MEAN VARIANCE OPTIMISATION, STOCHASTIC SIMULATION MODELLING AND PASSIVE FORMULA STRATEGIES FOR EQUITY INVESTMENTS

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I declare that the thesis which is hereby submitted for the qualification Philosophiae Doctor in Business Management at the University of the Free State, is my own independent work and has not been handed in before for a qualification at another university.

Mark Gary Pawley

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GLOSSARY

Active Investments: Investments that seek to outperform the market, through the use of techniques such as technical and fundamental analysis.

Asset Allocation: The process of dividing investment resources amongst competing asset classes.

Asset Class: A specific, identifiable portion of the investment spectrum that tends to respond, in a similar manner, to economic influences, and has common investment characteristics.

Autocorrelation: See serial correlation below.

Beta (β): A variable that is used by the capital asset pricing model. Determined by conducting a linear regression analysis. The slope of the linear regression line is beta. The market has a beta of one. A portfolio with risk levels higher than the market should manifest a beta higher than one, and vice versa.

CAPM: The capital asset pricing model is a theoretical asset pricing model that attempts to explain expected returns for a security or portfolio relative to the market, by using beta as a measure of risk, to determine the exposure to non-diversifiable risk.

Collective Investment: Synonymous with mutual fund and unit trust fund. Were known in South Africa as mutual funds until 1980, thereafter unit trust funds until 2002.

Correlation Coefficient (R): A statistic, ranging from –1 to +1, to measure the strength of a linear relationship between two variables. A positive result implies a positive relationship, and a negative result implies an inverse relationship. A relationship does not imply a causal link.

Covariance: A measure that determines the degree of co-movement between a portfolio's assets, and is derived by multiplying the standard deviations of the assets by the correlation coefficients of the assets.

Determination Coefficient (R^2): A statistic, ranging from zero to one, indicating the percentage contribution one variable has on the variation of a second variable. Commonly used to interpret correlation coefficients.

Diversification: The process of investing in a broad range of asset classes, markets, currencies and over time, in order to minimise risk.

Dollar Cost Averaging: Synonymous with Rand Cost Averaging below.

Efficient Frontier: An upward-sloping curve reflecting the trade-off between return and risk, comprising optimal portfolios. The portfolios along the efficient frontier reflect the allocation between competing assets.

Equity: Synonymous with share and stock, and used interchangeably.

Exchange Traded Fund (ETF): Synonymous with Mutual Fund and Unit Trust Fund, but which is traded like a stock and not subject to fund legislative requirements.

Formula Strategy: Any predetermined plan that will mechanically guide your investing. See Rand Cost Averaging below.

Geometric Rate of Return: The annualised rate of return pertaining to more than one period, otherwise known as the compound rate of return.

Index: A statistical measurement of the collective investment performance of an asset class, culminating in a market index.

Index Tracker Fund: A fund that attempts to provide the investment performance of an underlying market index, by holding all (or a sample) of the individual stocks that constitute the index.

Internal Rate of Return: The discount rate which, when applied to future cash flows, will make them equal to the initial outlay.

Linear Regression: The mathematical determination of a line of best fit that comes closest to the data points, by minimising the sum of the squared deviations of the pairs of observations from the line.

Market Capitalisation: The market price for a listed company, calculated by multiplying the stock in issue by the current market price per share.

Mean: A measure of average.

Mean Reversion: Future returns tend to be closer to their mean. Any significant movement away from the mean can be expected to be closer to the mean in future time periods.

Mean-Variance Model: A three dimensional investment portfolio construction process which accommodates volatility, returns and the interrelationship amongst assets within a portfolio. The model derives the critical line, otherwise known as the efficient frontier.

Mean-Variance Optimisation: The process of applying the mean-variance model through the use of a mean-variance optimiser.

Mean-Variance Optimiser: An algorithm that performs the mean-variance model calculations, thereby deriving the efficient frontier.

Modern Portfolio Theory: Refers to the mean-variance model, the capital asset pricing model and the Sharpe ratio.

Monte Carlo Simulations: An algorithm, acting as a stochastic simulator, that simulates the random functioning of a dataset based on mean returns, and standard deviations.

Mutual Fund: A fund that invests in a broad range of individual stocks, and other asset classes. The fund is, in turn, offered to investors by dividing the assets into units, which have a market price. Synonymous with unit trust funds, which are known in the U.S. as mutual funds.

Passive Formula Strategy: A strategy that allows for passive, automatic movement of money into, and out of, the stock market.

Passive Investments: Investments that seek to replicate the performance of an index or an appropriate benchmark with the minimum of activity.

Probability: A statistical measure that measures the likelihood of an event occurring.

Rand Cost Averaging: A method of investing the same amount each time period regardless of the asset price.

Rebalancing: The process of maintaining an asset allocation percentage of a portfolio over time, or the adjustment thereof.

Regression Analysis: See linear regression above.

Resampling: The process of redetermining data inputs.

Risk: The likelihood of receiving a return on an investment that is different from the return expected.

Sampling Error: The variation between a sample and a population.

Serial Correlation: The degree to which the return of a given series is related from period to period.

Share: A unit of ownership of a public company, which is held by a shareholder.

Sharpe Ratio: A statistical performance measure which calculates a risk adjusted return.

Standard Deviation: A statistical measure that measures volatility of a dataset compared to its average, and serves as a measure of total risk.

Stochastic Simulations: A simulation procedure based on randomness. See Monte Carlo simulations above.

Stock: Synonymous with share and equity, and used interchangeably.

Unit Trust Fund: See Collective Investments above.

Value Averaging: A Passive Formula Strategy which pursues a predetermined portfolio value.

CHAPTER 1

RESEARCH INTRODUCTION

1.1 INTRODUCTION

This study is a report culminating from the quantitative study of equity portfolio optimisation, where research was conducted pertaining to the selection of future portfolio asset allocations and asset classes, and the ongoing portfolio management, using a long term investment horizon. The imperative was to seek investment wealth maximisation, by applying a sustainable strategy that would yield ongoing optimal results, and that would be applicable to all forms of equity investments, both retirement and otherwise.

The first chapter of the thesis presents the background of the study, describes the problem requiring research, outlines the significance of the research, and presents an overview of the methodology used.

1.2 RESEARCH BACKGROUND

Economics is widely accepted as being the study of rational market participants seeking optimal choices (Campbell and Viceira, 2002, p. viii). Insofar as equity markets and investors are concerned, optimal portfolio choices are dependant on future risks and returns, and how these may change over time. In addition to seeking the optimal portfolio there is the inter-relationship between assets, both foreign and domestic, and how these may change over time, necessitating the management of an investment portfolio for optimum results.

Given the opening statement it is prudent to suggest that it is an investor's imperative to maximise investment returns generally, however, within this framework one of the most important investment decisions, facing anybody, is the maximisation of retirement savings. In this regard investors may hold retirement assets in the form of a defined benefit or defined contribution pension scheme (Campbell and Viceira, 2002, p. 1), or investors may hold some other general form of savings vehicle. In recent years employers in the U.S. have increasingly offered defined contribution plans, whilst decreasing the availability of defined benefit plans (Muller, 2003, p. 76).

Defined contribution plans offer the employee more choice, with investment risks and decisions resting with the employee. Therefore, the asset allocation the investor makes can have a substantial impact on the resultant retirement income.

Bernstein (2000c, p. 4) indicates that the majority of people, whether investing for retirement or otherwise, do not have a clear idea of the differences between asset classes, and have no grasp of their standard deviations or returns. Insofar as portfolio management is concerned, Bernstein indicates that there is no convincing evidence that investors have the knowledge or discipline to perform optimally. In this regard consider some evidence cited by McClatchey and Vandenhul (2003, p. 2):

- 26 percent of recent survey respondents invest their entire retirement assets in cash.

- 60 percent of recent survey respondents never rebalance their retirement accounts.

Within a South African context, given that equities have produced the largest inflation beating performance over the long term[1] one could deduce that the many market participants must have experienced significant gains. In this regard, with reference to Table 1.1, of the potential return of 14.93 percent that could have been realised, in an inflationary environment of 9.50 percent, it is observed that the average realised return was a mere 8.33 percent.

Table 1.1 Active investor performance (1988 – 2002)

	Active Investors	Market Portfolio
Geometric Return (net of costs)	8.33%	14.93%
Inflation	9.50%	

Source: Derived using data accessed at the Association of Collective Investments, www.aut.co.za (Archive Reference: Thesis Data I/Active vs Passive.xls).

[1] See Annexures 64, 69 and 70. Alternative asset classes do ephemerally outperform, however outperformance seems not to be sustainable, and not on an after tax basis.

What is immediately apparent is that, in an environment that produces returns in excess of inflation, investors seemingly do not realise these. This divergence of outcomes between the potential returns and the returns realised by investors leads one to ponder whether there is an approach to investment management that would significantly reduce the performance gap between current realised returns and potential returns, where potential represents the maximum return (net of costs) that is realistic and achievable.

No matter the form of the investment vehicle, the array of investable assets and the combination thereof, both foreign and domestic are seemingly endless[2], limited only by the number of investable assets, and therefore it is imperative that the issue of asset allocation and selection, and portfolio management be significantly addressed by the investor, and this is the focus of this research.

1.3 PROBLEM STATEMENT

A problem is defined as an ‘experience we have when an unsatisfactory situation is encountered’ (Locke, Spirduso and Silverman, 2000, p. 45). Once the problem is clearly defined, with all the related questions that may arise, it is the unsatisfactory situation that becomes the target of a study.

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[2] The number of combinations is a function of the number of investable assets to the power of itself.

Investors, both retirement and otherwise, experience a significant negative divergence in investment outcomes, relative to the potentially achievable result. This negative divergence is a result of the lack of a strategic approach to, and an understanding of asset allocations, and the lack of a sustainable approach to the management of a portfolio, as indicated quantitatively by McClatchey and Vandenhul (2003, p. 2). This propensity for sub-optimal investment outcomes is contrary to the rational market participants theory (Campbell and Viceira, 2002, p. viii), and is the problem that needs to be overcome. Furthermore, seeking a solution to the problem of sub-optimal performance is not just a matter of enhancing investment returns. With sub-optimal performance levels currently below the levels of inflation, as set out in Table 1.1, there is no incentive for investors to forego consumption where levels of real wealth are declining. The macro implications of this are a populace that becomes increasingly dependant on state welfare assistance, which in turn places taxation pressures on the working populace. Investment outcome optimisation is therefore a micro and macro imperative.

Investment outcome optimisation can be achieved through the effective application of a strategy that includes the integration of the mean-variance model through the use of a mean-variance optimiser, using resampled data inputs, the mean reversion of markets, passive investment management, appropriate asset class selection and the ongoing management of a portfolio, using both calendar and contingent rebalancing techniques, and passive formula strategies.

1.4 PURPOSE OF THE STUDY

Earlier attention was drawn to the issue of asset allocation and selection, and poor investment management resulting in inferior returns relative to what may be realised.

‘To ignore all efforts to rationally determine allocations is defaulting 100 percent to chance’ (Evensky, 1997, p. 237).

Therefore the primary purpose of the study will be to address the issue of asset allocation by seeking an understanding of whether mean-variance optimisation, through the use of stochastic simulation modelling, can be used to build optimal, forward-looking investment portfolios using passive investment instruments. The challenge is accordingly to develop a reliable (where reliability depends on the stability of outcomes) asset allocation model that accommodates past performance, and which is stable enough to produce optimised forward-looking investment portfolios, which are able to address the issue of optimal asset allocation and selection, within a global context, and which produce optimal investment outcomes, either in the form of higher returns, or reduced risk, taking cognisance of the fact that the future is unknowable and dynamic.

The secondary purpose of the study will be to explore whether the optimal, forward-looking portfolio can be managed parsimoniously using rebalancing techniques, and passive formula strategies in the form of value averaging to achieve relatively enhanced investment outcomes on a sustainable basis.

Passive investment instruments form the investment product universe, thereby removing the need for any form of active investor involvement or techniques; therefore the research focuses on asset allocation and ongoing strategic portfolio management.

Finally, the study seeks to condense the findings in a manner that can be understood and implemented by a broad section of the investor populace, and which will incorporate both national and international markets.

The objectives are:

a) to establish whether a mean-variance optimiser is an effective investment tool that can produce a forward-looking portfolio that is stable enough to produce maximised rates of return for a given level of risk, and is optimally diversified utilising the most appropriate data inputs;

b) to establish an approach to mean-variance optimisation that eliminates outcome instability through utilising resampled data inputs, over an extended period, to minimise the impact of data volatility;

c) to establish the resampled forward-looking optimal portfolio, using passive investment products exclusively;

d) to establish an approach to mean-variance optimisation that is responsive to changing relationships between the variables, namely geometric rates of return, cross-correlations and standard deviations, thereby keeping optimised portfolios optimal;

e) to establish a parsimonious approach to investing, that is effective, sustainable, and holistic and which provides broad investor benefits;

f) to establish optimal asset classes that can make up an effective investment portfolio; and

g) to establish an optimal approach to inter-market diversification that accommodates multiple intra-market assets.

1.5 RATIONALE FOR THE STUDY

There are both practical and theoretical reasons for conducting the study. From a practical perspective, knowing the benefits of the effective use of a mean-variance optimiser, the appropriate asset allocations and asset classes, the effective ongoing management of a portfolio and how investment outcomes may be enhanced will provide a valuable tool for investors that can lead to more informed, and intuitive investment decisions and ultimately improve wealth creation efforts for a broader South African investor base.

On a theoretical level, this study will address many of the primary issues that pertain to the use of a mean-variance optimiser, namely whether the required data inputs can be sufficiently accurate to produce significantly predictive future asset allocations between competing asset classes and markets in order to capture a significant portion of the expected return. Since the future is unknowable, with little probability of being a mere extrapolation of the past, this would require determining data inputs using a technique known as stochastic (Monte Carlo) simulation modelling, which simulates market behaviour, using historical data, based on the premise that markets are mean reverting and over the long term approximate past performance. Solutions to the question of data input determination will produce a framework within which practitioners can effectively utilise a mean-variance optimiser without the fear of generating widely divergent outcomes from a realised future portfolio.

Research conducted by Michaud (1998) and Jobson and Korkie (1980) supports the core hypothesis that mean-variance optimisation can lead to performance enhanced outcomes. It is possible, therefore, to test whether resampled data can lead to the establishment of optimal forward-looking portfolios. The results of this study will substantially contribute to an understanding of how complex theoretical models may be applied in the practical world of investing, in addition to highlighting potential new areas for asset class product development.

1.6 HYPOTHESES

With the research objectives established in Section 1.4 specific research hypotheses can be formulated which may later be accepted or rejected, based on the findings of research. These specific hypotheses are presented below.

a) Mean-variance optimisation, using resampled data, leads to performance enhanced outcomes, as a result of:

   i) effective asset class selection;

   ii) effective allocation of assets amongst competing asset classes;

   iii) effective allocation of assets amongst competing global markets; and

   iv) effective risk management through diversification.

b) An effective investment policy leads to performance enhanced outcomes as a result of:

   i) effective rebalancing techniques, to maintain asset allocations;

ii) effective investment management, using passive formula strategies.

c) Stochastic simulations are an effective data resampling technique, when deterministic linear extrapolation of historical data is unsuitable for use in a mean-variance optimiser.

d) Asset Allocations based on style and size, relative to investing in the broad market, lead to performance enhanced outcomes.

By applying a methodology that sets out to lend support for, or rejects, the hypotheses, the research objectives will be addressed. In this regard the overview of the methodology will provide, not only an overview of the methodology but an indication of which hypotheses will be addressed by which theoretical precepts.

1.7 OVERVIEW OF METHODOLOGY

The research is a quantitative study that makes a positivist assumption that something exists and can be numerically tested for. This positive assumption is based on the core hypothesis that mean-variance optimisation, using passive investment strategies, leads to an optimal investment outcome. The methodology to be applied will be in the form of constructing various portfolios in accordance with the theoretical precepts, and comparing these to control portfolios constructed in accordance with a predetermined methodology[3]. This comparison will include the asset allocation and risk-adjusted returns. The outcomes of such comparatives will either accept or reject the hypotheses.

The theoretical precepts to be applied in the construction of the proxy portfolios will include the mean-variance model through the use of a mean-variance optimiser, resampled data inputs, utilised by a mean-variance optimiser, using stochastic simulation modelling techniques. These theoretical precepts are discussed in Chapters 3 and 4, and with relevant methodologies established in Chapter 2 will go about lending support for, or disproving hypotheses a), c) and d).

Specifically, with regards to the hypotheses established around the proxy portfolios, testing will be conducted as set out below.

In order to test hypothesis a) i), the methodology will be to compare the weightings of the various asset classes within a portfolio to determine the effectiveness of asset class selection. These weightings, in turn, will be compared from period to period.

With regards to hypothesis a) ii), the resampled asset allocations will be compared to the actual efficient frontier asset allocations for the period under

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[3] See Section 2.4.8 (p. 28).

review, to determine whether resampled asset allocations are effective relative to the actual outcome.

Regarding hypothesis a) iii), the risk-adjusted outcomes for the inter-market portfolio will be compared to the domestic intra-market portfolio to determine whether the allocation of assets across global markets is effective. Additionally, the percentage of assets allocated to foreign markets, derived using the resampling process, will be compared to the percentage of assets allocated to foreign markets, using broad market indices only. This will be to establish whether exposure to the foreign market is increased or decreased relative to the broad market approach, thereby indicating the levels of risk exposure.

To test hypothesis a) iv), the effectiveness of risk reduction will be examined by contrasting the Sharpe ratios for the rebalanced redetermined resampled portfolio with the rebalanced broad market index portfolio.

Regarding hypothesis c), the asset allocations derived during the resampling process will be compared to the asset allocations derived from the actual efficient frontiers. This comparison is to search for diversification effectiveness; thereby establishing whether stochastic data input determination is an optimal approach to determining data inputs for the mean-variance optimiser.

Finally, with regards to hypothesis d), the weightings of the asset classes within the resampled portfolios will be analysed, as well as the returns for the resampled portfolios relative to the broad market portfolios. In this manner the research can determine whether the returns advantage, if there is one, is as a result of the allocation of assets to dominant asset classes.

With regards to the management of the portfolio, the theoretical precepts consist of mean reversion of the asset class returns, rebalancing techniques and the application of passive formula strategies. These theoretical precepts are discussed in Chapter 5, and with relevant methodologies established in Chapter 2 will go about lending support for, or disproving hypothesis b).

Specifically, with regards to the hypotheses established around the management of the portfolio, testing will be conducted as set out below.

In order to test hypothesis b) i), the redetermined resampled portfolio returns are to be compared to the resampled portfolio returns. In so doing the process of rebalancing can be evaluated for effectiveness.

Regarding hypothesis b) ii), the internal rate of return of a value averaged portfolio will be compared to the internal rate of return for the non-value averaged portfolio. The outcome will establish the effectiveness of value averaging as a portfolio management tool.

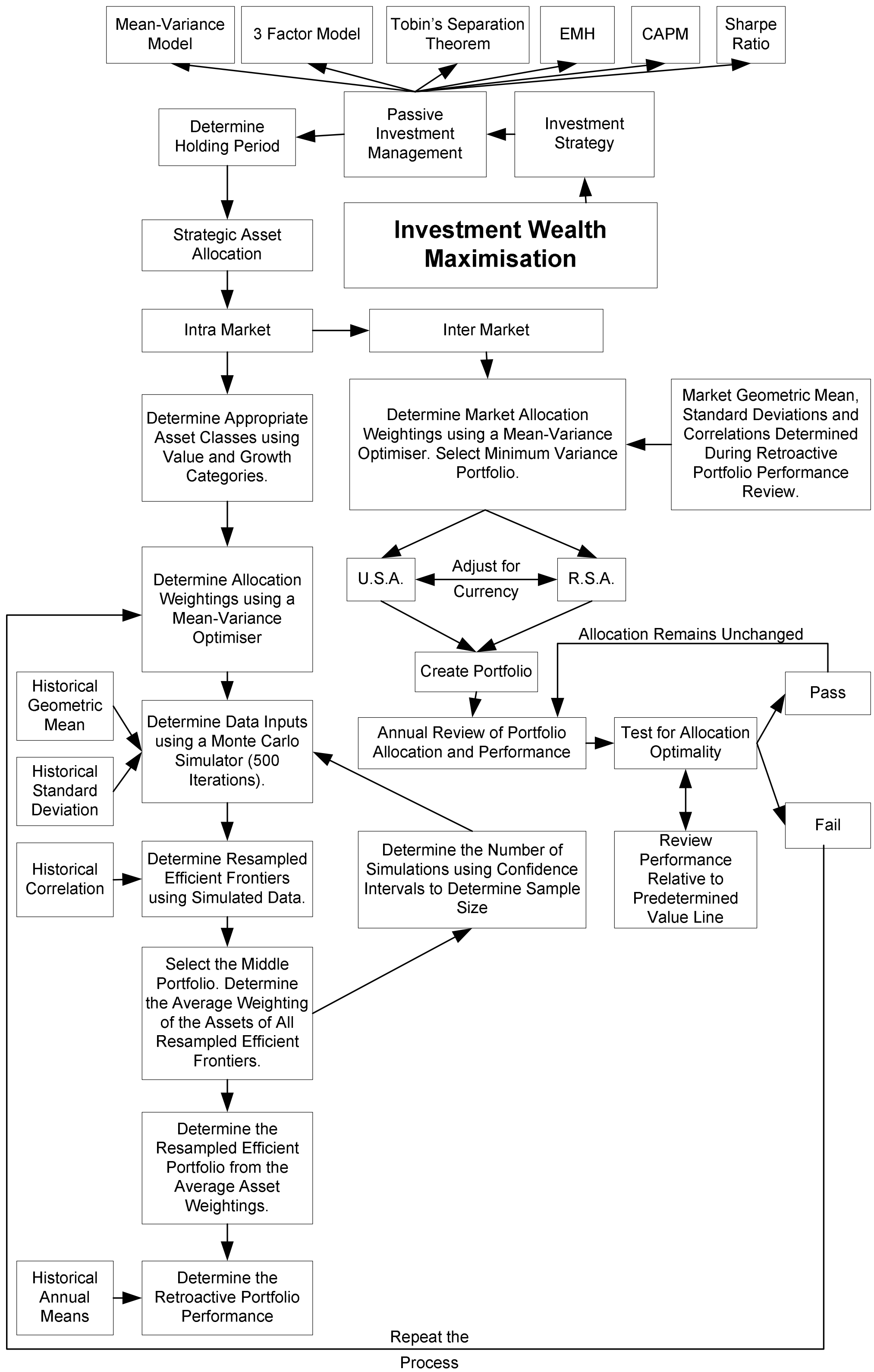
The asset classes used in the proxy and control portfolios are based on the literature in this regard. The appropriate market indices are selected as proxy passive investment products, and in the absence of such indices they are derived from primary data sources.

The investment approach applied is long-term in nature, and assumes a passive investment approach, namely that the asset classes mechanically track selected indices without the need for active investor involvement. The time period used for the research is 20 years for the data determination, and 10 years for the portfolio management.

1.8 RESEARCH CONCEPTUAL MAP

Given the complexity of the research, with reference to Figure 1.1, it was deemed to be prudent to provide a flow diagram, in order to contextualise the processes. The reader would be aided by the diagram in that any complex process can immediately be seen within the context of the broader model. It is recommended that the reader make use of the diagram during the study of the research.

Figure 1.1 Investment processes flow diagram



1.9 SUMMARY

The introduction to the research emphasised the importance of asset allocation and ongoing portfolio management. Thereafter, having identified the problems, proposed solutions to these were noted, and form the crux of the research. Prior to the application of the quantitative proposed solutions it is imperative to establish the methodology and review current literature that may provide support for the research.

It has been determined that the section dealing with the research methodology will precede the literature review in order to place the review of secondary literature sources within the correct context, thereby creating a more logical structure. Thereafter, in the ensuing chapters, the sections exploring the fundamental precepts of asset allocation, passive investing based on the efficient market hypothesis, modern portfolio theory and the related use of a mean-variance optimiser, mean reversion of asset classes and the capital asset pricing model and related issues, will be examined.

CHAPTER 2

RESEARCH DESIGN AND METHODOLOGY

2.1 INTRODUCTION

This study was a quantitative study, where hypotheses were tested using a combination of both primary and secondary data. According to Jankowicz (1995, p. 174) a quantitative approach is complementary to a positivist assumption that something exists and can therefore be numerically measured. The positive assumption in this case was that mean-variance optimisation, using passive investment strategies, leads to an optimal investment outcome. This assumption could then be numerically tested in order to cast doubt upon, or provide support, for the theory. Jankowicz (1995, p. 89) asserts that verification is provisional, in that a theory can never be proven to be true merely by proving the hypotheses. Karl Popper (1985 p. 102) espoused the view that the best that a researcher can achieve is a tentative acceptance of a theory, which may be rejected on the basis of new evidence, without necessarily discarding the old evidence.

2.2 OVERCOMING THE PROBLEM OF INDUCTION

The problem of induction is the risk of inferring a general conclusion from specific observations, since a single contradictory observation is all that is required to refute the original finding.

‘Most scientific inquiry deals not in the heady stuff of truth… but in hard-won increments of probability’ (Locke, Spirduso and Silverman, 2000, p. 88).

The answer in addressing the problem of induction was not whether acting in accordance with alternative hypotheses were able to produce higher investment returns, but whether on the balance of probabilities acting in accordance with the proposed hypotheses were superior in producing optimised investment outcomes. As proclaimed by the philosopher Pascal, the optimal strategy for all humans is to believe that God exists, since if God exists the believer is rewarded. If God does not exist then the believer has nothing to lose. The same logic can be applied to the problem of induction (Taleb, 2001, p. 109).

The literature review acted as the theoretical basis for the study, which addressed the core issues relating to mean-variance optimisation, passive formula strategies and stochastic simulation modelling in order to derive resampled data inputs for a mean-variance optimiser. The quantitative findings were derived using primary data, and covered the period 1973 – 2002.

2.3 GEOMETRIC VERSUS ARITHMETIC RATES OF RETURN

The issue of appropriateness arises with regards to rates of return. What is necessary is an understanding of the different measures applied to rates of return. Gibson (2000, p. 65) provides an example in Table 2.1. The arithmetic mean will always be greater than, or equal to, the geometric mean. The disparity between the two figures arises as a result of the high variability of variables. Gibson indicates that the arithmetic mean is the appropriate measure when analysing a single time period. The geometric mean, in turn, is the appropriate measure when analysing multiple time periods as it represents the compounded growth rate for an investment.

As is evident from Table 2.1 it would have been inappropriate to deduce that the rate of return on the portfolio is 2.5 percent, when the terminal value of the portfolio would have remained unaltered.

In light of the above, given that the research was conducted over multiple time periods, unless stated otherwise, the rates of return refer to the geometric rates of return.

