	3

Administrators, too, have wide teaching and administrative experience (see Table 5.3.4 and Table 5.3.5). Their teaching experience ranges from 6 to 20 years. Only one of them has been an administrator for only about five years. Two have been administrators for about ten years and one has worked as an educational administrator for mathematics for more than ten years.

Table 5.3.4 Teaching experience of administrators in years

6-10 years of teaching experience	**2**
11-15 years of teaching experience	**1**
16-20 years of teaching experience	**1**

Table 5.3.5 Time in years as administrator

1-5 years	**1**
6-10 years	**2**
More than 10 years	**1**

As pointed out before (see 5.2), general and biographic details of respondents are not the major pursuit of this research study. Noteworthy data relevant to the questions of this research study are established from the responses of the sample subjects to items in Section A, Section B, and Section C of Part 2 of Appendices 4, 5, and 6 of the respective questionnaires. The raw data from these sections are summarised and analysed in the following parts of this chapter.

5.3 ANALYSIS AND INTERPRETATION OF DATA WITH RESPECT TO RESEARCH OBJECTIVES

The present subsection of this Chapter 2 seeks to analyse and interpret data in relation to the specific objectives of the study. Each research question is examined and results are deduced from evidence portrayed in the administrators' data.

5.3.1 Objective 1: Current didactical practices relating to mathematics

As indicated in Section 1.4 of Chapter 1, one of the research objectives of this study is: to determine the present didactical practices relating to mathematics in secondary and high schools in Maseru.

This question is explored by separately examining the perspectives of secondary school students, high school students, teachers, and administrators on the issue. To this effect, students', teachers', and administrators' responses to Section C of Part 2 of the questionnaires in Appendices 4, 5, and 6 along with their responses to in-depth interviews facilitate the researcher to assess the solution to the question.

In Section C of the questionnaire, the researcher asked respondents to rank 15 didactical practices relating to mathematics in order of most frequently used, rarely used or never used. In coding the responses, the item that had the most frequency was ranked "first" and the next frequently chosen was "second", and so forth. As in all ordinal scales, the position of ranked item does not give a numerical or quantitative measurement, but only a rank ordering of items (Borg and Gall 1974:293, Coleman and Briggs 2002:235-236, Dyer 1979:54, Tuckman 1988:178).

On the strength of the frequency rank order of the item, further verified through interviews by triangulation for reliability, the researcher is guided to identify current didactical practices relating to mathematics in Lesotho.

5.3.1.1 Secondary school students' perspective on current didactical practices relating to mathematics

Table 5.4 displays the frequency of secondary school students who ranked the 15 didactical practices relating to mathematics in Section C of the questionnaire as "frequently practised", "rarely practised", or "never practised".

Table 5.4 Frequency of secondary school students in ranking each of the 15 didactical items per school

	Frequency of frequently used item ranked in the 5 schools					**Frequency of rarely used item ranked in the 5 schools**					**Frequency of never used item ranked in the 5 schools**				
School	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
Item															
1	16	20	19	20	23	5	4	6	5	2	4	1	0	0	0
2	14	13	12	15	15	8	10	13	10	10	3	2	0	0	0
3	0	0	0	0	0	0	0	0	0	0	25	25	25	25	25
4	5	10	8	15	13	17	9	7	10	7	3	8	10	0	3
5	2	0	0	0	0	3	1	0	0	0	20	24	25	25	25
6	19	5	15	15	13	6	9	8	10	12	0	11	2	0	0
7	5	2	2	7	6	6	10	9	10	9	14	13	14	8	10
8	16	10	8	13	12	9	15	15	8	10	0	0	3	4	3
9	10	9	7	14	13	13	12	8	11	12	1	4	10	0	0
10	0	0	0	0	0	0	10	0	2	3	25	15	25	23	22
11	2	1	2	0	0	10	0	0	0	0	13	24	23	25	25
12	10	13	12	15	14	12	7	12	6	6	3	5	1	4	5
13	18	20	22	22	21	5	5	3	3	4	2	0	0	0	0
14	20	19	18	19	18	4	6	7	6	7	1	0	0	0	0
15	16	17	18	16	14	6	7	5	7	8	3	1	2	2	3

The raw data in Table 5.4 need to be further refined to obtain a total frequency of students rating the same didactical item. This yields the results in Table 5.4.1. The first row in Table 5.4.1 represents the respective didactical item in question in Section C of the questionnaire in Appendices 4, 5, and 6. Each figure in the second, third, and fourth rows of the table represents the total frequency of secondary school students who indicate the didactical practice as "frequently practised", "rarely practised", or "never practised".

Table 5.4.1 Overall secondary school students total frequency in ranking of each of the 15 didactical items

Item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Frequently practised	98	69	0	51	2	67	22	58	53	0	5	64	102	94	81
Rarely practised	22	51	0	50	4	45	44	57	56	15	10	43	20	30	33
Never practised	5	5	125	24	119	13	59	10	16	110	110	18	2	1	11

Table 5.4.1 actually facilitates the researcher to compute the specific rank of each didactical practice as placed by secondary school students as in Table 5.4.2.

Table 5.4.2 The rank of each of the 15 items by question number as placed by secondary school students

Rank position of item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Item numbers ranked frequently practised	13	1	14	15	2	6	12	8	9	4	7	11	5	3, 10	
Item numbers ranked rarely practised	8	9	2	4	6	7	12	15	14	1	13	10	11	5	3
Item numbers ranked never practised	3	5	10 11		7	4	12	9	6	15	8	1, 2		13	14

Examining the positions of didactical items presented in Table 5.4.2 above seems to manifest a reliable rating by respondents. For instance, Item 13 is rated number 1 for being "frequently practised" and it is placed at number 14 as "never practised". Items 8 and 9 are rated numbers 1 and 2, respectively, as "rarely practised". These same items are in the middle of the rating scale of both the "frequently practised" items and the rating scale of the "never practised" didactical items. With assistance from Table 5.4.2, the researcher is in a position to translate the item number into the actual didactical practice that appears in Section C of the questionnaire in Part 2 of Appendix 4 and obtain Table 5.4.3 that embodies the currently practised didactical practices in secondary schools as perceived by students.

Table 5.4.3 Rank of didactical practice relating to mathematics assigned by secondary school students in order of mostly used

Rank	**Didactical Practice frequently used**	**Item number**
1	We do a lot of practice on the exercises that the teacher gives us.	13
2	We have explanations and questions directed by the teacher, followed by students working on given exercises.	1
3	We are made to understand mathematical concepts, rules, procedures, and processes.	14
4	The teacher creates chances for us to enjoy learning mathematics.	15
5	Students do their exercises either individually or in groups.	2
6	We follow the set textbook.	6
7 8	We connect what we learn in mathematics to other school subjects. We have problems that include applications to everyday situations.	12 8
9	Students do their exercises either individually or in groups.	9
10	We are led to discover mathematical ideas on our own.	4
11	We are given mathematical areas to investigate.	7
12	We have time to form mathematical expressions from the real world around us.	11
13	We are given mathematically based practical work that we are required to work on and complete over a long period of time.	5
14	• We have computer aided teaching/learning practices. • We work with problems that do not have one single correct answer.	3 10
15		

After analysing the responses from secondary school students, the researcher randomly selected two secondary school completed questionnaires from each school in order to conduct interviews that were aimed at obtaining in-depth information. Each respondent used a coded name for the purposes of this study as pointed out in Chapter 4 and also indicated in questionnaires 4, 5 and 6. The completed questionnaires were returned to the interviewees during the interview and the researcher used photocopies of each respective completed questionnaire in directing questions. In particular, the interviews sought to further explore the extreme ends of respondents' opinions: the "mostly used" and the "never used" didactical practices. The researcher observed that the "rarely used" items that respondents rank as first in being "rarely used" are, in fact, in the middle rank for both the "mostly used" and the "never used" items. Hence, Tables 5.4.4 and Table 5.4.5 address questions related to the extreme ends of the respondents' views in the questionnaire and the explanation and justification of these views in the interviews.

Table 5.4.4 Summary of responses of secondary school students to interview questions on mostly used didactical method

Question	**Response**
Describe five methods your teacher uses during the teaching/learning of mathematics in class.	• Explanations followed by students working on given exercises • Understanding of concepts, rules, and procedures • Lots of drill exercises • Individual or group work • Follow set textbooks
Why do you think your teacher uses those methods?	Have examinations to prepare for so we need to understand and to drill on what may come in the examinations

Table 5.4.5 Summary of responses of secondary school students to interview question on never used didactical method

Question	Response
Describe five methods your teacher never uses during the teaching/learning of mathematics in class.	• Mathematical modelling • Open-ended problems • Use of computers • Projects work • Extensive investigation
Why do you think your teacher does not use those methods?	• Every problem has one answer only • Do not know why we do not use computers and calculators; it could be fun

With regard to open-ended problems, the researcher gathered from responses in the interviews that secondary school students strongly believe that any mathematics problem has only one solution. Examining Table 5.4.4 and Table 5.4.5, it is evident that responses to interview questions by secondary school students concur with their responses to the ordinal scale whose results are displayed in Table 5.4.3. At the same time, in their innocence, the students are actually pointing out a need to address the preclusion of these didactical practices in the classroom. On the same issues, high school students are more rational in their views as indicated in Table 5.5 given in the following subsection.

5.3.1.2 High school students' perspective on current didactical practices relating to mathematics

Table 5.5 Frequency of high school students in ranking each of the 15 didactical items per school

	Frequency of frequently used item					**Frequency of rarely used item**					**Frequency of never used item**				
School	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
Item															
1	18	22	21	22	24	7	3	4	3	1	0	0	0	0	0
2	20	11	13	13	16	5	14	12	12	9	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	25	25	25	25	25
4	15	9	8	16	14	7	10	6	9	7	3	6	11	0	4
5	0	0	0	0	0	2	0	0	0	23	25	25	25	25	25
6	19	5	15	15	13	6	9	8	10	12	0	11	2	0	0
7	0	4	3	9	7	9	9	11	13	10	16	12	11	3	8
8	18	9	8	12	11	7	13	13	7	9	0	3	4	6	5
9	15	12	9	15	14	10	11	11	10	11	0	2	5	0	0
10	0	0	0	0	0	5	17	0	2	4	20	8	25	23	21
11	2	1	2	0	0	10	0	0	0	0	13	24	23	25	25
12	16	15	14	16	15	8	6	8	7	7	1	4	3	2	5
13	20	18	20	23	22	5	7	5	2	3	0	0	0	0	0
14	22	18	16	20	17	3	5	9	5	8	0	2	0	0	0
15	15	14	16	16	13	3	8	4	6	7	7	3	5	3	5

When one further processes the raw data in Table 5.5 to obtain a total frequency of high school students rating the same didactical item, it yields Table 5.5.1 that follows. The first row of Table 5.5.1 represents the respective number of the didactical item as it appears in Section C of questionnaire in Appendix 4. Each figure in the second, third, and fourth rows of the table represents the total frequency of high school students who indicate the didactical practice as “frequently practised”, “rarely practised”, or “never practised”.

Table 5.5.1 Overall high school students’s total frequency in ranking each of the 15 didactical items

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Frequently practised	107	73	0	62	0	63	23	58	65	0	0	76	103	93	74
Rarely practised	18	52	0	39	2	45	52	49	53	28	11	36	22	30	28
Never practised	0	0	125	24	123	17	50	18	7	97	114	13	0	2	23

Table 5.5.1 actually helps the researcher to compute the specific rank of each didactical practice item as placed by high school students as in Table 5.5.2 below.

Table 5.5.2 The rank of each of the 15 items by question number as placed by high school students

Rank position of item	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Item numbers ranked frequently practised	1	13	14	12	15	2	9	6	4	8	7	3 5 10 11			
Item numbers ranked rarely practised	9	7 2		8	6	4	12	14	10 15		13	1	11	5	3
Item numbers ranked never practised	3	5	11	10	7	4	15	8	6	12	9	14	1 2 13		

The positions of didactical items presented in Table 5.5.2 above seem to indicate a reliable rating by respondents. For instance, item 13 is rated number 1 in being "frequently practised" and it is placed number 14 as "never practised". Items 8 and 9 are rated numbers 1 and 2 respectively, as "rarely practised". These same items are in the middle of the rating scale of both the "frequently practised" items and the scale of the "never practised" didactical items. With assistance from Table 5.5.2, the researcher is in a position to translate the item number into the actual didactical practice that appears in Section C of the questionnaire in Part 2 of Appendix 4. One then obtains Table 5.5.3 that embodies the currently practised didactical practices in high school as perceived by high school students.

Table 5.5.3. Rank of didactical practice relating to mathematics as assigned by high school students in order of mostly used

Rank	Didactical practice frequently used	Item Number
1	We have explanations and questions directed by a teacher, followed by students working on given exercises.	1
2 3	We do a lot of practice of the exercises that the teacher gives us. We are made to understand mathematical concepts, rules, procedures, and processes.	13 14
4	We connect what we learn in mathematics to other school subjects.	12
5	The teacher creates chances for us to enjoy learning mathematics.	15
6	Students do their exercises either individually or in groups.	2
7	We relate concepts and topics in mathematics to each other.	9
8 9	We follow the set textbook. Led to discover mathematical ideas on our own.	6 4
10 11	We have problems that include applications of mathematics to everyday situations. We are given mathematical areas to investigate.	8 7
12	• We have computer aided teaching/learning practices. • We are given mathematically based practical work that we are required to work on and complete over a long period of time. • We work with problems that do not have one single correct answer. • We have time to form mathematical expressions from the real world around us.	3 5 10 11
13		
14		
15		

As pointed out in Chapter 4, Section 4.4.1, and again explained in Section 5.3.1.1 of the present chapter, interviews were carried out in order to solicit, through in-depth probing questions, the opinions of high school students on the mostly practised didactical methods relating to mathematics. Table 5.5.4 and Table 5.5.5 that follow display a summary of the import of responses of high school students to questions during the interviews translated for consistence into didactical indicators in Section C of the questionnaire in Part 2 of Appendix 4.

Table 5.5.4 Summary of responses by high school students to interview questions on mostly used didactical methods

Question	Response
Describe five methods your teacher uses during the teaching/learning of mathematics in class.	• Explanations followed by students working on given exercises • Understanding of concepts, rules, and procedures • Lots of drill exercises • Individual or group work • Follow set textbooks
Why do you think your teacher uses those methods?	Have examinations to prepare for so we need to understand and to drill on what may come in the examinations

Table 5.5.5 Summary of responses of high school students to interview questions on never used didactical method

Question	Response
Describe five methods your teacher never uses during the teaching/learning of mathematics in class.	• Use of computers • Project work • Extensive investigation • Open-ended problems • Mathematical modelling
Why do you think your teacher does not use those methods?	• Maybe the school does not have money for computers • Maybe it is not necessary for our exams • It is not in the syllabus (projects/modelling/extensive investigation) • Wastes time on none examinable material

Examining high school students' responses to interview questions reveals that these students hold and justify their views on what didactical practices they have for their mathematics lessons. The researcher deduces that results in Table 5.5.3 are confirmed. At the same time, high school students' views reflect their resolute target on needing to succeed in examinations. That success is without regard of whether they have acquired competences in the subject that equip them with mathematics, as an academic discipline, as well as a body of knowledge with skills that enable them to function in a world imbued with advanced technology, as discussed in Section 3.2.3 of Chapter 3.

However, in comparing the responses of secondary and high school students (see Tables 5.4.3, 5.4.4, 5.4.5, 5.5.3, 5.5.4, and 5.5.5), the researcher observes that the views of students correspond with regard to didactical practices relating to mathematics that are employed in their schools. A summary of all students' views is given in a later section of this chapter (see 5.3.1.5).

The results in Table 5.4.4, Table 5.4.5, Table 5.5.4, and Table 5.5.5 are also used later in this chapter to explore issues pertaining to the reliability and validity of the findings of the study. Having arrived at what secondary and high school students perceive to be the mostly practised, rarely practised, and never practised didactical practices relating to mathematics, the next subsection examines perceptions of teachers on the same issues.

5.3.1.3 Teachers' perspective on current didactical practices relating to mathematics

Table 5.6 Total frequency of teachers in ranking each of the 15 didactical items per school

	Frequency of fre-quently practised					**Frequency of rarely practised**					**Frequency of never practised**				
School	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
Item 1	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
2	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3
4	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3
6	2	3	2	3	3	1	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	3	3	3	3	3	0	0	0	0	0
8	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
9	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	3	3	3	3	3
11	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2
12	0	0	0	0	0	2	2	3	2	2	1	1	0	1	1
13	3	3	3	2	3	0	0	0	1	0	0	0	0	0	0
14	3	3	3	3	3	0	0	0	0	0	0	0	0	0	0
15	2	3	2	2	2	1	0	1	1	1	0	0	0	0	0

Using the results in Table 5.6, the researcher processed the data thereof in a similar manner to that followed for students (see Tables 5.4, 5.4.1, 5.4.2, 5.5, 5.5.1, and 5.5.2). Responses of teachers recorded in Table 5.6 encapsulate teachers' ranking of didactical items relating to mathematics in Section C of the questionnaire in Part 2 of Appendix 5.As teachers are fewer in number than students in the sample of the study, it is easy to analyse the frequency by which they rate the didactical items (see Table 5.6). The analysis of teachers' rating of how often a particular didactical practice is employed, is displayed in Table 5.6.1.