Table 2.1 Geometric versus arithmetic rates of return

Year 1 Return	+ 25%
Year 2 Return	- 20%
Sum	+ 5%
Arithmetic Mean (Sum/2)	+ 2.5%
Geometric Mean	0%

Source: Gibson, 2000, p. 65.

2.4 RESEARCH METHODOLOGY

The methodology applied was of an archival nature, where earlier literature and studies were reviewed, and reflected upon, thereby forming the foundation for further research.

With a theoretical foundation, as well as reviewing the outcomes of numerous studies, the researcher was able to gather pertinent primary data that was used to construct proxy portfolios in order to back-test theoretical models in the search for an optimal solution to the identified research problems as set out in Section 1.3 (p. 4).

Recommendations were based on the outcome of the primary data analysis, and combined with the researcher's judgement.

2.4.1 Primary data

The primary data was of a raw quantitative nature, obtained from archived printed sources, namely the Financial Mail, South African Reserve Bank Quarterly Bulletins and the Johannesburg Stock Exchange Handbook for the period 1972 – 2002.

2.4.2 Secondary data

The secondary data was sourced from subject specific books and journal articles were acquired in the U.S. since the existing theories, and researchers highlighted in the study, emanate from that country. These books and journal articles were sourced by using the internet to research the various subjects, and by using reader reviews to establish who the leading authors were on the various subjects. Additional research and studies were researched using the bibliographies provided in the subject specific books and journal articles.

The most recent domestic publications (Grieve, 2001, Kruger, De Kock and Roper, 2001, Minton, 2001, Magliolo, 2002, and Swanepoel, 2002) were used to benchmark current domestic practitioner thinking, and it was found that they do not include significant references to any of the research material covered in this study, and seem somewhat out of step with recent developments as highlighted in this study, in many instances preferring to adhere to heuristically determined solutions. An example by way of a quote is:

" … it was mooted in the press that all South Africans should have at least 20 - 30 percent … invested offshore. Current recommendations are closer to 70 – 80 percent" (Kruger, De Kock and Roper, 2001, p. 1).

Of the publications that do make mention of the latest investment philosophies, (Bradley, Higgins and Abey, 2000 and Marx, Mpofu and Van De Venter, 2003), the coverage is brief, theoretical and generalised, falling short of setting out a position for optimised investing. In short, the latter publications are merely academic in nature, with little value for the broader investor populace.

2.4.3 Comparison methodology

For the purposes of the research, individual stock selection, in order to construct a portfolio, was ignored. The reasons are presented below.

a) Optimal diversification requires investing across a broad spectrum of equities[4]. It is highly probable that selecting individual stocks will not lead to ample or appropriate diversification from a sector perspective.

b) The costs involved in acquiring individual equities incur costs at the individual transaction level, unlike collective investment products such as unit trusts. In this regard costs at the individual level would be prohibitive and therefore such a portfolio would be at a disadvantage compared to an equivalent collective investment portfolio.

c) Due to the sensitive nature of mean-variance optimiser inputs, the greater the number of assets the higher the error amplification.

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[4] See Chapter 3 (p. 65).

d) Since one of the variables required by the mean-variance optimiser is the cross-correlations between the assets, this would require approximately 500^{500} calculations[5] for the South African market alone, which is not practical.

The research made use of indices, or proxy indices, representing a homogenous set of equities, which in aggregate made up an asset class defined by a particular set of characteristics. These asset classes were selected or derived based on the literature, which favours style type assets, based on size and value factors. The U.S. market largely replicates the U.S. indices in the form of investment products. The South African market does not have a broad range of products that replicate indices. In this regard the indices were derived and used as proxy investment products.

A fundamental aspect of the research is that the asset classes used for the research replicated an index, or proxy index, therefore all the asset classes were considered to be passively managed. In this respect, outcomes from previous research, as updated, as well as secondary evidence was proffered in support thereof.

[5] Being the approximate number of equities listed on the JSE Securities Exchange.

2.4.4 Beta testing and CAPM

The capital asset pricing model was rejected as a determinant of asset allocation as a result of the findings set out in the literature review. In this regard a rudimentary test was conducted, with the results set out in the literature review[6]. This test involved determining the relationship between beta and the selected asset classes for the research.

The test was conducted as set out below.

a) Regression analyses were conducted between the relevant market portfolio and the asset classes identified for the research, for the period 1973 - 1992. The market portfolio was set as the independent variable, and the alternative asset class as the dependant variable.

b) Beta was derived from the slope of the regression line.

c) The derived beta for the asset classes was coupled to the relevant geometric mean for the period under review.

d) A correlation coefficient was derived for the beta and geometric mean combination to test for effectiveness of beta.

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[6] See Tables 3.1 and 3.2 (pp. 93 – 94).

e) The correlation coefficient was interpreted using the coefficient of determination to identify the proportion of commonality. Correlation coefficient critical value tables were not used since the datasets could not be expanded and were a function of the number of asset classes under review. Any statistical interpretation based on critical values may be flawed due to the dataset constraint.

2.4.5 Mean-variance optimiser[7]

The research made make use of a software package, namely MvoPlus, which is a stand-alone mean-variance optimisation package, developed by Efficient Solutions Inc, incorporating the Markowitz algorithm. As a result of core competencies a commercially available solution was selected.

MvoPlus functions both as a conventional single period optimiser and as a multi-period optimiser and back-tester of portfolios, which were rebalanced to a specified allocation at the end of each period.

2.4.6 Stochastic (Monte Carlo) simulations[8]

The research made use of a software package, namely XLSim, which is a plug-in Microsoft Excel Monte Carlo simulator, developed by AnalyCorp Inc. Again,

[7] See Section 3.10 (p. 106).

[8] See Section 4.2.3 (p. 120).

as a result of core competencies a commercially available solution was selected.

XLSim functions as a stochastic simulator to provide decision-making assistance under conditions of uncertainty, by producing a number of possible future outcomes. The outcomes of such stochastic simulations are listed by percentile, as well as providing simulated standard deviation and arithmetic mean results. The resultant simulation outcomes, if required, can be represented graphically in the form of a histogram.

2.4.7 Offshore market selection[9]

The offshore component was restricted to the U.S. market. The reasons are set out below.

a) As at year-end 2002 the U.S. market comprised 52 percent of the world stock market capitalisation, as compared to the U.K. market at 9 percent and the European market at 21 percent (Ibbotson Associates, 2003, p. 208).

b) Over the period 1993 – 2002 the U.S. market returned a geometric mean of 9.3 percent compared to a cumulative world market geometric mean of 6.7 percent (Ibbotson Associates, 2003, p. 219).

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[9] See Section 1.4, objective g) (p. 8).

c) The U.S. Dollar, amongst others and within this context, is a benchmark currency for South Africans.

d) For the period 1970 – 2002 the U.S. market returned a geometric mean of 10.8 percent, with the world achieving 9.8 percent and Europe (including U.K.) 10.7 percent (Ibbotson Associates, 2003, p. 219). Therefore over the long-term it was not prudent to assume that alternative developed markets would outperform the U.S. market. Furthermore, alternative higher risk markets, such as emerging markets carry the additional risk of currency volatility relative to the U.S. Dollar. In this regard there was no certainty that these higher risk markets would produce returns to offset the potential currency risk.

2.4.8 Portfolio construction

A comparison was made between proxy and control portfolios for various indices representing equity size and investment styles. Where such indices did not exist, these proxy indices were created for the purposes of comparison.

The primary methodology was to use mean-variance model principles by combining different unconstrained resampled asset classes, to establish the optimal investment portfolio, comprising both South African and U.S. asset classes. Such combinations revealed the percentage split between the two markets.

The secondary methodology was again to use mean-variance model principles, but constrained unresampled asset classes were applied. These constraints were heuristically determined based on what seemed rational and intuitive as espoused by Evensky (1997, p. 253), and were as follows:

a) a constraint of 25 percent allocation to foreign equities;

b) a constraint of 20 percent allocation to domestic large-cap equities (13.2 percent large-cap value, 6.8 percent large-cap growth);

c) a constraint of 60 percent allocation to foreign large-cap equities (40 percent large-cap value, 20 percent large-cap growth);

d) a constraint of 40 percent allocation to domestic mid-cap equities (26.4 percent mid-cap value, 13.6 percent mid-cap growth);

e) a constraint of 40 percent allocation to domestic small-cap equities (26.4 percent small-cap value, 13.6 percent small-cap growth);

f) a constraint of 40 percent allocation to foreign small-cap equities (26.4 percent small-cap value, 13.6 percent small-cap growth); and

g) the split between value/growth is 66 percent/34 percent respectively for all asset classes.

It was prudently determined that absolute constraints were to be used, as opposed to allowing asset allocations to vary within a minimum and maximum constraint range, in order to reduce the computational complexity, and mean-variance optimiser instability.

The proxy portfolios were automatically rebalanced back to the original asset allocations, by means of the MvoPlus software, unless such asset allocations were redetermined.

The portfolios on the efficient frontiers, derived using the mean-variance model, theory, were divided into three portfolios, namely maximum return, middle portfolio and minimum volatility. The market-specific portfolio of choice was the middle portfolio which provided a more appropriate allocation of assets relative to the remaining two extreme portfolios. When combining the two market specific middle portfolios, to derive the inter-market efficient portfolio, the minimum volatility portfolio was selected from the outcome.

2.4.9 Control portfolio construction

The portfolios constructed in accordance with the mean-variance model principles were measured relative to control portfolios.

There were three control portfolios.

a) The first was a naïve portfolio consisting of an equal weighting between asset classes. The portfolio was rebalanced annually to maintain the original portfolio characteristics.

b) The second was the same as for a), except that broad market indices were used as the asset classes, namely the S&P 500 (USA), and the ALSI (South Africa).

c) The last one was the same as for b) except that the portfolio was not rebalanced.

Finally, all the portfolios (both control and proxy) were compared to the actual terminal efficient frontier portfolio as at 2002, using the asset allocation for the period 1983 – 2002, with assets invested for the period 1993 – 2002.

2.4.10 Proxy index construction

There is a significant body of international research that suggests that style investing results in optimised outcomes relative to sector investing (Fama and French, 1992, p. 445). Therefore the research is premised on the style investing hypothesis. Sector investing is sector specific and therefore does not offer widespread diversification benefits. The JSE Securities Exchange does not construct style indices in the form of large-cap value, large-cap growth, mid-cap growth, mid-cap value, small-cap value and small-cap growth. For comparison purposes such indices were constructed based on an adapted version of the index qualification criteria as applied by the JSE Securities Exchange (2002a, p. 11 – 13).

The construction methodology is set out below:

a) The proxy indices were constructed annually, on the date coinciding with the share pricing.

b) The top 40 companies on the JSE Securities Exchange, as ranked by market capitalisation, were allocated to large-cap companies. The next 60 companies on the JSE Securities Exchange, as ranked by market capitalisation, were allocated to mid-cap companies. The remainder were allocated to small-cap companies.

c) Market capitalisation was calculated by multiplying the equity price by the ordinary shares in issue. The ordinary shares in issue were those listed at the time of the printing of the primary data source, which was as close to the date of the equity price as was achievable.

d) Once the shares were ranked by capitalisation, and divided into their respective groupings (as per b) above), the shares were then ranked by price-to-earnings ratio, from the largest to the smallest.

e) The price-to-earnings ratio ranked shares were then split by dividing the capitalisation of the various groupings in half, i.e. 50 percent of the groupings were growth shares, and the remainder were value shares.

f) The proxy indices began at a nominal value of 100. This figure was derived by taking the respective total market capitalisation for the derived groupings, and dividing by a number (divisor) so that the resulting figure was 100 (JSE Securities Exchange, 2002b, p. 4)[10].

g) The dividend and earnings yields for the respective proxy indices were derived by multiplying the constituent dividend and earnings yields by the respective weightings the shares had within the index. The total of such multiplications yielded the respective dividend and earnings yields for the proxy index.

h) The opening dividend and earnings yields for each constructed year were deemed to be the pending yields.

i) Since the price-to-earnings ratio is the inverse of the earnings yield, this was calculated by inverting the derived earnings yield for the respective proxy indices.

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[10] $I = \frac{\sum_{i=1}^{n}(p_i s_i)}{d}$, n = number of equities in index, p_i = price of the i-th equity, s_i = shares in issue for the i-th equity and d = the divisor.

This aspect of the research was discretionary as there are an infinite number of ways to calculate indices. In this regard the objective of the research was to establish whether style and size investing, using the simplest form of construction, produced return premiums as suggested by the literature, and to construct such indices in a manner that may later be replicated by practitioners, hence the adherence to the JSE Securities Exchange classification of size. The sub-classification based on value was in accordance with the criteria established by the seminal work of Fama and French (1992).

2.4.11 Proxy indices and allocation of primary data

There were anomalies in the construction of the historical proxy indices that were addressed in the ways which follow.

a) The market constituents were determined using historical Financial Mail records as the primary source. This allowed for the inclusion of equities that were subsequently delisted.

b) The market price of the proxy index constituents was recorded using historical primary source records. These prices were at, or near, the end of each calendar year.

c) Where data such as shares in issue, earnings per share and dividends per share, were missing or not included in the primary source, these were gathered from final company financial statements published in the JSE Handbook (secondary source). The financial statements closest to the end of the calendar year were used.

d) Only issued ordinary shares, primary and secondary, were used in the construction of the proxy indices. All alternative listings such as preference shares, debentures, venture capital, loan stock and development capital listings were excluded due to their esoteric nature, as well as the lack of data thereon.

e) Where ordinary issued shares were indicated as delisted in a JSE Handbook, and therefore did not include data, although data was present in the primary source, the data from the primary source was used. Where any relevant data of a less variable nature (shares in issue) was missing from the primary and secondary source, this was gathered from the closest available year.

f) Where ordinary issued shares appeared in the secondary source, but not in the primary source, these shares were ignored as if they did not form part of the original population of data.

g) Where ordinary issued shares appeared in the primary source, but not in the secondary source, and the primary source data appeared complete, the shares were included, otherwise not. Any missing data of a less variable nature was gathered from the closest available year. If such missing data could not be sourced in previous years then the shares were excluded.

h) Where the secondary source reported financials either exceeded, or fell short of a 12 month period, and the primary source had data, this data was used on the assumption that adjustments were made. Where such data was missing from the primary source, the secondary source data was adjusted to reflect a pro-rata 12 month period.

i) Where earnings, and dividends were reflected as zero in both the primary and secondary data sources, these shares were ignored as being anomalous.

j) Where earnings and dividends were reflected as zero in the primary source, but not the secondary source, the secondary source data was used, and vice versa.

k) Where ordinary issued shares, with outstanding data, were included in the primary source, and could not be identified in the secondary source, these were ignored unless they could be identified in a previous year. In such instances only the data of a less variable nature was gathered from the closest available year.

l) Where no dividend was reflected in the primary source, and the secondary source could not verify this, it was assumed that no dividend was declared.

m) In some instances where neither the primary nor the secondary sources could clarify a position, researcher discretion was applied.

n) Where company results were reported in a foreign currency, the primary source data was used on the assumption that this was correctly converted at the time of publishing. Where the primary source data was anomalous, or missing, the results were calculated using an appropriate exchange rate. Where such an exchange rate could not be determined the share was excluded.

o) All South African Reserve Bank entries were excluded due to the legislative restrictions placed on it regarding dividend payouts and share capital, and the impact this had on market pricing.

p) Where the shares in issue were missing from both the primary and secondary sources, these were derived by dividing the attributable income by the earnings per share, or alternatively by dividing the dividend distribution by the dividend per share, and compared to previous years for consistency.

q) Where a share was listed during a year this was not included for that year, but was included thereafter. This was to prevent the artificial inflation of market capitalisation.

r) Where a share had been discontinued during a year this was excluded from that year. This was to prevent the artificial deflation of market capitalisation. A calculation, used for control purposes, reflecting the amount of market capitalisation so removed was completed to reveal the effect of this methodology.

s) Where the shares in issue differed from the opening year in the index calculation, the ensuing year's index was calculated using the equivalent number of shares in issue. This was to prevent the artificial inflation, or deflation, of market capitalisation.

t) Where there was no overt mention of a share split, or consolidation, however the shares in issue varied by at least 100 percent, an adjustment was made to the pricing for comparative purposes. This adjustment was based on a factor, derived by dividing the two sets of shares in issue.

2.4.12 ALSI index relativity

Extensive discussions with Dr J. Immelman of the JSE Securities Exchange revealed that the methodology used to calculate the ALSI index over the period

1973 – 2002 is unknown, and highly probable that such methodology varied over the period.

Furthermore, there are no records of the data utilised to compile the constituents of the index. The implications are that the existing ALSI index cannot be replicated, and therefore researchers have to resort to the next best alternative, namely data reconstruction, which is what has been applied.

The result is that there may be a tracking error relative to the original index. For research purposes the original ALSI index was deemed as being correct. Since the proxy index was constructed using a methodology unique to the research, any perceived tracking error was adjusted for. This adjustment process was conducted to align the proxy ALSI index with the original ALSI index, and was conducted as set out below.

a) Established a correlation coefficient between the original and the proxy ALSI indices. This correlation coefficient had to be highly positive, namely higher than 95 percent, otherwise the proxy ALSI index, and the related sub-indices would not have been representative. This allowed for the data to be tested for reliability thereby assisting in assuring the validity of any findings.

b) Determined the coefficient of determination to establish the likely size of the tracking error.

c) Adjusted the proxy ALSI index and the proxy sub-indices for the tracking error. This adjustment process was conducted on a market capitalisation basis, namely that the larger equities received the largest adjustments as follows:

   i) sum the market capitalisations of the six proxy sub-indices. This equalled the proxy ALSI index market capitalisation;

   ii) determined the individual sizes of the proxy sub-indices relative to the proxy ALSI index, expressed as a percentage;

   iii) determined the annual return differentials between the original ALSI index and the proxy ALSI index;

   iv) adjusted the proxy sub-indices annual returns by the return differential determined in iii), multiplied by the size factor determined in ii); and

   v) the same process applied in iv), was applied to dividend and earnings yields.

2.4.13 Indices tracking error

It would not be intuitive to calculate a tradable index based on the above methodology; however tradable index construction was not the purpose of this study. As a result there would be a tracking error relative to an applied method of index construction. The areas of error would result from the following:

a) the delay in adjusting for shares in issue throughout a year;

b) the delay in index rebalancing; the methodology conducted this process annually; however it would seem prudent to conduct this process quarterly; and

c) the delay in adjusting for shares delisted during a year.

This, however, had little bearing on the research as all indices, including the proxy ALSI index, were constructed using the same methodology. Additionally, since the research was premised on the principle of long-term investing the short-term unadjusted movements in the indices as a result of share issues, listings and delistings were irrelevant. Movements of this nature are bound to be in both directions, and over the long term should have a cancelling out effect.

2.5 ACTUAL EFFICIENT FRONTIER DETERMINATION

The process involved the application of the Markowitz algorithm, otherwise known as mean-variance optimisation, and resulted in the determination of the optimal, retroactive asset allocation. The calculations were determined using the software MvoPlus. The steps of this procedure appear below.

a) Input the real geometric mean data, with inflation removed, for the various asset classes under review, using the data that was relevant to the period under review.

b) Input the standard deviation for the asset classes under review, using the data that was relevant to the period under review.

c) Input the cross-correlation data for the asset classes under review, using the data that was relevant to the period under review.

d) The time period utilised was 20 years.

e) Allow the software to compute the outcome.

f) The results were presented as a minimum variance, middle and maximum return portfolio.

g) Determine the intra-market asset allocations by selecting the middle portfolio. This was known as the actual intra-market efficient portfolio for the respective markets.

h) Determine the inter-market asset allocations by means of the steps listed below.

i) Calculate the retroactive respective actual intra-market portfolio returns, using the relevant actual data for the period under review and the portfolio weights as determined in g) above.

ii) Calculate the cross-correlation between the portfolios.

iii) Adjust the U.S. portfolio for currency volatility.

iv) Calculate the geometric mean and the standard deviation for the respective portfolios.

v) Input the mean, standard deviation and cross-correlation variables into the mean-variance optimiser.

i) The results were presented as a minimum variance, middle and maximum return portfolio.

j) Determine the inter-market asset allocations by selecting the minimum variance portfolio. This will be known as the actual inter-market efficient portfolio.

2.6 DATA INPUTS

2.6.1 Historical data

Historical market data for the period 1973 - 2002 was utilised, covering both the U.S. and South African markets. There was no speculation on future market performance, using alternative methods to determine market data. All rates of return were real geometric unless indicated otherwise.

2.6.2 Stochastic data simulation

This procedure utilised the historical data inputs and the XLSim software where numerous alternative outcomes were generated using the historical real geometric rate of return and standard deviation. The simulation was subjected to 500 trials (Michaud, 1998, p. 35).

The significance of 500 trials was that the real geometric means of the simulated outcomes were significantly broader, due to the relatively high standard deviation of the asset classes (as determined by confidence interval[11] calculations). In this regard refer to the bold figures in Table 2.2. The higher the number of trials, the lower the disparity between simulated outcomes and historical data. This is undesirable as the data inputs are a best guess, hence it

[11] CI = $z_{\frac{\alpha}{2}}(\frac{s}{\sqrt{n}})$, where: z = z score, α = areas in a two tail distribution, s = standard deviation of sample and n = size of sample (Wisniewski, 1997, p. 202).

was necessary to allow for a spread of resampled outcomes to budget for the variability in future returns, thereby allowing for the inclusion of outlier portfolios, which would lead to more effective diversification by providing an allocation that might otherwise not have been included.

Table 2.2 Confidence intervals

Standard Deviation		25.28%	
Geometric Mean		6.31%	
No. of Trials		10 000	500
95% Confidence Level		0.50%	2.22%
Sampling Error		7.85%	35.12%
Expected Range	Min	5.81%	**4.09%**
	Max	6.81%	**8.53%**

Source: Derived using absolute ALSI Return for 1973 – 1992, adjusted for inflation.

2.6.3 Data resampling and resampled efficient frontier determination

A derivative of the technique developed by Michaud (1998) was used to resample data, and the appropriate resampled data inputs were used to construct proxy portfolios as at 1992.

The technique is described as follows:

a) The data inputs were stochastically simulated, using 500 trials.

b) The simulated resampled data were used as data inputs for the optimiser, thereby establishing a resampled efficient frontier.

c) Steps a) – b) were repeated as many times as required to satisfy the confidence interval at the 98 percent level, with a sampling error of less than two percent.

d) The asset allocation percentages for the resampled efficient frontiers were listed by minimum variance portfolio, middle portfolio and maximum return portfolio respectively.

e) The listed asset allocation percentages were averaged for the respective portfolios.

f) The resampled efficient portfolio was determined by selecting the results of the averaged middle portfolio. This portfolio was known as the resampled intra-market efficient portfolio.

g) The steps used to determine the resampled inter-market asset allocations are set out below.

i) Calculate the retroactive respective resampled intra-market portfolio returns, using the relevant actual data for the period under review and the resampled portfolio weights as determined in point f) above.

ii) Calculate the cross-correlation between the resampled portfolios.

iii) Adjust the U.S. resampled portfolio for currency volatility.

iv) Calculate the geometric mean and the standard deviation for the respective resampled portfolios.

v) Input the mean, standard deviation and cross-correlation variables into the mean-variance optimiser.

vi) The results are presented as a minimum variance, middle and maximum return portfolio.

vii) Determine the inter-market asset allocations by selecting the minimum variance portfolio. This is known as the resampled inter-market efficient portfolio.

viii) Substitute the resampled efficient portfolio asset allocations for the asset allocations in the actual efficient portfolio.

ix) Compare the outcome of point viii) to the actual efficient portfolio to establish whether the resampled efficient portfolio is intuitively better in terms of asset class diversification.

2.7 PERIODIC REVIEW OF RESAMPLED ASSET ALLOCATIONS

It is counter-intuitive to continuously rebalance to the established asset allocations due to the dynamic nature of asset classes as indicated by the literature. The characteristics of, and the correlation between, asset classes may change over time, leading to different asset allocation weightings within a portfolio.

It was therefore imperative that the established asset allocations be periodically tested to ascertain that they remained optimal.

The test steps are listed below.

a) The actual efficient portfolios using the same methodology[12] for the respective markets were determined using the actual returns for the asset classes for the 20 year period preceding the year under review.

b) The middle portfolio from each market's efficient frontier was selected.

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[12] See Section 2.5 (p. 41).

c) Using the actual returns for the assets classes for the 20 year period under review and the asset allocations previously established, as well as the asset allocations determined in b) above, annual returns were calculated for the two sets of asset allocations for the respective markets.

d) The outcomes of the annual returns for the actual efficient frontier portfolio versus the previously established resampled efficient frontier portfolio were compared using the coefficient of determination (R^2).

e) Where the coefficient of determination exceeded 99 percent, the previously established assets allocations were deemed to be optimal, and therefore did not require resampling.

f) Where the coefficient of determination did not exceed 99 percent, the previously established assets allocations were deemed to be sub-optimal, and therefore required redetermination using the established methodology[13].

g) The newly established asset allocations determined in f) were used for rebalancing purposes from the ensuing year.

This section of the methodology was based on the view expressed in the literature that an optimal approach to rebalancing would be to test whether a

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[13] See Section 2.6.3 (p. 45).

portfolio requires rebalancing. In this regard it was prudent to test whether the asset allocations remained optimal. The test for optimality was to measure how much of the portfolio return was determined by the actual returns by using the coefficient of determination (R^2). This methodology was derived using the logic applied to the Brinson, Hood and Beebower (1986, p. 136) study, namely a portfolio that has similar assets, and asset weightings, to the broad market should manifest a high coefficient of determination to the market.

2.8 PORTFOLIO REBALANCING

Portfolio rebalancing to the previous or newly established asset allocations was conducted once annually.

2.9 MEAN REVERSION TESTING

Testing was conducted using the Fama and French (1988, p. 247 - 248) methodology, namely serial-correlations[14] where a series of data points were correlated against the same number of data points from the previous period. The data points were rolling geometric rates of return for the periods 1 - 10 years. Testing was only conducted on periods of 1 - 10 years, since data was not available for shorter periods.

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[14] SC = $\left(\frac{Cov(x, y)}{\sigma_x \sigma_y}\right)$, where Cov (x,y) = the covariance of x and y, σ_x = standard deviation of x and σ_x = standard deviation of y (Ibbotson, 2003, p. 109). A.k.a. first-order autocorrelation.

Cyclical behaviour was be indicated by a negative serial-correlation. A serial-correlation of, or around, zero indicated stochastic type behaviour. Trend-like behaviour was indicated by a positive serial-correlation.

2.10 INVESTMENT POLICY

A lump sum investment (R1,000,000) was implemented at the outset of the research in accordance with the techniques discussed below.

a) The investment was allowed to advance and decline stochastically without intervention, subject to the performance of the underlying asset classes, with portfolios rebalanced annually.

b) Value averaging (Bernstein, 2002a, p. 283) was applied to the relevant portfolios, which involved ensuring the value of an investment increased by a predetermined amount at regular intervals, and included purchasing or selling equities to attain the predetermined return. This had a tendency to minimise the stochastic advances and declines of a portfolio.

c) The predetermined value averaging amount was determined for the first year using the historical real rate of return for the defined portfolio for the period 1973 – 1992. Thereafter the historical real rate of return for ensuing periods was applied, and reviewed annually. Therefore, the value averaging amount became a dynamic variable, subject to change as historical real rates of return changed.