Table 5.6.1 The rank of each of the 15 didactical items by question number as placed by teachers

Rank position of item	Item number of didactical method frequently practised	Ite m number of didactical method rarely practised	Item number of didactical method never practised
1 2	1, 2, 4, 8, 9, 14	7 12	3, 5, 10
3		15	
4		6	12
5		11	11
6		13	1, 2, 4, 6, 7, 8, 9, 13, 14, 15
7	13	1, 2, 3, 4, 5, 8, 9, 10, 14	
8	6		
9	15		
10	3, 5, 7, 10, 11, 12		
11			
12			
13			
14			
15			

Table 5.6.1 indicates that teachers unanimously agree about the practice of particular didactical methods. This is shown by having clusters of didactical practices that share the same rank position. At the bottom ranking scale of both the “rarely practised” and the “never practised” didactical practices are those items that are ranked at the top of being frequently used. Although didactical items 7, 11, and 12 are at the bottom end of “frequently used” along with items 3, 5, and 10, examining the “rarely practised” and “never practised” responses reveals that these didactical methods are at the top end of the “frequently used” didactical practices. However, items 3, 5, and 10 are not practised at all. The following Table 5.6.2, in words, spells out each didactical practice in rank order placed by teachers as frequently employed.

Table 5.6.2 Rank of didactical practice relating to mathematics assigned by teachers in order of mostly used

Rank of item	**Didactical practice frequently used**	**Item number**
1	• Explanations and questioning directed by teacher, followed by students working on given exercises. • Students do exercises either individually or in groups. • Students are led to discover mathematical ideas. • Problems include applications of mathematics to everyday situations. • Mathematical concepts and topics are linked to each other. • Students are made to understand mathematical concepts, rules, procedures, and processes.	1 2 4 8 9 14
2		
3		
4		
5		
6		
7	Students drill and practice skills, processes, and operations in exercises that teachers give.	13
8	Follow the set textbook	6
9	Teacher creates opportunity for students to enjoy mathematics	15
10	• There is computer aided teaching/learning practices. • Mathematical projects and practical work are part of the teaching/learning practices. • Open-ended problems are given to students. • Mathematical modelling is practised. • Students do mathematical investigations • Mathematics is linked to other school subjects.	3 5 7 10 11 12
11		
12		
13 14		
15		

Interviews were again used to seek illuminating explanations and justification for responses in Tables 5.6 and 5.6.1. The researcher, again, took completed questionnaires to the respective teachers. Each respondent used coded names for the purposes of this study as pointed out in 5.4 of this chapter (also see Chapter 4, Section 4.4, and questionnaires in Appendices 4, 5, and 6). Responses obtained supported the findings depicted in Table 5.6. and Table 5.6.1. The interview responses displayed in Tables 5.6.2 and 5.6.3 are teachers' opinions about didactical practices that they ranked "mostly used" or "never used".

Table 5.6.3. Summary of responses of teachers to interview questions on mostly used didactical methods

Question	**Response**
Describe five didactical approaches that you use often.	• Explanations and questioning directed by teachers, followed by students working on given exercises • Students do exercises either individually or in groups • Problems include the application of mathematics to everyday situations • Mathematical concepts and topics are linked to each other • Students are made to understand mathematical concepts, rules, procedures, and processes
Why do you use those methods? Are the practices in line with those recommended by the curriculum planners? What are your views about the use of textbook in your teaching and also about connecting maths to other school subjects?	• Used because they work in a situation driven by expectations and pressure from school administration, parents, and community to produce good results in mathematics • We are the people on the ground and know what is appropriate in the context within which we work • Didactical practices used are by and large in line with the recommended ones; it agrees with those common to mathematics education • The textbook is an indispensable tool; to cut expenses, one good, relevant textbook is enough; we can use our bank of questions for practice • The "connecting" is not deliberate, but occasionally when need arises

Table 5.6.4 Summary of responses by teachers to interview questions on never used didactical methods

Question	Response
Describe five methods you never use during the teaching/learning of mathematics in class.	• Computer aided teaching/learning practices • Mathematical projects and practical work are part of the teaching/learning practices • Open-ended problems are given to students • Mathematical modelling is practised • Students do mathematical investigations
Why do you not use those methods? Are the practices not in line with those recommended by the curriculum planners?	• Extensive investigation, projects, real mathematical modelling, and open-ended problems are time consuming in a syllabus where the only final assessment is a written exam; we use these in preparation for maths and science fairs • Finances and a lack of experience in teaching maths with computers make that not feasible; calculators are optional • The mention of "investigation" and "project" in the recommended didactical practices does not imply that typical and real forms of investigational work and projects are actually carried out.

From responses by teachers in interviews, the researcher deduces that there is a tendency to let didactical practices be governed by what and how to teach for success in examinations. "Meaningful explanations of concepts and procedures" followed by "students doing relevant exercises" seems to be inevitably the vehicle to teaching for success in an examination oriented system of education..

Not all of the last six didactical approaches in Table 5.6.2 are equally practised. Table 5.6.1, supported by interviews, indicates that only the following practices are not employed at all:

- mathematical modelling,
- the use of open-ended problems, and

- computer aided teaching/learning practices.

Furthermore, from the interviews, it seems that mathematical projects and extensive mathematical investigations are also rarely practiced. When it is practiced, it is one by only a few students in preparation for regional, national, and international mathematics and science fairs.

More of the interview responses and justification of collected data are addressed in the section that considers the validity and reliability of the research data.

5.3.1.4 Administrators' perspective on current didactical practices relating to mathematics

There were four administrators in the sample group. In part 2, Section C of the questionnaire (see Appendix 6, Questionnaire to administrators), the administrators were asked to put in rank order 15 didactical practice items according to how they ranked the item as "frequently used", "rarely used", or "never used". Their responses to this second part of the questionnaire were analysed, and results are displayed in Table 5.7.

Table 5.7 Total frequency of administrators in ranking each of the 15 didactical practice items

Item number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Frequency of frequently practised item	4	4	0	4	0	1	2	4	4	0	0	0	2	4	4
Frequency of rarely practised item	0	0	0	0	0	1	2	0	0	0	1	2	2	0	0
Frequency of never practised item	0	0	4	0	4	2	0	0	0	4	3	2	0	0	0

Table 5.7, like Table 5.4, Table 5.5, and Table 5.6, displays the frequency of respondents in rating the practice of the given 15 didactical practices relating to mathematics. From the table, it is seen that the administrators are in total agreement about the use of a number of didactical practices. The table also indicates that administrators do not unanimously agree about items 6, 7, 11, 12, and 13. After spelling out (see Table 5.7) which didactical practices are represented numerically by the items in Table 5.7, a brief explanatory discussion of supporting interview information sheds more understanding of the responses of administrators displayed in Table 5.7.

Table 5.7.1 Rank of each of the 15 didactical practices relating to mathematics assigned by administrators in order of mostly used

Rank of item	**Didactical practice frequently used**	**Item number**
1	• Explanations and questioning directed by teachers, followed by students working on given exercises • Students do exercises either individually or in groups • Students are led to discover mathematical ideas • Problems include applications of mathematics to everyday situations • Mathematical concepts and topics are linked to each other • Students are made to understand mathematical concepts, rules, procedures, and processes • Teachers creates opportunity for students to enjoy mathematics	1 2 4 8 9 14 15
2		
3 4		
5		
6		
7		
8	• Students do mathematical investigations • Students drill and practice skills, processes, and operations in exercises that teachers provide	7 13
9		
10	Follow the set textbook	6

11	• Computer aided teaching/learning practices • Mathematical projects and practical work are part of the teaching/learning practices • Open-ended problems are given to students • Mathematical modelling is practised • Mathematics is linked to other school subjects	3 5 10 11 12
12		
13		
14		
15		

After analysing the administrators' responses in ranking the given 15 didactical methods, the researcher observed that administrators assign the same rank to a number of didactical practices. In the search for explanations and justification, the researcher took back the completed questionnaire to the respective respondents and interviewed them. Tables 5.7.2 and 5.7.3 displays a summary of the questions and responses to those questions in the interviews that were conducted with the administrators.

Table 5.7.2 Summary of responses by administrators to interview questions on rarely used didactical practices

Question	**Response**
Describe five didactical approaches that you recommend to be used often.	All are important, but should always include: • Explanations and questioning directed by teachers, followed by students working on given exercises • Students are led to discover mathematical ideas • Problems include applications of mathematics to everyday situations • Mathematical concepts and topics are linked to each other • Students are made to understand mathematical concepts, rules, procedures, and processes
Why do you list those methods?	They enable the student to acquire required concepts, skill, and general competence in maths

Table 5.7.3 Summary of responses of administrators to interview questions on never used didactical practices

Question	Response
Describe five methods that are never used during the teaching/learning of mathematics in your schools?	"Not yet used" could better describe it because it would be ideal to use them: • Mathematical projects and practical work are part of the teaching/learning practices • Open-ended problems are given to students • Mathematical modelling is practised • Students do mathematical investigations • Use of computers
Are the practices not in line with those you recommended? How do you expect teachers to use the textbook?	• We are aware that investigation, modelling, open-ended problems, and projects cannot be fully integrated in our classrooms because the syllabus is not by course work, but we expect teachers to find time for these so as to make maths relevant to real life situations • Finances limit the use of computers in all our schools • Teachers should use a multiple of relevant resources to build understanding in students and not stick to one textbook

5.3.1.5 Summary of current didactical practices relating to mathematics in secondary and high schools

From the discussion of respondents' perceptions in Sections 5.3.1.1, 5.3.1.2, 5.3.1.3, and 5.3.1.4, along with Tables 5.4, 5.4.1, 5.4.2, 5.4.3, 5.5, 5.5.1, 5.5.2, 5.5.3, 5.6, 5.6.1, 5.6.2, 5.7, and 5.7.1, the currently employed didactical practices relating to mathematics in both secondary and high schools are put in tabular form according to the responses of subjects in the sample group. Tables 5.7.3 and 5.7.4 that follow display comparisons between responses by secondary and high school students and between responses by teachers and administrators on currently employed didactical practices relating to mathematics.

Table 5.7.4 Summary of rank of current didactical practices assigned by students

Rank	Didactical practice ranked by secondary school students as frequently used	Didactical practice ranked by high school students as frequently used
1	We do a lot of practice on the exercises that the teacher gives us	We have explanations and questions directed by teachers, followed by students working on given exercises
2	We have explanations and questions directed by teachers, followed by students working on given exercises	We do a lot of practice on the exercises that the teacher gives us
3	We are made to understand mathematical concepts, rules, procedures, and processes	We are made to understand mathematical concepts, rules, procedures, and processes
4	The teacher creates chances for us to enjoy learning mathematics	We connect what we learn in mathematics to other school subjects
5	Students do their exercises either individually or in groups	The teacher creates chances for us to enjoy learning mathematics
6	We follow the set textbook	Students do their exercises either individually or in groups
7	We connect what we learn in mathematics to other school subjects	We relate concepts and topics in mathematics to each other.
8	We have problems that include applications to everyday situations.	We follow the set textbook
9	Students do their exercises either individually or in groups	We are led to discover mathematical ideas on our own
10	We are led to discover mathematical ideas on our own	Students do their exercises either individually or in groups
11	We are given mathematical areas to investigate	We are given mathematical areas to investigate
12	We have time to form mathematical expressions from the real world around us	• We have computer aided teaching/learning practices • We are given mathematically based practical work that we are required to work on and complete over a long period of time • We work with problems that do not have one single correct answer • We have time to form mathematical expressions from the real world around us

13	We are given mathematically based practical work that we are required to work on and complete over a long period of time	
14	• We have computer aided teaching/learning practices • We work with problems that do not have one single correct answer	
15		

Table 5.7.5 Summary of rank of current didactical practices assigned by teachers and administrators

Rank	**Didactical practice ranked by teachers as frequently used**	**Didactical practice ranked by administrators as frequently used**
1	• Explanations and questioning directed by teachers, followed by students working on given exercises • Students do exercises either individually or in groups • Students are led to discover mathematical ideas • Problems include applications of mathematics to everyday situations • Mathematical concepts and topics are linked to each other • Students are made to understand mathematical concepts, rules, procedures, and processes.	• Explanations and questioning directed by teachers, followed by students working on given exercises • Students do exercises either individually or in groups • Students are led to discover mathematical ideas • Problems include applications of mathematics to everyday situations • Mathematical concepts and topics are linked to each other • Students are made to understand mathematical concepts, rules, procedures and processes • Teacher creates opportunity for students to enjoy mathematics
2		
3		
4		
5		
6		

7	Students drill and practice skills, processes, and operations in exercises that teachers give	
8	Follow the set textbook	• Students do mathematical investigations • Students drill and practice skills, processes, and operations in exercises that teachers give
9	Teacher creates opportunities for students to enjoy mathematics	
10	• There is computer aided teaching/learning practices • Mathematical projects and practical work are part of the teaching/learning practices • Open-ended problems are given to students • Mathematical modelling is practised • Students do mathematical investigations • Mathematics is linked to other school subjects • Mathematical modelling is practised • Students do mathematical investigations • Mathematics is linked to other school subjects	Follow the set textbook
11		• Computer aided teaching/learning practices • Mathematical projects and practical work are part of the teaching/learning practices • Open-ended problems are given to students • Mathematical modelling is practised • Mathematics is linked to other school subjects
12		
13		
14		
15		

Comparing the ranks that students, teachers, and administrators place on didactical practices in order of “frequently used”, it can, thus, be deduced that the currently used didactical practices relating to mathematics in secondary and high schools in Maseru include:

- drill and practice of skills, processes, and operations in exercises that teachers give,
- explanations and questions directed by teachers, followed by students working on given exercises,
- students do exercises either individually or in groups,
- students are led to discover mathematical ideas,
- problems include applications of mathematics to everyday real situations;
- mathematical topics are linked to each other,
- students are made/led to understand mathematical concepts, rules, procedures, and processes,
- students are led to discover ideas on their own,
- opportunities are created for students to enjoy mathematics, and
- teaching/learning follows set textbooks.

However, there are some didactical practices that are used rarely or to a limited extent and others which are not practised at all. These include:

- limited investigational work,

- use of calculators is optional (there are calculator and none calculator examination versions),
- limited connection of mathematics to other school subjects;
- limited learning through project work,
- mathematical modelling is almost absent,
- there is no computer aided teaching/learning; and
- there is no open-ended problem solving.

This summary of results from the analysis of respondents' responses to the administered questionnaire also agree with respondents' answers to questions during interviews. Table 5.7.5 displays responses to interview questions by students, teachers, and administrators.

Table 5.7.6 Comparative summary of responses of students, teachers, and administrators to interview questions

Question	**Secondary school students' answers**	**High school students' answers**	**Teachers answers**	**Administrators' answers**
Describe five didactical approaches that are/you use(d) often.	Explanations followed by students working on given exercises Understanding of concepts, rules, and procedures	Explanations followed by students working on given exercises Understanding of concepts, rules, and procedures Lots of drill	Explanations and questioning directed by teachers, followed by students working on given exercises Students do exercises either individually or in groups	Explanations and questioning directed by teachers, followed by students working on given exercises Students are led to discover

	Lots of drill exercises Individual or group work Follow set text books	exercises Individual or group work Follow set text books	Problems include applications of mathematics to everyday situations Mathematical concepts and topics are linked to each other Students are made to understand mathematical concepts, rules, procedures, and processes	mathematical ideas Problems include applications of mathematics to everyday situations Mathematical concepts and topics are linked to each other Students are made to understand mathematical concepts, rules, procedures, and processes
Why are those methods used? (Why do you use those methods?) Are the practices in line with those recommended by the curriculum planners? What are your views about the use of textbooks in your teaching and about connecting maths to other school subjects?	Have examinations to prepare for so we need to understand and to drill on what may come in the examinations	Have examinations to prepare for so we need to understand and to drill on what may come in the examinations	Used because they work in a situation driven by expectations and pressure from school administration parents, and the community to produce good results in mathematics We are the people on the ground and know what is appropriate within the context in which we work Didactical practices used are by and large in line with the recommended ones; it agrees with those	They enable the student to acquire the required concepts, skill, and general competence in maths

			common for mathematics education The textbook is an indispensable tool; to cut expenses, one good, relevant textbook is enough; we can use our bank of questions for practice The connecting is not deliberate, but occasional when need arises	
Describe five methods your teacher never uses during the teaching/ learning of mathematics in class	Mathematical modelling Open-ended problems Use of computer Projects work Extensive investigation	Use of computers Projects work Extensive investigation Open-ended problems Mathematical modelling	Computer aided teaching/learning practices Mathematical projects and practical work are part of the teaching/learning practices Open-ended problems are given to students Mathematical modelling is practised Students do mathematical investigations	Mathematical projects and practical work are part of the teaching/learning practices Open-ended problems are given to students Mathematical modelling is practised Students do mathematical investigations Use of computers

Why do you think your teacher does not use those methods?	Every problem has one answer only Do not know why we should not use computers and calculators; it could be fun	Maybe the school does not have money for computers Maybe it is not necessary for our exams It is not in the syllabus (projects/modelling /extensive Wastes time on none examinable material	Extensive investigation, projects, real mathematical modelling, and open-ended problems are time consuming in a syllabus where the only final assessment is a written exam; we use these in preparation for maths and science fairs Finances and lack of experience in teaching maths with computers makes that not feasible; calcula-tors are optional The mention of “investigation” and “project” in the recommended didactical practices does not imply that typical and real forms of investigational work and projects are actually carried out.	We are aware that investigation, modelling, open-ended problems, and projects cannot be fully integrated in our classrooms because the syllabus is not by course work, but we expect teachers to find time for these so as to make maths relevant to real-life situations Finances limit the use of computers in all our schools Teachers should use a multiple of relevant resources to build understanding in students and not stick to one textbook

From Table 5.7.6, the researcher deduces that responses by administrators to the interview questions, in essence, concur with responses of teachers on the same issues. Further, when probed during interviews with regards to their response to the didactical practice, “follow the set textbook only”, where a few of the teachers and administrators indicated in the questionnaire (see Tables 5.4, 5.5, 5.6, 5.7) that the textbook is frequently followed whilst a majority indicated that it is rarely or never used, the interview response from administrators was that teachers are encouraged to use a multiple set of relevant resources and not to stick to one source only. Teachers indicated that one good and relevant textbook is cheaper and more user friendly for students. Moreover, teachers' bank of worksheets, exercises, test questions, and past examination questions often supplement work from the textbooks.