2.11 PRODUCT FEES, TRADING COSTS AND TAXATION

Product fees, trading costs and taxation are crucial factors in determining the success of an investment strategy; however, since the purpose of the study was not to research costs, these have been ignored. This should not be construed as minimising the negative impact costs have on investment returns.

2.12 RISK ANALYSIS

During the research, standard deviation was used as a measure of variability around the mean return. Such standard deviation was used as a proxy for risk for a particular asset class or portfolio and used for comparison purposes. Furthermore, the efficiency of asset class or portfolio returns was tested using the Sharpe ratio.

2.13 DESCRIPTION OF THE TIME HORIZON

A time horizon of 20 years was selected for the purposes of determining historical data. A time horizon of 10 years was selected for the purposes of evaluating a portfolio. Although this is somewhat longer than is anticipated to be the normal holding period for the average investor, it is more pragmatic than applying a steady 20 year holding period, which is rarely achieved, and is approximate to the holding period suggested by the literature.

2.14 ROLLING TIME PERIODS

Risk and returns were calculated for a 20 year time horizon starting from 1973 – 1992, and were calculated each year thereafter until 2002, which provided a rolling period of review from 1992 – 2002. This allowed the researcher to verify whether the long-term risk and returns were mean consistent.

2.15 THE TREATMENT OF INCOME

Dividend income, based on historical dividend yields, was included.

2.16 ADJUSTING FOR EXCHANGE RATES

Since the research takes the view that markets are mean reverting, namely that market averages over the long term are mean consistent, the same methodology was applied to exchange rates. Therefore it was assumed that the South African Rand would continue to depreciate by at least its long-term average relative to the U.S. Dollar.

2.16.1 Exchange rate adjustment to U.S. asset classes

For the purposes of determining the data inputs for the mean-variance optimiser it was prudently determined that the foreign geometric rate of return and standard deviation were converted to South African Rand based assets. The methodology is shown below.

a) The stochastically generated foreign asset return was adjusted by adding a stochastically generated exchange rate variable.

b) The converted stochastically generated foreign asset was then adjusted to a real return by deducting the long-term South African geometric inflation variable.

The above methodology was based on the findings of a serial correlation test as reflected in Table 2.3, where exchange rate movements manifested stochastic behaviour, and inflation had trend-like behaviour and therefore was more predictable. Although exchange rates[15] are determined as a result of the intervention of numerous variables, the occurrence of variables and the magnitude of the occurrences are random, which result in the stochastic behaviour pattern manifested by exchange rates.

[15] See Section 5.5 (p. 150).

Finally, for data input purposes, a linear addition of annual exchange rate differentials in the applicable year would imply a link between market performance and currency movement, which there was not, as both the South African and the U.S. markets displayed a stochastic return pattern.

Table 2.3 Serial-correlation test results (1973 – 2002)

SAR:USD Exchange Rate	-0.13
South African Inflation Rate	0.84
South African Market	-0.04
U.S. Market	0.13

Source: Derived using data for the period 1973 – 2002.

2.16.2 Standard deviation for U.S. asset classes

Since the historical data time period was deemed to be 20 years, the standard deviation applied to the simulation data was the actual U.S. asset class standard deviation. However, since the U.S. asset classes were converted to South African Rand, the effect was an increase in the standard deviation for the U.S. asset classes. This would manifest itself in higher standard deviation figures during the simulations. The reason for the added volatility was the stochastic nature of currency movements, as depicted in Table 2.3.

2.16.3 Geometric rate of return for U.S. asset classes

The historical geometric rate of return for U.S. Asset Classes was calculated using a derived index (assumed to be an arbitrary U.S. Dollar 100 at the outset in 1972), which was adjusted annually by the relevant U.S. asset class return. The terminal amount was the U.S. Dollar amount multiplied by the prevailing exchange rate at the end of the period.

This method yielded the rate of return for the entire period, thereby reflecting the compounding effect due to the exchange rate.

The alternative method, namely the annual addition of the U.S. asset class return to the cross-currency rate of decline, yielded a lower geometric rate of return, as shown in Table 2.4. In other words the full rate of return was not reflected.

Table 2.4 U.S. asset class geometric rates of return (1973 – 1992)

	Method 1	Method 2
FF L Cap Growth	17.58%	14.00%
FF L Cap Value	23.87%	19.51%
FF S Cap Growth	16.98%	12.89%
FF S Cap Value	27.11%	22.09%

Source: Derived using data for the period 1973 – 1992.

2.16.4 Cross-correlation data for U.S. asset classes

The cross-correlation, unadjusted for currency, for U.S. asset classes was used, as set out in Table 2.5. This was because adjusting for currency movements had the effect of moving all correlations closer to one, which would have a detrimental effect on the asset allocation selection process during the application of the mean-variance optimiser.

The reason why the cross-correlations would tend to one is that the currency movements are the same for all the assets, and add a return to that which already exists. This equal addition of a return has the effect of increasing the correlations since all the assets move in the same direction. For this reason it is prudent to use the unadjusted cross-correlations which would allow for the appropriate selection of asset classes.

Table 2.5 Cross-correlation data for U.S. asset classes (1973 – 1992)

	FF Large-Cap Growth		FF Large-Cap Value		FF Small-Cap Growth		FF Small-Cap Value	
	USD	SAR	USD	SAR	USD	SAR	USD	SAR
FF Large-Cap Growth	1.00	1.00	0.75	0.90	0.83	0.87	0.65	0.84
FF Large-Cap Value	0.75	0.90	1.00	1.00	0.71	0.79	0.85	0.93
FF Small-Cap Growth	0.83	0.87	0.71	0.79	1.00	1.00	0.84	0.88
FF Small-Cap Value	0.65	0.82	0.85	0.93	0.84	0.88	1.00	1.00

Source: Derived using data from the period 1973 - 1992.

2.17 RESEARCH DELIMITATIONS

The purpose of the delimitations section will be to highlight the boundaries for the research, as well as any possible areas where the findings may not be generally applicable and to indicate any difficulties arising that may have a bearing on the interpretation of the findings.

2.17.1 Proxy indices

Where a period under review occurred prior to the existence of the identified passive instruments, the instruments were constructed using the relevant methodologies. These methodologies were based on the review of the literature; however the applicable methodologies are not universal. They are a reasonable representation of what may be expected from equivalent passive instruments.

2.17.2 Proxy index anomalies

The selected construction methodology may produce additional tracking errors based on the time delay effect with the release of company financials. In this regard the method of utilising financials that correspond with the calendar year under review was to allow for consistency.

Alternative methods would not be prudent since it was impossible to determine what the lag was between a company's financial year-end and the release of the final financial statements, and therefore which financials were appropriate. This, however, was only an issue where the primary source was missing data, which was in a minority of instances.

Example: A company has a financial year ending 31 August, and the index construction date is 31 December. Would one use the latest 31 August financial figures on the assumption that they were released, or would one use the previous financial figures?

This however is a minor problem, as this may result in inter-year variations, which would be diluted if not altogether eliminated, over an extend time horizon.

In addition to the above there was the difficulty of establishing the correct shares in issue as a result of share splits and consolidations, new issues, missing data and share buy-backs. The impact of this was minimised by adjusting for such movements in the shares in issue, as explained in the research methodology. Moreover, the shares in issue from the secondary source (the only source up until 1993) were verified against data from the primary source (available from 1994) for accuracy. Finally, the large-cap shares were closely scrutinised, and the shares in issue verified for consistency. Since the large-cap growth indices are closely correlated to the market index, with a comparatively similar rate of return, it can be assumed that the mid-cap and small-cap indices are equally accurate, however it is intuitive to accept that the smaller the capitalisation of an index, the greater the probability of tracking error. Since the original index cannot be replicated, or verified for accuracy, it is counter-intuitive to conclude that should there be a variation that the constructed proxy indices are not accurate.

In regards the above anomalies it is worth noting Figure 6.1 (p. 158) which represents the minimal tracking error of the proxy index relative to the actual index.

2.17.3 Index methodology amendment

The previous methodology applied to the JSE Securities Exchange indices has recently been altered to allow for more liquid indices, namely that the constituents making up the index are selected and weighted according to their tradability, and not strictly according to market capitalisation, as was previously the case.

With reference to Table 2.6, although the back-tested data differs somewhat using the new methodology, over the long term the impact on returns can be expected to be negligible. The reason for this is clear. The correlation between the new methodology and the old is almost a perfect positive, at 99.59 percent, indicating virtually identical behaviour between the indices, although six years of data is hardly statistically broad enough to be conclusive. Additionally, the R^2 suggests that 99.18 percent of the new index can be ascribed to the returns of the old index. From this it can be inferred that there is a mere 0.82 percent which can be ascribed to the new methodology.

Since market capitalisation continues to play a role in index calculations, irrespective of how the constituents are weighted, the index will continue to produce returns that are closely correlated to the aggregate market index.

Table 2.6 ALSI index analysis

Date	New Index	Return	Old Index	Return
2-Jan-96	5620.26		6228.42	
2-Jan-97	5955.42	5.96%	6657.53	6.89%
5-Jan-98	5460.99	-8.30%	6202.31	-6.84%
4-Jan-99	4989.21	-8.64%	5430.48	-12.44%
4-Jan-00	8337.41	67.11%	8542.79	57.31%
2-Jan-01	8128.78	-2.50%	8326.19	-2.54%
2-Jan-02	10668.59	31.24%	10361.28	24.44%
Statistics			New Index	Old Index
Geometric Rate of Return			11.27%	8.85%
Arithmetic Rate of Return			14.15%	11.14%
Standard Deviation			29.88%	26.05%
Correlation Coefficient (R)			99.59%	
Determination Coefficient (R^2)			99.18%	
Regression Equation*			Y = 1.1423X + 0.0142	
The old index was selected as the independent variable, with the new index being the dependent variable thereby allowing for interpolation if required.				

Source: JSE Securities Exchange (Primary Data) Statistics (Researcher).

What can be expected, and does seem to be the case, is that the new index, moving forward, may manifest a higher standard deviation, due to the increased liquidity. Provisional calculations reveal an increase from 26.05 percent to 29.88 percent. This would indicate a potential for higher risk and return, with the regression equation seeming to support this premise.

Furthermore, the absolute index returns are not what is important, since there are an inordinate amount of ways to construct an index. What is important is the consistent application of index construction methodologies, particularly for research purposes, and whether, by applying such methodologies, certain asset classes consistently yield a premium.

2.17.4 Period specificity

Since the research covers the period 1972 – 2002 it is possible, with no South African data available for earlier periods, for the asset classes represented within the research, and based on the inherent uncertainty of equity markets, that the findings may be either period specific, or that the dataset used is not sufficient to be representative of market behaviour. In this regard the findings may only be tested by future studies where more extensive data would become available.

2.17.5 Asset class selection

The research solves for risky equity asset classes, and ignores cash and its equivalents as well as any alternative asset classes. This is not necessarily representative of an investor's asset allocation portfolio. In this regard the research contends that no position is taken as to the effectiveness of the findings as related to non-equity based investments.

2.17.6 Annuity investing

The research has been based on a lump sum investment and therefore is directly applicable to this. It is anticipated that the same approach could be taken for periodic additions to an investment, although this has not been verified.

2.18 SUMMARY

With the methodology preceding the literature review creates a logical structure for the examination of prior studies. The ensuing section explores the fundamental precepts of asset allocation, passive investing and the contributions thereto in the form of the efficient market hypothesis, Tobin's separation theorem, the capital asset pricing model and the mean-variance model. Included is the discussion on the use of a mean-variance optimiser, the mean reversion of asset classes and related issues.

CHAPTER 3

ASSET ALLOCATION, PASSIVE INVESTMENT MANAGEMENT AND RELATED ISSUES

3.1 INTRODUCTION

The review of relevant literature will be divided into three chapters which collectively will serve to provide the knowledge base upon which the research is built. In this regard the hypotheses will be placed in the context of previous work in such a way as to explain and justify the methodology. This will be accomplished in a step-by-step manner, punctuated by reference to studies that support the ongoing argument.

Chapter 3 begins with an overview of the current popular investment philosophy that the correct allocation of assets accounts for the bulk of investment returns, followed by an overview of the evidence which supports the hypothesis that passive forms of investment are superior to active investment management. The section will then discuss modern portfolio theory and mean reversion. This will be followed by an overview of the use of an optimiser to replicate, by means of a computer, the Markowitz algorithm, and which constitutes the mean-variance model. Thereafter the literature will be reviewed regarding the shortcomings of the use of a mean-variance optimiser, as well as the reasons for seeking an optimal approach to its use.

3.2 ASSET ALLOCATION THEORY AND PRACTICE

Asset allocation can be defined as the process of dividing investment resources amongst competing asset classes (Marx, Mpofu and Van de Venter, 2003, p. 222). Asset allocation is not a contemporary theory, as is evidenced by the ensuing quote:

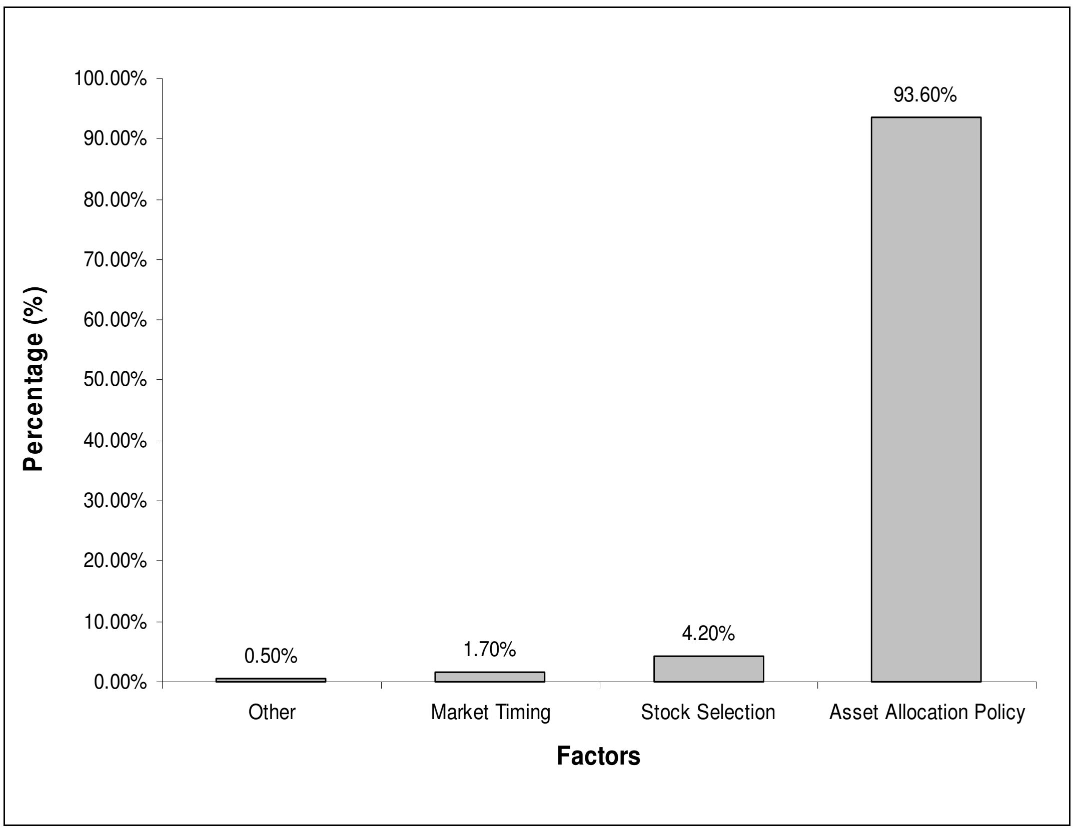
“Let every man divide his money into three parts, and invest a third in land, a third in business, and a third let him keep in reserve - Talmud (Circa 1200 B.C. – 500 A.D.)” (Gibson, 2000, p. 1).

In this regard the quote espouses equal allocations to the competing asset classes, indicating that the management of risk through diversification was an imperative even 2000 years ago. Contemporary academic thinking considers an equal weighted portfolio to be naïve diversification (Shefrin, 2002, p. 136), where investors have a tendency, when faced with a plethora of asset classes, to divide their assets equally, known as the $\frac{1}{n}$ rule.

In this regard it becomes clear that when an investor is faced with a plethora of asset classes, some with similar characteristics and others not, the investor may construct a portfolio, naively, which results in either too little return or excessive risk relative to their desired outcome. As a result it is every investor’s imperative to concentrate on the appropriate allocation of assets.

The industry standard for emphasising the importance of asset allocations is founded on the hypothesis established by Brinson, Hood and Beebower (1986) (Jahnke, 1997, p. 109), where it was postulated that 93.6 percent of a portfolio's variability is a result of asset allocation decisions (Brinson, Hood and Beebower, 1986, p. 137), and that a little less than six percent of the variance can be attributed to market timing, stock selection and other active investment techniques, as set out in Figure 3.1.

Figure 3.1 Factors that determine variance in investment returns



Source: Brinson, Hood and Beebower (1986, p. 137)

Contrary to the findings of the study there is a universal misunderstanding between the relationship between asset allocation and performance, thereby creating the impression that asset allocation decisions are the panacea for superior investment outcomes. Several of these are presented below.

a) Gibson (2000, p. 12) states that asset allocations are the primary determinant of portfolio performance.

b) Scott Simon (1998, p. 150) states that the Brinson, Hood and Beebower study provides ‘real-world confirmation’ of the importance of asset allocation, and the irrelevance of stock picking and market timing.

c) Swedroe (1998, p. 10) states that the Brinson, Hood and Beebower study demonstrates that ‘fully 94 percent of returns’ result from asset allocations.

d) Marx, Mpofu and Van de Venter (2003, p. 222 – 223), South African academics, provide a formula:

   Total Portfolio Performance (100 percent) = Asset Allocation Decision (90 percent) + Security Selection (10 percent).

e) Nofsinger (2002, p. 167 – 169), asserts that asset allocation ‘explained 93.6 percent of the return difference’.

f) Evans (1999, p. 50), proclaims that 92 percent of the return comes from asset allocation.

This widespread misinterpretation of the findings could be attributed to the esoteric nature of statistics.

The study by Ibbotson and Kaplan (2000, p. 26) sets about clarifying the Brinson, Hood and Beebower study. What the Brinson, Hood and Beebower study actually showed was that 93.6 percent of a portfolio's variation, or movement relative to the broad market, was as a result of asset allocations, rather than the amount of total return that results from asset allocation policy, with 75 percent of financial professionals (Ibbotson, 2000, p. 3) misinterpreting this.

What Brinson, Hood and Beebower were measuring was the relationship between the movement of a portfolio relative to the movement of the market as a whole, and that the 93.6 percent variability is due to the movement of the asset classes that make up the portfolio. This does not mean that these movements are necessarily superior in outcome.

The Ibbotson and Kaplan (2000) study set about clearing up many of the misunderstandings. Here it was determined that, when comparing two portfolios, with differing asset allocations, on average 40 percent of the return variation was a result of asset allocations, the remaining 60 percent was a result of equity selection, timing and fee differentials (Ibbotson and Kaplan, 2000, p. 30). What the Brinson, Hood and Beebower study shows, according to Ibbotson and Kaplan (2000, p. 29), is that participation in similar asset classes to the benchmarks yields a high coefficient of determination[16] due to the participation in the market in general, not as a result of asset allocation decisions. When the impact of asset allocation on return levels was analysed, it was determined that on average 100 percent of the return was a result of the asset allocation, which is intuitive (Ibbotson and Kaplan, 2000, p. 32) since in aggregate the combined efforts of investors must equal the market.

The implications, as highlighted by Ibbotson (2000, p. 7), are largely dependant on the investor. With reference to Figure 3.2, a long-term passive investor can be assured that 100 percent of the portfolio returns will be as a result of the asset allocations. In this regard the imperative is to select appropriate asset classes. As regards the short-term investor, asset allocation is less of an imperative.

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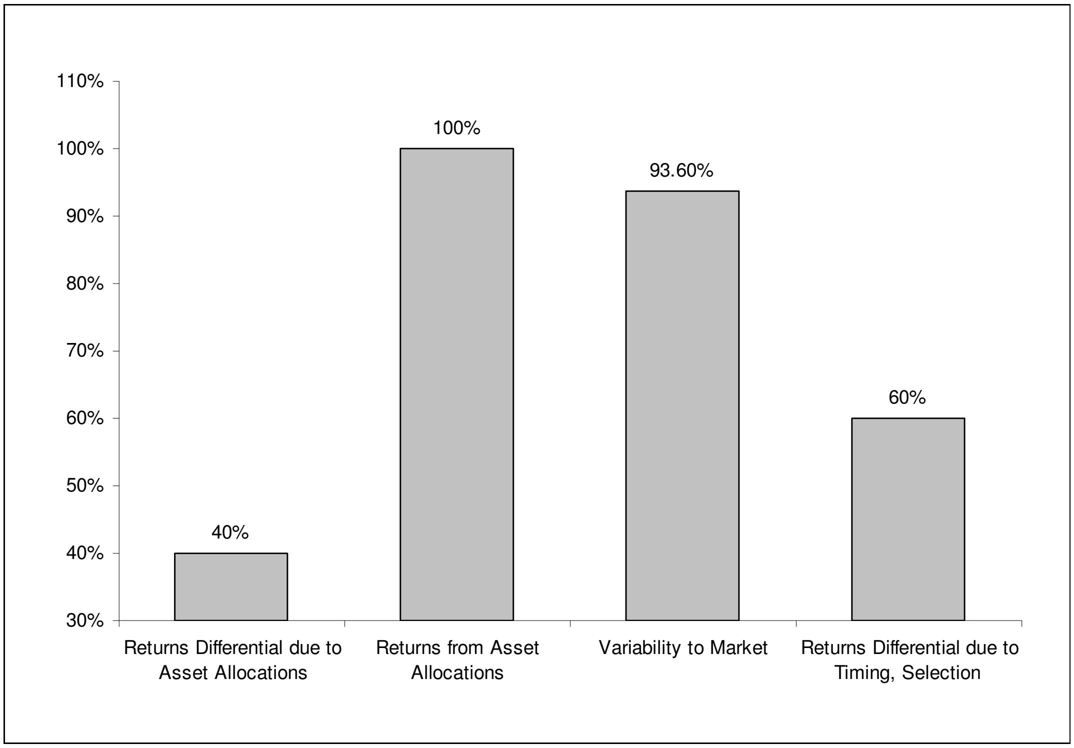
[16] $R^2 = \frac{\sum(\hat{y} - \overline{y})^2}{\sum(y - \overline{y})^2}$, where $\sum(y - \overline{y})^2$ = Total variation, and $\sum(\hat{y} - \overline{y})^2$ = Explained variation (Wisniewski, 1997, p. 331).

Figure 3.2 Returns due to asset allocations explained



Source: Derived using data from Brinson, Hood and Beebower (1986) and Ibbotson (2000).

Jahnke (1997, p. 109) opposed the Brinson, Hood and Beebower study by claiming that the focus had been on portfolio volatility as opposed to the investor imperative of portfolio returns. Jahnke asserts that Brinson, Hood and Beebower drew the wrong conclusions from the research by implying that long-term fixed asset allocation weights were paramount. Jahnke (1997, p. 111) postulates that, only if expected returns are fixed, should asset allocation weights remain static. Since capital markets are dynamic he asserts that any asset allocation policy should be dynamic in nature, and that an investor's imperative should be the integration of quantitative financial planning, expectations-based asset allocations and investment selection. Jahnke (1997, p. 112) goes on to assert that any form of fixed asset allocation based on historical data is often a perverse indicator of the future and is inferior to a methodology of linking forward-looking strategic asset allocation solutions with financial planning. Jahnke concludes that asset allocation is no doubt important; however the investor needs to address how to do this by answering the ensuing questions, many of which will receive attention in the research.

a) What are the appropriate asset classes?

b) Should asset class weights be static or dynamic?

c) How should asset allocation be determined?

d) What are the costs of implementation?

Loeper (2001, p. 6) indicates that ‘it is no surprise’ that a broadly diversified portfolio displays a high degree of fit to the selected benchmarks, as evidenced by the Brinson, Hood and Beebower 93.6 percent finding. Any well diversified portfolio will manifest these findings, and the only contrary outcome would have been to ignore the prudence of diversification (Loeper, 2001, p. 8).

In this regard, Bernstein (2002a, p. 107) acknowledges the controversy and postulates that it is irrelevant how much of a return is determined by stock selection or market timing. These are uncontrollable variables. Since asset allocation is the only area that can be managed, this should be the major focus of an investor's attention.

Statman (2001, p. 129) sets about indicating the difference between tactical and strategic asset allocation. He postulates that tactical asset allocation is what the Brinson, Hood and Beebower study call market timing. Tactical asset allocation is the short-term altering of allocation weights in an attempt to exploit market anomalies. Strategic asset allocation involves dividing a portfolio amongst competing asset classes, and maintaining the weightings unless the investor changes asset class preferences for risk and return. Significantly Statman (2001, p. 132) goes on to demonstrate, using hypothetical examples, that the 93.6 percent variability finding in the Brinson, Hood and Beebower study can be manipulated to yield outcomes that do not reveal anything regarding the significance of strategic or tactical asset allocation. He postulates, however, that they are both important but in different ways. Strategic asset allocations involve movement on the efficient frontier[17] while tactical asset allocations involve movement of the efficient frontier (Statman, 2001, p. 133).

By and large the dispute revolving around asset allocations is one of investment philosophy. Active investors rely predominantly on stock selection and market timing; therefore, any research that may indicate the futility of such an approach is bound to be controversial. In this regard, with reference to Figure 3.3, it is

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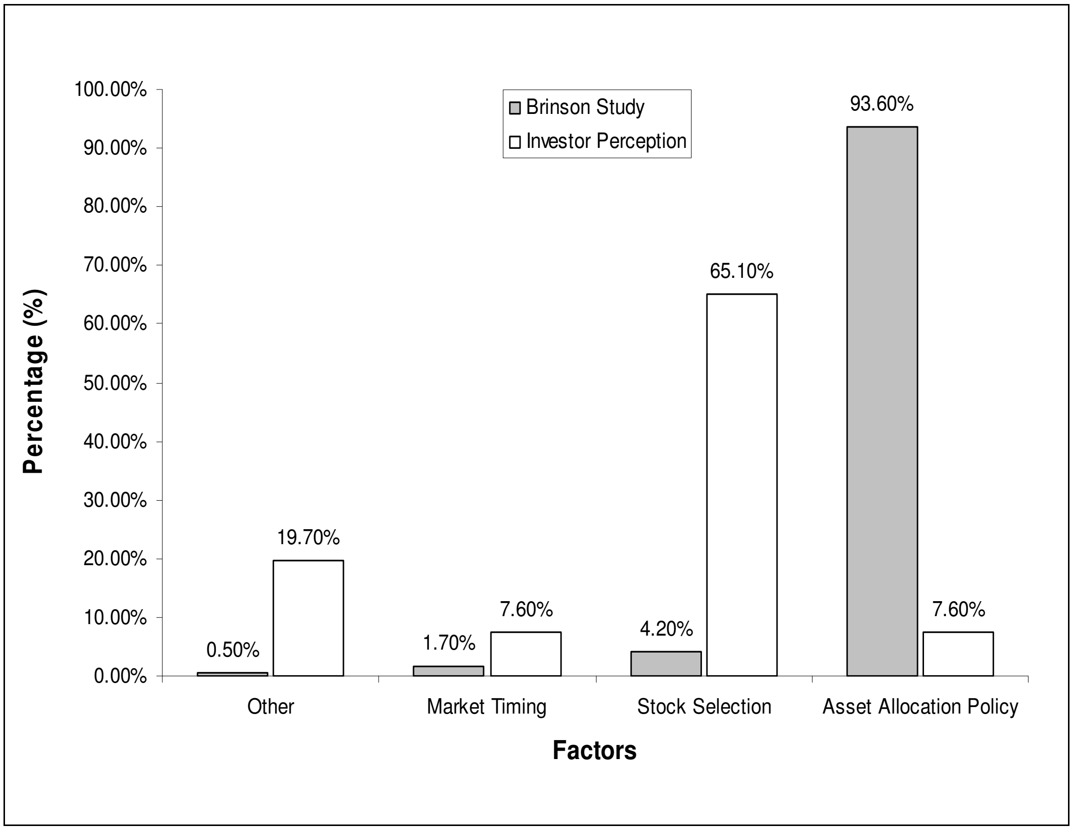
[17] See Section 3.4 (p. 79).

seemingly apparent that investors perceive active investing to be superior thereby validating the dispute.

Figure 3.3 Brinson study and investor perception



Source: Derived from Scott Simon (1998, p.152).

Finally, McClatchey and Vandenhul (2003, p. 2) cite some statistics that reflect the manner in which investors actually allocate assets.

a) 66 percent of retirement investors invest into a single asset class.

b) 26 percent of retirement investors invest entirely in cash.

c) 24 percent of retirement investors hold no equity.

d) 20 percent of retirement investors cannot recall how many investment assets they allocate resources to.

e) 50 percent of investment decisions are research or advice based.

f) 14 percent select the best performing asset class at the time.

g) 11 percent obtain advice from friends and colleagues.

h) Nine percent applied naïve diversification.

i) Eight percent guess.

These statistics, presented by McClatchey and Vandenhul (2003), seem to suggest that the asset allocation imperative, in reality, is grossly ignored. Therefore, in light of the afore-mentioned discussion on asset allocation, investors will not optimise their investment returns unless a scientific approach to asset allocation is integrated into the eventual investment strategy.

Given the broad views on asset allocations, there is no denying the importance of asset allocations within a portfolio. The issue of the size of the returns attributable to asset allocations seems somewhat of a pedant argument. Since the research is premised on passive investment management, both market timing and stock selection are less of an imperative. In fact, the research takes the view espoused by Bernstein (2002a, p. 107), namely the return contribution due to various factors is irrelevant. Given that equity markets are inherently unstable, which makes market timing and stock selection very difficult, if not impossible. The only area that an investor can manage is the asset allocation. Furthermore, since the research is premised on passive investing, thereby acknowledging the dynamic nature of equity markets, any attempts to apply tactical asset allocations are deemed to be attempts at market timing, and therefore are rejected as being futile. In this regard the approach to asset allocations, as set forth by Statman (2001), will be strategic, with the occasional adjustment for the dynamic nature of equity markets (Jahnke, 1997, p. 111).

In the ensuing section the research will examine the argument for passive investing.

3.3 PASSIVE INVESTMENT MANAGEMENT

By adopting a passive investment strategy an investor does not actively pursue investment returns that exceed the market, as indicated by a broad market index. Rather an investor seeks to replicate the performance of the market or a predetermined benchmark.

Passive investors believe that the analysis of market information will not yield anything more than normal returns. Adherents of this strategy do not perform stock selection or market timing, but rather construct a portfolio that tracks a specific market index (Kirzner, 2000, p. 10), or a combination thereof, and is more commonly known as indexing. This method of investing eliminates considerable research and transaction expenses, allowing for the construction of a highly cost efficient investment portfolio.

Damodaran (2003, p. 7) distinguishes between passive investing and indexing by proclaiming that passive investing may also apply to an investment in stocks or asset classes that are not indexed, where the investor implements an investment strategy and waits for the 'pay off'.

As early as 1933 Alfred Cowles postulated that the market was much too dynamic for any one person to figure it out (Ferri, 2002, p. 34), however it was not until 1973, when Professor Burton Malkiel suggested that an investor would be better off buying and holding an index fund, rather than actively trading (Malkiel, 1999, p. 13) that mainstream attention was drawn to passive investing. Yet it was not until 1976 that index funds were available to the private investor through the efforts of John Bogle, founder of the Vanguard Group (Ferri, 2002, p. 32).

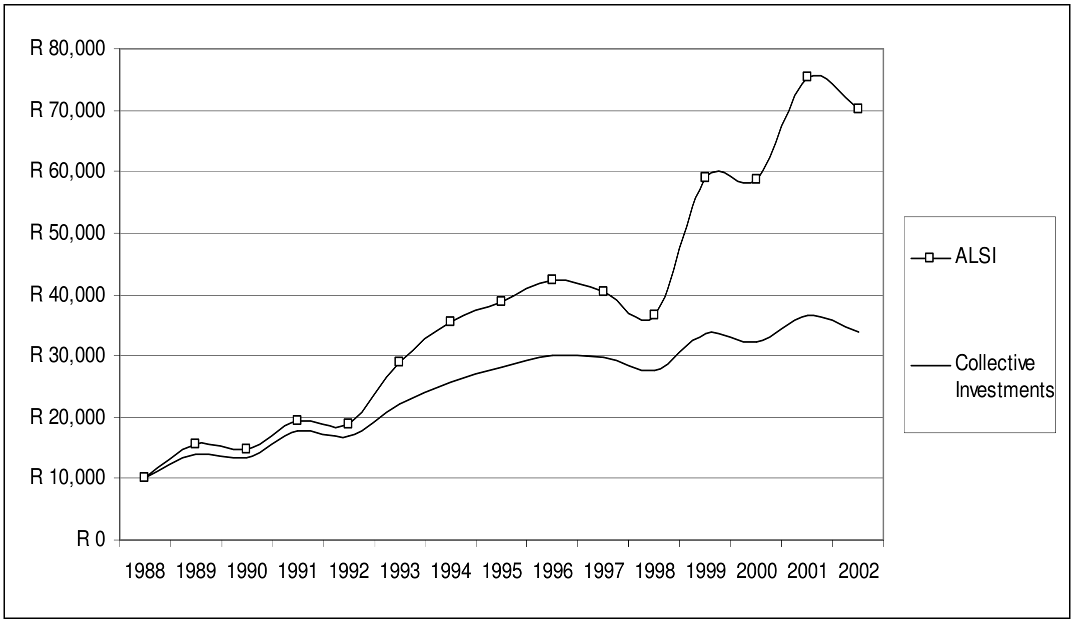
The crux of the argument is that, on average, active investing has been unable to produce investment returns that are superior to a passive form of investing.

For the period 1969 – 1998, the S&P 500 increased an investment of $10,000 to $311,000, as compared to the same $10,000, invested in the average general equity fund, increasing to $171,950 (Malkiel, 1999, p. 13 - 14), an outperformance (net of costs) of 80.87 percent. The South African market has performed similarly. For the period 1976 – 2001, the ALSI increased an investment of R10,000 to R475,085, as compared to the same R10,000, invested in the average general equity fund, increasing to R233,987 (Pawley, 2002, p. 89), an outperformance (net of costs) of 103.04 percent.

In an update of the Pawley (2002, p. 89) findings, with reference to Figure 3.4, it is evidenced that the ALSI index outperformed the collective investment terminal value by 108.11 percent, manifesting a geometric rate of return of 13.87 percent versus 8.44 percent for the period. What is most interesting about these updated findings is that the average collective investment included all allowable offshore investments and passively managed funds, therefore even a digression, allowing the introduction of alternative asset classes, does not seem to enhance the collective investors predicament.

Given that passively managed funds replicate market performance, and given that passively managed funds constitute a small share of overall funds invested (Pawley, 2002, p. 95), it can be deduced that the findings for Figure 3.4 are a result of active management performance. Moreover, the collective investment performance would be slightly positively exaggerated due to the inclusion of passive investments.

Figure 3.4 Active versus passive investment performance (1988 – 2002)



Source: Derived using data from Annexure 65.

It would seem prudent to investigate the theoretical foundations for passive investment management given the performance outcomes of this approach to investing. The hypotheses upon which passive investing are premised are the efficient market hypothesis, the James Tobin separation theorem and the Markowitz mean-variance model.

3.4 THE MEAN-VARIANCE MODEL

One of the fundamental precepts of the research is what is known as the mean-variance model. The hypothesis, developed by Markowitz in 1952, argued that return alone would lead to absurd results (Markowitz, 2000, p. 52), and that a portfolio should be assessed by mean and standard deviation on the portfolio as a whole, as opposed to the weighted average of the mean and the standard deviation. Markowitz determined that portfolio risk[18], as measured by standard deviation, is a function of covariance (Evensky, 1997, p. 186).

The key insight from Markowitz's work is that the risk of a portfolio is usually less than the weighted average risk of the individual assets, and is the key to diversification.

Additionally, with reference to Figure 3.5, Markowitz derived the 'critical line algorithm' which identifies all feasible portfolios, from a given set of assets, that minimise risk for a given level of return and maximises return for a given level of risk, which is known as the efficient frontier. To derive the efficient frontier requires three variables (Markowitz, 2000, p. 4), namely:

a) the expected return of the asset;

b) the expected standard deviation of the asset; and

c) the cross-correlation between the asset classes.

---

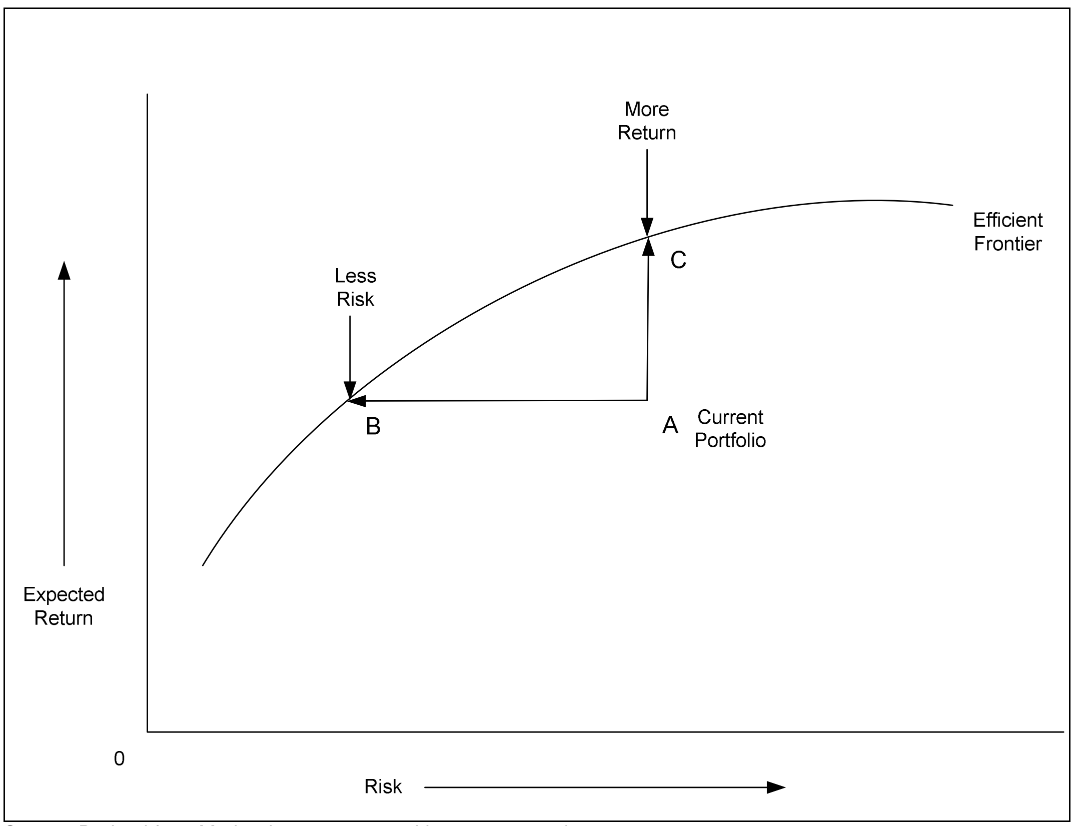
[18] $\sigma Port = \sqrt{\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N}{}_{i \neq j} \sum_{j=1}^{N} W_i W_j Cov_{ij}}$

σPort = standard deviation of the portfolio
W_i = weight of the individual assets in the portfolio
σ_i = standard deviation of asset i
COV_{ij} = covariance between returns for assets i and j, where $COV_{ij} = \sigma_i \times \sigma_j \times r_{ij}$
r_{ij} = correlation coefficient for assets I and j

Initially the process of deriving the ‘critical line’ involved solving for corner portfolios along the line. These corner portfolios included the maximum return portfolio, the minimum variance portfolio and any number of portfolios in-between. Computing power is now able to derive the multitude of portfolios that make up the ‘critical line’, otherwise known as the efficient frontier.

This analysis of risk and return became known as mean-variance analysis, and later the hypothesis, together with the capital asset pricing model, became known as modern portfolio theory (Kirzner, 2000, p. 14).

Figure 3.5 Mean-variance portfolio efficiency



Source: Derived from Markowitz, 2000, p. 5, with axes reversed.

The efficient frontier is an upward-sloping curve reflecting the trade-off between return and risk, as measured by standard deviation[19]. With reference to Figure 3.5, it is clear that for a movement along the x axis (standard deviation) there would be a portfolio that yields the highest possible rate of return. In this regard an efficient portfolio would be any portfolio found along the efficient frontier, as indicated by B and C, and this is the most important property of the efficient frontier. Portfolio A is less than efficient and would require an adjustment to the asset allocation in order to move closer to the efficient frontier, known as strategic asset allocation (Statman, 2001, p. 133).

The second important property of the efficient frontier is that it is curved, not straight, and is the key to diversification. With reference to Figure 3.6, assume two assets D and E. Assume a 50 percent allocation to each asset. With reference to Figure 3.7, assume further that the cross-correlation[20] between the two assets is negative. Based on the assumptions, the standard deviation of the 50/50 portfolio will be less than the average of the standard deviations of the separate assets. Graphically this moves the possible asset allocations to the left of the straight line. The area between the straight line and the efficient frontier

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[19] SD = $\sqrt{\frac{\sum (x - \bar{x})^2}{n}}$, where x = individual data item, $\bar{x}$ = data set mean and n = number of data items (Wisniewski, 1997, p. 98).

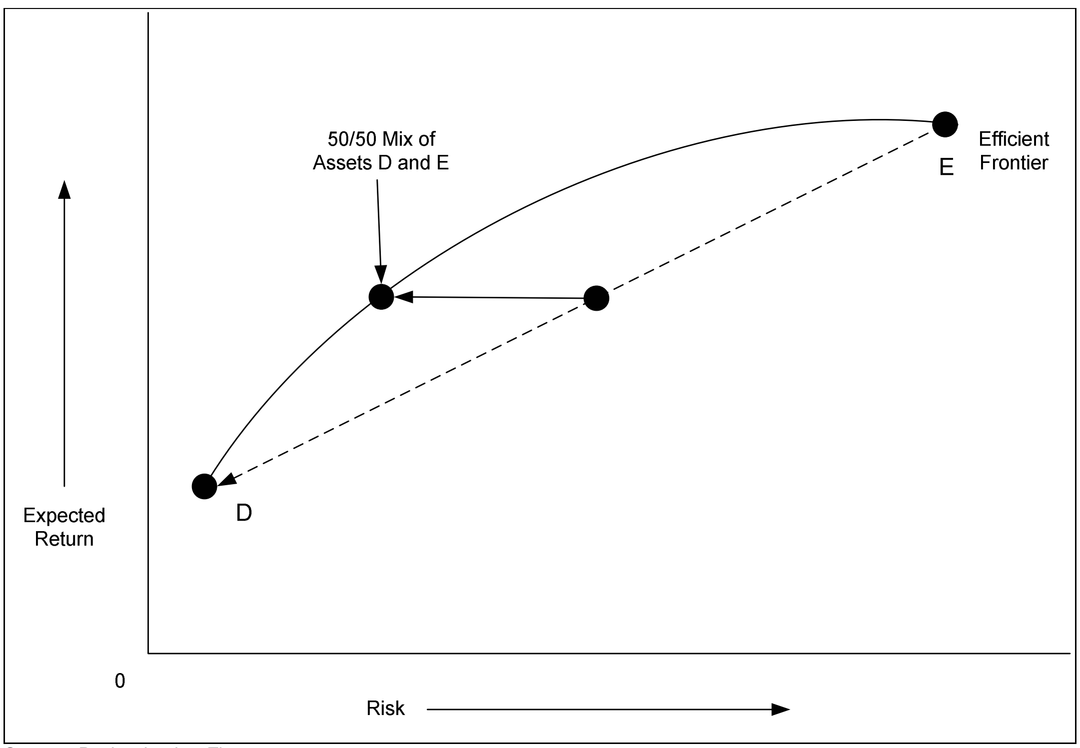
[20] $R = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2}\sqrt{\sum (y - \bar{y})^2}}$

(Wisniewski, 1997, p. 322)

indicates the benefit due to diversification, and is the contribution made by Markowitz based on the cross-correlation of the assets.

The magnitude of the benefit due to diversification, ceteris parabis, is due to the covariance coefficient, which is derived using the correlation between assets. The lower the correlation, the higher the diversification benefit.

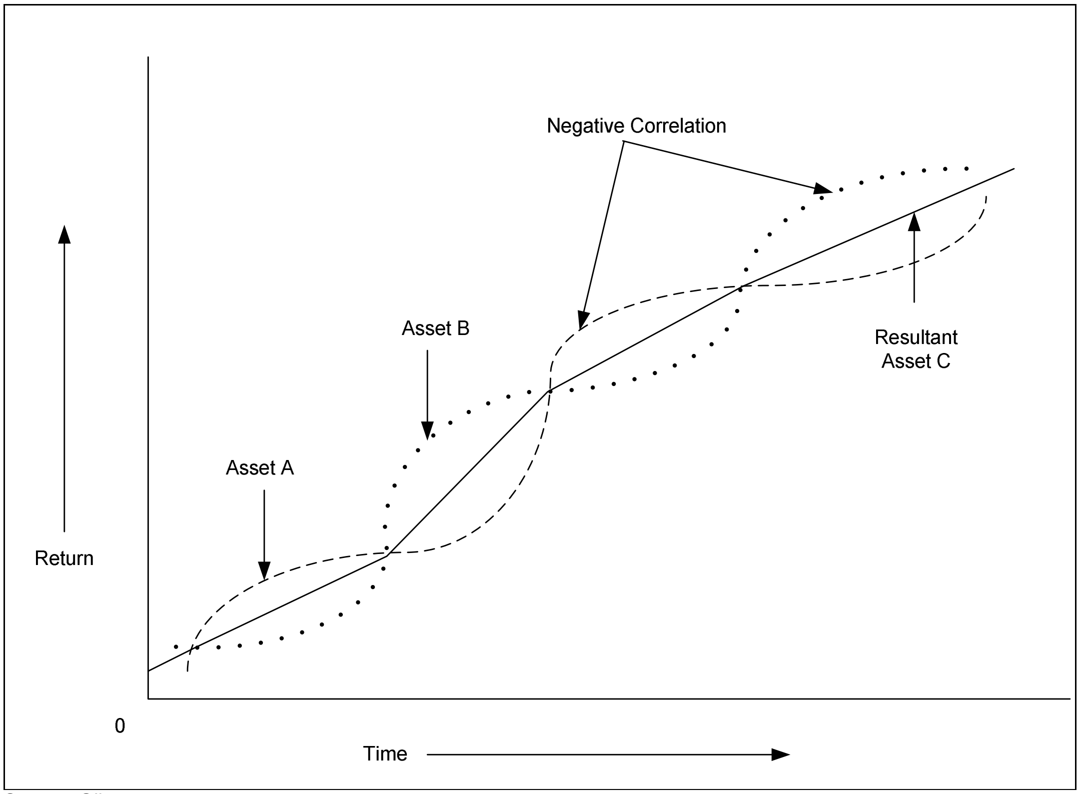
Figure 3.6 Mean-variance portfolio diversification



Source: Derived using Figure 2.4.

In this regard Ferri (2002, p. 186) explained that by combining different assets, risk could be lowered, and returns increased. This led to the assertion that it is not the risk of a portfolio, but rather how a portfolio fits together (asset allocation) that leads to enhanced returns for a given level of risk.

Figure 3.7 Cross-correlation explained

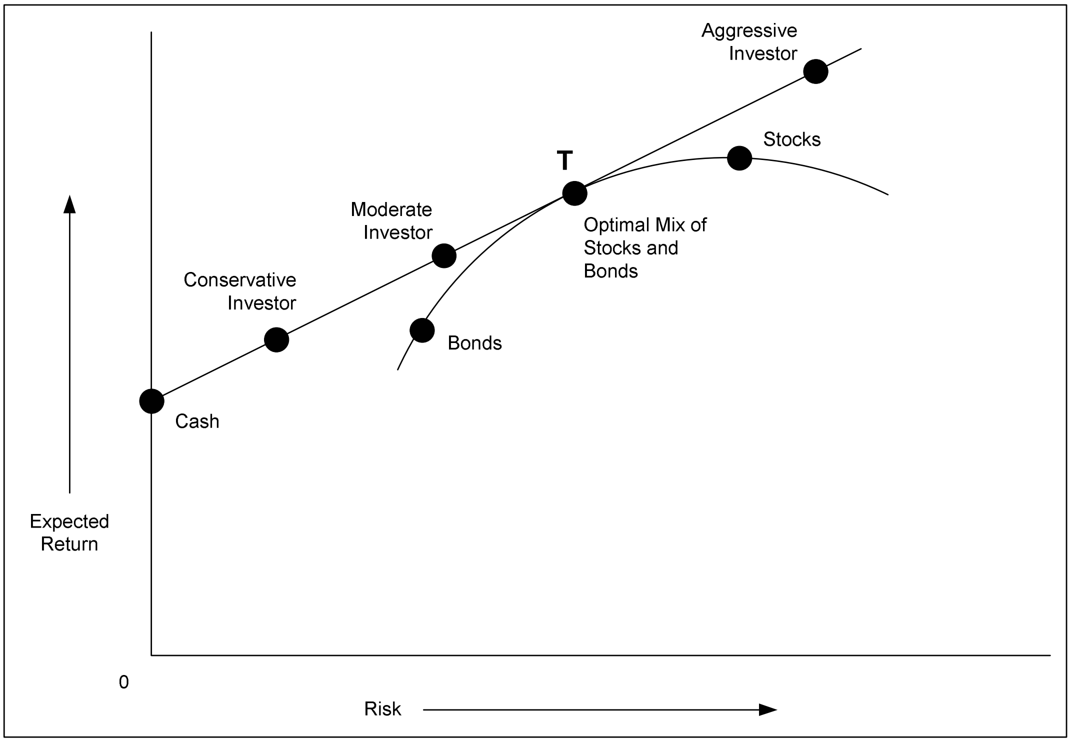


Source: Gibson, 2000, p. 125.

3.5 THE JAMES TOBIN SEPARATION THEOREM

The next development occurred when Tobin (1958) developed a model, premised upon the mean-variance model that led to the identification of a tangency portfolio, later to be known as the market portfolio, along the efficient frontier (Fama and French, 2004, p. 4). Tobin's model had a key assumption, namely that cash was riskless (Tobin, 1958, p. 67).

Figure 3.8 Tobin's separation theorem explained



Source: Derived from Tobin, 1958, p. 73 with axes reversed.

With reference to Figure 3.8 it is noted that the curved line denotes the combination of risky assets, otherwise known as the efficient frontier. Assuming cash has a fixed rate, and no risk, this would be represented as the intersect on the vertical axis. Therefore when cash is added to a portfolio, the asset allocation would be a straight line. Assuming that an investor is only concerned about the rate of return and the risk, an optimal portfolio would be somewhere along the straight line (Campbell and Viceira, 2002, p. 3). What is most striking, based on Tobin's assumptions, is that the point at which the straight line touches the curved line, as indicated by T, is known as the 'tangency portfolio', and is the optimal mix of risky assets.

Given the assumptions, all investors would hold the same combination of risky assets. All that would happen is that investors would manage their risk exposure by adding, deducting or lending cash. If all investors held the same risky portfolio then the answer would be a simple case of passively investing in the optimal mix.

Tobin's model is referred to as the separation theorem since the allocation of resources amongst risky assets is seen as a separate decision to the level of cash within a portfolio. Cash is added to the portfolio after the risky asset allocation to moderate risk, based on an investor's tolerance therefore.

Given Tobin's separation theorem, it would seem that all investors should hold the same risky portfolio. There are criticisms however, which are largely centred on the assumption (Campbell and Viceira, 2002, p. 3). With reference to Figure 3.8, it is observed that the cash component, over the short term, is plotted on the vertical axis, which implies zero risk[21]. One of the primary concerns is that over the long term cash is not a riskless asset, with future interest rates, and inflation, producing a return variance. This variability implies risk, as measured by standard deviation, and would result in the cash component forming part of the efficient frontier, thereby no longer being plotted on the vertical axis. This would imply that over the long term the investor would select an optimal portfolio based on the mean-variance model precepts, which could have asset

[21] In the presence of inflation the nominal investments are not riskless in real terms, however over one time period this risk is small (Campbell and Viceira, 2002, p. 2).

allocations significantly different from the short term investors ‘tangency portfolio’, and is a reasonable argument to dispute the efficacy of passive investing given that investors are likely to have differing investment horizons. Further to this argument Fama and French (2004, p. 7) contend that since risk free cash, and short selling are unrealistic, investors would still choose efficient portfolios. In this regard the selected portfolio would not necessarily be the market portfolio since the algebra of portfolio efficiency does not default to the market portfolio.

Given the criticisms, passive investing advocates have tended to move away from theoretical hypotheses, towards statistical models confirming that passive investing is hard to outperform. In this regard, given the assumption of a common market, Sharpe (1991, p. 8) asserts that the weighted average active return will equal that of the passive return before costs, since in aggregate they comprise the market. Therefore, since active investors incur higher costs due to trading and research, it follows that returns for the passive investor must be higher than the aggregate after cost return from active investing.