Mathematical modelling is another didactical practice of which teachers indicated in the questionnaire that it is rarely done, whilst the administrators said it is not done. However, during interviews, teachers said that a few students are exposed to this when they prepare projects for mathematics and science fairs. Hence, a few of the students admitted that they have experienced mathematical modelling.

Connecting mathematics to other school subjects is yet another practice that had low positive responses. Some of the administrators said it is rarely done whilst a large number of the teachers said it is rarely done. Students indicated that it is done. However, during interviews, teachers pointed out that this practice is not overtly done, but when occasions arise, they link mathematics to other school subjects.

5.3.2 Objective 2: The extent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy

One of the research objectives of this study is "To establish the extent to which the current didactical practices relating to mathematics in Maseru's secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematical literacy as reflected in literature?" (See Chapter 1, Section 1.4).

To explore this question, the scores of respondents on items in Sections A and B are correlated through statistical computation, using the Karl Pearson product moment formula in section 4.4.1 (Naiman, Rosenfeld, and Zirkel 1977:208-212). As discussed in Chapter 4, section 4.4.1, Upton and Cook (1997:551) refer to this formula as the population product moment since characteristics in the population are merely sample characteristics taken to the extreme. The sample can be small or large. The product moment formula is, therefore, an ideal statistical measure for this part of the study as it accommodates both small and large numbers of pairs of variables to be correlated.

Naiman, Rosenfeld, and Zirkel (1977:208, 211-212) point out that Pearson's formula for computing the correlation produces a numerical coefficient of correlation. This coefficient, r, has a minimum possible value of -1 (for perfect negative correlation) and a maximum possible value of +1 (for perfect positive correlation); otherwise r is a fraction. Should the value of r be zero, no linear correlation exists between the two measured aspects (Ibid).

As discussed in Chapter 4, Section 4.4, the quality and characteristics of items in Section A of each questionnaire to each group of respondents, by design, delineate the content representations of common didactical practices relating to mathematics presumably also practised in Maseru. In the same vein, Section B of each questionnaire contains didactical items that are purported to reinforce mathematical literacy in learners. Although the number of question items is 30 in Section A and only 25 in Section B, correlating scores of respondents in these sections does not pose a problem since the researcher uses percentage scores and not the raw scores.

Each of the items in both sections of the questionnaire has a maximum score of 5. Hence, the total maximum score on items in Section A is 150 and that on items in Section B is 125. Responses of all members in the sample group are displayed, processed, and analysed in order to arrive at a solution to the research question. Table 5.8 summaries all respondents' raw scores in each category as coded from the completed questionnaires that were administered to the sample group. A brief analysis of the data is also given.

5.3.2.1 Secondary school students' perspective on the extent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy

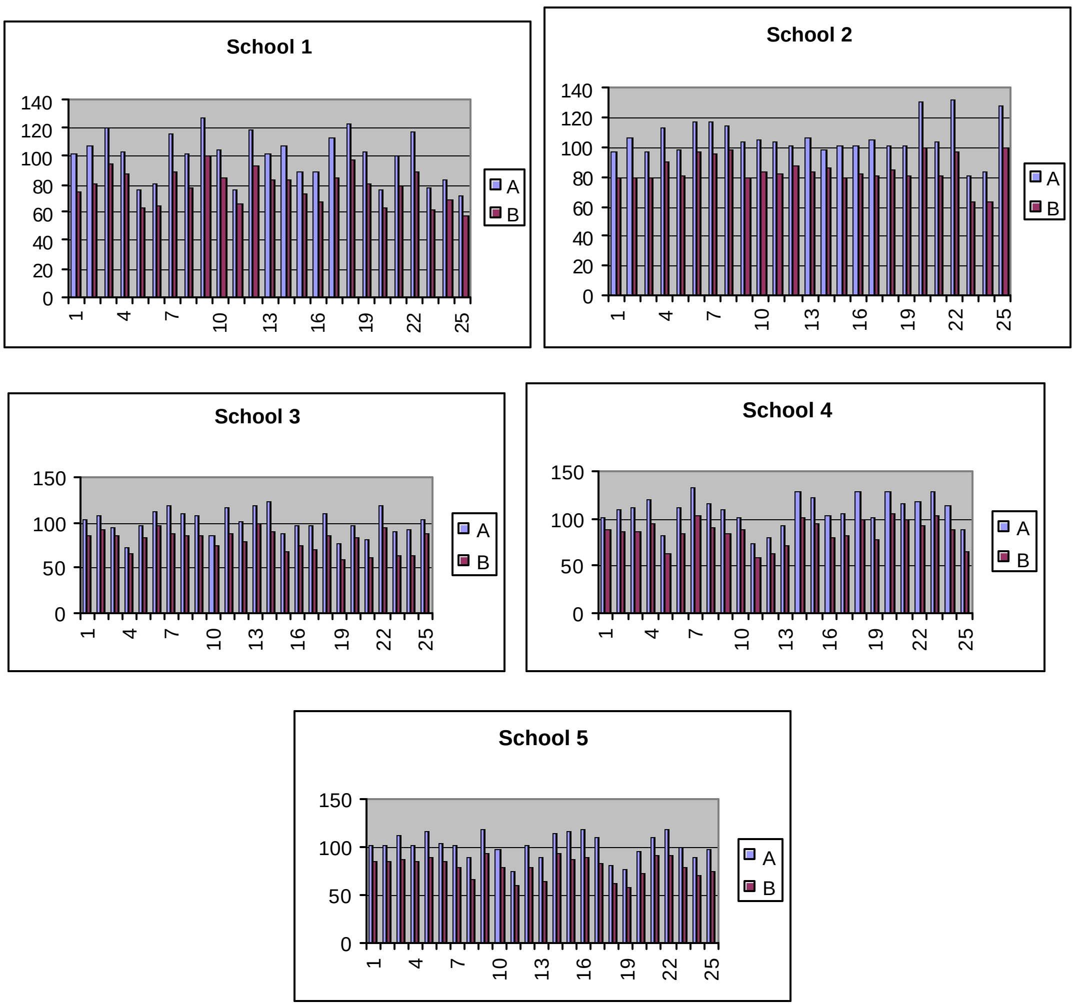
In this section, the data for secondary school students are displayed and analysed. Table 5.8 below displays the raw scores of secondary school respondents on Section A and Section B of the questionnaire in Appendices 4, Part 2, per school.

Table 5.8 Summary of total raw scores of individual secondary school students on Section A and Section B of questionnaire per school

School	**1**		**2**		**3**		**4**		**5**	
Section	A	B	A	B	A	B	A	B	A	B
Student										
1	102	75	97	80	104	85	101	88	103	85
2	107	81	107	80	109	92	109	87	103	86
3	120	95	98	80	95	86	113	87	113	87
4	104	88	113	91	73	66	120	96	102	85
5	77	63	99	81	97	83	83	63	116	90
6	81	65	118	97	113	98	111	85	105	85
7	116	90	117	96	119	88	133	103	102	80
8	102	78	115	99	111	87	116	90	89	66
9	128	100	104	80	108	86	110	85	119	94
10	105	85	105	84	87	74	101	88	99	79
11	77	66	104	83	116	89	75	58	75	60
12	119	94	102	88	102	80	81	63	102	79
13	102	84	107	84	120	99	93	73	89	65
14	108	83	99	86	123	91	129	101	115	93
15	90	73	102	80	89	69	123	96	116	88
16	89	68	102	83	97	76	104	80	119	90
17	113	85	105	81	96	70	106	83	110	84
18	123	98	102	85	110	85	128	100	81	63
19	104	81	101	81	77	60	102	79	77	59
20	77	63	131	100	97	84	129	105	95	73
21	101	80	104	81	81	61	117	100	111	91
22	117	90	132	98	120	95	119	94	118	91
23	78	62	81	63	90	63	129	104	101	79
24	84	70	84	64	92	65	114	88	90	71
25	72	58	129	100	104	89	89	66	99	76

Another way of summarising the information in Table 5.8 is by using a bar chart as portrayed in Figure 5.1. Naiman, Rosenfeld, and Zirkel (1977:31) point out that bar charts organise and give shape to data in a clear, visible, and easy to compare pattern. In figure 5.1, for each school, a respondent's scores are represented by two adjacent bars: the first bar is the respondent's score in Section A and the second bar is his/her score in Section B. Although this could be noted in Table 5.8 from the bars, it can be visibly ascertained whether high scores on Section A go with high scores on Section B or vice versa as pointed out in Section 5.3.2 of the present chapter.

Figure 5.1 Bar charts for scores of secondary school students per school



The correspondence is affirmed by actually computing the coefficient of correlation between percentage scores of secondary school students in Section A and those on Section B using the Pearson's product-moment formula (given in section 4.4.1). The resulting calculated coefficients of correlation were found to be:

$$r_{11} = 0.977611$$
$$r_{12} = 0.929352$$
$$r_{13} = 0.880877$$
$$r_{14} = 0.969795$$
$$r_{15} = 0.956252$$

where the first 1 in the subscript refers to secondary schools and the second numeral refers to school number 1, 2, and so forth.

Therefore, with this notation we have that:

r_{11} = correlation coefficient between percentage scores in Section A and scores in Section B for secondary students in School 1

r_{12} = correlation coefficient between percentage scores in Section A and scores in Section B for secondary students in School 2

r_{13} = correlation coefficient between percentage scores in Section A and scores in Section B for secondary students in School 3

r_{14} = correlation coefficient between percentage scores in Section A and scores in Section B for secondary students in School 4

r_{15} = correlation coefficient between percentage scores in Section A and scores in Section B for secondary students in School 5.

All the coefficients obtained in the calculations for each of the sample schools above represent the degree to which common didactical practices in secondary schools in Maseru corresponds to didactical practices that enhance mathematical literacy. From the discussion in 5.3.2, perfect correspondence is not reflected here since none of the coefficients is +1 or -1. However, since all coefficients are fractions nearer to 1 than nearer to zero, there is sufficient indication that strong linear correlation correspondence exits. Didactical practices that limit perfect correlation were assessed qualitatively by first inspecting the responses to identify those didactical practice items where respondents obtained low scores that might have an effect on the correlation, and then the validity and reliability of the gathered data were checked through interviews.

As Leedy and Ormrod (2005:267) posit, we can "...find substantial correlation between two characters only if you can measure both characters with a reasonable degree of validity and reliability."

Therefore, the calculated coefficients all indicate a strong existence of positive correlation between the two variables that are measured in the two subsections of the questionnaire, we will not necessarily take this as trustworthy information until factors of validity and reliability are established. This issue is discussed later in Section 5. 4 when the validity and reliability of the research findings is addressed.

The calculation of the correlation coefficient between percentage scores on Section A and those on Section B for all secondary school students yields

$$r_s = 0.947558$$

where r_s = correlation coefficient between percentage scores in Section A and scores in Section B for all secondary school students in the sample group.

The coefficient, 0.947558, reflects a strong correlation between scores by respondents in Section A and their scores in Section B. Again, the coefficient is much nearer to +1 than to zero. This indicates a very strong positive correlation between percentage scores of secondary school respondents in Section A and their percentage scores in Section B. Assessing this correlation relationship in the light of the research question explored in this section, the results point out that a strong positive correspondence exists between didactical practices followed in secondary schools and didactical practices that enhance mathematical literacy.

5.3.2.2 High school students' perspective on theextent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy

As was done with secondary school students in Section 5.3.2.1, in the present section, an analysis of high school students' responses to Sections A and B of the questionnaire in Appendices 4, Part 2 is carried out. Table 5.9 below displays raw scores of each high school student in the sample group per school.

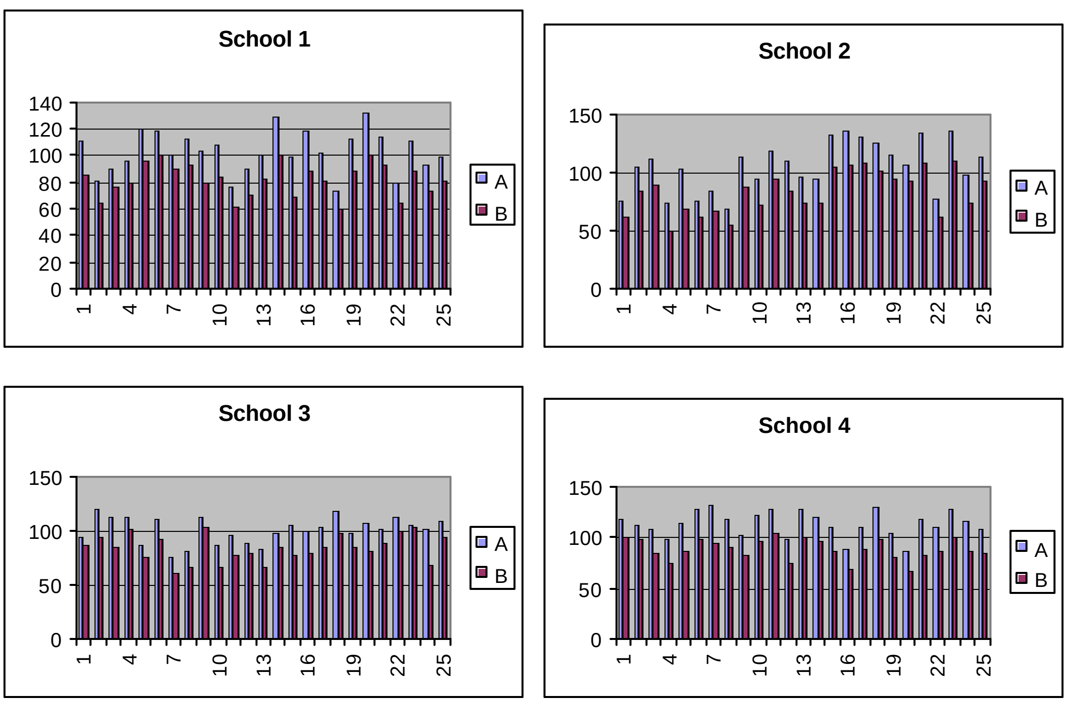
Table 5.9 Summary of total raw scores of individual high school students in Section A and Section B of the questionnaire per school

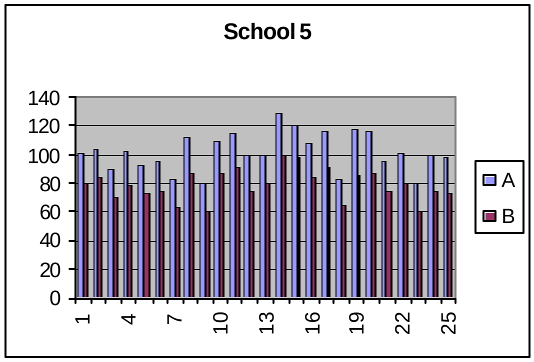
School	**1**		**2**		**3**		**4**		**5**	
Section	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
Student										
1	111	86	76	62	94	87	118	101	102	81
2	81	65	106	85	120	94	113	98	105	85
3	90	76	113	91	113	85	108	85	90	71
4	96	79	75	50	113	102	99	75	104	79
5	120	96	105	70	88	76	115	88	93	73
6	119	100	76	62	111	92	129	98	96	75
7	101	90	85	68	76	61	132	95	84	64
8	113	93	69	55	81	67	119	91	113	88
9	104	79	115	88	114	104	102	83	81	60
10	108	84	95	73	87	66	123	96	110	88
11	76	62	120	95	97	78	129	105	115	91
12	90	70	111	85	90	79	99	76	101	75
13	101	83	97	75	84	66	128	100	99	80
14	129	100	95	75	99	85	120	96	129	101

15	99	69	134	106	106	78	110	88	120	98
16	119	88	136	108	101	80	90	69	108	85
17	102	81	131	109	104	86	111	89	117	91
18	74	60	126	103	119	99	130	99	84	66
19	113	88	116	95	99	85	105	81	118	86

The data in Table 5.9 is portrayed in bar charts in Figure 5.3 below. As for the charts in Figure 5.1 that indicated the scores of secondary school students, for each pair of adjacent bars in Figure 5.2, the first bar represents the score of the respective respondent in Section A of the questionnaire, whilst the second one stands for the score in Section B.

Figure 5.2 Bar charts for scores of high school students per school





Again, correspondence between scores of high school students on section A and on section B of the questionnaire was established through computation of the coefficient of correlation. For each school, Pearson's product moment formula was used to compute the coefficient of correlation between percentage scores of respondents on Section A with their percentage scores on Section B. The coefficients obtained are:

$$r_{21} = 0.972924$$
$$r_{22} = 0.973952$$
$$r_{23} = 0.820555$$
$$r_{24} = 0.952882$$
$$r_{25} = 0.975987$$

where the first 2 in the subscripts refer to high school and the second numeral refers to school number, i.e. 1, 2, and so forth. The notation used denotes that:

r_{21} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for high school students in School 1

r_{22} = correlation coefficient between percentage scores in Section A and p percentage scores in Section B for high school students in School

r_{23} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for high school students in School 3

r_{24} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for high school students in School 4

r_{25} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for high school students in School 5.

However, in calculating the correlation coefficient between percentage scores in Section A and those in Section B for all high school students, we find that:

$$r_h = 0.929118$$

where r_h = correlation coefficient between scores in Section A and scores in Section B for all high school students in the sample group.