The Tobin separation theorem is elegant in that it is parsimonious in regards to the explanation of risky and less risky assets, and the combining thereof. The research contends that the model is not entirely superfluous based on the fallacious assumption of risk free cash. Although outside the ambit of this research, the research takes the position that an efficient market portfolio can be derived, that is applicable to all time periods. The only aspect requiring adjustment, as set forth by Tobin (1958), would be the amount of cash included in a portfolio, to mitigate risk. This position would simply be achieved by solving for the efficient market portfolio first, which, by definition, would mean solving for the risky assets. Once a market portfolio is established, the resultant outcome can be combined with cash to solve, based on the investment horizon that may be unique to the investor.

3.6 CAPITAL ASSET PRICING MODEL

The capital asset pricing model[22] is what its name suggests, a hypothesis developed by William Sharpe and John Lintner to price assets (Fama and French, 2004, p. 1), and is an extension of Tobin's separation theorem.

Like Tobin, Sharpe and Lintner's model had a few key assumptions. Firstly, that there is complete agreement, given the market clearing asset prices, the distribution of asset returns and that the distribution is the correct one. Secondly, borrowing and lending takes place at the risk free rate, which is the same for all investors regardless of amount (Fama and French, 2004, p. 3).

As with Tobin's separation theorem, all investors will hold the same mix of risky assets, known as the market portfolio, and since the market has to clear, the market portfolio has to be based on the efficient frontier.

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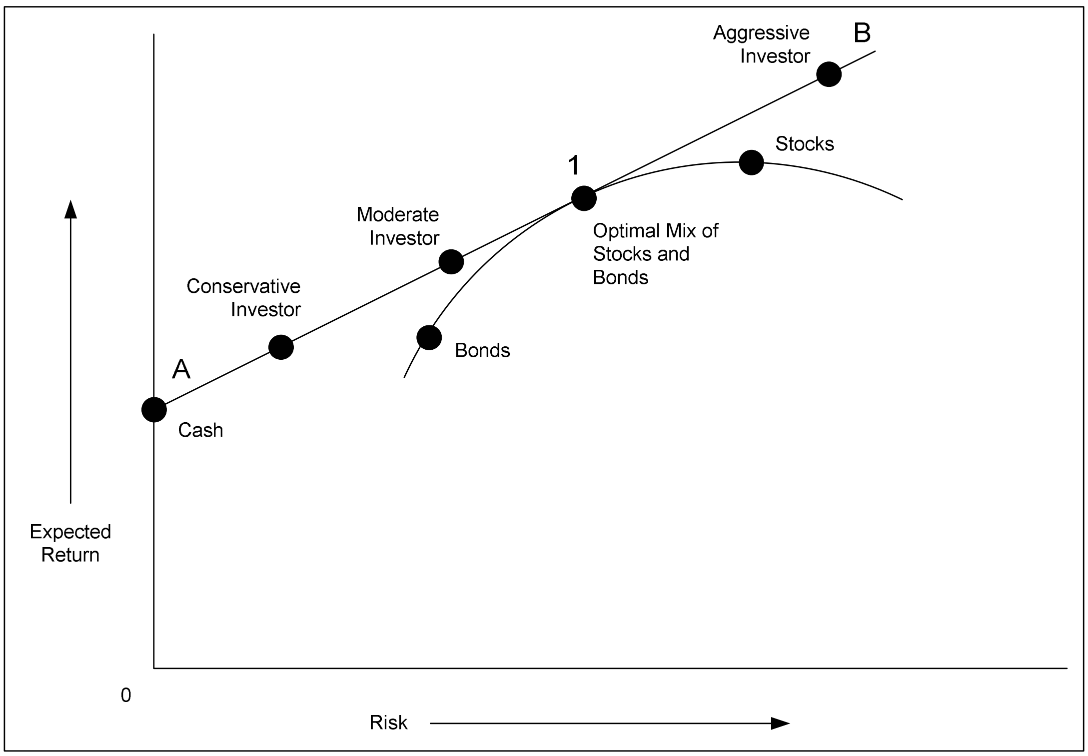
[22] $CAPM = r_f + \beta_i(k_m - r_f)$, where r_f = risk-free rate, β_i = beta and k_m = portfolio return (Fama and French, 2004, p. 6).

With reference to Figure 3.9, the model measures the risk of an asset, or portfolio, relative to the market, where all feasible assets or portfolios are found along the AB line. The AB line is also commonly referred to as the capital market line. The measure of risk is called beta. Beta is determined by conducting a linear regression[23] analysis.

Figure 3.9 Capital asset pricing model explained



Source: Derived from Figure 2.5.

The slope of the linear regression line is measured by beta. Therefore the market would have a slope, or beta of one, as indicated. It is assumed that a portfolio with a higher rate of return than the market would have to incur higher

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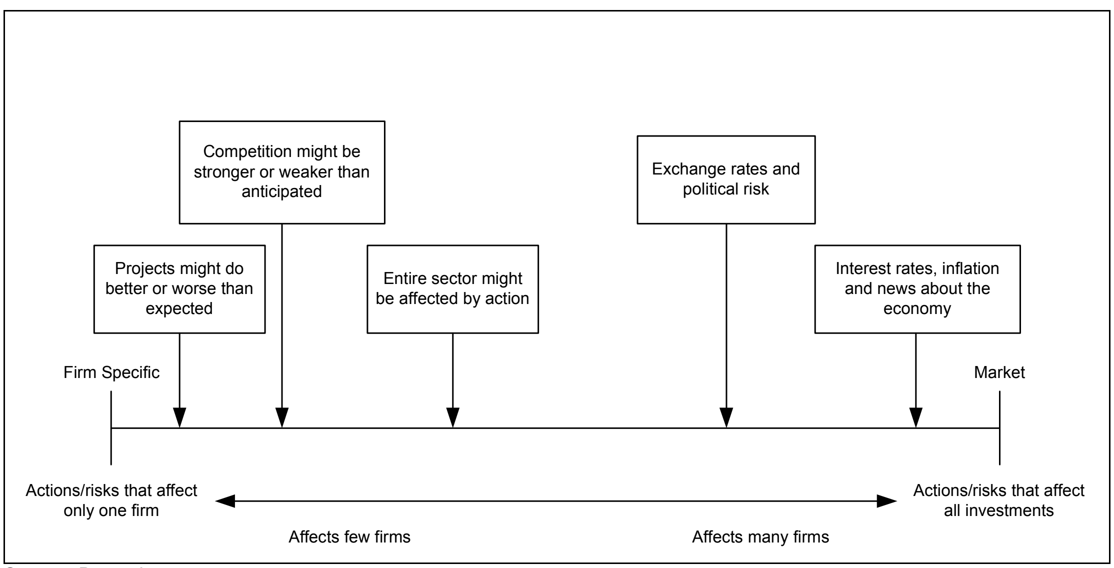
[23] $Y = a + \beta X$, where a = alpha (Y axis intercept) and β = beta (slope of the linear function) (Wisniewski, 1997, p. 318).

risk (specifically diversifiable risk), as measured by beta. This would result in the slope of the regression line being greater than one. Furthermore, a portfolio of risk free assets would have a beta of zero, which implies zero correlation to the market portfolio.

One of the functions of the capital asset pricing model is to explain diversification. The hypothesis revolves around two forms of risk, diversifiable and non-diversifiable, or market risk (Damodaran, 2003, p. 18 - 19). The premise is that investors would not assume a risk that could be diversified away, and that there should not be an expectation of a higher reward should such a diversifiable risk be accepted (Loeper, 2001, p. 2). With reference to Figure 3.10 it is clear that diversifiable risk can be diversified away and hence should not be rewarded.

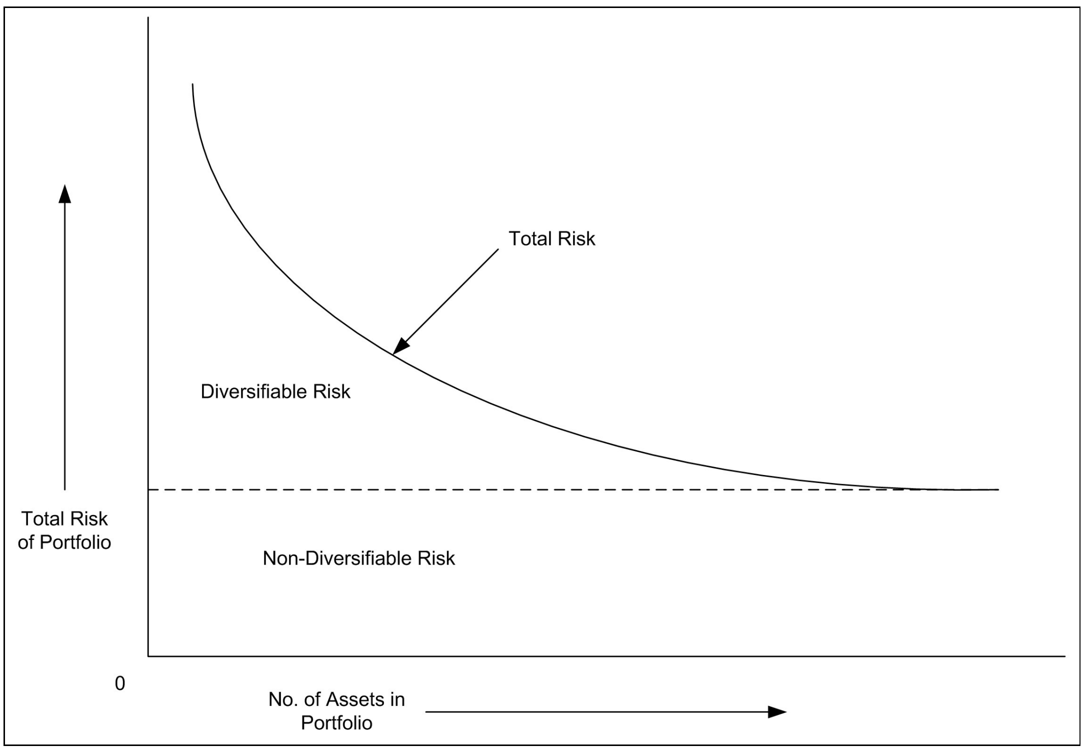
Figure 3.10 Risk breakdown



Source: Damodaran, 2003, p. 19.

Diversifiable risk is that component of risk that is easily diversified away by holding more than one investment, in more than one asset class, across more than one market. Figures 3.10 and 3.11, graphically demonstrate how an investor should hold more than one asset, across an array of classes, markets and currencies in order to be optimally diversified.

Figure 3.11 Diversifiable and non-diversifiable risk



Source: Adapted from Marx, Mpofu and Van de Venter (2003, p. 38).

The component of risk that cannot be diversified away, otherwise known as non-diversifiable risk, is reflected by beta. Given the beta component, theoretically an investor could calculate the expected rate of return on a portfolio for an assumed level of risk.

Therefore, if the investor accepts the premise of the capital asset pricing model, diversification can be explained. Assuming an investor was risk averse and was only willing to accept a beta of 0.5, this could be achieved by selecting a single equity that yields a beta of 0.5. However, the investor would be exposed to excessive diversifiable risk in the form of event risk (Loeper, 2001, p. 3). The capital asset pricing model assumes that such excessive risk is not rewarded since a more appropriate way of constructing a portfolio with a beta of 0.5 would be to combine the market portfolio, with a beta of one, with asset classes with lower betas. In this way the same expected return is achieved with the maximum of diversity.

In the world of academia the capital asset pricing model is parsimonious, and therefore elegant, however not without criticism. The foremost critics, Fama and French (1992, p. 464), proclaim that the capital asset pricing model is not a good method of calculating a value or size premium, since beta shows no correlation to the returns manifested for alternative asset classes relative to the market. In other words, asset classes that yield sustainably higher returns than the market should yield a higher beta. In Fama and French (2004, p. 1) the assertion is more forceful. It is proclaimed that the capital asset pricing model empirical results are so poor as to invalidate the way in which the model is used, and that the model is not a good proxy for expected returns.

In a period specific replication of the Fama and French (1992) methodology this is tested for within the U.S. market, with reference to Table 3.1, where the correlation coefficient of -0.5793 indicates a low correlation between beta and returns. If anything this tends to indicate a slight inverse relationship. An analysis of the determination coefficient yields that a mere 33.56 percent of returns are attributable to beta.

Table 3.1 U.S. market beta correlation analysis (1973 – 1992)

	Beta	Geometric Returns
S&P 500	1.0000	11.33%
FF L Cap Growth	1.0741	9.81%
FF L Cap Value	0.9160	15.69%
FF S Cap Growth	1.3969	9.25%
FF S Cap Value	1.0327	18.71%
Correlation Coefficient	-0.5793	
Determination Coefficient	0.3356	

Source: Derived using U.S. data for the period 1973 – 1992. (Archive Reference: Thesis Data I/Beta Analysis.xls)

Fama and French (1997, p. 16) upheld their view with regards to international markets as well. This is demonstrated within a South African context, with reference to Table 3.2, where a correlation coefficient of -0.6519 again indicates a low correlation between beta and returns, indicating an inverse relationship. An analysis of the determination coefficient yields that a mere 42.50 percent of returns are attributable to beta.

Although Tables 3.1 and 3.2 are not an exhaustive replication of the Fama and French studies, the period specific results do lend support to the hypothesis that beta is a poor predictor of returns. In support of the above findings Van Rensburg and Robertson (2003, p. 15), using a comprehensive methodology to test the capital asset pricing model on the JSE Securities Exchange, concluded that their study provided an 'unambiguous empirical contradiction'.

Table 3.2 South African market beta correlation analysis (1973 – 1992)

	Beta	Geometric Returns
ALSI	1.0000	19.98%
MP L Cap Growth	1.2034	14.10%
MP L Cap Value	0.9924	20.39%
MP M Cap Growth	0.8539	16.40%
MP M Cap Value	0.7740	24.20%
MP S Cap Growth	0.7150	18.35%
MP S Cap Value	0.6798	24.07%
Correlation Coefficient	-0.6519	
Determination Coefficient	0.4250	

Source: Derived using South African for the period 1973 – 1992. (Archive Reference: Thesis Data I/Beta Analysis.xls)

Sharpe (Burton, 1998, p. 24), the founder of the capital asset pricing model, responds to the Fama and French (1992) findings by insisting that Fama and French have measured historical returns relative to beta, whereas the capital asset pricing model was designed to measure expected returns. Sharpe adds that the Fama and French findings were period specific, and are subject to change.

In further contrast, however, the capital asset pricing model measures risk relative to the market, by using the slope of a linear regression line. The variability is therefore the positioning of the data points of a portfolio relative to the market's data points. Therefore the linear regression line, which determines beta, results in a 'best fit' line between the data points (Loeper, 2001, p. 6). With reference to Tables 3.1 and 3.2, using the coefficient of determination (R^2) to measure how well the data points fit the linear regression line, it is observed that in some instances the 'fit' is not all that good. The lower the coefficient of determination (R^2) the poorer the 'fit', and therefore the lower the forecasting ability of beta (Loeper, 2001, p. 6).

Nawrocki (n.d.b, p. 8) proclaims that the capital asset pricing model, as a tool for practitioners, is too simplistic. Loeper (2001, p. 5) postulates that to conclude that investors use the capital asset pricing model to measure risk is erroneous. He points out, affirming Nawrocki's view, that if all it took to outperform the market was a higher beta, many more managers would achieve this, which seems to be elusive[24]. Loeper concludes that the value of the capital asset pricing model is in the premise that it ‘makes sense to diversify’.

Finally, Fama and French (2004, p. 21) conclude, expectedly, that their three factor model is a better predictor of expected returns since it accommodates variables that seem to explain market pricing more effectively than the capital asset pricing model.

The true test of any model is in its application, irrespective of the ubiquity of a model. In this regard the capital asset pricing model is not successful, albeit that the model is viewed favourably by academia. The research does not suggest that the capital asset pricing model is flawed due to its assumptions merely that the model, through the use of beta, is unable to explain much of the returns realised by the market. For this reason the capital asset pricing model is rejected as an effective asset pricing model, in light of more favourable alternatives, namely the three factor model and the mean-variance model.

3.7 THE SHARPE RATIO

Sharpe (n.d., p. 1) indicates that he introduced a performance measure for mutual funds known as the reward-to-variability ratio, which has become

[24] See Section 3.3 (p. 76).

ubiquitously known as the Sharpe Ratio[25], and is simply a risk-adjusted rate of return measure. He states that the Sharpe Ratio builds upon the Markowitz algorithm where the mean and standard deviation are considered sufficient statistics for evaluating a portfolio. In this regard he indicates that the Sharpe Ratio does not take correlation into account, is not independent of the time period and therefore cautions that any information derived from the Sharpe Ratio should be supplementary (Sharpe, n.d., p. 14).

Sharpe (n.d., p. 4) points out that the ubiquitous use of the Sharpe Ratio is subject to confusion, and therefore misused. The primary area of confusion surrounds the return variable in the equation. This variable must be net of the riskless rate of return, otherwise known as the differential return. He explains by way of an example (Sharpe, n.d., p. 5).

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[25] SR = $\frac{(r_p - r_f)}{\sigma_p}$ where: $(r_p - r_f)$ = return in excess of the risk free rate and σ_p = standard deviation of the portfolio (Marx *et al.*, 2003, p. 234).

Table 3.3 Sharpe ratio explained

	Fund X	Fund Y
Riskless Rate	3%	
Return	5%	8%
Standard Deviation	10%	20%
Incorrect Sharpe Ratio	0.5	0.4
Correct Sharpe Ratio	**0.2**	**0.25**

Source: Sharpe (n.d., p. 5).

With reference to Table 3.3, it is noted that using the incorrect Sharpe Ratio, without adjusting for the riskless rate of return, fund X seems superior to fund Y with a Sharpe Ratio of 0.5 to 0.4. When the differential rate of return is utilised, it is noted that fund Y is in fact superior to fund X, as indicated by the bold text.

Sharpe (n.d., p. 13) concludes that the merits of the ratio are that the measure accounts for both risk and return, and therefore is inherently superior to any measure of return only.

The Sharpe ratio is a parsimonious manner in which to evaluate risk-adjusted returns. The research contends that the Sharpe ratio is an effective method of measuring performance and for this reason uses the Sharpe ratio to assist in selecting the appropriate portfolio found along an efficient frontier.

3.8 EFFICIENT MARKET HYPOTHESIS

The efficient market hypothesis is a theory that describes a situation where competition amongst many intelligent participants leads to a market whose pricing reflects all available information. Therefore at any point in time the market prices are a good estimate of intrinsic value (Fama, 1995, p. 76). This does not mean that the intrinsic value is known exactly, merely that the intrinsic value is a good estimate. Since the world is uncertain, the actual intrinsic value is never truly known, therefore pricing will wander randomly about, hence the term a random walk.

The efficient market hypothesis evolved from what was known as the random walk in stock prices, and although earlier works had alluded to the stochastic nature of equity prices it was not until Eugene Fama produced a definitive synthesis on market efficiency that it became a mainstream hypothesis (Evensky, 1997, p. 199).

Given the definition, the efficient market hypothesis holds profound implications for active investors. These are presented below.

a) Equity research and valuation is a costly task that provides no benefits.

b) A strategy of indexing, or diversifying across assets, with no information and little transaction costs would be superior to any other strategy.

c) Minimal trading is superior to active trading.

It appears evident that active investors[26], on average, seemingly do not produce market outperformance. The primary reason given for the failure of active investors to produce sustainable investment returns in excess of the market is a result of the efficient market hypothesis.

Ross (2002, p. 55) points out that it is not important that the majority of investors have access to, or are even aware of, all available information at all times, merely that the information is available. What this means is that the price investors pay is not related to their level of sophistication. The information relating to the price of an investment does not reside with the investor but rather in the price. For this reason it takes a 'relatively small fraction' of participants, amongst a large investor population, to ensure an efficient market (Ross, 2002, p. 55).

Importantly it must be noted that market efficiency does not mean (Damodaran, 2003 p. 142) that:

a) prices cannot deviate away from true value. This may occur, however these deviations need to be stochastic in nature, and therefore unpredictable;

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[26] See Section 3.3 (p. 76).

b) no investor will beat the market in any given time period. In fact it is expected that approximately half of all investors will beat the market in any time period; and

c) no investor will beat the market over the long term. The condition is that no disproportionate number of investors should beat the market using the same investment strategy.

Fama (1995, p. 76) proclaims that it is unlikely that the efficient market theory provides an exact description of market behaviour, and for this reason the hypothesis was developed to reflect three levels, with different definitions of information.

a) The strong form of the efficient market hypothesis. At this level all information both public and non-public (including insider information) are reflected in the market prices.

b) The semi-strong form of the efficient market hypothesis. At this level all publicly available information is reflected in the market prices.

c) The weak form of the efficient market hypothesis. At this level market prices reflect all stock market information only, and can be explained by the random walk theory.

Empirical tests on the three forms generally support the weak form, reject the strong form and are undecided on the semi-strong form (Evensky, 1997, p. 200). In this regard, with reference to the JSE Securities Exchange, Gilbertson and Roux (2002, p. 5) conclude that their results were 'unequivocal: neither trading rules nor mutual fund managers were able to outperform the market'. They conclude that their findings are consistent with the efficient market hypothesis.

Interestingly the efficient market hypothesis is not universally accepted. The behavioural finance school of thought espouses that markets are inefficient (Thaler, 1992, p. 153), and investors are irrational. Therefore pricing anomalies could be exploited by a rational investor. In this regard the poor track record by active investors is a result of the lack of 'smart money' (Shefrin, 2002, p. 89). Since investor irrationality is ubiquitous there is a cancelling out effect, leading the behaviouralists to conclude that investors would be better off 'holding a well diversified set of securities, mainly in index funds' rather than trying to outperform the market (Shefrin, 2002, p. 89).

Given the difficulty that active investment managers' experience in beating[27] the market there is certainly a level of efficiency prevalent in the market. For this reason the research contends that the semi-strong form of efficiency is more likely to be representative of market behaviour over the short term. This implies

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[27] See Section 3.3 (p. 76).

that any attempts at market timing as well as stock selection are futile, which is tantamount to a rejection of active investment management.

3.9 MEAN REVERSION

It is prudent to infer that in order to construct forward looking investment portfolios, using historical asset allocations that future returns, although volatile and unknowable, should display movement around a known average, or there would be no value in using historical data. This is counter to the efficient market hypothesis that postulates that, due to the stochastic nature of returns, their movement is unpredictable (Fama, 1995, p. 76) therefore any movement away from the mean would not predict a pending movement towards the mean.

Malkiel (1999, p. 244) suggests that markets overreact in the short term, although, due to the stochastic nature of the markets, the rationale investor would not be able to identify such reactions in order to capitalise on, offering the ensuing quote, “Although the market may not always be rational in the short-run, it always is over the long haul” (Malkiel, 1999, p. 242).

Interestingly Eugene Fama synthesised the theory of efficient markets and yet in later research (Fama and French, 1988, p. 247), through the use of serial correlation tests, Fama and French suggest that there is ‘mounting evidence that stock returns are predictable.’ At first glance this seems paradoxical. If there is a level of predictability then this would refute the efficient market hypothesis, providing an opportunity for rational investors to exploit the predictability in order to achieve above market rates of return. Thaler (1992, p. 153), a behavioural economist highly critical of Fama, adds that taking a long term view (three to seven years) equities show significant mean reversion characteristics and therefore suggests that the efficient market hypothesis is not valid.

This view, that the efficient market hypothesis is not valid, is what active investors base their strategies of stock selection and market timing on, through the application of technical and fundamental analysis.

Although it is acknowledged that popular media sources are not considered reliable for academic purposes, in light of the fact investment managers are not considered to be academics, a gauge on their sentiment will not be garnered from academic journals. For this purpose alone a quote is included from a South African market practitioner:

“Indexation essentially relies on the efficient market hypothesis, that at any given time, asset prices fully reflect all available information. Active portfolio managers, on the other hand, believe they can beat the market by using market timing and exploiting anomalies and behavioural finance patterns. In SA, in particular, where markets are relatively inefficient and illiquid, there is a strong case for active portfolio management” (Financial Mail, 4 January 2002, p. 34).

When both arguments are analysed it is apparent that both hypotheses are valid. Markets may display efficiency and mean reversion. In the short term stochastic pattern behaviour makes markets inherently unpredictable, and therefore efficient. A longer term view provides evidence of predictability (Edleson, 1997 p. 150). This predictability however is only available to longer term investors, and the exact timing of such predictability remains elusive. In this regard John Maynard Keynes made an interesting observation, “Markets can remain irrational longer than you can remain solvent” (Lowenstein, 2002, p. 123).

Importantly, the implication is that long-term passive investors may enjoy the benefits of mean reversion, with Malkiel (1999, p. 360) emphasising that mean reversion is applicable to markets and asset classes but not individual stocks. This acts as a caveat to investors seeking extraordinary investment returns through stock picking.

Finally, the ultimate test regarding efficiency is whether active investors are consistently able to beat the market, thereby capitalising on what appears to be predictable market behaviour. The answer appears to be no, and may suggest that active investors are too focused on the short term.

3.10 MEAN-VARIANCE OPTIMISER

With the advent of computer power, the 'critical line algorithm' (Kaplan, 1998, p. 2) was incorporated into a software application, known as a mean-variance optimiser (Bernstein, 2000a, p. 64 - 65). The mean-variance optimiser, in turn, would produce the efficient frontier[28], developed during the mean-variance analysis process. This efficient frontier, generated by the mean-variance optimiser, of which Annexure 72 is an example thereof, is made up of a number of efficient portfolios. These portfolios, in turn, are made up of the given assets in accordance with the determined allocations. Therefore, the mean-variance analysis process, by means of a mean-variance optimiser, solves the asset allocations for a given portfolio found on the efficient frontier. With regards to Annexure 72 the critical line algorithm solved for the middle portfolio, indicated on the efficient frontier. The results thereof are shown in Annexure 73. For this process to occur requires the inputs of three sets of data, namely:

a) the asset return;

b) the standard deviation of the asset; and

c) the correlations between the assets, otherwise known as the cross-correlations.

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[28] See Section 3.4 (p. 79).

Therefore the positive feature, regarding the application of a mean-variance optimiser, is the simplicity of use. The process of establishing the efficient frontier, and solving the asset allocations for a selected portfolio, is a simple matter of entering the data.

It is prudent to note that the application of mean-variance analysis is not dependant on a mean-variance optimiser. The mean-variance optimiser acts as a tool to reduce the computational complexity and the propensity to commit errors in the computational process.

3.11 MEAN-VARIANCE OPTIMISER SHORTCOMINGS

Application of the mean-variance analysis process[29], through the use of a mean-variance optimiser, is not without serious shortcomings however. The process displays considerable sensitivity to changes in the data inputs, which significantly alters the asset allocations within a portfolio (Bernstein, 2000a, p. 69), and thus has a tendency to favour assets with a recent history of high returns.

'… the most important limitations to MV *(mean-variance)* optimisation are instability and ambiguity' (Michaud, 1998, p. 3).

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[29] The terms mean-variance optimiser, and mean-variance optimisation are to be used inter-changeably, with both referring to the mean-variance analysis process.

Evensky (1997, p. 230) states that the crux of the objections to mean-variance optimisation are lodged in the 'arrogance of absolutes'. Mean-variance optimisers are too exact, and any statistical errors regarding the computing of the inputs will be maximised. This then leads to portfolios that appear counter-intuitive.

Lummer, Riepe and Siegel (1994, p. 3) state that, if the data inputs were free of estimation error, an optimiser, or optimisation, would be guaranteed to find the optimal portfolio asset allocations, and this is the imperative, since because the data inputs are estimates, typically based on historical performance, they cannot be devoid of error. Lummer, Riepe and Siegel (1994, p. 9) postulate further that the principal cause of mean-variance optimiser instability is the inclusion of highly correlated assets. In this regard Bernstein (2000a, p. 46) states that effective portfolio construction is premised on the use of poorly correlated assets. Therefore by utilising poorly correlated assets some of the optimiser instability can be eliminated.

Michaud (1998, p. xiii) postulates that a primary shortcoming in the use of a mean-variance optimiser is that there is a relatively 'low level of analytical sophistication', or mean-variance optimiser competence, in the culture of institutional investors, which is a result of the lack of a statistical understanding of how the optimiser utilises data inputs. The incorrect use of data often leads to portfolios having little investment value, where a naïve portfolio is often closer to an optimal portfolio determined by using mean-variance optimisation (Michaud, 1998, p. 3).

Finally, it is apparent that the instability pertaining to mean-variance optimisers is largely a result of the incorrect use of data, where the mechanical use of historical data may be inappropriate. In this regard Bernstein (2000a, p. 71) cautioned that optimising raw historical returns is a 'one-way ticket to the poor house' thereby suggesting that the data inputs require attention if a mean-variance optimiser is to be of any value.

3.12 BEYOND THE SHORTCOMINGS

Although there are many mean-variance optimiser shortcomings, the limitations largely arise as a result of viewing a mean-variance optimiser as providing an absolute answer. This view was asserted by Michaud (1998, p. 40) who postulated that a paradigm shift could result in new procedures that can reduce or eliminate many of the deficiencies.

At this juncture it would be prudent to briefly review the literature regarding the alternative approaches to asset allocation. Evensky (1997, pp. 224 – 233) astutely sets out the alternatives, as summarised below.

a) Model portfolios, which are the simplest solution for investors, and consist of asset allocation portfolios developed by investment product providers, using models that are designed using the product providers assumptions. These portfolios generally lack customisation, and the portfolio may deviate from the underlying model. Moreover, the investor may be implementing an asset allocation strategy without knowing how the allocations were determined.

b) Judgemental intuition is an approach that relies solely on intuition. In this regard Evensky (1997, p. 226) acknowledges that although the process may be based on intuition, judgement should be based in science, which should form the foundation of the process. The intuitive process is subject to heuristic bias, and is an inadequate defence in the event of investor asset allocation failure.

c) Multiple scenario analysis is based on developing a best, worst and most probable scenario. This approach requires significant amounts of intuition, as well as the estimation of an inordinate amount of input variables for each scenario. This process is therefore prone to significant amounts of error.

Evensky (1997, p. 233) concurs with Michaud (1998, p. 23) by proclaiming that the knowledgeable use of a mean-variance optimiser is superior to any alternative, and that although a mean-variance optimiser has tremendous shortcomings there is a need to pursue all efforts to rationally determine asset allocations, otherwise the alternative is solely dependant on 'judgement and intuition' (Evensky, 1997, p. 237).

3.13 SUMMARY

Asset allocation and diversification are clearly investor imperatives. The level of contribution to returns, provided by diversification, seems less important than the selection of the appropriate asset classes, which provides optimal risk diversification. The mean-variance model provides an appropriate approach to asset allocation since, in this regard, cross-correlation coefficients are included as a variable, which gives rise to a return enhancement, as indicated by Figure 3.6 (p. 83). The capital asset pricing model seems not to be a good indicator of returns, based on beta. Moreover, the determination of asset allocations is subjective, being solely reliant on the management of beta. In this regard the mean-variance model is more robust, allowing for asset class selection, as well as the allocation between the selected assets.

The success of diversification is in the elimination of diversifiable risk, through the investment in multiple assets that provide maximum diversity, thereby reducing event risk that is firm, sector or market specific. Insofar as the accrual of benefits, due to diversification, is concerned, these should be treated as a windfall gain since cross-correlation coefficients are dynamic in nature. The magnitude of such a windfall gain would remain unknowable, and is merely an additional benefit as a result of diversification. It is merely the future magnitude thereof that is unknowable due to the dynamic nature of equity markets.

With reference to Figure 3.2 (p. 71) it is noted that asset allocation decisions are less of an imperative for active investors, however even if this is the case, active investors seem unable to produce sustainable market beating performances. In this regard the literature provides support for the theory that a passive investment approach that minimises the amount of trading activity predominantly produces superior returns relative to any active alternative.

With passive investment management showing a seeming tendency to prevail, the ensuing imperative becomes one of determining a rational method of selecting the appropriate asset classes and the allocation of assets between such classes. In this regard the mean-variance model provides an alternative in the form of the efficient frontier which comprises portfolios at the one end that minimise risk for a given level of return, and at the other maximises return for a given level of risk. The portfolios, in turn, comprise the allocation of assets between asset classes.

With the advent of computing power the mean-variance model algorithm has been incorporated into a software system, known as a mean-variance optimiser, thereby making the use of the mean-variance model algorithm far more accessible. The literature however highlights the risks of using a mean-variance optimiser, by indicating that the incorrect use of data may lead to portfolios that are less than efficient, and appear counter-intuitive. On the converse side theoreticians are of the view that a mean-variance optimiser is superior to alternative approaches to asset allocation, however that attention should be focused on the methodology applied to the selection of appropriate data for use in the mean-variance optimiser.

The ensuing section will review the imperatives of data and asset class determination. Since sufficiently accurate data inputs are fundamental to the successful use of a mean-variance optimiser, it is imperative that approaches to the appropriate use of data are examined. Related to this will be the discussion involving the use of stochastic simulation modelling, as well as the identification of appropriate asset classes.

CHAPTER 4

DATA AND ASSET CLASS DETERMINATION

4.1 INTRODUCTION

The imperative of building a portfolio which contains the appropriate asset classes to which there are the appropriate asset allocations is subject to the correct application of input data, as indicated in the previous section. Therefore, it is of primary importance that the use of data is understood, as the portfolios derived using the mean-variance model are an integral part thereof.

Chapter 4 will focus primarily on the review of the literature regarding the determination of data inputs for the mean-variance optimiser, as well as the selection of appropriate asset classes. The section begins with an overview of the various methodologies proposed in the literature regarding input data determination, followed by a review of the literature on data and asset class constraints.

Finally, the section will conclude with a review of the literature regarding asset class determination based on size and style characteristics.

4.2 DATA INPUT DETERMINATION

Data determination is the imperative if the mean-variance optimiser is to produce optimal asset allocation outcomes. In this regard Nawrocki (n.d.a, p. 3) suggests seeking solutions that:

a) reduce the impact of estimation errors;

b) lead to more diversified portfolios; and

c) provide stable portfolios that change allocations slowly over time.

In order to overcome the issue of 'absolutes' (Evensky 1997, p. 230) Nawrocki (n.d.a, p. 1) suggests that instead of seeking the 'very best solution' using a set of data inputs, it is better to seek out 'an approximately good solution' by using what he terms portfolio heuristics, which simplistically is inputs derived through an intuitive trial-and-error process.

A search of the literature regarding data input determination for use in a mean-variance optimiser yields a number of alternatives for forecasting future returns, the most important data input required by the mean-variance optimiser. These methods are summarised below.

a) The first is historical data, on the premise that returns, over the long term, are mean reverting (Bogle, 1999, p. 225, Fama and French, 1988, p. 247, Thaler, 1992, p. 153 and Edleson, 1993, p. 150).

b) DR (Market Return) = Dividend Yield + Dividend Growth (or Earnings Growth) (Bernstein, 2002a, p. 52).

c) Forecasted Expected Return = Historical Risk Premium + Current Risk-Free Rate, is a derivative of the capital asset pricing model without the use of beta, where the current risk-free rate is added to a historical market risk premium for the particular asset class, where the risk-free rate is the treasury bond with a maturity equal to the optimiser time horizon, and the historical risk premium is the historical risk-free rate subtracted from the asset class return, over a long-term time horizon (Evensky, 1997, p. 242).

d) Another method is data input constraints, whereby data inputs are adjusted subjectively using 'art' (Evensky, 1997, p. 235) thereby referring to intuition, experience and common sense in order to arrive at a set of data inputs that would provide an 'an approximately good solution' (Nawrocki, n.d.a, p. 1).

e) Finally, there are data resampling techniques, whereby data inputs are determined using stochastic simulation methods (Jobson and Korkie, 1980, p. 547, and Michaud, 1998, p. 35).

4.2.1 Historical data techniques and their derivatives

Methods a) – c) in Section 4.2 are dependant on historical data to some extent and in this regard there are relevant caveats in the literature.

Bernstein (1998, p. 1) draws attention to the fact that the use of raw historical data will produce asset allocations that were relevant in the past, and are therefore no longer relevant, and adds that the rational use of a mean-variance optimiser would be to seek out the 'reasonable allocation' (Bernstein, 1998, p. 2). An additional anomaly attached to the use of raw historical data is the period from which the data is derived. The performance characteristics of a particular period may have no resemblance to ensuing periods. In this regard Bernstein (2000a, p. 70) draws attention to the mean reversion hypothesis where assets have a tendency to mean revert over time, thereby resulting in inordinately different portfolio asset allocations from one period to the next. In this regard is the selection of an appropriate time period that would reduce the asset allocation volatility. Further support for the risk of using raw historical data is provided by Evensky (1997, p. 235) where it is postulated that the unquestioning acceptance of asset allocations derived from using a mean-variance optimiser is 'likely to be a threat' to one's financial well being.

Since the use of input data is intricately linked to the final asset allocation, it is worth re-examining some of the comments on asset allocation. The most vociferous has been in Jahnke (1997, p. 111) where it was postulated only if future returns were expected to remain static, in other words were an exact replica of past performance, that long-term asset allocations should remain static. This is an important issue since capital markets are dynamic; therefore Jahnke asserts that any asset allocation policy should be dynamic in nature, which implies that the data inputs should be dynamic. This is a strong argument against the use of pure historical data, and for this reason the research takes the view that historical data will act as a starting point only, being used by a stochastic data input determination process, thereafter new data inputs will be used.

4.2.2 Data input constraints

In this regard Evensky (1997, p. 236 - 247) espouses the use of input constraints to tame the unstable outputs of a mean-variance optimiser, and postulates certain concepts.

a) Mean-variance optimiser inputs should be based on five year estimates, with the exception of standard deviation, which should be based on a one year estimate, which clearly exaggerates the expectant risk, but is based on the investor's downside 'investor psychology', namely, that investors are unable to stay the course for longer periods during declining markets.

b) For cross-correlations, historical inputs should be used (10 years), not because they are more stable, but because the mean-variance optimiser is less sensitive to changes in cross-correlations. When in doubt Evensky (1997, p. 249) prudently assumes the assets will be more positively correlated.

c) For standard deviation, a combination of historical data (arithmetic average for 10 years), and judgement are suggested. If any changes are made it is prudently to adjust standard deviation upwards.

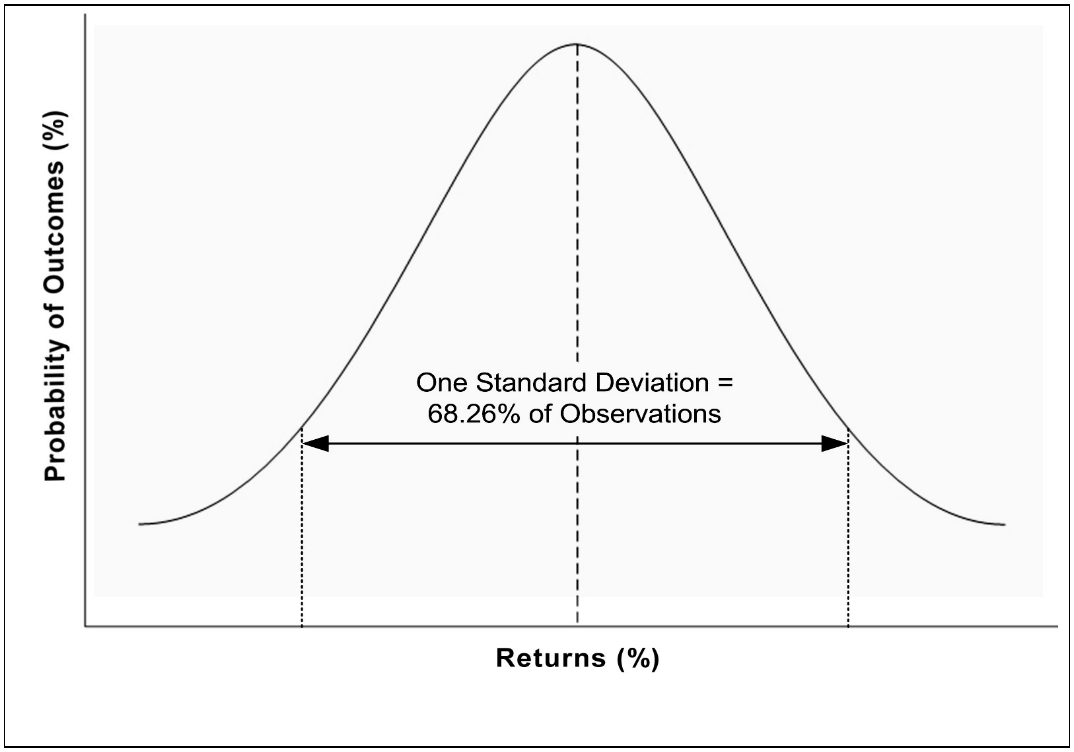
In contrast is the view propounded by New Frontier Advisors (2001, p. 6) that investors tend to manage mean-variance optimiser inputs to obtain portfolios, and asset allocations that seem reasonable to the investor. New Frontier Advisors conclude that such constraints are often subjective, and time consuming to implement resulting in limited investment value. This is an important point since it may be that investors are inclined to constrain inputs so as to derive asset allocations that appear intuitive by past standards. In this regard there is the risk of constructing a portfolio that is specific to a past period and therefore of no investment value.

The setting of constraints is intuitive from a data instability perspective; however constraints may tend to be biased thereby producing outcomes that seem intuitive to the investment manager. In line with the literature in this regard the research takes the view that data constraints are wholly inappropriate especially in light of the fact that an alternative approach to data input determination is considered.

4.2.3 Stochastic (Monte Carlo) simulations

The process of stochastic simulations, otherwise known as Monte Carlo simulations, is a mathematical technique that factors in randomness (Kautt, 2001, p. 72). A stochastic simulator is a simple computational concept of deriving random variables based on uncertainty, where uncertainty is measured by standard deviation. In a deterministic environment there is no uncertainty therefore, with regards to equity market returns, the future return variable would simply be a straight line extrapolation of past performance. With reference to Tables 6.1 (p. 159) and 6.2 (p. 160), it is apparent that the levels of uncertainty inherent in the equity markets are high, therefore future returns are unlikely to reflect the past. Given this uncertain environment, a probabilistic approach is recommended (Michaud, 1998, p. 62) to determine a more realistic future rate of return. In a probabilistic environment two variables are required, the rate of return (geometric mean) and the standard deviation. With these two variables the process of stochastic simulations can occur. During this process there is the repetitive calculation of outcomes based on the parameters (Wisniewski, 1997, p. 495). At the end of the process the calculated outputs are represented in a probabilistic form since the primary assumption with stochastic simulations is that the results are normally distributed.

Figure 4.1 Normal distribution



Source: Derived from Wisniewski, 1997, p. 159.

With reference to Figure 4.1, given that probabilistic analysis is premised on the precept of a normal distribution, the resultant outcomes would be arranged around the mean in a random fashion. Such simulations can be conducted endlessly, and the higher the number of iterations the closer the simulated average would be to the initial parameters. The number of iterations would be determined by the user of the model.

4.2.3.1 Input data resampling

With techniques involving the use of historical data not being without critics, (New Frontier Advisors, 2001, p. 6, Bernstein, 1998, p. 1 and Evensky, 1997, p.

235), data input determination requires a more scientific approach. In this regard Michaud (1998, p. 62) suggests that data input resampling leads to asset allocations that are more robust and intuitive relative to classic mean-variance analysis using historical data. New Frontier Advisors (2001, p. 6) add further that resampled portfolios are more stable and have the added benefits of simplifying the management of a portfolio and reducing the need to trade. In this regard data resampling is integrally associated with stochastic simulation modelling.

As regards the determination of mean-variance optimiser inputs Evensky (1997, p. 171) proposed that stochastic simulations be used to develop optimiser inputs, and although stochastic simulations had been experimented with in a study conducted by Jobson and Korkie (1980), it was Michaud (1998) who researched the concept further, developing an approach to equity markets.

In this regard resampling is based on a stochastic simulation procedure where resampled returns and standard deviations are derived stochastically using the original historical optimiser inputs (New Frontier Advisors, 2001, p. 7). These stochastically derived inputs are, in turn, used as inputs into the mean-variance optimiser. This procedure is repeated numerous times, with the resultant statistically associated resampled efficient frontier asset allocations being averaged to arrive at the resampled efficient portfolio (New Frontier Advisors, 2001, p. 14). Given the level of uncertainty inherent in determining inputs, the resampling process leads to many alternative outcomes based on the original inputs. The resampled efficient frontiers are statistically associated since they are associated by means of the original set of data inputs, used in the stochastic simulation process. The association of resampled portfolios is due to their positioning on the efficient frontier. All minimum variance, middle and maximum return portfolios are, by definition, respectively associated (Michaud, 1998, p. 46).

Insofar as the appropriate selection of the time period is concerned regarding the original mean-variance optimiser inputs, Jobson and Korkie (1980, p. 553) postulated, from their study using differing lengths of time, that the simulated variances and mean returns were comparable to the actual variances and mean returns for periods in excess of 25 years (300 monthly observations). Michaud (1998, p. 35) replicated the Jobson and Korkie (1980) experiment and concluded that the Sharpe ratio, a risk-adjusted performance measure, was lower for smaller time periods. The implications of these findings are that data input errors are elevated and overall portfolio performances reduced the smaller the time periods, ceteris parabis, thereby implying that the longer time periods are inclined to produce more stable results.

Another suggested advantage (New Frontier Advisors, 2001, p. 12) of stochastic simulations is that the portfolio outcomes produce a best average, and therefore typically avoid the period dependant character of deterministic strategies, since the stochastic simulator, by its very nature, given underlying assumptions, will continue to produce a broad spread of outcomes, which when averaged will provide a best average for most uncertain events.

In light of the fact that the research rejects the data input determination approaches suggested in Section 4.2 a) – d), the only alternative, given the current state of knowledge, that can accommodate equity market uncertainty is the method of resampling data inputs, as espoused by Michaud (1998), and is the method fundamental to the research.

4.3 ASSET CLASS DETERMINATION

Given that the universe for asset classes is restricted only by the imagination, namely that new asset classes can be created at will by combining equities that have some or other common characteristic, it is imperative that the literature be studied in order to review the position thereon.

The predominant research findings in this arena, albeit international, suggest that value equities, defined as being a high book value relative to market capitalisation, or a derivative thereof, produce higher returns than growth equities, defined as being a low book value relative to market capitalisation, or a derivative thereof (Fama and French, 1992, p. 445). Coupled to this is the finding that equity size has an influence on equity returns, with small-cap equities showing a tendency to produce higher returns than larger-cap equities (Fama and French, 1992, p. 445).

The crux of this seminal research, known as the Fama and French three factor model[30], is that a U.S. diversified portfolio's performance is determined by three factors (Fama and French, 1992, p. 451 – 452):

Factor 1: exposure to the market itself;

Factor 2: small-cap equities; and

Factor 3: value orientation.

These findings are explained using the concept of risk. It is hypothesised that value equities have 'poor prospects' (Fama and French, 1992, p. 446) relative to growth equities, resulting in low stock prices and a high book to market ratio. Similarly large-cap stocks are more likely to be companies that have 'stronger prospects' (Fama and French, 1992, p. 446) relative to smaller firms, resulting in higher stock prices and a low book to market ratio.

[30] $R - r_f = \beta_{im}(k_m - r_f) + \beta_{is}(SMB) + \beta_{ih}(HML)$, (Fama and French, 2004, p. 20).

These seminal findings were later found to be applicable to developed international markets as well. Furthermore, the findings were also found to be applicable to emerging markets, where it was revealed that small-cap equities produce larger average returns than large-cap equities, and that value stocks have higher average returns than growth stocks (Fama and French, 1997, p. 15). Moreover, there has been research with a South African context to lend support to this notion.

Graham and Uliana (2002, p. 16), using the methodology established by Fama and French (1992), state that post-1992 value shares listed on the JSE Securities Exchange outperformed growth shares. The period prior to 1992, namely 1987 – 1991, growth shares outperformed. Although the study lends support to the value out-performance hypothesis, the research has many fundamental flaws.

a) As indicated (Graham and Uliana, 2002, p. 12), the data sample is too restrictive with a mere 58 companies forming the dataset.

b) The suggestion of out-performance based on static time periods of three, five and 10 years (Graham and Uliana, 2002, p. 12) are too restrictive and unrealistic. Rolling time periods of 20 years would have been more appropriate.

Van Rensburg and Robertson (2003, p. 7) cite studies that indicate no small size effect on the JSE Securities Exchange, however indicate that these findings may be flawed based on the limited size of the datasets used. Importantly, Van Rensburg and Robertson (2003, p. 8) conducted studies using a full dataset for a 10 year period, and conclude that small firms do earn a return premium (Van Rensburg and Robertson, 2003, p. 10).

Finally, Van Rensburg and Robertson (2003a, p. 10) proclaim that size and price-to-earnings explain the cross-section of returns on the JSE Securities Exchange.

Based on the findings of Fama and French, it is widely suggested that an optimal portfolio includes value components, specifically amongst the small-cap equities.

At this juncture what seems most interesting is that the South African academics have partially tested for the size and value phenomenon albeit restrictively, yet the discussions seem to be limited. The findings, although commendable, fall short of stating a position, and are not practical from an investor's perspective. In this regard the current research supports the view that size and style are important factors in determining asset classes, however the research endeavours to test for the size and style phenomenon using a combination of JSE Securities Exchange size classifications and classifications based on the price-to-earnings ratio. The results would be demonstrably more useable than previous research. Furthermore, the current research uses data for the entire period 1972 – 2002, a significantly larger dataset than previous research.

Although the Fama and French findings seem to be universally applicable they are not without their caveats. Ferri (2002, p. 230) points out that the small-cap and value premiums, although evident, are only realised over the long term. There are occasions when extended time periods have not yielded a small-cap or value premium as evidenced in the U.S. for the period 1985 – 2000. Swedroe (1998, p. 116) reinforces the view that the investor needs to exercise patience. In his view if the investor has a short investment time horizon, vis-a-vis active investors, it is all the more likely that the small-cap and value premium will not be evident. Swedroe (1998, p. 116) cites research conducted by Heartland Advisors for the period 1976 -1996 where various time periods were examined to determine how likely a value strategy was to outperform an index or growth strategy. In this regard when the holding period was at least eight years, a value strategy outperformed an index strategy 'over 97 percent of the time', and outperformed a growth strategy 'almost 91 percent of the time'.

Of course the persistence of a phenomenon is imperative if an investor is to capitalise on the small-cap and value premiums. In this regard Bernstein (2002b, p. 6) revisited the Fama and French (1992, p. 446) findings to confirm that for the period 1993 – 2002 the premium was evident, although 'no small amount of patience' was required.

Since the style and size phenomenon may require significant time periods to manifest, the research has taken the position that datasets should be constructed using a minimum of 20 years. For this reason the findings as set forth by Graham and Uliana (2002, p. 12), may have been flawed.

Since the strength of the mean-variance model lies in the combination of assets that are not directly correlated, it can be deduced that the pursuit of a maximum expected absolute return, to the exclusion of asset class cross-correlations, will not produce the maximum optimised realised return, and is the reason U.S. mid-cap asset classes are ignored by efficient frontier practitioners (Bernstein, 2000b, p. 2). Mid-cap assets tend to display a strong correlation to both large and small-cap asset classes and therefore tend to be excluded by the mean-variance optimiser.

In light of the diversification imperative, which includes investment across markets, and the imperative of selecting asset classes that are not highly correlated, in order to reap the maximum benefits from a mean-variance optimiser, it seems prudent to include the foreign market aspect of asset class selection. In this regard Statman (2000, p. 1) states that the primary inclusion of foreign stocks is due to their low levels of correlation relative to domestic stocks.

4.4 ASSET CLASS CONSTRAINTS

In Section 4.2.2 (p. 118) the literature regarding data input constraints was reviewed. An additional area of review pertains to the constraining of asset class allocations. In this regard Evensky (1997, p. 253) postulates that since errors in input data are inevitable, these errors will lead to an inappropriately allocated portfolio. To overcome this Evensky posits that both sensitivity analysis, by way of rerunning the mean-variance optimiser, as well as asset class constraints be applied. In this manner it is suggested that the final portfolio will be intuitive and rational, but not optimal. In support of this theory Lummer, Riepe and Siegel (1994, p. 5) also propose the constraining of the optimisation process, by setting maximum and/or minimum allocation targets, thereby seeking to prevent overly favourable data inputs from dominating a portfolio to the extent that it 'violates common sense'.

These views are not without their detractors. Nawrocki (n.d.a, p. 3) makes mention that asset class constraints are an option to consider. However problems are encountered in this regard, primarily what he terms 'degrees of freedom'. In this regard he refers to the spread between the minimum and/or maximum percentages that may be allocated to an asset class. The mean-variance optimiser tends to be forced so far away from an optimal portfolio that the result tends to be no better than a 'heuristic non-optimal solution', with the 'added computational complexity' (Nawrocki, n.d.a, p. 5).

New Frontier Advisors (2001, p. 6) adds to the Nawrocki position by asserting that the manipulation of asset allocations, although seemingly reasonable, would be as perceived by the investor. New Frontier Advisors conclude that such constraints are often subjective, and time consuming to implement, resulting in limited investment value.

4.5 SUMMARY

Given that the data inputs are of primary importance for the effective use of a mean-variance optimiser, it stands to reason that a scientific approach to the determination of such inputs needs to be sought. Coupled to this is the need to identify which asset classes seem appropriate for inclusion in a portfolio. Therefore the successful application of a mean-variance optimiser seems to have two imperatives:

a) the selection of appropriate data inputs; and

b) the selection of appropriate asset classes.