Again, as was found for secondary school students, the results for high school students depict a strong correlation correspondence between percentage scores of respondents in Section A and their percentage scores in Section B. Similarly, the coefficients are nearer to +1 than to zero, indicating a very strong positive correlation between the scores of respondents in Section A and their scores in Section B.

Relating these scores to the respective research question explored in this section implies that some positive correspondence exists between didactical practices followed in high schools and didactical practices that enhance mathematical literacy.

5.3.2.3 Mathematics teachers' perspective on the extent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy

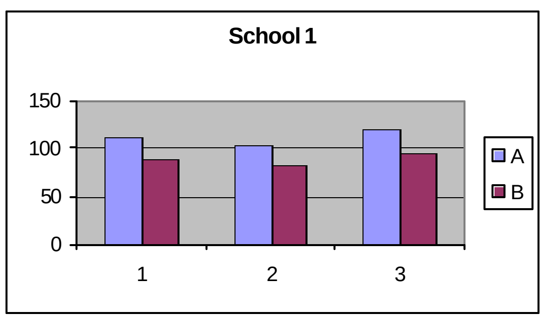
The research also includes the perspective of teachers in exploring the question of the extent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy in secondary and high schools in the district of Maseru, Lesotho. Response scores of teachers on items in Section A and Section B of the questionnaire in Part 2 of Appendices 5, are the major tool in investigating the question from the teacher's point of view. Table 5.10 displays total raw scores by individual teachers (per school) on Section A and B of questionnaire referred to in the previous sentence.

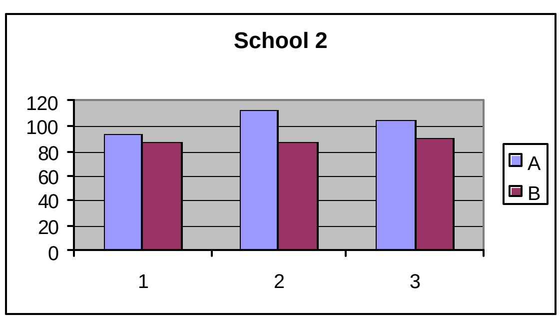
Table 5.10 Summary of total raw scores of individual teachers on Section A and Section B of questionnaire per school

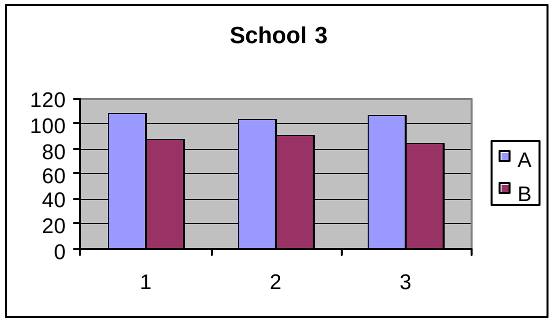
School	1		2		3		4		5	
Section	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
Teacher										
1	113	88	94	87	108	88	123	98	110	86
2	104	83	112	88	103	91	81	62	113	88
3	120	95	105	90	106	84	133	100	132	103

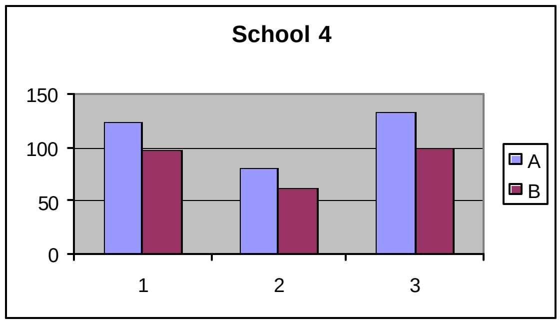
Figure 5.3 that follows depicts the summary of the scores of teachers per school in each of the sections in bar charts.

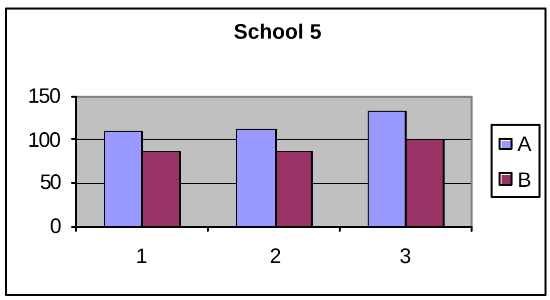
Figure 5.3 Bar chart for scores of teachers in each school











Pearson's product moment formula was also used in this case to compute the coefficient of correlation between percentage scores of teachers on section A and section B of the questionnaire. By calculation, the correlation coefficient for teachers in the respective sample schools were found to be:

$$r_{31} = 0.986021$$
$$r_{32} = 0.445633$$
$$r_{33} = -0.526742$$
$$r_{34} = 0.993736$$
$$r_{35} = 0.999837$$

where the first 3 in the subscripts refer to teacher and the second numeral refers to school number 1, 2, and so forth. Similarly, the notation also implied that:

r_{31} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for teachers in School 1

r_{32} = correlation coefficient between percentage scores in Section A and scores in Section B for teachers in School 2

r_{33} = correlation coefficient between percentage scores in Section A and scores in Section B for teachers in School 3

r_{34} = correlation coefficient between percentage scores in Section A and percentage scores in Section B for teachers in School 4

r_{35} = correlation coefficient between percentage scores in Section A and scores in Section B for teachers in School 5.

However, it should be pointed out here that, for the first time in the collected data, a negative correlation coefficient exists between the percentage scores in Section A and the percentage scores in Section B for teachers in School 3. Furthermore, the correlation coefficient between scores in Section A and scores in Section B for teachers in School 2 is the lowest of all the positive coefficients in the gathered data. The researcher followed up this discrepancy in the interviews where data was gathered that will be discussed again under validity and reliability of the data.

However, when all the data from teachers are put together and the product moment formula is used to compute the coefficient of correlation for teachers between teachers' scores in Section A and their scores in Section B, we obtain

$$r_t = 0.716854$$

Although the teachers' perspective on this issue is represented by a smaller fraction when compared to those coefficients for students' opinions, the result still shows that a positive correspondence exists between didactical practices followed in secondary and high schools and didactical practices that enhance mathematical literacy.

5.3.2.4 Administrators' perspective on the extent to which current didactical practices relating to mathematics correlate with indicators of teaching mathematics for mathematical literacy

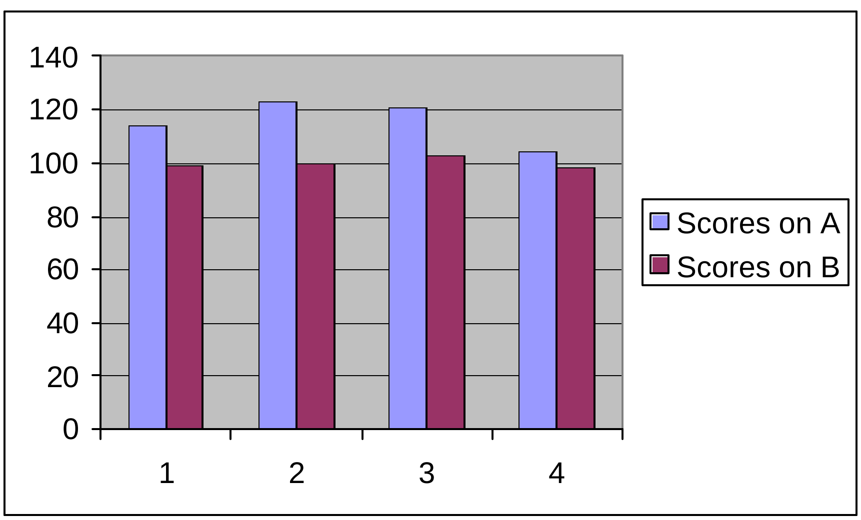
Table 5.11 below displays the total raw scores of administrators in Section A and Section B of the questionnaire in Part 2 of Appendices 6.

Table 5.11 Summary of total raw scores of administrators in Section A and Section B of questionnaire

Administrator	Scores in Section A	Scores in Section B
1	114	99
2	123	100
3	121	103
4	104	98

From Table 5.11, it is evident that high scores in Section A correspond to high scores in Section B. The bar charts in Figure 5.4 show this aspect diagrammatically. As pointed out at the beginning of this section and also in 4.4.2 of chapter 4, the bar charts assist in noting the difference in scores of the respondents on the two sections of the questionnaire.

Figure 5.4 Bar chart for scores of administrators



In computing the coefficient of correlation between percentage scores of administrators in Section A and Section B, we find that

r_a = 0.737334 (where r_a is the correlation coefficient between scores in Section A and scores in Section B for administrators).

Once again, the calculation yields a positive correlation coefficient which is nearer to +1 than to zero. The results, therefore, indicate that some positive correspondence exists between didactical practices followed in secondary and high schools and didactical practices that enhance mathematical literacy as viewed by administrators.

5.3.2.5 Summary of the extent to which current didactical practices relating to mathematics concurs with indicators of teaching mathematics for mathematical literacy

From results discussed in Sections 5.3.2.1, 5.3.2.2, 5.3.3.3, and 5.3.2.4, the researcher concludes that a large degree of correspondence exists between current didactical practices employed in secondary and high schools in Maseru and indicators of teaching mathematics for mathematical literacy. This is reflected by the values of the respective coefficients of correlation, which were found to range from 0.716854 to 0.947558. According to Cohen and Manion (1992:168-169), correlations ranging from 0.65 to over 0.85 indicate a "close relationships between the two variables correlated.

5.3.3 Objective 3: Assessment of whether the nature of the mathematics curriculum offered concurs with that suggested in literature on teaching mathematics for mathematical literacy

In this section of the chapter, the researcher explores one of the objectives of this research study which is "to examine and assess whether the nature (content, objectives, and recommended didactical practices) of the mathematics curriculum offered in Maseru's secondary and high schools concurs with that suggested in literature on teaching mathematics for mathematical literacy (see chapter 1.1.4).

To explore this objective, the researcher carried out a documentary analysis of school syllabuses, set textbooks, and some teachers' schemes of work. In line with the literature on documentary analysis, (Borg and Gall 1974:251-257, Dyer 1974:183-185, Shaw 1999 154-155, Travers 1965:312-329, Tuckman 1988:397-398), the researcher had to look for some main category of characteristics pursued in the research. Hence, the nature of didactical approaches reflected in the documents was the major category of interest in the documents.

Since the question pursued compares the content, objectives, and recommended didactical practices in Maseru's secondary and high schools with that suggested in literature on teaching mathematics for mathematical literacy, the researcher categorically compares (using tables) the two aspects by qualitative analysis. The categories in literature on teaching/learning for mathematical literacy in terms of which the practices in the district of Maseru are examined are taken from Chapter 3 where the following aspects are addressed:

- indicators of mathematical literacy (see Section 3.2.4 of Chapter 3),
- didactical practices that entrench mathematical literacy (see Section 3.4 of Chapter 3),
- the nature of didactical practices that entrench mathematical literacy (see Section 3.4.1 of Chapter 3), and
- recommended didactical practices that entrench mathematical literacy (see Section 3.4.2 of Chapter 3).

However, data from documents in Maseru are grouped as follows:

- data from mathematics syllabuses for secondary and high schools in Lesotho,
- data from mathematics textbooks used in secondary and high schools in Lesotho and

- data from teachers' schemes of work.

5.3.3.1 Data from mathematics syllabuses for secondary and high schools in Maseru

According to literature (Fennema and Romberg 1999:1-17, Hoyles, Morgan, and Woodhouse 1999:76-88), mathematics curricula and syllabuses throughout the world embody purposes, content, and the didactical practices that should be used to facilitate learners to acquire knowledge of that subject. Lesotho is no exception. Therefore, in this research, it would be a mistake not to assess the syllabuses for secondary and high school students in Maseru.

Examining mathematics syllabuses for both secondary and high schools in Maseru reveals that the main aims of teaching mathematics in schools are to:

- provide students with knowledge and skills by enhancing their abilities to think logically and analytically, and

- promote a positive attitude towards the subject as mathematics provides an investigative environment that stimulates curiosity to investigate and solve problems (Kingdom of Lesotho, Secondary and High School Mathematics Syllabuses 2004:1).

The content of mathematics in the syllabuses and textbooks for Lesotho reflects the usual school mathematics (for usual school mathematics, see Hoyles, Morgan, and Woodhouse 1999:90-100, Kaiser, Luna, and Huntley 1999:21-22, Tanner and Jones, 2000:143-150) stressing themes classified as:

- knowledge and skills,
- application and problem solving, and
- appreciation of the environment (Kingdom of Lesotho, Secondary and High School Mathematics Syllabuses 2004:1).

In order to facilitate learning and meet the aims for education in mathematics and, at the same time, succeed to impart the content areas to learners, the recommended didactical approaches relating to mathematics for secondary and high schools in Maseru include the following:

- teaching should be pupil-centred (i.e. more activities for pupils),
- the application of concepts should be emphasised (this means that, when developing a concept, one should start with examples in the immediate environment of pupils, i.e. from the known to the unknown),
- understanding of mathematical concepts is encouraged as opposed to memorisation of a collection of formulas,
- learner's experience and interest should be taken into account in the development of concepts, and
- hands-on activities are mostly encouraged to aid recall understanding and a better attitude towards the subject; pupils should be made aware of the need to study a given topic by the application of the topic in their everyday life (Kingdom of Lesotho, Secondary and High School Mathematics Syllabuses 2004:1).

The above recommended didactical practices, in a way, briefly state the summary of findings on current didactical practices in Maseru that are discussed in Section 5.3.1.5. However, the nature of mathematics content in the syllabuses is indicated in Table 5.13 in Section 5.3.3.4 of this chapter.

5.3.3.2 Data from mathematics textbooks used in secondary and high schools in Maseru

As Kaiser, Luna and Huntley (1999:63) point out, the analysis of textbooks has to examine the entire textbook in order to establish a "truly curricula sensitive" analysis. The main textbooks used in Lesotho's secondary and high schools are the Project in

Secondary Mathematics (PRISM) 2000 Plus Book 1, Book 2, Book 3, and Book 4 along with a corresponding Pupil Workbook (1, ..., 4) and a Teachers Guide Book 1-4 (with answers to all corresponding exercises in the respective textbooks). PRISM 2000 Plus Book 1 to Book 3 cover all the work for secondary school students and lay a foundation for high school work, which is completed in PRISM 2000 Plus Book 4. The PRISM 2000 Plus book series are a joint production of authors from the Ministries of Education of Lesotho and Swaziland under the auspices of co-ordinators from both ministries. Macmillan is the publishers of the series.

On examining the PRISM 2000 Plus, the researcher found evidence of the following factors:

- There is a similar pattern of chapter presentation for all pupil textbooks and a different, but similar, one for the respective Teachers' Guides,

- Topics are connected to each other as far as possible and cross reference to previous work in the book series is evident,

- The problem content and the application of concepts are based on real-world contexts and, as far as possible, they are set in Sotho contexts. Some problems are not solidly set in real-world contexts, but are mostly contrived exercises with single answers,

- In PRISM 2000 Plus books 1 and 2, readers are led, mostly deductively, to the meaning of concepts, formulae or procedures,

- In PRISM 2000 Plus books 3 and 4, readers are led, either inductively or deductively, to the meaning of concepts, formulae, or procedures,

- All textbooks have ample worked examples followed by plenty of exercises for learners,

- No reference to calculators or computers is made in any of the books. Rather, in Book 3, there are tables of squares, square roots of numbers, and tables for the sine, cosine, and tangent of angles from 0 to 90 degrees,

- Some chapters tend to move away from discovery, inductive/deductive exploration, to more algorithmic learning procedures, particularly in Books 3 and Book 4,

- There is evidence of some short investigation leading to discovering either a formula or meaning of a concept,

- Textbooks do not show evidence of open-ended problems, meaningful mathematical modelling, and long project assignments,

- There is no direct connection between mathematics and other disciplines. Rather, mathematics is presented as an abstract entity with reference to contrived real-life situational contexts,

- There is no mathematical modelling work for readers.

5.3.3.3 Data from teachers' schemes of work

Schemes of work of teachers are aligned to the set syllabuses of year groups with which they deal. From teachers' schemes of work, the researcher found the following evidence of work:

- Topics are connected,
- The nature of content is displayed in Table 5.13 in Section 5.3.3.4,
- Students are involved in lessons,
- Group work and individual drill exercises are performed,
- Varied methods are used,
- Frequent topic tests and occasional comprehensive tests on several covered topics are written.

5.3.3.4 Summary of the results on the concurrence of the nature of the mathematics curriculum offered with that suggested in literature on teaching mathematics for mathematical literacy

In this section, the concurrence of the mathematics curriculum offered in Maseru's secondary and high schools with suggested didactical practices that entrench mathematical literacy is summarised in Table 5.12. Didactical practices that entrench mathematical literacy are taken from Chapter 3, Section 3.4.2. Didactical practices evidenced in syllabuses, textbooks, and in schemes of work are taken from the above Sections 5.3.3.1, 5.3.3.2, and 5.3.3.3.

Table 5.12 Summary of results on the concurrence of the mathematics curriculum offered in Maseru's secondary and high schools with that suggested in literature on teaching mathematics for mathematical literacy

Didactical practices for mathematical literacy	**Evidence of didactical practices in syllabus**	**Evidence of didactical practices in textbooks**	**Evidence of didactical practices in teachers' schemes of work**
Active involvement of students	Active involvement of students	Active involvement of students exists since there are activities either in the learner's workbook on in the text of the pupil's book	Active involvement of students
Technology to aid in concept development	None	None	None
Problem solving and multi-step problems and answers	Mention of problem solving in real-life situations	Single-answer contrived problems	Mention of problem solving in contrived textbook problems
Mathematical reasoning	Mathematical reasoning	Mathematical reasoning with mention of recall of previous work in preceding books in series	Mathematical reasoning
Conceptual understanding	Conceptual understanding	Conceptual understanding	Conceptual understanding

Realistic problem encountered by adults	To the extent that is possible, realistic problem encountered by adults	To the extent that is possible, realistic problems that are encountered by adults are captured in contrived word problems	To the extent that is possible, realistic problem that are encountered by adults are captured in contrived textbook word exercises
An integrated curriculum with ideas developed in real-life contexts	An integrated curriculum and to the extent that it is possible, ideas are developed in real-life contexts	An integrated curriculum and to the extent that it is possible, ideas are developed in real-life contexts	An integrated curriculum and to the extent that it is possible, ideas are developed in real-life contexts
Multiple approaches to problem solving	Mention of multiple approaches to problem solving	Particular method for solving a problem	Reference to exercises that require a particular method for solving a problem
Diverse and frequent assessment both in class and outside class	Frequent assessment, then final exam	None	Frequent assessment in class and then final exam
Open-ended problems	None	None	None
Oral and written communication to explain solution	Limited oral, but ample written communication to explain solution	Limited oral, but ample written communication to explain solution	Limited oral, but ample written communication to explain solution
Variety of teaching strategies Extended projects and investigation	Variety of teaching strategies Mention of investigation	Variety of teaching strategies Short investigation leading to discovering either a formula or meaning of a concept	Variety of teaching strategies Reference to textbooks with short investigation leading to discovering either a formula or meaning of a concept
Opportunity for students to have fun in maths Mathematical modelling	Mention of students having to enjoy maths None	None None	Students enjoyment of maths is built–in in the didactical approach teachers use None

Table 5.12 reveals that some positive aspects exists in most didactical practices relating to mathematics that are employed in secondary and high schools in Lesotho. In some instances, the practised didactical methods do not concur with didactical practices that are purported to entrench mathematical literacy in students. From the table, the following are didactical practices that are not (or not fully) practised:

- the use of technology,
- the use of mathematical modelling,
- textbooks to include mathematical fun in order to motivate learners to enjoy mathematics,
- the use of extended projects and mathematical investigation as a didactical approach that enhances mathematical literacy in learners,
- communication to overtly include both oral and written work,
- the use of open-ended problem questions,
- the use of multiple approaches in problem solving, and
- less seeking of particular solutions to particular contrived problems.

Some shortfalls exist between requirements of the kind of mathematics that literature purports to enhance mathematical literacy and the kind of mathematics offered in secondary and high schools in Maseru as revealed in school mathematics syllabuses, teachers' schemes of work, and in set textbooks. The table reflects that there are some areas where the kind of secondary and high school mathematics does not match that suggested for mathematical literacy. The following are such areas:

- the use of technology,
- mathematical modelling,

- real-life data in statistics and probability, and

- mathematical problems related contrived real-life problems and not to actual, real-life situations (part of modelling).

5.3.4 Objective 4: Didactical practices relating to mathematics to be improved/ embraced/redefined in order to achieve mathematical literacy

As discussed in Chapter 1, Section 1.4, one of the four objectives of the study is to "assess which didactical practices relating to mathematics (if any) still need to be improved/embraced/redefined in order to achieve mathematical literacy in Maseru's secondary and in high school students.

In the present chapter, the discussion in Section 5.3.2 leading to results indicated in Section 5.3.2.5 pointed out that some didactical practices indeed exist that need to be either improved, embraced, or refined in order to effect mathematical literacy in secondary and high school students. With further discussion and exploration in Section 5.3.3 that resulted in the findings displayed in Table 5.12, and specifically indicated in Sections 5.3.3.1 to 5.3.3.4, the researcher deduces the identification of such didactical practices relating to mathematics to include:

- the use of extended projects and mathematical investigations as a didactical approach that enhances mathematical literacy in learners,

- mathematical modelling of real-life problems,

- use of real-life data in statistical problems,

- overt mathematical communication that includes both oral and written work,

- use of an open-ended, problem-solving approach to real-life problems with a little less seeking of particular solutions to particular contrived problems,

- use of a multiple approach to problem solving,
- less dependence on the textbook approach,
- matrix algebra to be set in real-life situations as a means of organising and analysing data, and
- connecting mathematical concepts and skills to other school subjects.

5.4 QUALITY OF DATA: RELIABILITY AND VALIDITY

Sections 5.2 and 5.3 display and describe data collected for this study. How truthful and trustworthy are the data and with how much confidence can one draw conclusions and inferences based on the data? These questions take the researcher to issues of the validity and reliability of the research data.

As was discussed in chapter 4, the actual justification of validity and reliability employs both qualitative and quantitative means. Validity and reliability of the data and findings are qualitatively verified by triangulation. Data collected through questionnaires are validated by comparing it to data from a documentary analysis by responses gathered during interviews. Quantitative data is examined for its reliability by obtaining the correlation coefficient between relevant scores on appropriate sections of the questionnaire.

5.4.1 Validity of the data

As indicated in Section 4.5.1 of Chapter 4, the validity of findings in this study is checked through triangulation between methods of data collection (questionnaires, interviews, and documentary analysis) and through triangulation within the same method (Cohen and Manion 1992:269-275, 278). By inspecting the responses of subjects to items in questionnaires and to probing questions in the interviews, it is observed that content correspondence exists between the data produced. For instance, responses to questionnaires, responses from interviews, and observations noted from documents all concur on at least the following didactical practices:

- promoting understanding and logical thinking skills,
- explanations by teachers and the use of a variety of teaching/learning methods with discovery methods highly esteemed,
- linking concepts and topics,
- application to real-life situations, where possible,
- drill exercises done individually or in groups,
- use of set textbooks,
- there are no open-ended problems as a teaching approach,
- no use of computer as a teaching/learning tool, and
- there is no extensive mathematical modelling, nor long mathematical investigations and examinable project work.

According to the discussion in Section 4.5.1 of Chapter 4 of this study, this shows, amongst other things, that "different measures of the same construct converge" to the same didactical trait and, thus, shows evidence of construct validity (Dyer 1979:132). At the same time, content validity is reflected since the data obtained true samples of didactical methods that are practised in Maseru as evidenced and confirmed from different responses to different measuring instruments. Furthermore, these didactical methods concur with those purported in literature as discussed in Chapter 2.

5.4.2 Reliability of the data

According to Chapter 4, Section 4.5.2, a research instrument is considered reliable when it has stability and internal consistency. According to Wiersma and Jurs (1985:67), this type of reliability is established by checking the consistency of measurement of the variable across two or more equivalent forms of the instrument. As indicated and justified in Section 4.5.2 of Chapter 4, the researcher chooses to use equivalent forms of split halves of the items in each section of the questionnaire. Each item in the first half corresponds to a particular item in the second half such that both items are, in essence measuring, similar didactical quality. The following tables display how items in Section A and in Section B of the questionnaire were split in two equivalent parts. Table 5.13 also displays the total scores on each item obtained by students, and teachers and administrators combined.

Table 5.13 Items in and scores on the split halves of Section A of the questionnaire

Items in first half	**Scores of all students**	**Scores of teachers and administrators**		**Item in second half**	**Scores of all students**	**Scores of teachers and administrators**
1	1173	85		10	1187	90
2	1200	95		23	987	61
3	1158	82		7	696	65
4	1211	80		16	998	80
5	574	90		22	300	31
6	1140	90		27	1028	82
8	565	41		29	1210	89
9	1109	82		30	1198	78
11	1107	90		28	989	80
12	965	72		21	717	82
13	262	26		20	986	45
14	997	60		24	642	25
15	986	85		19	906	76

17	402	20		18	879	38
25	1024	95		26	1024	95

According to Tuckman (1988:174), the reliability of the split instrument is judged using the Spearman-Brown coefficient, r (given in section 4.5.2). It is purported that the Spearman-Brown coefficient gives a measure of the extent to which the whole instrument is reliable (Ibid). Thus, the researcher used the Spearman-Brown formula to calculate the respective reliability coefficients on section A and that on section B.

The researcher obtained the respective reliability coefficients in Section A of the questionnaire to be:

Reliability coefficient on Section A for Students = 0.976760
Reliability coefficient on Section A for teachers and administrators = 0.892153

However, on separating the scores into scores of secondary school students, scores of high school students, scores of teachers, and scores of administrators on each item and computing the respective reliability coefficient for each category of respondents, the researcher obtained the following:

Reliability coefficient in Section A for secondary school students = 0.969789
Reliability coefficient in Section A for high school students = 0.979656
Reliability coefficient in Section A for teachers = 0.807535
Reliability coefficient in Section A for administrators = 0.953456.

The above reliability coefficient results show that the teachers' coefficient is lower than the rest. On checking the scores per item for teachers compared to the rest of the respondents, the researcher observed that teachers' scores are, on the whole, comparatively low. Taking this minor discrepancy up in interviews with teachers, the responses were:

- we are the people on the ground and know what is happening,

- students may feel that they are appraising their teachers, and
- administrators are the policy keepers and uphold the set educational expectations.

However, computations were carried out with respective item scores in Section B of the questionnaire. Items in Section B were split into two equivalent parts of items that measure similar didactical traits. This section of the questionnaire has an odd number of items. Item 17, however, does not measure a didactical characteristic that is similar to any in the group. Hence, it was left out in the pairing of items in the equivalent halves. Table 5.14 displays a summary of the items in each split half.

Table 5.14 Items in the split halves of Section B of the questionnaire

Items in first half	1	2	3	4	5	6	7	8	9	10	11	21
Items in second half	13	12	19	25	18	14	20	16	15	22	24	23

Computations with the scores of the respective respondents on each item gave the following corresponding results:

Reliability coefficient in Section B for students = 0.965237
Reliability coefficient in Section B for teachers and administrators = 0.929875
Reliability coefficient in Section B for secondary school students = 0.975678
Reliability coefficient in Section B for high school students = 0.956873
Reliability coefficient in Section B for teachers = 0.812356
Reliability coefficient in Section B for administrators = 0.949878.

On enquiring from teachers and administrators why scores on items in Section B tend to be higher for most items than for items in Section A, the justification centred on “the many workshops where current mathematical child centred didactical practices are emphasised”.

Considering the reliability coefficients obtained for items in the main quantitative part of the instrument of research for this study, the researcher is confident that the responses of subjects in the sample group are reliable. Validity was incorporated during the construction of the instruments (see Chapter 4 of this study) and it was further examined and justified in Section 5.4.1 of the current chapter. As literature purports, “valid measures must be reliable ones” (Dyer 1979:135).

5.5 CONCLUSION

In this chapter, the raw data collected, as it originally appears in the responses of the different subjects, has been displayed, described, and analysed. Firstly, data about the biographic details of respondents were discussed. The purpose of the discussion was to acquaint the reader with the respondents. Secondly, the subsequent discussion focused on establishing how far the data from other parts of the research instruments and the research questions themselves were cogent.

In the discussion, analysis and interpretation of data (which was obtained from students, teachers, and administrators) were explored in the light of the research questions. This exercise culminated in assisting the researcher to explore, establish or identify the following:

- the current didactical practices relating to mathematics in Lesotho’s secondary and high schools,
- the extent to which current didactical practices relating to mathematics in Maseru’s secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematical literacy;

- the degree to which the nature of the mathematics curriculum offered in Maseru's secondary and high schools concurs with that suggested in literature for mathematical literacy, and

- the didactical practices relating to mathematics in Lesotho that still need to be embraced, improved, or redefined.

These results and findings would be meaningless if the quality of the data were not valid and reliable. Hence, in Section 5.4 of the chapter, validity and reliability issues were examined and justified. Based on the results of the assessment of reliability and validity of the data, the researcher concluded that, as far as possible, results of the study are valid and reliable.

The next chapter discusses the findings, conclusions and recommendations of the study on each of the four research questions. Conclusions about the exploration of each research question are drawn in the same chapter. Limitations, recommendations, and forecasts for further research studies arising from this research are explored and indicated.

CHAPTER 6

FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

6.1 INTRODUCTION

The challenge of teaching mathematics for mathematical literacy in secondary and high schools in Maseru is the main focus of this study. In Chapter 1, this problem was raised, motivated, and broken up into sub-problems (see 1.1, 1.2, 1.3, and 1.4). Enlightened by the literature about mathematics education and mathematical literacy that was reviewed in chapters two and three, the researcher described the methodology of the study (see Chapter 4) and explored the problem through guiding research questions and focused objectives that were stated in Section 1.4 of Chapter 1. Research instruments as described in 4.4, 4.4.1, 4.4.2 of chapter 4 were constructed and administered to the respective research subjects. The data from the responses of the subjects (students, teachers and administrators in the sample group) to questionnaires (see Appendices 4, 5 and 6) and interview questions were used to explore the research questions.

In Chapter 5, the researcher displayed and described the collected raw data for this research study as it originally appeared in the responses of subjects to various items in the research instruments. As was indicated in Section 5.5 of that chapter, the data itself was examined, analysed, and interpreted in relation to the objectives of the study (see 5.3). According to the proposition in 4.5, 4.5.1, 4.5.2 of chapter 4, in Section 5.4, of chapter 5, the results from the exploration were tested for both validity and reliability in order to prove the worthiness and truthfulness of the findings before they can be generalised to the whole of the district of Maseru. The researcher found the results both valid and reliable (see 5.4.1 and 5.4.2 of chapter 5).

The current chapter looks at the findings, conclusions, and recommendations at which the study arrives based on the results of investigations into the research objectives. In particular, the findings, conclusions, and recommendations with respect to each research question are delineated. Suggestions and recommendations with regard to teaching mathematics for mathematical literacy in Maseru's secondary and high schools are outlined. The chapter also points out limitations of the study along with areas for further research that originate from this study.

6.2 OBJECTIVE 1: CURRENT DIDACTICAL PRACTICES RELATING TO MATHEMATICS IN MASERU'S SECONDARY AND HIGH SCHOOLS

The first objective of this study is to find out the current didactical practices relating to mathematics in Maseru's secondary and high schools (see chapter 1 section 1.4 and chapter 5 section 5.1). A discussion of common didactical practices in mathematics education as purported in literature supported by actual practices in various countries in the world at large was recorded in Section 2.6 and elucidated in Sections 2.6.1 to 2.6.2 of Chapter 2 of the study. Through a scientific enquiry, formulated in Chapter 1, motivated in Chapter 4, carried out with results analysed and documented in Chapter 5, Section 5.3.1 (see also 5.3.1.1, 5.3.1.2, 5.3.1.3 and 5.3.1.4) the researcher found that the current didactical practices relating to mathematics in both secondary and high schools in Lesotho are as presented in Section 6.2.1 that follows.

6.2.1 Findings on didactical practices relating to mathematics

The findings about the question of which didactical practices are currently employed in Maseru secondary and high school mathematics classes are based on the results which were found from analysing responses of students, teachers and administrators to questionnaire items (see Part 2 Section C of Appendices 4, 5 and 6). Actually, the findings also include perspectives from responses of the respective subjects to interview questions (see Tables 5.4.4, 5.4.5, 5.5.4, 5.5.5, 5.6.3, 5.6.4, 5.7.2, 5.7.3 and 5.7.6). From the results of the analysis of all responses of the research subjects, the researcher grouped the findings under three categories, namely: the didactical practices which respondents considered to be "frequently practised", those they deemed to be "rarely practised" and others they thought as "never practised". For convenience and easy accessibility of the information the researcher compiled Table 6.1 that follows to portray the didactical practices that the study found in each category.

Table 6.1 Didactical practices relating to mathematics the study found in Maseru's secondary and high schools

Frequently practiced	**Rarely practiced**	**Never practiced**
• Teacher directs explanations and questioning followed by students working on given exercises • Students do exercises either individually or in groups to drill learnt work • Students are led to discover mathematical ideas • Problems include contrived applications to everyday life • Mathematical concepts are linked to each other • Students are made to understand mathematical concepts, rules, procedures, and processes • Teacher creates opportunities for students to enjoy mathematics • Use of textbook • Drill and practice of skills, processes, and operations in exercises that teachers give	• Mathematical investigation • Linking concepts to other school subjects • Linking concepts to real-life situations, not to contrived ones • Overt mathematical communication that includes both oral and written work • Multiple approach to problem solving	• Extended mathematical projects and practical work as part of the teaching/ learning practices • Open-ended problem questions • Mathematical modelling giving rise to real-life problem solving and not contrived real-life problems • Use of computer-aided teaching/learning practices • Extensive mathematical investigations • Cross-curricular connections

6.2.2 Conclusion about didactical practices relating to mathematics in Lesotho

From Table 6.1, the didactical practices relating to mathematics that the study found to be frequently used in the district of Maseru have some instructional positive attributes according to the literature reviewed in chapter 2 sections 2.3, 2.3.1, 2.3.2 and 2.3.2. In particular, the didactical practices are based upon positive influence from psychological learning theories and philosophies of people like Piaget, Bruner, Dienes, Skemp, Gahne and Vygotsky (see 2.3.3.1, 2.3.3.2, 2.3.3.3, 2.3.3.4, 2.3.3.5 and 2.3.3.6). These practices include didactical aspects such as:

- the constructivism approach where construction of understanding by learners is the key note (see 2.3.3),
- the discovery method, exposition of concepts (see 2.6, 2.6.1);
- individual and group work (see 2.6.2),
- enjoyment of mathematics for intrinsic motivation (Grobler 1998:9, Lacombe 1985:1-24: see 2.6.1), and
- integration of mathematical concepts (Coxford 1995: 3, International Baccalaureate Middle Years' Programme Fundamental Concepts 2000:21, and NTCM Standards 1989:32: see 2.6.2).