With regards the data inputs the review of the literature highlighted the extensive caveats against using raw historical data by suggesting that the selected data may either be from an inappropriate time period, or may reflect too short a time horizon. It therefore seems apparent that any methodology requiring the use of raw historical data would be inappropriate for a mean-variance optimiser. This resulted in the review of the literature on input and asset constraints, and data resampling. In this regard data resampling seemed to offer the desired solution in the form of providing data inputs that allude to a best average (New Frontier Advisors, 2001, p. 12) solution, without the subjective interventions of investors.

The review of the literature on asset class selection seems to weigh decidedly on the side of value assets with over-weighting in small-cap equities, as set out in Section 4.3 (p. 124). The research suggests that the value and size criterion prevails over all time periods, however that the time periods need to be sufficiently long. In this regard it is suggested that the requisite time period is at least eight years (Swedroe, 1998, p. 116), as indicated in a study in the U.S. by Heartland Advisors covering the period 1976 – 1996, where a value strategy outperformed an index strategy 'over 97 percent of the time', and outperformed a growth strategy 'almost 91 percent of the time'.

Once the best average portfolios (New Frontier Advisors, 2001, p. 12) have been determined, using the appropriate asset classes and data inputs, there remains the imperative of managing the portfolio, on a sustainable basis, to continue to ensure optimised performance. In this regard imperatives that require analysis include the issues of rebalancing, investment horizons and passive formula strategies that may enhance the concept of buy-low sell-high. These concepts form the basis for the discussions in the ensuing chapter.

CHAPTER 5

APPLIED PORTFOLIO STRATEGIES

5.1 INTRODUCTION

Having considered asset allocation, asset class selection, passive investing and modern portfolio theory, the task of portfolio optimisation would be incomplete without seeking an approach to portfolio management. In this regard Jahnke (1997, p. 111) indicates that the quantitative integration of financial planning and expectations-based asset class selection is rarely observed amongst institutional or private investors alike. Therefore this chapter will examine financial planning issues such as the investment horizon, portfolio rebalancing and passive formula strategies.

Finally, since diversification across markets and across currencies is an imperative, the section will conclude with a review of the impact currency volatility may have on foreign equity investments.

5.2 PORTFOLIO REBALANCING

Rebalancing is the process whereby a portfolio is rebalanced periodically in accordance with a predetermined asset allocation percentage. Since different asset classes yield different returns over time, at different levels of risk as measured by standard deviation, if a portfolio is left unrebalanced, the asset allocation percentages alter, which alters the standard deviation risk profile of the portfolio.

According to Perold and Sharpe (1995), cited in Bernstein and Wilkinson (1997, p. 2), a rebalanced portfolio will underperform an unrebalanced portfolio, based on absolute returns, using a buy-and-hold strategy, when there are no return reversals. Bernstein and Wilkinson (1997, p. 3), suggest that return reversals are only part of the answer. They suggest that another important criterion is the similarity of returns between asset classes. In this regard Bernstein and Wilkinson indicate that excess returns will be experienced by a rebalanced portfolio when the asset class return differences are small. Conversely, rebalancing penalises the investor when the difference between asset class returns is large. Moreover, using data for the period 1970 – 1996, Bernstein and Wilkinson (1997, p. 13) postulate that, where the long-term asset class returns are similar to their historical averages a rebalanced portfolio will always be superior to an unrebalanced portfolio.

Bernstein (2002a, p. 287) provides the most succinct example of rebalancing.

a) Assume two assets, A and B.

b) Assume two possible return outcomes for both asset A and B, either a gain of 30 percent, or a loss of 10 percent per asset, with a probability of 50 percent, which equates to an expected geometric rate of return of 8.17 percent.[31]

c) Assume an asset allocation of 50 percent A, and 50 percent B.

Table 5.1 Rebalancing advantages

Year	1	2	3	4
Asset A	+ 30%	+ 30%	- 10%	- 10%
Asset B	+ 30%	- 10%	+ 30%	- 10%
50:50	**+ 30%**	**+ 10%**	**+10%**	**- 10%**

Source: Bernstein (2002a, p. 287)

With reference to Table 5.1, the geometric rate of return is 9.08 percent[32]. The implications of the experiment are that the portfolio realised 0.91 percent more than the expected rate of return due to rebalancing to a 50 percent allocation between the assets. In other words, had the portfolios only consisted of either asset A or B, the rate of return would have been 8.17 percent. Moreover, through diversification, losses occurred 25 percent of the time (once in four years) instead of 50 percent of the time (once in two years).

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[31] 100% – 10% = 90% and 100% + 30% = 130%, therefore to solve for the arithmetic return over two years is as follows: 90% x 130% = 117% (17 percent arithmetic return for two years), which equates to a geometric return of 8.17 percent (Bernstein, 2002a, p. 286).

[32] 130% x 110% x 110% x 90% = 141.57% (41.57 percent arithmetic return for four years), which equates to a geometric return of 9.08 percent (Bernstein, 2002a, p. 287).

Furthermore, rebalancing produces a phenomenon known as buying low and selling high, namely that assets that may be overvalued relative to their mean averages are sold to rebalance the portfolio, and conversely assets that are undervalued relative to their mean averages are bought. The benefits of this are brought about by mean reversion.

Evensky (1997, p. 265) draws attention to the issue of asset class differentials. He postulates that over time, due to the different innate characteristics of different asset classes portfolios, without adjustment will not remain normal. Without intervention, the portfolio will manifest risk characteristics, as measured by standard deviation, unlike those originally intended, which may result in significantly different outcomes to those projected. In this regard Evensky (1997, p. 266) proposes two approaches to rebalancing:

a) calendar rebalancing, which is conducted periodically; and

b) contingent rebalancing, which is triggered by a predetermined event, namely that rebalancing take place only when the equity allocations move away from the original allocation by a predetermined percentage or measure. The actual predetermined target is debatable, since it is dependant on variability, and therefore may be different for different markets.

If rebalancing does produce a premium, as suggested in the literature, then the next imperative would be one of frequency. In this regard there is no conclusive approach. Evensky (1997, p. 268) suggests that this decision is dependant on factors such as portfolio performance, risk tolerance, taxation consequences, and transaction and management costs. Bernstein (2002a, p. 290) suggests that the correct frequency would be 'once every few years' stating that efficient markets do not allow for extraordinary gains over the short term. In this regard it is worth noting that rebalancing is a bet that out of favour assets will perform better in ensuing periods. Infrequent rebalancing may be optimal, and may be in line with the market's mean reversion time frame, as proffered by Thaler (1992, p. 153), as being somewhere between three and seven years.

The primary caveat, with regards to rebalancing, is the issue of 'normality' of the asset allocations (Evensky, 1997, p. 265). Rebalancing to predetermined asset class weights is to presuppose that such weights remain optimal. In this regard attention needs to be drawn to the issue of input data errors where it is highly probable that the identified asset class weights may not be suitable for a future portfolio. In this regard a contingent rebalancing process that requires rebalancing based on the breach of a particular measure would be more desirable.

Michaud (1998, p. 41) suggests that a portfolio should be examined to determine whether rebalancing is required, thereby potentially avoiding unnecessary costs and consequences. Michaud indicates that there are many ‘statistically equivalent portfolios’ and as a result a portfolio that is consistent with mean-variance efficiency would not require revision. In this regard he proposes that a portfolio’s asset allocations be examined relative to the statistically equivalent efficient resampled portfolios derived during the resampling process. Should the portfolio fall within the 90 percent region, namely that the asset allocations fall within the range of 90 percent of the statistically equivalent portfolios, then resampling may not be required (New Frontier Advisors (2001, p. 9).

Due to the inherent uncertainty of the data inputs, stochastically derived inputs may result in widely divergent resampled portfolios, which have a broad spread of asset allocations thereby including virtually all future portfolios. Given that the original inputs may not be appropriate (Evensky, 1997, p. 265), and therefore any resampled portfolios may result in a further move away from future optimum portfolios, it seems prudent to develop a distance function that measures how optimal a past resampled portfolio continues to be relative to the actual realised asset allocation, and adjust accordingly. In this regard Jahnke (1997, p. 111) indicates that asset allocations are dynamic, and consequently, by definition the data inputs should be dynamic, and therefore continuously revised.

Throughout the review of the literature the issue of market dynamism has been discussed. In this regard, at every juncture, the research has taken the view that all processes need to remain flexible in order to accommodate the dynamic nature of equity markets. Rebalancing is no exception.

5.3 PASSIVE FORMULA STRATEGIES

A passive formula strategy is a predetermined plan that will automatically and mechanically guide one's investing (Edleson, 1993, p. 1), without active involvement. Examples of such passive formula strategies would be dollar and rand cost averaging[33], value averaging and rebalancing to fixed asset class weights. In the context of the research value averaging will be analysed.

5.3.1 Value average investing versus a stochastic alternative

Bernstein (2002a, p. 283) draws attention to the methodology of value averaging by citing a study by Edleson (1993). Edleson (1993, p. 39) postulates that making a portfolio rise by a predetermined amount, or percentage produces an internal rate of return[34] that is superior to a stochastic alternative. This is represented in Table 5.2 for the ALSI for the period 1973 – 2002.

Table 5.2 ALSI value averaged performance (1973 – 2002)

	Stochastic Portfolio	Value Averaged Portfolio
Internal Rate of Return	17.33%	20.16%

Source: Derived using data from Annexures 66 and 67.

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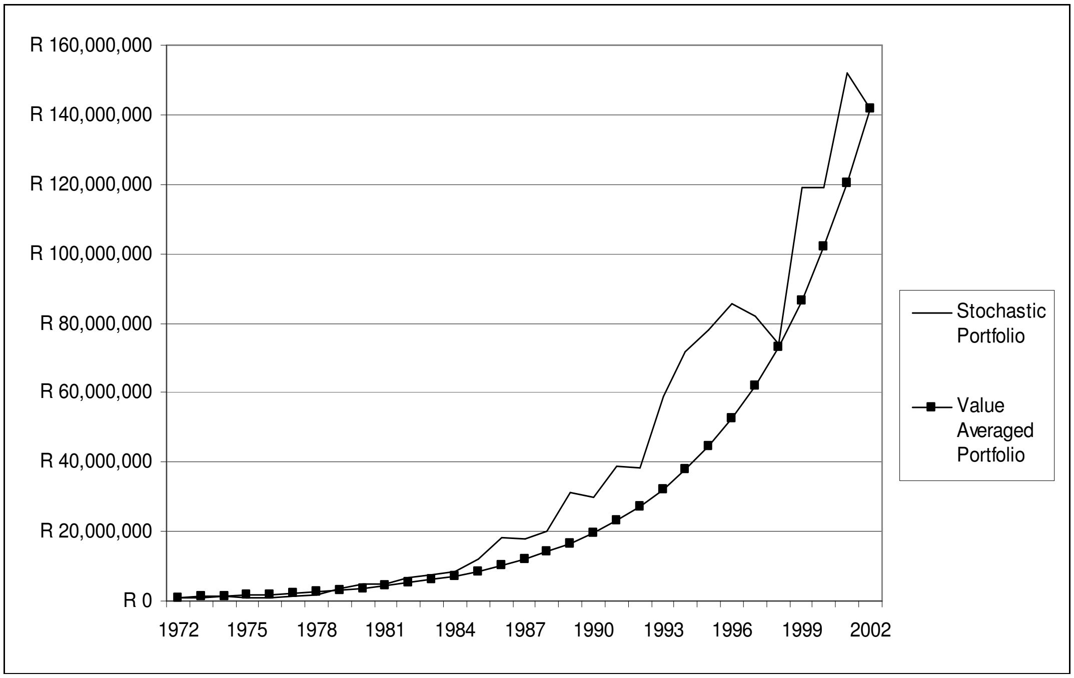
[33] Descriptive term is dependant on the currency in use.

[34] IRR = $r_1 + (r_2 - r_1)\frac{|NPV_1|}{|NPV_1| + |NPV_2|}$, r = rate of return, NPV = Net Present Value. Formula requires two rates of return and Net Present Values (Wisniewski, 1997, p. 512).

With reference to Figure 5.1, and given that markets seem to manifest mean reverting[35] characteristics, the predetermined amount is derived by utilising the historical real rate of return for the portfolio, given a set asset allocation.

Figure 5.1 Value averaged portfolio versus stochastic portfolio



Source: Derived using data from Annexures 66 and 67.

Of course it is apparent from Figure 5.1 that the value averaged portfolio manifests less volatility; however this was established in hindsight. A forward looking portfolio would seek to achieve a similar outcome, by establishing a value line based on past market performances.

Once the value line is determined, a portfolio either sells equities when the portfolio value exceeds the value line, as when the market is excessively high,

[35] See Section 3.9 (p. 103).

placing the surplus resources into cash, or purchases equities when the portfolio falls short of the value line, as when the market is excessively low, utilising the accumulated cash resources. Additional benefits include avoiding large purchases in overvalued markets, as well as avoiding panic selling in undervalued markets (Edleson, 1993, p. 43), thereby enhancing the sell-high buy-low phenomenon. The imperative with value averaging is selecting the appropriate value to be achieved. Too high a value would require constant topping up by the investor, whereas too low a figure would result in less than an optimal outcome. Moreover, historical rates of return are subject to variation from period to period, and may need to be revised periodically.

Edleson (1993, p. 93) provides a cautionary in this regard. He notes that the risk is when there is a protracted bear market towards the end of a goal, where a sizable portfolio has been established, thereby reducing the likelihood of success. In this regard he suggests that an investor should rather select a predetermined rate of return that is somewhat more conservative than market expectations. This would allow for ease in achieving predetermined targets, and would result in surplus funds to meet any eventual goal shortfalls. Bernstein (2002a, p. 283) indicates that the benefit of value averaging is that the portfolio is not left to the stochastic ebb and flow of the market, as it would otherwise be.

Given the position taken by the research, namely that all processes are required to be able to accommodate the dynamic nature of equity markets, the value line determined for the value averaging process will be redetermined annually as required in order to assure that the portfolio remains aligned with the desired returns over time.

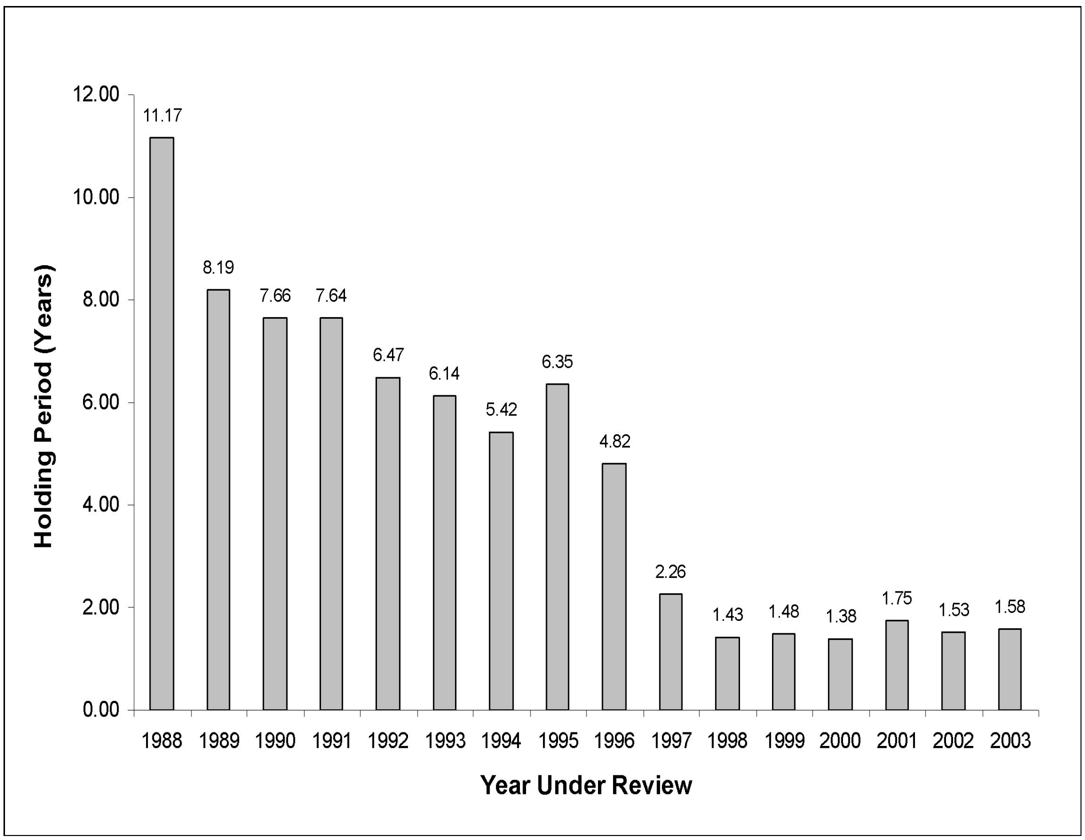
5.4 INVESTMENT TIME HORIZONS

Seemingly, from research in related areas, there is an allusion to the ‘long-term’, but specific literature with regards to appropriate investment time horizons is surprisingly sparse. In this regard few theoreticians commit to a precise time period. Therefore the prudent approach is to review the literature, where time has been mentioned, and from this to deduce an appropriate investment time horizon.

Damodaran (2003, p. 242) indicates that all research on value investing uses time periods of five years or greater. It is advisable to recall the findings (Swedroe, 1998, p. 116) by Heartland Advisors for the period 1976 -1996 where various time periods were examined to determine how likely a value strategy was to outperform an index or growth strategy. In this regard when the holding period was at least eight years, a value strategy outperformed an index strategy ‘over 97 percent of the time’, and outperformed a growth strategy ‘almost 91 percent of the time’.

Bogle draws attention to the fact that ‘everybody talks about long-term investing, but nobody does anything about it’ (Bogle, 1999, p. 20), indicating that investors have the intention of investing for an extended period, however rarely achieve this objective. In this regard, using collective investors as a proxy for investors, Pawley (2002, p. 78) highlights that the average South African collective investment investor has significantly reduced the average investment holding period. With reference to Figure 5.2, the holding period has changed from 11.17 years in 1988 to 1.58 years in 2003, with a turnover of 63.17 percent, seeming to indicate a strong tendency to reduce investment holding periods.

Figure 5.2 South African investor holding periods

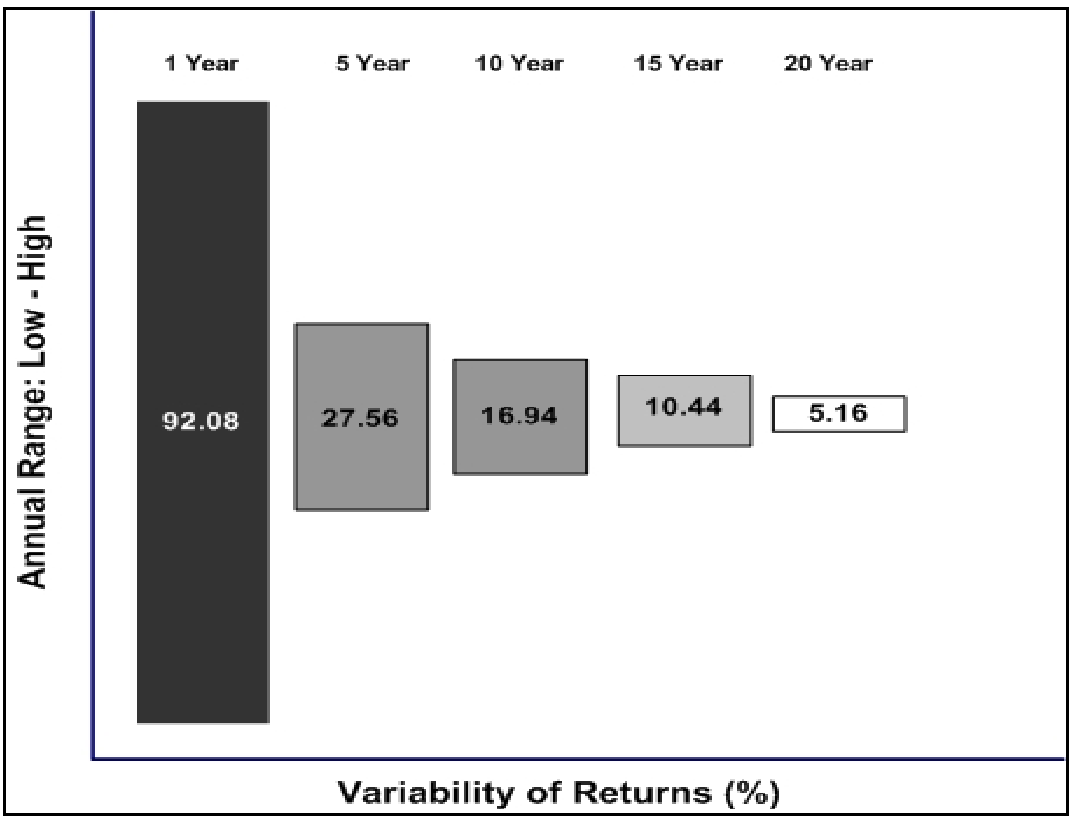


Source: Pawley, 2002, p. 78 and updated using data from the Association of Collective Investments.

Looking to foreign markets, Bogle (1999, p. 24) asserts that collective investment fund investors, within the U.S., have become short-term traders, holding their investments for barely three years, with a turnover rate of 31 percent, which is in line with the trend established by South African investors.

Malkiel (1999, p. 354) directs attention to the fact that equity investment volatility is substantially reduced, the longer the investment holding period.

Figure 5.3 South African risk and time analysis



Source: Pawley, 2002, p. 118.

With reference to Figure 5.3, Pawley (2002, p. 118) replicated these findings within a South African context, where 100 percent of the volatility for the period

1976 – 2001 was represented graphically, clearly indicating the risk diversification potential of a long-term holding period. In this regard it is noted that the majority of the portfolio variability, ceteris parabis, is diversified away within a time period of five years.

Bernstein (2000a, p. 63) provides some insights by postulating that actual returns can deviate significantly from expected returns for periods of up to 10 years.

Significantly, Evensky (1997, p. 240) postulates that for long-term projections, using an optimiser, 20 – 30 year historical rates of return may be viable. This is supported by the Michaud study (1998, p. 35) which indicated that results seem to be more stable, the longer the time period.

Gibson (2000, p. 85) provides the most comprehensive view on investment time horizons, indicating that equity investment volatility, as evidenced by standard deviation, can be extremely damaging to a portfolio over the 'short term', (2000, p. 85). In this context the short term is viewed as one year or less, in this regard, with reference to Tables 6.1 (p. 159) and 6.2 (p. 160), it is evident that the South African and U.S. markets respectively, display standard deviations of 25.83 percent and 29.53 percent. Given the levels of volatility Gibson (2000, p. 87) proceeds to discuss the point at which a rational risk averse investor would consider accepting such levels of volatility relative to a riskless asset, such as a treasury bill.

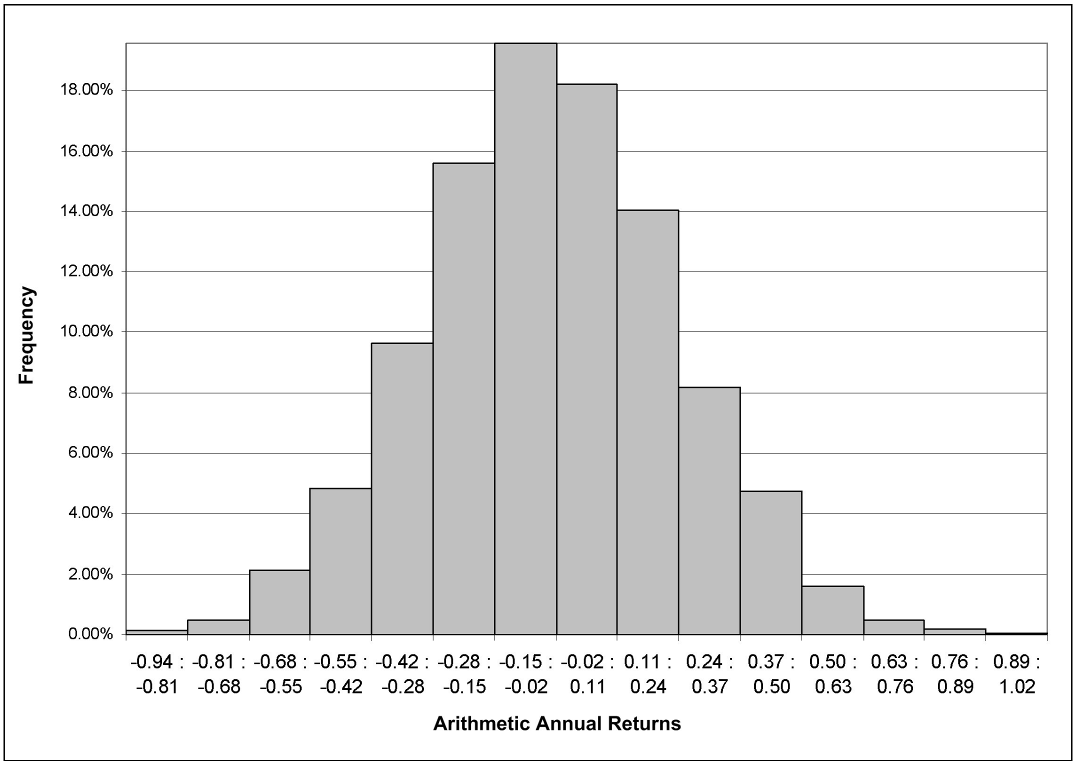
Table 5.3 **South African treasury bills and equity premiums (1973 – 2002)**

	Arithmetic Mean
Treasury Bills	11.76%
Risk Premium	8.55%
ALSI Standard Deviation	26.73%

Source: Derived using data from the period 1973 – 2002.

Using the data within Table 5.3, a probability distribution can be derived comparing the return outcome of the risk premium relative to a treasury bill. With reference to Figure 5.4, the probability of a South African risk premium outperforming the treasury bills in any given year is 45 percent.

Figure 5.4 **Histogram showing South African equity risk premium versus treasury bills**



Source: Derived using data in Table 4.3 and a stochastic simulator using 1000 iterations.

Gibson (2000, p. 87) asserted that the probability of this occurrence, within the U.S. market, was approximately 50 percent. Therefore it seems apparent that there is no incentive to assume such high levels on the basis of a one year, or lower time horizon. When the actual market returns were compared to the riskless asset over rolling time periods, Gibson (2000, p. 88) found the results for the U.S. market as set out in Table 5.4.

Table 5.4 **U.S. market probability comparison (1926 – 1998)**

	Actual Probability
1 Year Time Horizon	59%
5 Year Time Horizon	75%
10 Year Time Horizon	78%
20 Year Time Horizon	94%

Source: Gibson, 2000, pp. 90 – 91.

Replicating for the South African market, using bonds which produced a slightly higher geometric rate of return, the results derived are set out in Table 5.5.