There are also some didactical practices that are frequently used in Maseru's secondary and high schools that are discouraged in literature (see discussion in 2.4 and 3.3.1). These include:

- drill and practice of skills, processes, and operations (Davies, Maher, and Noddings 1990:57-63), and
- problems with contrived applications to real-life situations (Hoyles *et al*, 1999:8, 220).

At the same time, there are also other didactical practices in which Maseru needs to consider improving. These are portrayed in Table 6.1 as being rarely practiced or never practised; yet, literature strongly recommends them as imperative for the teaching mathematics for mathematical literacy (see 2.7, 3.3.2, 3.3.3 and 3.4). These didactical practices include:

- the use of technology in the didactical environment (Fey & Hirsch 1992:9-29, 91-98; Heid & Baylor 1993:198-200; Tanner & Jones 2000:179- 195),
- connecting mathematical concepts both across the school curriculum and to meaningful real-life situations through mathematical modelling,
- the use of open-ended problem-solving techniques,
- mathematical investigation,
- linking concepts to other school subjects,
- linking concepts to real life, not to contrived situations,
- overt mathematical communication that includes both oral and written work,
- multiple approach to problem solving,
- extended mathematical projects and practical work as part of the teaching/learning practices,
- open-ended problems questions and
- mathematical modelling, giving rise to real-life problem solving and not using contrived real-life problems.

6.2.3 Recommendations about didactical practices relating to mathematics in Maseru

If learners are to acquire mathematical literacy, didactical practices that are discouraged in the literature on teaching mathematics for mathematical literacy should not be used in Maseru's secondary and high schools. To this end, the acquisition of mathematical skills, processes, and operations through memorisation and drill should not be used. Instead, didactical practices relating to mathematics that promote a meaningful understanding of mathematics in learners should be practised.

The study strongly recommends the use of didactical practices that literature purports to entrench mathematical literacy in learners (see 3.4.2 of Chapter 3). If learners in Maseru are to be mathematically literate, these didactical practices will need to be employed in all mathematics classrooms in all secondary and high schools.

6.3 OBJECTIVE 2: FINDINGS, CONCLUSIONS AND RECOMMENDATIONS ABOUT THE EXTENT TO WHICH CURRENT DIDACTICAL PRACTICES RELATING TO MATHEMATICS CORRESPOND TO AND CORRELATE WITH INDICATORS OF TEACHING MATHEMATICS FOR MATHEMATICAL LITERACY

The second objective of this study is to establish the extent to which current didactical practices relating to mathematics correspond to and correlate with indicators of teaching mathematics for mathematical literacy. Related to this objective, the research question which the study sought to answer was: Do the present didactical practices and mathematics curriculum offer students mathematical education that is necessary for mathematical literacy?

6.3.1 Findings about the extent to which current didactical practices relating to mathematics correspond to and correlate with indicators of teaching mathematics for mathematical literacy

In Section 5.3.2 of Chapter 5, a statistical analysis was carried out to explore the extent to which didactical practices relating to mathematics followed in Maseru secondary and high schools corresponds to and correlates with indicators of teaching mathematics for mathematical literacy. This was confirmed by calculating the coefficients of correlation between percentage scores of the two (see 5.3.2 sections 5.3.2.1, 5.3.2.2, 5.3.2.3 and 5.3.2.4). In 5.3.2.5, the resulting coefficients for each group of respondents were found to be:

$$r_s = 0.947558$$

where r_s = correlation coefficient between scores in Section A and scores in Section B for all secondary school students in the sample group.

$$r_h = 0.929118$$

where r_h = correlation coefficient between scores in Section A and scores in Section B for all high school students in the sample group.

$$r_t = 0.716854$$

where r_t = correlation coefficient between scores in Section A and scores in Section B for all teachers in the sample group.

$$r_a = 0.737334$$

where r_a = the correlation coefficient between scores in Section A and scores in Section B for administrators.

6.3.2 Conclusion about the extent to which current didactical practices relating to mathematics correspond to and correlate with indicators of teaching mathematics for mathematical literacy

From the discussion in 5.3.2.5, it was pointed out that Cohen and Manion (1992: 168-169) purport that the coefficient of correlation which ranges between 0.65 to 0.85, is an indication of close correspondence between two variables that are being compared. In this light, the researcher concludes that the correlation coefficients found in this study (see 5.3.2.1, 5.3.2.2, 5.3.2.3, 5.3.2.4, 5.3.2.5 and 6.3.1) indicate that a strong correspondence exists between the didactical practices relating to teaching mathematics in Maseru and those that entrench mathematical literacy.

As much as the coefficients of correlation reflect strong correspondence between practised didactical methods in Maseru and those didactical indicators purported in literature to enhance mathematical literacy in learners, it needs to be pointed out that there seems to be a contradiction between what the study found in answer to this research question and what was found as solutions to similar questions and guiding objectives of this same study (see objective 1 in 6.2 and objective 3 in 6.4). To this end, results in 5.3.1, 5.3.3 and 5.3.4 also do not seem to agree with those in 5.3.2. This in turn results in effecting a contradiction between findings in 6.2.1 and those in 6.3.1.

6.3.3 Recommendations about the extent to which current didactical practices correspond to and correlate with indicators of teaching mathematics for mathematical literacy

The researcher recommends that research methods other than just a correlation study need to be conducted to the answers to this contradiction regarding the question under investigation.

6.4 OBJECTIVE 3: FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS ABOUT THE CONCURRENCE OF THE MATHEMATICS CURRICULUM WITH THAT SUGGESTED IN THE LITERATURE ON TEACHING MATHEMATICS FOR MATHEMATICAL LITERACY

The problem of whether the nature of the mathematics curriculum offered in Maseru's secondary and high schools concurs with that suggested in literature on mathematics education for mathematical literature was raised and formulated in Sections 1.3 and 1.4 of Chapter 1. Actually the focused question which the study sought to answer was: does the mathematics curriculum offered in Maseru secondary and high schools concur with that suggested in literature on mathematical literacy?

6.4.1 Findings about the concurrence of the mathematics curriculum with that suggested in the literature on teaching mathematics for mathematical literacy

The study examined the mathematics curriculum by documentary analysis as described in chapter 4 (see 4.4.1 and 4.4.2). The assessment was carried out and discussed in Section 5.3.3 of chapter 5 (see 5.3.3.1, 5.3.3.2, 5.3.3.3 and 5.3.3.4). The results of the documentary analysis was summarised in 5.3.3.4 and Tables 5.12 and 5.13. From evidence in the analysis, the researcher found a number of didactical practices that are undermining the concurrence of didactical practices relating to mathematics in Maseru and ideal didactical practices. These aspects are the same as those found in investigating objective one (see 6.2, 6.2.1 and 6.2.2).

6.4.2 Conclusions about the concurrence of the mathematics curriculum with that suggested in the literature on teaching mathematics for mathematical literacy

From the summary in 5.3.3.4 and Tables 5.12 and 5.13, the researcher observes that the mathematical aspects which are not yet practised Maseru secondary and high school mathematics have an important place in teaching mathematics for mathematical literacy. For instance, the position and influence of computers in the mathematical didactical environment is highly recommended (see 2.7 in chapter 2 and 3.3.2 in chapter 3). One of the reasons is pointed out by Tanner and Jones (2000:181-183) as they purport that computers are a necessary "resource for teaching and learning" mathematics since they are capable of developing higher order skills such as "monitoring the progress of problem solving activities" and are also used to " emancipate the mathematical mind" in the exploration of diverse functions.

Furthermore, the study found that the kind of mathematics offered in Maseru's secondary and high schools has to be adjusted in order to meet the requirements of the mathematics that entrenches mathematical literacy in learners (see 5.3.3.4). Actually, Table 5.13 in Chapter 5 portrays the kind of mathematics that literature purports to enhance mathematical literacy and the kind of mathematics in secondary and high schools in Maseru as revealed in school mathematics syllabuses, teachers' schemes of work, and in set textbooks. Inspecting the table reflects that there are some areas where the secondary and high school mathematics does not match with that suggested for mathematical literacy. From that examination, the researcher concludes that the areas that undermine concurrence are the same as those found in 6.2.2.

6.4.3 Recommendations about the concurrence of the mathematics curriculum with that suggested in the literature on teaching mathematics for mathematical literacy

In view of the discussion in 6.4.2, the study, therefore, strongly recommends that secondary and high school mathematics in Maseru should include mathematical modelling, project work that is related to real-life situations, and the use of technology as a didactical method as these elements assist in entrenching mathematical literacy in learners. Mathematical modelling, project work, mathematical investigations, computers/calculators, and open-ended problem questions should all be part of the multiple approaches in meaningful mathematical problem solving.

6.5 OBJECTIVE 4: FINDINGS, CONCLUSIONS AND RECOMMENDATIONS ABOUT DIDACTICAL PRACTICES RELATING TO MATHEMATICS IN MASERU, THAT STILL NEED TO BE IMPROVED/ EMBRACED/ REDEFINED IN ORDER TO EFFECT MATHEMATICAL LITERACY IN STUDENTS

With the exception of the statistical correlation findings, where the teaching of mathematics strongly corresponded with that suggested in literature for teaching for mathematical literacy, results of the study reflect that there are some didactical practices that are rarely employed or not employed at all but are very much influential in effecting mathematical literacy in learners.

6.5.1 Findings about didactical practices relating to mathematics in Lesotho, that still need to be improved/embraced/ redefined in order to effect mathematical literacy in students

From the results of the analysis in 5.3.4, there are some didactical practices necessary in developing mathematical literacy that showed up as needing address in terms of not ever being practised, only rarely practised, or not given attention. Evidence in the syllabus, textbooks, or teachers' schemes of work reflected this need. Such didactical practices also limited the concurrence between didactical practices currently practised in Maseru's schools and those recommended in literature on teaching mathematics for mathematical literacy (see 5.3.1.5).

6.5.2 Conclusions about didactical practices relating to mathematics in Lesotho, that still need to be improved/embraced/redefined in order to effect mathematical literacy in students

Chapter 3 of this study explored the literature on teaching mathematics for mathematical literacy. In the light of the recommended didactical practices which entrench mathematical literacy in learners (see 3.2, 3.2.1, 3.2.2, 3.2.3, 3.2.4, 3.4, 3.4.1 and 3.4.2), along with evidence from the results and findings of the study (see Tables 5.12 and 5.13, findings in 6.2.2) the researcher concludes the didactical practices that need to be improved/embraced/redefined are the following:

- the use of extended projects and extensive mathematical investigation as a didactical practice that enhances mathematical literacy in learners,

- the use of multiple approaches in pr oblem solving,

- textbooks to include mathematical fun/investigation that motivate learners to genuinely enjoy mathematics,

- connecting mathematical concepts and skills to other school subjects,

- the use of multiple approaches to problem solving,

- overt mathematical communication that includes both oral and written work;

- less dependence on the textbook approach,

- mathematical communication to overtly include both oral and written work; and

- less seeking of particular solutions to particular contrived problems.

Table 5.13, in particular, draws attention to the fact that the actual school syllabus should encompass the following kind of mathematics in order to enhance mathematical literacy:

- real-life data in statistics and probability; and

- mathematics proble ms to encompass actual, real-life problems rather than deal only with contrived problems related to real-life situations.

6.5.3 Recommendations about didactical practices that are to be embraced / redefined/improved

The literature review in chapters 2 (see 2.5, 2.6.2 and 2.7) and in chapter 3 (see 3.2.1, 3.3.2, 3.4, 3.4.1 and 3.4.2) indicated that the whole purpose of mathematical literacy in learners is to meaningfully participate in a technological society. The use of technology in a mathematical didactical environment is crucial. Hence, both calculators and computers need to be part of the mathematical didactical tools since they make "a dramatic impact on the way people are taught and the way they learn" (Dinkheller, Gaffney, and Vockell 1989:vii). During interviews (see Tables 5.4.4, 5.4.5, 5.5.4, 5.5.5, 5.6.3, 5.6.4, and 5.7.6) some of the respondents indicated that a lack of finances limits the use of computers in schools. This study, therefore, strongly recommends that funds should to be secured and set apart for supplying schools with computers, which will need to be made available to every student in secondary and high schools.

6.6 OVERARCHING SUGGESTIONS AND RECOMMENDATIONS

It is clear from the conclusions of the study that there are some didactical practices that are rarely employed or not employed at all. The study found that these didactical practices undermine the quest for entrenching mathematical literacy in learners. The use of technology, such as calculators and computers, is one of the practices that were rated either as "rarely used" (calculators) or "not used at all" (computers), yet, didactical practices are greatly enriched by employing such technology in the teaching/learning scenario. Educational benefits that influence mathematical literacy in learners who use this technology are motivated and discussed in chapter 3 of this study (see 3.3.2). In chapter 2 section 2.7, literature strongly recommends the use of technology in the mathematical didactical situation. It is, therefore, imperative to make computers and calculators available for use to every learner in secondary and high schools in Maseru.

However, Hoyles, Morgan, and Woodhouse (1999:25) argue that it can be counter productive in situations where technology is used by "unskilled or inadequately trained people". Hence, in cases where the teachers themselves are not fully computer literate, in-service training in computer literacy must be arranged to ensure that teachers are able to handle the didactical situation where computers are used.

According to the summary in Section 6.2.4 of the present chapter, only three content aspects need to be modified. These are:

- authentic project work and mathematical modelling,
- real-life data in statistics and probability; and
- mathematics problems to encompass actual, real-life problems rather than deal only with contrived problems related to real-life situations.

Projects, authentic mathematical modelling of real life situations, open-ended problem solving and investigations can naturally form part of examinable course-work with school-based assessment as is in the Cambridge International General Certificate of Secondary Education (see chapter 1 section 1.2.2). However, teachers will have to undergo training in project supervision if learning mathematics is to involve projects that are meaningful and examinable. The Ministry of Education will have to release funds for in-service training of teachers of mathematics.

Further, mathematical investigations, open-ended problems and project assignments can be incorporated into the textbooks which students use (Morris 1994: i-iii). Along with this, teachers could be requested to construct worksheets which have assignments that incorporate work with this type of mathematical content.

There are also some didactical practices in the list of those that "need to be redefined and modified" (see Section 6.2.4 of the current chapter) that are less expensive to practise than employing technology and incorporating examinable course-work in mathematics. These practices only need to be modified by teachers to their advantage. For example, in indicating links between mathematics and other school subjects, the teachers need to:

- use mathematical connectors such as functions, matrices, graphs, variables, and transformations (House and Coxford 1995:9-12); and

- use unifying themes across the school curriculum (House and Coxford 1995:4-9).

Another didactical practice that teachers can improve on is communication in the mathematics classroom. Silver and Smith (1996:20-28) posit that mathematical discourse communities can be built in mathematics classrooms through motivating students to participate with the teacher supporting the discourse by focussing it on worthwhile mathematical ideas. Chai, Lane & Jakabcsin (1996:137-145) also recommend the use of open-ended mathematical tasks as a didactical tool that enhances teacher-student interaction and communication in the classroom. This study, therefore, considers communication in mathematics classrooms to be imperative as it promotes mathematical literacy in students (see 3.2.2 of chapter 3).

6.7 LIMITATIONS OF THE STUDY

As mentioned and justified in chapter 4, there is no single research method that is without flaw and, even with the best of planning, research does not emerge without limitations. In that regard, the researcher identifies the use of cluster sampling (see Section 4.2, 4.2.1 and 4.2.3 of Chapter 4) and the fact that the results thereof had to be generalised for the whole of the district of Maseru as a minor limitation of the study. In spite of similar traits in didactical practices relating to mathematics in Maseru's secondary and high schools, it may be that respondents in other parts in the district of Maseru have slightly different perspectives in the data.

Another limitation is the correlation method that the researcher used to establish the correspondence between didactical practices relating to mathematics in Maseru's secondary and high school and indicators of teaching mathematics for mathematical literacy. The findings of the study using statistical coefficient of correlation were that there is a very strong correspondence; yet, the study also indicated that there are a number of didactical practices which are still to be embraced/redefined/improved This discrepancy motivates the researcher to think that this question could have been addressed by a comparative qualitative analysis of the indicators of mathematical literacy that are found in the didactical practices relating to mathematics in Lesotho's secondary and high schools.

6.8 RECOMMENDATIONS FOR FURTHER RESEARCH

On the basis of discussions in and findings of this study, a number of questions arise that indicate the need for further research. The biographic details of respondents were not a specific interest of the study. Yet, the question of interest in, or performance by, male or female students in mathematical literacy could be a topic of research.

Research procedures which can be used to explore the same questions of research may be an interesting base for considering further research study. It is evident from the findings, that only the high coefficients of correlation between the didactical practices which are employed in Maseru secondary and high schools and those suggested in literature for teaching mathematics for mathematical literacy, indicate a strong correspondence between the two (see findings and conclusions in 6.2.1, 6.2.2, 6.3.1, 6.3.2, 6.4.1, 6.4.2, 6.5.1 and 6.5.2). In fact, theoretically, the correlation approach to research is purported to be a "precise way of stating the extent to which one variable is related to another" (Borg and Gall 1974:318). According to Borg and Gall (1974:318, 322), "the level of planning" of the theoretical constructs that is to be measured and "the selection of a group of subjects" in the sample, are some of the factors one has to carefully consider in a relationship study. Further, Borg and Gall point out that "it is very important to select a group of subjects drawn from a narrowly defined population" in order to avoid obscuring the relationship by the presence of subjects who vastly differ from each other (Ibid). In view of this, further areas of study could look at the same relationship study either from a qualitative approach or still from a quantitative correlation study approach with a more careful level of planning which involves a narrowed homogeneous population.

An examination of the impact of the pressure of examinations on didactical practices relating to mathematics is pertinent to actual issues raised in this study, but these could not be fully addressed. This aspect was raised by both students and teachers during in-depth interviews (Table 5.5.3.1 and Table 5.6.2.1 in Chapter 5).

Furthermore, a number of research studies through action research in the classroom are possible in the area of didactical practices needing to be dropped, modified, or re-defined. Research studies could encompass questions such as: the effect of dropping/embracing some didactical practices relating to mathematics in raising correspondence between didactical practices relating to mathematics in Maseru's secondary and high schools and didactical practices that enhance mathematical literacy (see Section 6.2.4 of the current chapter).

A comparative didactical study is also possible. A research study could focus on comparing the effectiveness of common didactical practices relating to mathematics and didactical practices purported to entrench mathematical literacy in preparing students for examinations.

These are some of the possible research areas that are revealed in this study and may form the ground for further exploration.

6.9 SUMMARY OF THE STUDY AND CONCLUDING REMARKS

In this study, the researcher has addressed the question of teaching mathematics for mathematical literacy in Maseru's secondary and high schools. Teaching mathematics was defined as a didactical perspective in Section 1.6.1 of Chapter 1. To appropriately place the concept of didactical practice in school mathematics and mathematics education, a range of related areas in mathematics education was explored by using relevant literature (see Chapter 2). The concept of mathematics education and its neighbours were discussed in Section 2.2.1. In Sections 2.2.2 and 2.2.3, the researcher used support from literature to examine who are involved in mathematics education and what is the actual position of mathematics within the umbrella body of education.

Theories of learning in mathematics education were discussed, including some psychological theories and philosophies (that influence trends in didactical practices relating to mathematics) of people like: Piaget, Bruner, Dienes, Skemp, Gagne, and Vygotsky (see 2.2.3, 2.3.1-2.3.6 of Chapter 2). In Sections 2.5, 2.6, and 2.6.2, the purpose of mathematics education and common didactical practices reported from other countries in the world formed a premise in terms of which the didactical practices relating to mathematics in Maseru.

Mathematical literacy itself was defined in Chapter 1 and it was later motivated and elucidated in Chapter 3 where literature on mathematical literacy was reviewed to support the arguments that ensued. The researcher examined, by means of literature, the meaning of mathematical literacy and found it to consist of the following, amongst others:

- appreciating the utility and elegance of mathematics,
- understanding what is being learnt in mathematics,
- connecting mathematics to the real world and to other school subjects,
- using mathematics in a variety of situations and contexts,
- synthesising, analysing, and even evaluating mathematical thinking; and
- communicating by employing the rich wealth of the language of mathematics (see Sections 3.2, 3.2.1-3.2.4 of Chapter 3).

Thus, the study describes mathematical literacy as an individual's aggregate of mathematical skills and knowledge that enables the individual to engage meaningfully and make well-founded mathematical judgements in a technologically imbued society (see Section 3.3, 3.3.1- 3.3.6).

Didactical practices and the nature of mathematics that are required to entrench mathematical literacy in learners are discussed in Sections 3.4, 3.4.1, 3.4.2, and 3.5 of Chapter 3. The purpose of Chapter 3 in the study is to spell out mathematical literacy, its indicators, and its purported didactical practices so that the whole question of teaching mathematics for mathematical literacy in Maseru's secondary and high schools is meaningfully explored.

In the study, the problem of teaching mathematics for mathematical literacy in Lesotho's secondary and high schools is investigated under four research objectives, namely:

- to determine the didactical practices relating to mathematics that are currently being applied in Lesotho's secondary and high schools,

- to establish the extent to which current didactical practices relating to mathematics followed in Lesotho's secondary and high schools correspond to and correlate with indicators of teaching mathematics for mathematical literacy as reflected in literature,

- to examine and assess whether (content, objectives, and recommended didactical practices) the mathematics curriculum offered in Lesotho's secondary and high schools concurs with that suggested in literature on teaching mathematics for mathematical literacy, and

- to assess what didactical practices relating to mathematics in Lesotho (if any) still need to be improved/embraced/refined in order to affect mathematical literacy in students (see 1.4 of Chapter 1 and 5.1 of Chapter 5).

The objectives of the study are addressed in Chapter 5 using research instruments that were constructed and justified as explained in Chapter 4 of the study. Data relevant to addressing the objectives of the study were collected, displayed, and analysed in Chapter 5. Interpretation of the data and ensuing results of the study are discussed in the respective sections of Chapter 5. Findings, conclusions and recommendations about each question of the study are summarised in Sections 6.2, 6.3, 6.4 and 6.5 of the current chapter.

However, there are some issues emanating from the findings of the study that need further attention. These issues are highlighted in Section 6.6 of this chapter. The researcher draws attention to these aspects that need to be considered for improvement if didactical practices relating to mathematics in Maseru's secondary and high school are to enhance mathematical literacy.

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APPENDICES

APPENDIX 1: OPEN-ENDED QUESTIONNAIRE FOR STUDENTS

1. This questionnaire is divided into **two Sections.**
2. Section A requires you to give general information about yourself.
3. Section B is about the teaching and learning you get during mathematics lessons.
4. Section B has **four** open-ended questions that will take you less than 10 minutes to answer.
5. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for purposes of the study.
6. You are requested to be honest and truthful so that the research findings will be meaningful.

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THE STUDY.

Yours sincerely

Fungai Shava

SECTION A

PERSONAL GENERAL INFORMATION FOR STUDENTS

Please circle the correct number relevant to your personal information.

1. Gender

FEMALE	1
MALE	2

2. Age in years

11-12	1
13-14	2
15-17	3
18-20	4
Over 20	5

3. Which form are you studying?

Form 1 or 2	1
Form 3	2
Form 4 or 5	3

4. How long have you been at this school?

1 year	2 years	3 years	4 years	5 years	6 years

SECTION B

In the spaces provided, please give your response to each of the following statements:

1. Describe how you learn mathematics in class.

....

....

....

....

2. Describe the usual activities that you do as you learn mathematics in class.

....

....

....

....

3. Describe the usual activities that your teacher undertakes during the teaching-learning process that takes place in your class.

....

....

....

....

4. Please write down any further aspects that relate to the teaching and learning of mathematics in your class.

....

....

APPENDIX 2: OPEN-ENDED QUESTIONNAIRE FOR TEACHERS

1. This questionnaire is in **two Sections.**
2. Section A requires you to give general information about yourself.
3. Section B is about the teaching/learning processes in your mathematics lessons.
4. Section B has **four** open-ended questions that will take you less than 10 minutes to answer.
5. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for purposes of the study.
6. You are requested to be honest and truthful so that the research findings will be meaningful.

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THE STUDY.

Yours Sincerely,

Fungai Shava.

SECTION A

PERSONAL GENERAL INFORMATION FOR TEACHERS

Please circle the correct number relevant to your personal information.

1. Gender

FEMALE	1
MALE	2

2. Age in years

Less than 20	**1**
20-25	2
26-30	3
31-35	4
40-45	5
Over 45	6

3. Which forms are you teaching?

Form 1	1
Form 2	2
Form 3	3
Form 4	4
Form 5	5

4. What are your present qualifications?

Secondary Teacher's Diploma	1
B. Sc. Ed	2
Honours	3
Masters Degree	4
Other, specify	5

5. Your teaching experience in years:

1-2	1
3-5	2
6-10 11-15	3 4
16-20	5
Over 20 years	6

SECTION B

In the spaces provided, please give your response to each of the statements below.

1. Indicate the different instructional methods that you usually employ in the teaching-learning process that goes on in your mathematics lessons.

....

....

....

2. Describe the usual activities that your students engage in as they learn mathematics in your classes.

....

....

....

....

3. Describe the usual activities that you undertake during the teaching-learning process of mathematics that takes place in your mathematics lessons.

....

....

....

....

4. Please write down any further aspects that relate to the teaching –learning of mathematics in your class.

....

....

APPENDIX 3: OPEN-ENDED QUESTIONNAIRE FOR ADMINISTRATORS

1. This questionnaire is in **two Sections.**
2. Section A requires you to give general information about yourself.
3. Section B is about the teaching/learning processes that you recommend for mathematics lessons in your schools.
4. Section B has **four** open-ended questions that will take you less than 10 minutes to answer.
5. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for purposes of the study.
6. You are requested to be honest and truthful so that the research findings will be meaningful.

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THE STUDY.

Yours sincerely,

Fungai Shava.

SECTION A

PERSONAL GENERAL INFORMATION FOR ADMINISTRATORS

Please circle the correct number relevant to your personal information.

1. Gender

FEMALE	1
MALE	2

2. Age in Years

Less than 20	1
20-25	2
26-30	3
31-35	4
40-45	5
Over 45	6

3. What are your present qualifications?

Secondary Teacher's Diploma	1
B. Sc. Ed Honours	2 3
Masters Degree	4
Other, specify	5

4. Your teaching experience in years:

1-2	1
3-5	2
6-10	3
11-15	4
16-20	5
Over 20 years	6

5. How long have you worked as an administrator?

1-5 years	1
6-10 years	2
Over 10 years	3

SECTION B

In the spaces provided, please give your response to each of the statements below.

1. Indicate the different instructional methods that you usually observe (and/or would recommend) in the teaching-learning process that goes on in the mathematics lessons when you visit schools.

   ....

   ....

2. Describe the usual activities that you see students engage (and/or would recommend) in as they learn mathematics in the classes that you visit.

   ....

   ....

3. Describe the usual activities that you observe the teacher undertaking during the teaching-learning process of mathematics that takes place in the mathematics lessons that you visit.

   ....

   ....

   ....

   ....

4. Please write down any further aspects related to the teaching – learning of mathematics in the classes that you visit (and/or would recommend).

   ....

   ....

APPENDIX 4: QUESTIONNAIRE FOR STUDENTS

1. This questionnaire is divided into **TWO PARTS.** It will take you about 20 minutes to complete it.
2. **Part 1** requires you to give general information about yourself.
3. **Part 2** has **Three Sections, A, B** and **C** that deal with the teaching/learning of mathematics that you experience.
4. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for the purposes of the study.
5. When the study is complete, research findings will be provided on request to participating schools.
6. You are requested to be honest and truthful so that the research findings will be meaningful.
7. To help in meaningful statistical analysis of your answers, **in instruction number 8**, please use the coding "**NAME**" you will be given.
8. **NAME:STUDENT..........**

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THIS STUDY

PART 1

PERSONAL GENERAL INFORMATION FOR STUDENTS

Please circle the correct number relevant to your personal information.

1. Gender

FEMALE	1
MALE	2

2. Age in years

11-12	1
13-14	2
15-17	3
18-20	4
Over 20	5

3. Which form are you studying?

Form 1 or 2	1
Form 3	2
Form 4 or 5	3

4. How long have you been at this school?

1 year	2 years	3 years	4 years	5 years	6 years

PART 2

SECTION A

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. **Circle,** either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
   - 1 stands for "strongly disagree", **SD**
   - 2 stands for "disagree" **D**
   - 3 stands for "unsure" **U**
   - 4 stands for "agree" **A**
   - 5 stands for "strongly agree" **SA**

Statement	**SD 1**	**D 2**	**U 3**	**A 4**	**SA 5**
1. In class, we often learn mathematics in groups.	1	2	3	4	5
2. We are given time to really understand new topics.	1	2	3	4	5
3. We are encouraged to discover ways of doing mathematics on our own.	1	2	3	4	5
4. Sometimes we work as a whole class.	1	2	3	4	5
5. Our teacher gives us mathe matics problems that do not have specific answers.	1	2	3	4	5
6. New mathematics topics are taught from different directions.	1	2	3	4	5
7. We do all exercises on our own, with little help from the teachers.	1	2	3	4	5
8. Teacher does not explain/demonstrate procedures that are directly linked to the exercises that students are asked to work on.	1	2	3	4	5
9. The teacher uses problems that are directly linked to everyday life.	1	2	3	4	5
10. We do a lot of discussion of mathematics in class that	1	2	3	4	5

helps us to communicate our mathematical ideas.					
11. I enjoy mathematics.	1	2	3	4	5
12. Our teacher makes us see links between mathematics topics that we learn.	1	2	3	4	5
13. Computers are used in the teaching /learning of mathematics at our school.	1	2	3	4	5
14. I am able to use mathematics in other school subjects.	1	2	3	4	5
15. The teacher lets us discover mathematical principles on our own.	1	2	3	4	5
16. I see real life connections with the mathematics that I learn.	1	2	3	4	5
17. We do long pieces of mathematics (practical work) where we gather information to solve a problem in real life situations.	1	2	3	4	5
18. We do a lot of investigation in mathematics.	1	2	3	4	5
19. I am allowed to build my own understanding of mathematical concepts.	1	2	3	4	5
20. We use calculators in learning mathematics at our school.	1	2	3	4	5
21. I am able to see relationships among different topics in mathematics.	1	2	3	4	5
22. Some of the mathematics problems given to us have many possible answers.	1	2	3	4	5
23. Our teacher allows us to freely discuss and explore mathematical concepts until we have full understanding.	1	2	3	4	5
24. Mathematics is linked to other school subjects.	1	2	3	4	5
25. The teacher gives us work in mathematics that is interesting and challenging.	1	2	3	4	5
26. We have mathematics puzzles and games in class that make learning fun.	1	2	3	4	5
27. We do not follow only the set textbook in our learning of mathematics.	1	2	3	4	5
28. The teacher creates opportunities to let us enjoy	1	2	3	4	5

mathematics.				
29. Little of our learning activities in mathematics are drill and practice of concepts and procedures that the teacher has explained to us.	1	2	3 4	5
30. I am able to use the mathematics that I learn in my everyday activities.	1	2	3 4	5

SECTION B

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. Circle either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
   - 1 stands for "strongly disagree", **SD**
   - 2 stands for "disagree" **D**
   - 3 stands for "unsure" **U**
   - 4 stands for "agree" **A**
   - 5 stands for "strongly agree" **SA**

Statement	**SD 1**	**D 2**	**U 3**	**A 4**	**SA 5**
1. Our teacher makes sure that every student in the class is actively taking part in the lesson.	1	2	3	4	5
2. We are encouraged to use different ways to solve the same problem.	1	2	3	4	5
3. We use computers to help us develop our understanding of mathematics.	1	2	3	4	5
4. The teacher requires us to show clear and logical mathematical reasoning in solving given problems.	1	2	3	4	5
5. We are taught to see connections between mathematics topics that we learn.	1	2	3	4	5
6. Sometimes we are given one long piece of work in mathematics that requires us to use different	1	2	3	4	5

mathematics topics, procedures and skills in order to complete the work.					
7. I am sure of my mathematics and I am able to use it in real life situations. 8. Our teacher requires that we understand the mathematics that we learn.	1 1	2 2	3 3	4 4	5 5
9. We are given practical work in mathematics that we are required to complete over a long period of time.	1	2	3	4	5
10. Sometimes the teacher asks us to verbally explain our solutions to problems during class discussion to see if we have meaningfully understood the work.	1	2	3	4	5
11. Some of the problems that we deal with have no particular answer.	1	2	3	4	5
12. Our teacher uses different ways of teaching us.	1	2	3	4	5
13. We all actively take part in maths lessons.	1	2	3	4	5
14. Some problems that we deal are long, with have many steps to get to the solution, and we need to clearly show these steps.	1	2	3	4	5
15. We are given long pieces of work that require us to use trial and error to search for patterns and ways to complete the task.	1	2	3	4	5
16. We sometimes work on particular areas of mathematics with the aim of finding out a general mathematical expression that we did not know before. 17. In class, we sometimes work in groups.	1 1	2 2	3 3	4 4	5 5
18. We connect the mathematics that we learn to other school subjects.	1	2	3	4	5
19. We use calculators in learning mathematics.	1	2	3	4	5
20. Real world situations are found in most of the problems that we deal with.	1	2	3	4	5

21. Our teacher leads us to see and like the usefulness, power and beauty of mathematics.	1	2	3	4	5
22. The teacher stresses that we be sure of our mathematics and that we be able to express ourselves both in written form and verbally.	1	2	3	4	5
23. We are taught to see mathematics in real world situations and to be able to form mathematical expressions that represent those situations.	1	2	3	4	5
24. Some problems have many answers and we have to give reasons why those answers are correct.	1	2	3	4	5
25. We are taught in a way that makes us enjoy mathematics.	1	2	3	4	5

SECTION C

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. There are **15** statements about the teaching/learning of mathematics. Circle either 1, 2 or 3 to indicate how often you actually experience that teaching/learning method:
   - 1 stands for "frequently practised" **F**
   - 2 stands for "rarely practised" **R**
   - 3 stands for "never practised" **N**

Statement	**F** **1**	**R** **2**	**N** **3**
1. We have teacher directed explanations and questions followed by students working on given exercises.	1	2	3
2. Students do their exercises either individually or in groups	1	2	3
3. We have computer aided teaching/learning practices.	1	2	3
4. We are led to discover mathematical ideas on our own.	1	2	3
5. We are given mathematically based practical work that we are	1	2	3

required to work on and complete over a long period of time.			
6. We follow the set textbook only.	1	2	3
7. We are given mathematical areas to investigate.	1	2	3
8. We have problems that include application of mathematics to everyday situations.	1	2	3
9. We relate concepts and topics in mathematics to each other.	1	2	3
10. We work with problems that do not have one single correct answer.	1	2	3
11. We have time to form mathematical expressions from the real world around us.	1	2	3
12. We connect what we learn in mathematics to other school subjects.	1	2	3
13. We do a lot of practice of the exercises that the teacher gives us.	1	2	3
14. We are made to understand mathematical concepts, rules, procedures and processes.	1	2	3
15. The teacher creates chances for us to enjoy learning mathematics.	1	2	3

APPENDIX 5: QUESTIONNAIRE FOR TEACHERS

1. This questionnaire is in **TWO PARTS.** It will take you about 20 minutes to complete the questionnaire.
2. **Part 1** requires you to give general information about yourself.
3. **Part 2** has **three Sections, A, B** and **C,** and is about the teaching/learning practices in your mathematics lessons
4. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for the purposes of the study.
5. When the study is complete, research findings will be provided on request to participating schools.
6. You are requested to be honest and truthful so that the research findings will be meaningful.
7. To help in meaningful statistical analysis of your answers, **in instruction number 8**, please use the coding "**NAME**" you will be given.
8. **NAME:TEACHER..........**

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THE STUDY.

Yours Sincerely,

Fungai Shava.

PART 1

PERSONAL GENERAL INFORMATION FOR TEACHERS

Please circle the correct number relevant to your personal information.

1 Gender

FEMALE	1
MALE	2

2. Age in Years

Less than 20	**1**
20-25	2
26-30	3
31-35	4
40-45	5
Over 45	6

1. Which Forms are you teaching?

Form 1	1
Form 2	2
Form 3	3
Form 4	4
Form 5	5

2. What are your present qualifications?

Secondary Teacher's Diploma	1
B. Sc. Ed	2
Honours	3
Masters Degree	4
Other, specify	5

3. Your teaching experience in years:

1-2	1
3-5	2
6-10 11-15	3 4
16-20	5
Over 20 years	6

PART 2

SECTION A

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. Circle either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
    - 1 stands for "strongly disagree", **SD**
    - 2 stands for "disagree" **D**
    - 3 stands for "unsure" **U**
    - 4 stands for "agree" **A**
    - 5 stands for "strongly agree" **SA**

Statement	**SD 1**	**D 2**	**U 3**	**A 4**	**SA 5**
1. In class, I often let students learn mathematics in groups.	1	2	3	4	5
2. I give students time to really understand new topics.	1	2	3	4	5
3. I encourage my students to discover ways of doing mathematics on their own.	1	2	3	4	5
4. I allow whole class learning in my lessons.	1	2	3	4	5
5. I give students mathematics problems that do not have specific answers.	1	2	3	4	5

6. I teach new mathematics topics from different directions.	1	2	3	4	5
7. Most exercises are worked out by students on their own, with little help from me.	1	2	3	4	5
8. I do not explain/demonstrate procedures that are directly linked to the exercises that students are asked to work on.	1	2	3	4	5
9. I use problems that are directly linked to everyday life.	1	2	3	4	5
10. Students do a lot of discussion of mathematics in class that helps them to communicate their mathematical ideas.	1	2	3	4	5
11. I create situations for my students to enjoy mathematics.	1	2	3	4	5
12. I make students see links between mathematics topics that they learn.	1	2	3	4	5
13. Computers are used in the teaching /learning of mathematics at our school.	1	2	3	4	5
14. I facilitate my students to be able to use mathematics in other school subjects.	1	2	3	4	5
15. I facilitate students to discover mathematical principles on their own.	1	2	3	4	5
16. I provide students with situations that connect the mathematics that they learn with real life.	1	2	3	4	5
17. I engage students in mathematical project work as part of the teaching/learning of mathematics.	1	2	3	4	5
18. My students do a lot of investigation in mathematics.	1	2	3	4	5
19. I allow my students to construct their own understanding of mathematical concepts.	1	2	3	4	5
20. Calculators are used in the teaching/learning of mathematics at our school.	1	2	3	4	5
21. I make students see relationships among different topics in mathematics.	1	2	3	4	5
22. Some of the mathematics problems that I give to	1	2	3	4	5

students have many possible answers.					
23. I allow students to freely discuss and explore mathematical concepts until they have full understanding.	1	2	3	4	5
24. I facilitate students to connect mathematics to other school subjects.	1	2	3	4	5
25. I give my students work in mathematics that is interesting and challenging.	1	2	3	4	5
26. We have mathematics puzzles and games in class that make learning fun.	1	2	3	4	5
27. We do not follow only the set textbook in the teaching/learning of mathematics in my class.	1	2	3	4	5
28. I create opportunities to let students enjoy mathematics.	1	2	3	4	5
29. None of the learning activities for my students are drill and practice of skills, concepts and procedures that are directly linked to what I have just explained to students.	1	2	3	4	5
30. I make my students to be able to use the mathematics that they learn in their everyday activities.	1	2	3	4	5

SECTION B

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. Circle either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
   - 1 stands for "strongly disagree", **SD**
   - 2 stands for "disagree" **D**
   - 3 stands for "unsure" **U**
   - 4 stands for "agree" **A**
   - 5 stands for "strongly agree" **SA**

Statement	SD 1	D 2	U 3	A 4	SA 5
1. I make sure that every student in the class is actively taking part in the lesson.	1	2	3	4	5
2. I facilitate students to use different ways to solve the same problem.	1	2	3	4	5
3. We use computers to help students to develop their understanding of mathematics.	1	2	3	4	5
4. I require my students to show clear and logical mathematical reasoning in solving given problems.	1	2	3	4	5
5. I facilitate students to see connections between mathematics topics that they learn.	1	2	3	4	5
6. I assign to students mathematical investigation work that is done over a period of time.	1	2	3	4	5
7. I facilitate students to be sure of their mathematics so as to be able to use it in real life situations.	1	2	3	4	5
8. My teaching/learning activities aim to make students understand the mathematics that they learn.	1	2	3	4	5
9. I give practical project work in mathematics that is to be completed over a long period of time.	1	2	3	4	5
10. Sometimes I ask students to verbally explain their solutions to problems during class discussion to see if they have meaningfully understood the work.	1	2	3	4	5
11. Some of the problems that we deal with have no particular answer.	1	2	3	4	5
12. I use different teaching/learning strategies.	1	2	3	4	5
13. My students all actively take part in maths lessons that we have.	1	2	3	4	5
14. Some problems that we deal with have many	1	2	3	4	5

steps to get to the solution and students need to clearly show these steps.					
15. I give long investigation work that requires students to use trial and error to search for patterns and ways to complete the task.	1	2	3	4	5
16. Students sometimes work on particular areas of mathematics with the aim of discovering a general mathematical expression that they did not know before.	1	2	3	4	5
17. In class, students sometimes work in groups.	1	2	3	4	5
18. We connect the mathematics that students learn to other school subjects.	1	2	3	4	5
19. We use calculators in the teaching/learning of mathematics in all my classes.	1	2	3	4	5
20. Real world situations are found in most of the problems that we deal with.	1	2	3	4	5
21. I facilitate students to appreciate the usefulness, power and beauty of mathematics.	1	2	3	4	5
22. My teaching/learning strategies seek to empower students with knowledge in mathematics that they should be able to express both in written form and verbally.	1	2	3	4	5
23. My teaching/learning techniques enable students to see mathematics in real world situations and to be able to form mathematical expressions that represent those situations (modelling).	1	2	3	4	5
24. Some problems are open-ended and students have to justify why the many possible answers are correct.	1	2	3	4	5
25. My students enjoy mathematics.	1	2	3	4	5

<u>SECTION C</u>

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. There are **15** statements about the teaching/learning of mathematics. Circle either 1, 2 or 3 to indicate how often you actually practise that teaching/learning item:
   - 1 stands for "frequently practised" **F**
   - 2 stands for "rarely practised" **R**
   - 3 stands for "never practised" **N**

Statement	**F** **1**	**R** **2**	**N** **3**
1. I class, I have teacher directed explanations and questions followed by students working on given exercises	1	2	3
2. Students do their exercises either individually or in groups	1	2	3
3. We have computer aided teaching/learning practices.	1	2	3
4. I use the discovery method. 5. I give mathematical projects and practical work that students are required to work on and complete over a long period of time.	1 1	2 2	3 3
6. We follow the set textbook only.	1	2	3
7. I give my students mathematical areas to investigate.	1	2	3
8. We have problems that include application of mathematics to everyday situations.	1	2	3
9. We relate concepts and topics in mathematics to each other.	1	2	3
10. We work with problems that do not have one single correct answer.	1	2	3
11. We have time to form mathematical expressions from the real world around us (mathematical modelling).	1	2	3
12. I link mathematics topics that I teach to other to other school subjects.	1	2	3
13. I let students do a lot of the exercises in order to make them practise the taught skills.	1	2	3
14. I make students understand mathematical concepts, rules, procedures and processes.	1	2	3
15. I create opportunities for students to enjoy learning mathematics.	1	2	3

APPENDIX 6: QUESTIONNAIRE FOR ADMINISTRATORS

1. This questionnaire is divided into **TWO PARTS.** It will take you about 20 minutes to complete it.
2. **Part 1** requires you to give general information about yourself.
3. **Part 2** has **three Sections, A, B** and **C,** and is about the teaching/learning processes in the schools that you administer lessons.
4. Your responses will be handled with **complete confidentiality.** Only the researcher will have access to the completed questionnaire for the purposes of the study.
5. When the study is complete, research finding will be provided on request.
6. You are requested to be honest and truthful so that the research findings will be meaningful.
7. To help in meaningful statistical analysis of your answers, **in instruction number 8**, please use the coding '**NAME**" you will be given.
8. **NAME:ADMINISTRATOR.................................**

THANK YOU FOR KINDLY ACCEPTING TO PARTICIPATE IN THIS STUDY

Yours sincerely,
Fungai Shava

PART 1

PERSONAL GENERAL INFORMATION FOR ADMINISTRATORS

Please circle the correct number relevant to your personal information.

1. Sex

FEMALE	1
MALE	2

2. Age in years

Less than 20	**1**
20-25	2
26-30	3
31-35	4
40-45	5
Over 45	6

3. What are your present qualifications?

Secondary Teacher's Diploma	1
B. Sc. Ed Honours	2 3
Masters Degree	4
Other, specify	5

4. Your teaching experience in years:

1-2	1
3-5	2
6-10	3
11-15	4
16-20	5
Over 20 years	6

5. How long have you worked as an administrator?

1-5 years	1
6-10 years Over 10 years	2 3

PART 2

SECTION A

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. Circle either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
   - 1 stands for "strongly disagree", **SD**
   - 2 stands for "disagree" **D**
   - 3 stands for "unsure" **U**
   - 4 stands for "agree" **A**
   - 5 stands for "strongly agree" **SA**

Statement	**SD 1**	**D 2**	**U 3**	**A 4**	**SA 5**
1. When we visit classes, we see that students often learn mathematics in groups.	1	2	3	4	5
2. We observe that students are given time to really understand new topics.	1	2	3	4	5
3. We observe that students are made to discover ways of doing mathematics on their own.	1	2	3	4	5
4. Sometimes we find that whole class learning is facilitated.	1	2	3	4	5
5. We observe that teachers give students open-ended mathematics problems. 6. We observe that new mathematics topics are taught from	1 1	2 2	3 3	4 4	5 5

different directions.					
7. We observe that all exercises are done by students on their own, with little help from the teacher.	1	2	3	4	5
8. Teachers explain/demonstrate procedures that are directly linked to the exercises which students are asked to work on.	1	2	3	4	5
9. Teachers use problems that are directly linked to everyday life.	1	2	3	4	5
10. We see that a lot of discussion of mathematics in class is done that helps students to communicate their mathematical ideas.	1	2	3	4	5
11. We observe that students enjoy mathematics.	1	2	3	4	5
12. Our teachers make students see links between mathematics topics that they learn.	1	2	3	4	5
13. Computers are used in the teaching /learning of mathematics in our schools.	1	2	3	4	5
14. We see that students link their knowledge of mathematics to other school subjects.	1	2	3	4	5
15. Students are made to discover mathematical principles on their own.	1	2	3	4	5
16. Real life connections with the mathematics are incorporated in the teaching/learning process.	1	2	3	4	5
17. We see that mathematical project work is part of the teaching /learning process of mathematics.	1	2	3	4	5
18. Students are made to do a lot of mathematical investigation as a teaching/learning activity in mathematics.	1	2	3	4	5
19. Students are allowed to work out and build up their own understanding of mathematical concepts.	1	2	3	4	5
20. Calculators are used in the teaching/learning activities of mathematics in our schools.	1	2	3	4	5
21. Students are made to be able to see relationships among different topics in mathematics.	1	2	3	4	5
22. Some of the mathematics problems given to students have many possible answers.	1	2	3	4	5

23. Teachers allow students to freely discuss and explore mathematical concepts until they have full understanding.	1	2	3	4	5
24. Mathematics is linked to other school subjects.	1	2	3	4	5
25. Teachers give students work in mathematics that is interesting and challenging.	1	2	3	4	5
26. Mathematics puzzles and games are given to students in order to make learning fun, challenging and interesting.	1	2	3	4	5
27. Not only the set textbooks are used in the teaching/learning of mathematics.	1	2	3	4	5
28. Teachers create opportunities that make students enjoy mathematics.	1	2	3	4	5
29. None of the learning activities in mathematics are drill and practice of concepts and procedures that the teacher has explained to students.	1	2	3	4	5
30. Teachers make students to be able to use the mathematics that they learn in everyday activities.	1	2	3	4	5

SECTION B

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. Circle either **1, 2, 3, 4, or 5** to indicate how strongly you disagree or agree with the given statement.
   - 1 stands for "strongly disagree", **SD**
   - 2 stands for "disagree" **D**
   - 3 stands for "unsure" **U**
   - 4 stands for "agree" **A**
   - 5 stands for "strongly agree" **SA**

Statement	SD 1	D 2	U 3	A 4	SA 5
1. Our teachers make sure that every student in the class is actively taking part in the lesson.	1	2	3	4	5
2. We observe that teachers use multiple approaches to problem solving.	1	2	3	4	5
3. Computers are used in our schools to help students to develop conceptual understanding of mathematics.	1	2	3	4	5
4. We observe that teachers instruct students to show clear and logical mathematical reasoning in solving given problems.	1	2	3	4	5
5. We see that an integrated maths curriculum is taught such that students should be able to see connections between mathematics topics that they learn.	1	2	3	4	5
6. We observe that teachers assign to students multi-step problems that afford students an opportunity to use different mathematics topics, procedures and skills in order to complete that task.	1	2	3	4	5
7. We observe that teachers make students to be both confident and competent in using mathematics in real life situations. 8. We find that the mathematics teaching/learning practices in our schools build up conceptual understanding of the mathematics in students.	1 1	2 2	3 3	4 4	5 5
9. We see that teachers make students do practical project work in mathematics that is to be complete over a long period of time.	1	2	3	4	5
10. Sometimes in class discussion, we see that teachers request students to communicate orally	1	2	3	4	5

or in written form the explanation of solutions to problems.					
11. Some of the problems that are dealt with are open-ended.	1	2	3	4	5
12. Our teachers use a variety of teaching/learning strategies.	1	2	3	4	5
13. Interactive approaches to teaching /learning of mathematics takes place in maths lessons in our schools. .	1	2	3	4	5
14. Some problems that students deal with have many steps to get to the solution and students need to clearly show these steps.	1	2	3	4	5
15. We find that students are given long investigational work that requires the students to use trial and error to search for patterns and ways to complete the task.	1	2	3	4	5
16. We observe that students sometimes investigate particular areas of mathematics with the aim of finding out a general mathematical expression that they did not know before.	1	2	3	4	5
17. In class, students sometimes work in groups.	1	2	3	4	5
18. We see that teachers make students to make links between the mathematics that they learn and other school subjects.	1	2	3	4	5
19. Calculators are used in learning mathematics in our schools.	1	2	3	4	5
20. We find that teachers give students maths problems based on real world situations.	1	2	3	4	5
21. Our teachers lead students to appreciate the usefulness, power and beauty of mathematics.	1	2	3	4	5
22. We observe that our teachers stress that students should be confident and competent in mathematics such that the students are able to	1	2	3	4	5

express themselves both in written form and verbally.					
23. We find that teachers instruct students to see mathematics in real world situations and to make them to be able to form mathematical expressions that represent those situations.	1	2	3	4	5
24. We observe that teachers give some open-ended maths problems where students have to justify why the many answers are correct.	1	2	3	4	5
25 We find that students are taught in a way that makes them enjoy mathematics.	1	2	3	4	5

SECTION C

Instructions:

1. Please answer **all** questions as truthfully as you can.
2. There are **15** statements about the teaching/learning of mathematics. Circle either 1, 2 or 3 to indicate how often you actually observe that teaching/learning item practised in the classes that you visit:
   - 1 stands for "frequently practised" **F**
   - 2 stands for "rarely practised" **R**
   - 3 stands for "never practised" **N**

Statement	**F** **1**	**R** **2**	**N** **3**
1. In the maths classes that we observe, we find that teacher explanations and instructive questions, are followed by students working on given exercises.	1	2	3
2. We observe that students do their work either individually of in groups	1	2	3
3. We have computer aided teaching/learning practices.	1	2	3
4. Students are led to discover mathematical ideas on their own.	1	2	3
5. Mathematical projects and practical work are part of the teaching/learning practices in our maths classes.	1	2	3
6. Not only the set textbooks are used.	1	2	3

7. Mathematical investigational work is given to students.	1	2	3
8. Teachers give maths problems that include application of mathematics to everyday situations.	1	2	3
9. Linking and connecting concepts and topics in mathematics to each other are carried out.	1	2	3
10. Open-ended problems are given to students.	1	2	3
11. Mathematical modelling is practised and examined.	1	2	3
12. Mathematics is connected and linked to other school subjects.	1	2	3
13. Students are given drill and practice of skills, processes and operations that teachers have just explained/demonstrated.	1	2	3
14. Students are made to understand mathematical concepts, rules, procedures and processes.	1	2	3
15. Teachers create chances for students to enjoy learning mathematics.	1	2	3