Table 5.5 **South African market probability comparison (1973 – 2002)**

	Actual Probability
1 Year Time Horizon	60.00%
5 Year Time Horizon	76.92%
10 Year Time Horizon	76.19%
20 Year Time Horizon	100.00%

Source: Derived using data from Annexures 68 and 69.

Both Tables 5.4 and 5.5 lend support to the time diversification theory represented graphically in Figure 5.3 (p. 144), and indicate more clearly the appropriate investment holding periods. Gibson (2000, p. 93) concludes by indicating that the volatility inherent in equities is undoubtedly excessively risky over a one year, or lower time horizon, and not a risk worth taking, however such volatility is the basis for higher returns, and dissipates with time.

Siegel (2002, p. 29) explains that most investors underestimate their investment holding period. Siegel explains that a holding period is not asset specific but rather is the holding period for any assets, no matter how many changes occur to the asset allocation.

In summary, to arrive at a deduction, it is noted that, in order for size and style investing to produce a premium, the holding period seems to suggest a high probability of success from about eight years (Swedroe, 1998, p. 116). For the benefits of time diversification, the holding period seems to be at least five years (Pawley, 2002, p. 118). Mean reversion studies seem to indicate a reversion of between three and seven years (Thaler, 1992, p. 153), and Bernstein (2000a, p. 63) indicates that there may be a return divergence for periods up to 10 years. Finally, it seems that actual holding periods for both markets are between two and three years (Pawley, 2002, p. 78 and Bogle, 1999, p. 24).

Given the numerous indicated time horizons, an analysis thereof seems appropriate. With reference to Figure 5.2 (p. 143), actual time horizons are clearly well below any of the times indicated by the literature. It is therefore prudent to suggest that universally investors need to lengthen their time horizons, given the benefits to be derived as indicated by the above-mentioned literature. With reference to the literature, there is no convergence around a particular time horizon. Therefore, in this regard an element of discretion is required. Given that the majority of risk, as measured by standard deviation, seems to be reduced after five years, and given that markets display mean reversion characteristics at between three and seven years and that size and style investing display outperformance characteristics from around eight years, it seems prudent to suggest a holding period of 10 years.

5.5 EXCHANGE RATES AND EQUITY INVESTMENTS

Finally, given the importance of determining the data inputs[36] for use in the mean-variance optimiser, it is imperative that the consequences of investing across markets, and the determinants of exchange rates, need to be understood in terms of the currency impact on investment returns, and the need to make prudent adjustments for future currency movements.

Flight and Lee-Swan (1988, p. 15) postulate that currency movements have become the primary determinant of investment values internationally, and summarise that the long-term (three years or more) outlook for any exchange rate is dependant on a number of variables, both statistical and emotive (Flight and Lee-Swan, 1988, p. 40), as set out below.

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[36] See Section 4.2 (p. 115).

a) The relative price changes, or the inflation rate differential between countries (McConnell, 1981, p. 849).

b) The effect of money supply figures on inflation, where high levels of money supply growth are normally consistent with high inflation (Flight and Lee-Swan, 1988, p. 59).

c) The current account balances (Carter and Partington, 1984, p. 334), where a continuous and increasing deficit is a strong indication that a country is becoming uncompetitive, leading to likely currency depreciation (Flight and Lee-Swan, 1988, p. 62).

d) The capital account funds flow and the resultant dependence on foreign capital investments (Aron, Elbadawi and Kahn, 1997, p. 14). The capital account is influenced in turn by several factors:

   i) interest rate differentials (McConnell, 1981, p. 849);

   ii) corporate investment abroad, otherwise known as foreign direct investment (Flight and Lee-Swan, 1988, p. 67); and

   iii) portfolio investments, where capital flows are based on the relative attractiveness of the bond and stock markets (Flight and Lee-Swan, 1988, p. 68).

e) The productivity growth rate, where a sustained below average rate is likely to indicate a loss of competitiveness and a weakening currency (Flight and Lee-Swan, 1988, p. 69).

f) The domestic savings rate, which has a bearing on a country's level of consumption; low savings rates and high levels of consumption generally lead to higher imports (Roux, 1999, p. 106), and are associated with economies with large deficits and weak currencies (Flight and Lee-Swan, 1988, p. 71).

g) The national budget balance[37] which has a significant impact on whether a currency will be weak or strong. The national budget balance is determined by incorporating the current account, the capital account and savings, and the impact on the exchange rate is determined by the level at which capital account transactions offset current account transactions, and vice versa (Carter and Partington, 1984, p. 310).

h) The intervention of the central bank through the purchase or sale of its own currency (Carter and Partington, 1984, p. 334).

i) The political climate based on the confidence foreign investors have in a domestic government (Flight and Lee-Swan, 1988, p. 75).

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[37] National Budget Deficit/Surplus = Current Account Deficit/Surplus + (Savings + Investments) (Flight, 1988, p. 71).

j) Market sentiment and momentum which cause short-term currency volatility (Flight and Lee-Swan, 1988, p. 75).

k) Major ad hoc events which result in the movement of capital in anticipation, or as a result thereof (Flight and Lee-Swan, 1988, p. 76).

In the short term, the movement of exchange rates may mean the difference between a profit and a loss. Traditional equity analysis concentrated on past earnings performance of equities, this view is now widely accepted as being myopic within a foreign investment environment. Exchange rate movements now determine the magnitude of the profit or loss within many globalised conglomerates (Flight and Lee-Swan, 1988, p. 100), which in turn determines the movement of stock markets, therefore the investment time horizon is crucial, due to the short-term currency volatility. The most prudent approach is to spread currency risk as well as equity risk, thereby maximising the investment benefits. Invariably competitive economies manifest strong currencies (Flight and Lee-Swan, 1988, p. 106), therefore when combining two markets in a portfolio, the imperative becomes one of relative currency performance. In this regard the exchange rate factor is of importance, and any expected depreciation must be added to any projected equity market valuations (Flight and Lee-Swan, 1988, p. 104).

In summary, the investor should develop a long-term view of the relative currency performances, of the markets being considered for investment, based on the fundamentals highlighted in points a) – k), and make the necessary valuation adjustments.

5.6 SUMMARY

Portfolio rebalancing is clearly an imperative advocated by many theoreticians and practitioners alike. There are a number of approaches, namely to rebalance to a fixed asset allocation or to rebalance only when required based on a measure of optimality. The measures of optimality can take the form of determining how close a portfolio's asset allocation is to a predetermined optimal asset allocation (New Frontier Advisors, 2001, p. 9), or the asset allocation process can be treated as dynamic (Jahnke, 1997, p. 111), which means that the rebalancing process should be treated as dynamic. It seems clear that any rebalancing on the basis of historical asset allocations may not yield optimal results. The prudent choice seems to be to apply both a calendar and a contingent rebalancing strategy where a portfolio's asset allocation is assessed periodically, relative to the relevant actual efficient frontier asset allocation to ascertain how much of the return is being captured, and in this regard whether rebalancing is required. This would not only ensure approximate optimality, given the unknowable nature of future outcomes, but would address the issue of frequency and the dynamic nature of asset classes. The methodology proposed by Michaud (1998) may indeed reduce the need to rebalance frequently, however, although a portfolio may seem optimal relative to the resampled efficient portfolio, this resampled efficient portfolio was derived using data inputs that may no longer be appropriate, and therefore over time there may be a reduction in how optimal the portfolio continues to be.

In line with the rebalancing imperative is the concept of value averaging, which allows for the maximisation of the buy-low sell-high phenomenon and yields higher returns in the form of the internal rate of return relative to allowing the portfolio to move stochastically with the market. The beauty of value averaging is that it prevents the investor from over-committing resources during an over-valued market, and allows for the accumulation of equities in times of market weakness. Also, it provides a clear plan in the form of values to be attained annually, thereby allowing the investor to monitor progress towards a given terminal value.

Given that many of the suggested approaches to investing, such as asset allocation, mean reversion, rebalancing, value averaging and asset class selection, are premised on the understanding that the benefits of equity investing only accrue over the long term, it is somewhat disconcerting that there is very little literature about appropriate time horizons. What literature there is seems to suggest the longer the holding period the better. It seems prudent to suggest a holding period of 10 years[38].

Finally, the effect of exchange rate volatility can be profound, not only can it distort the data inputs that are utilised in the mean-variance optimiser, which in

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[38] See Section 5.4 (p. 142).

turn has consequences for asset allocation, but it can also affect the overall returns realised by a portfolio. In this regard though, it is evident that diversification across markets assists in the reduction of both market and currency risk, and again these benefits only accrue to the long-term investor.

CHAPTER 6

RESEARCH DATA ANALYSES AND FINDINGS

6.1 INTRODUCTION

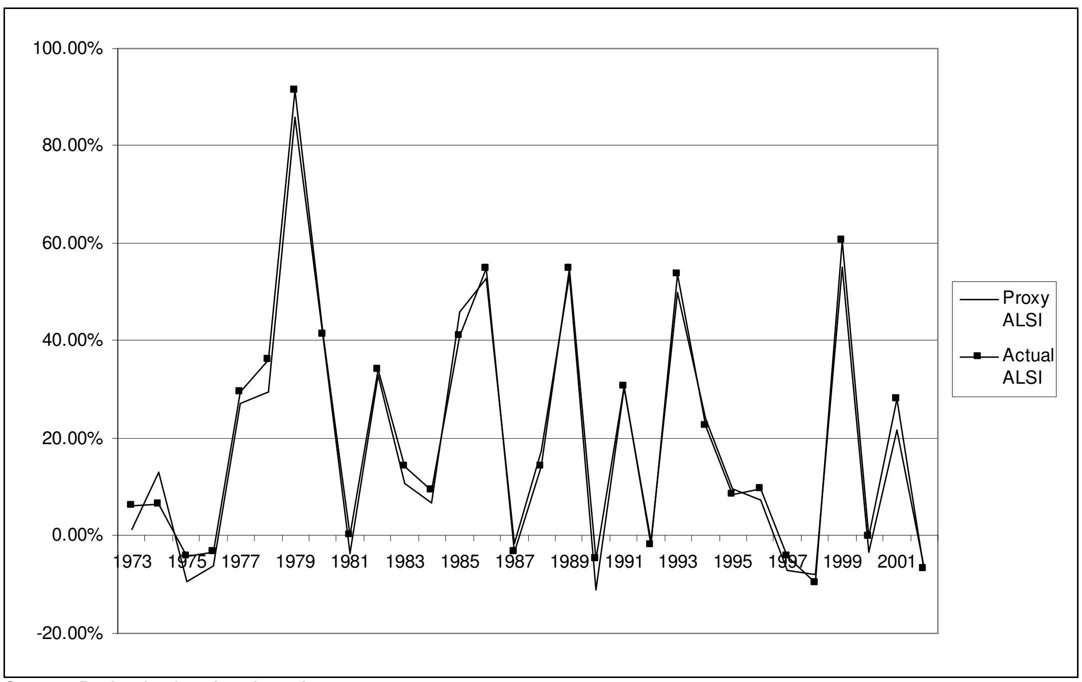
This chapter presents the detailed results and findings derived from analysing the data gathered for the study. The results and findings are presented within the parameters set by the literature review and the methodology. Any additional findings that may arise during the analyses, for which evidence have been gathered, are presented under the appropriate headings.

6.2 ALSI INDEX RELATIVITY

At the outset it was imperative to ascertain that the proxy indices approximate the historical indices, otherwise the outcomes would not accurately reflect the retrospective behaviour of the identified asset classes. In this regard, as set out in the methodology, the actual ALSI index, as supplied by the JSE Securities Exchange, was compared to the proxy ALSI index using the correlation coefficient to determine whether the market movements were closely associated. A correlation coefficient of 0.9912 was obtained indicating extremely close correlation. The two indices, as represented in Figure 6.1, are virtually indistinguishable from each other.

This was then converted to the coefficient of determination at 0.9826. This seemed to indicate that 98.26 percent of the proxy index could be ascribed to the actual ALSI, or conversely that there was a tracking error relative to the actual ALSI of 1.74 percent. These results were satisfactory and indicated that the proxy ALSI was a close substitute for the original ALSI. The implication was that the adjustment process of aligning the original and proxy ALSI indices was not too dramatic, and that the post-adjusted proxy sub-indices would be a good replication of historical performance.

Figure 6.1 Proxy ALSI versus actual ALSI (1973 – 2002)



Source: Derived using data from Annexure 63.

6.3 ASSET CLASS HISTORICAL CHARACTERISTICS

The analysis begins by presenting the historical characteristics for the various asset classes, for the 20 year period 1973 – 1992. This data will form the foundation for the research.

Table 6.1 South African equities (1973 – 1992)

	Geometric Return*	Geometric Real Return* **	Standard Deviation	Sharpe Ratio
MP L Cap Growth	14.10%	0.63%	33.46%	0.02
MP L Cap Value	20.39%	6.92%	28.24%	0.25
MP M Cap Growth	16.40%	2.92%	24.02%	0.12
MP M Cap Value	24.20%	10.73%	23.06%	0.47
MP S Cap Growth	18.35%	4.88%	22.23%	0.22
MP S Cap Value	24.07%	10.60%	22.80%	0.47
ALSI Index	19.98%	6.50%	25.83%	0.25

* Returns are inclusive of income and South African Rand based.
** Real Returns are adjusted for the South African geometric inflation variable @ 13.47%.

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

The names allocated to the South African asset classes were arbitrarily determined, using the researcher's initials. This provides uniformity relative to the U.S. asset classes constructed by Eugene Fama and Kenneth French, hence the prefix FF, as set out in Ibbotson Associates (2003, p. 156 - 169).

Table 6.2 U.S. equities (1973 – 1992)

	Geometric Return*	Geometric Real Return* **	Standard Deviation	Sharpe Ratio
FF L Cap Growth	17.58%	4.11%	29.38%	0.14
FF L Cap Value	23.87%	10.40%	34.85%	0.30
FF S Cap Growth	16.98%	3.51%	35.64%	0.10
FF S Cap Value	27.11%	13.64%	37.53%	0.36
S&P 500 Index	19.21%	5.74%	29.53%	0.19

* Returns are inclusive of income and South African Rand based.
** Real Returns are adjusted for the South African geometric inflation variable @ 13.47%.

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

The data analysis, for the period 1973 – 1992, reveals some interesting anomalies when reviewing the two markets relative to each other. With reference to Table 6.3, the U.S. market reveals that, small-cap assets have a larger standard deviation than the large-cap assets, indicating higher volatility. When the U.S. asset classes are adjusted for risk, using the Sharpe ratio, the small-cap value and large-cap value assets seem to prevail. Therefore the characteristics of the U.S. market seem to suggest that small-cap assets are more volatile than large-cap assets. However on a risk-adjusted basis value assets are predominant, with small-cap value assets being the highest followed by large-cap value assets. This clearly indicates the phenomenon of risk and reward, the riskier assets, namely value and small-caps, are rewarded with higher rates of return.

Table 6.3 Asset class comparison (1973 – 1992)

South African Assets	Standard Deviation	Sharpe Ratio	U.S. Assets	Standard Deviation	Sharpe Ratio
MP L Cap Growth	33.46%	0.02	FF L Cap Growth	29.38%	0.14
MP L Cap Value	28.24%	0.25	FF L Cap Value	34.85%	0.30
MP M Cap Growth	24.02%	0.12			
MP M Cap Value	23.06%	0.47			
MP S Cap Growth	22.23%	0.22	FF S Cap Growth	35.64%	0.10
MP S Cap Value	22.80%	0.47	FF S Cap Value	37.53%	0.36
ALSI Index	25.83%	0.25	S&P 500 Index	29.53%	0.19
* Returns are inclusive of income and South African Rand based. ** Real Returns are adjusted for the South African geometric inflation variable @ 13.47%.					

Source: Derived using Tables 6.1 and 6.2.

For the period 1973 – 1992, the South African market reveals that, remarkably, small-cap assets have a smaller standard deviation than the large, and mid-cap assets, indicating lower volatility. When the South African assets are adjusted for risk, using the Sharpe ratio, the value assets seem to prevail. Therefore the characteristics of the South African market seem to suggest that small-cap assets are less volatile than both the large, and mid-cap assets, and on a risk-adjusted basis, value assets are more predominant, with small-cap value assets being the highest, followed by mid, and large-cap value assets respectively. This phenomenon was unexpected since it seems to suggest that there is an excessive reward for high risk amongst the small-cap assets, or conversely that the large-cap assets are excessively risky for the reward received.

Finally, it is noted that U.S. asset classes, having been adjusted for currency, display significantly higher standard deviations relative to the South African asset classes as a result of currency volatility, as set out in Table 6.4, which will have implications for inter-market asset allocations, makes diversification an imperative and raises the risk of an offshore investment focus due to the higher volatility of the offshore component.

These anomalous findings will be examined more thoroughly in the ensuing section.

Table 6.4 Impact of currency on U.S. asset risk levels (1973 – 1992)

*Pre-Adjustment	Standard Deviation	**Post-Adjustment	Standard Deviation
FF L Cap Growth	19.37%	FF L Cap Growth	29.38%
FF L Cap Value	18.86%	FF L Cap Value	34.85%
FF S Cap Growth	29.21%	FF S Cap Growth	35.64%
FF S Cap Value	24.72%	FF S Cap Value	37.53%
S&P 500 Index	17.50%	S&P 500 Index	29.53%
* Standard deviations are based on nominal returns, inclusive of income and are U.S. Dollar based. ** Standard deviations are based on nominal returns, inclusive of income and are South African Rand based.			

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

6.4 ACTUAL INTRA-MARKET ASSET ALLOCATIONS (1973 – 1992)

The allocation of assets is determined using the mean-variance optimisation algorithm[39] as incorporated in the MvoPlus software.

The first step in the process is to derive the actual asset allocations that would have resulted in the optimal outcome, for the respective markets, using the identified asset classes. The data inputs used are as per Tables 6.1 and 6.2, as

[39] See Section 2.5 (p. 41).

well as the cross-correlation matrix between the identified asset classes, as per Annexure 1, for the respective markets.

Table 6.5 Actual South African asset allocations (1973 – 1992)

	Minimum Variance Portfolio	**Middle Portfolio**	Maximum Return Portfolio
MP L Cap Growth	0.00%	**0.00%**	0.00%
MP L Cap Value	0.00%	**2.71%**	0.00%
MP M Cap Growth	14.23%	**0.00%**	0.00%
MP M Cap Value	28.04%	**41.57%**	57.41%
MP S Cap Growth	34.83%	**12.72%**	0.00%
MP S Cap Value	22.90%	**43.00%**	42.59%
Standard Deviation	20.90%	**21.36%**	21.81%
Geometric Return	7.92%	**10.13%**	10.89%
Sharpe Ratio	0.38	**0.47**	0.50

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

It is noted that the middle portfolio, as highlighted in Tables 6.5 and 6.6, provides better asset class diversification than the maximum return portfolio, with very little trade-off in returns, and significantly better returns, on a risk-adjusted basis, than the minimum variance portfolio. The middle portfolio will be used in ensuing calculations as set out in the methodology[40].

Table 6.6 Actual U.S. asset allocations (1973 – 1992)

	Minimum Variance Portfolio	**Middle Portfolio**	Maximum Return Portfolio
FF L Cap Growth	81.08%	**19.44%**	0.00%
FF L Cap Value	3.75%	**17.34%**	0.00%
FF S Cap Growth	0.00%	**0.00%**	0.00%
FF S Cap Value	15.17%	**63.21%**	100.00%
Standard Deviation	28.94%	**33.24%**	37.53%
Geometric Return	6.26%	**11.82%**	13.64%
Sharpe Ratio	0.22	**0.36**	0.36

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

The implications of the findings, discussed in Section 6.3, are highlighted in Table 6.7 where it is apparent that the varying standard deviation has impacted on the asset allocations.

When factoring in standard deviation and return, the U.S. market, with the lower volatility amongst the large-cap assets, displays a 36.78 percent allocation to the large-cap sector, with the remainder being allocated to the small-cap sector.

[40] See Section 2.5 g) (p. 42).

When the sectors are analysed by style it is noted that 80.55 percent of the U.S. assets are allocated to the value sector.

When factoring in standard deviation and return, the South African market, with the lower volatility amongst the small-cap assets displays a 55.72 percent allocation to the small-cap sector, followed by 41.57 percent allocation to the mid-cap assets, with the remainder being allocated to the large-cap sector (2.71 percent). When the sectors are analysed by style it is noted that 87.28 percent of the South African assets are allocated to the value sector.

Most notable, relative to the U.S. market, is the lack of allocation to the large-cap assets within the South African market, and the significant allocation to mid-cap assets. The implications of these two findings are significant.

In the first instance it needs repeating that recent research has showed that active investment managers have been unable to outperform the broad market, as represented by the ALSI index, over the period 1976 – 2001, which has a strong correlation to the large-cap assets. Given that the large-cap assets are the worst performing assets over the period 1973 – 1992, this may indicate the lack of active investment managers ability to identify value creation assets.

In the second instance there is a notable allocation of assets to the mid-cap assets indicating that the theory relating to mid-cap assets, namely their strong correlation to both large and small-cap assets, seems not to apply across markets, which will be discussed further in the ensuing chapter.

Table 6.7 Asset allocation comparison (1973 – 1992)

	Actual Asset Allocations		
South African Assets	Middle Portfolio	U.S. Assets	Middle Portfolio
MP L Cap Growth	0.00%	FF L Cap Growth	19.44%
MP L Cap Value	2.71%	FF L Cap Value	17.34%
MP M Cap Growth	0.00%		
MP M Cap Value	41.57%		
MP S Cap Growth	12.72%	FF S Cap Growth	0.00%
MP S Cap Value	43.00%	FF S Cap Value	63.21%
Standard Deviation	21.36%	Standard Deviation	33.24%
Geometric Return	10.13%	Geometric Return	11.82%
Sharpe Ratio	0.47	Sharpe Ratio	0.36

Source: Derived using Tables 6.5 and 6.6.

6.5 ACTUAL INTER-MARKET ASSET ALLOCATIONS (1973 – 1992)

Following the determination of asset allocations for the respective markets the final step is to derive the allocation per market. This process is achieved first by using the data in Tables 6.5 and 6.6, the inter-year rates of return in Annexure 2 and retro-actively calculating the optimal portfolios inter-year rates of return. These inter-year rates of return for the respective markets are then used to derive the cross-correlations between the respective markets, as well as the standard deviation and geometric rates of return for the period 1973 – 1992, as reflected in Annexure 3. This data is then used for the mean-variance optimisation algorithm to derive the split of assets per market by way of the efficient frontier, as reflected in Annexure 4.

Table 6.8 Actual inter-market asset allocations (1973 – 1992)

	Minimum Variance Portfolio	Middle Portfolio	Maximum Return Portfolio
United States Market	**19.92%**	50.08%	63.84%
South African Market	**80.08%**	49.92%	36.16%
Standard Deviation	**20.20%**	22.77%	25.35%
Geometric Return	**11.18%**	12.02%	12.11%
Sharpe Ratio	**0.55**	0.53	0.48

Source: Derived using data from the multiple asset classes for the period 1973 – 1992.

It is noted, with reference to Table 6.8, that the minimum variance portfolio provides the best solution on a risk-adjusted basis, and provides minimal exposure to the foreign market without significantly compromising returns. The minimum variance portfolio will be used in ensuing calculations as set out in the methodology[41].

Similar to the findings in Table 6.8 is the derivation of the inter-market allocation using only the broad based market indices that will form part of the control portfolios. This process is conducted using the inter-year rates of return for the respective markets, as per Annexure 5, and deriving the cross-correlations between the respective markets, as well as the standard deviation and geometric rates of return for the period 1973 – 1992, as per Annexure 6. This data is then used for the mean-variance optimisation algorithm to derive the split of assets per market by way of the efficient frontier, as reflected in Annexure 7.

[41] See Section 2.5 j) (p. 43).

Table 6.9 **Actual indices inter-market asset allocations (1973 – 1992)**

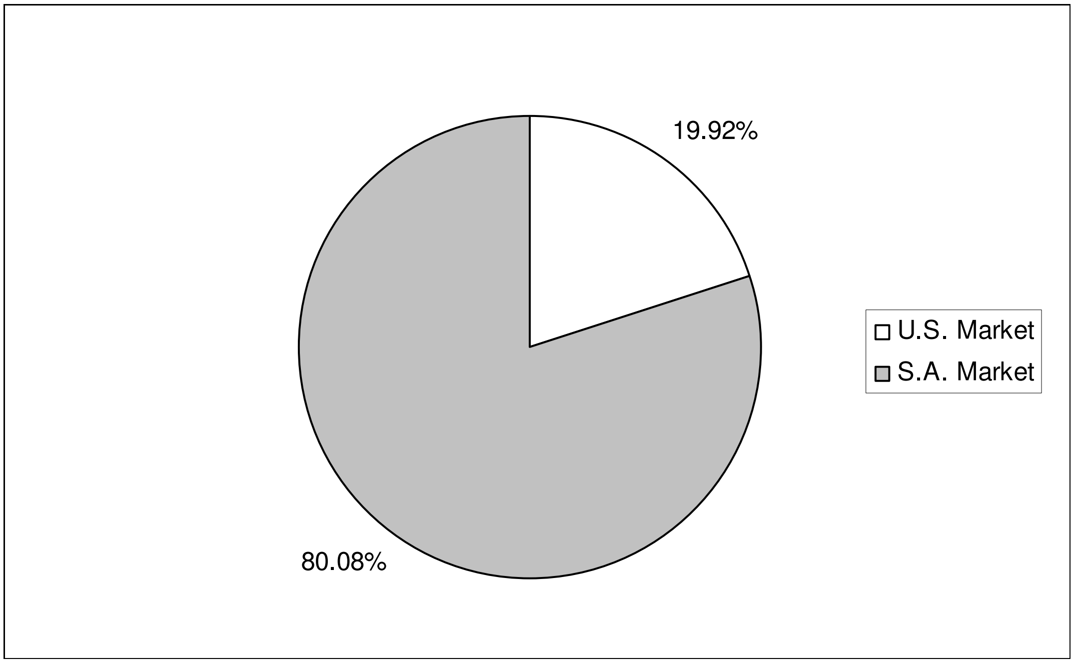
	Minimum Variance Portfolio	Middle Portfolio	Maximum Return Portfolio
United States Market	**39.45%**	40.23%	40.55%
South African Market	**60.55%**	59.77%	59.45%
Standard Deviation	**22.72%**	22.72%	22.72%
Geometric Return	**7.18%**	7.18%	7.18%
Sharpe Ratio	**0.32**	0.32	0.32

Source: Derived using data from the market indices for the period 1973 – 1992.

What is immediately apparent is the disparity between Tables 6.8 and 6.9 when comparing the minimum variance portfolios, as indicated in bold. These findings will be discussed in the ensuing section.

Following the findings in Section 6.3 (p. 159), with reference to Figure 6.2, it is noted that the increased volatility due to the currency adjustment manifests itself by reducing the allocation of assets to the U.S. market.

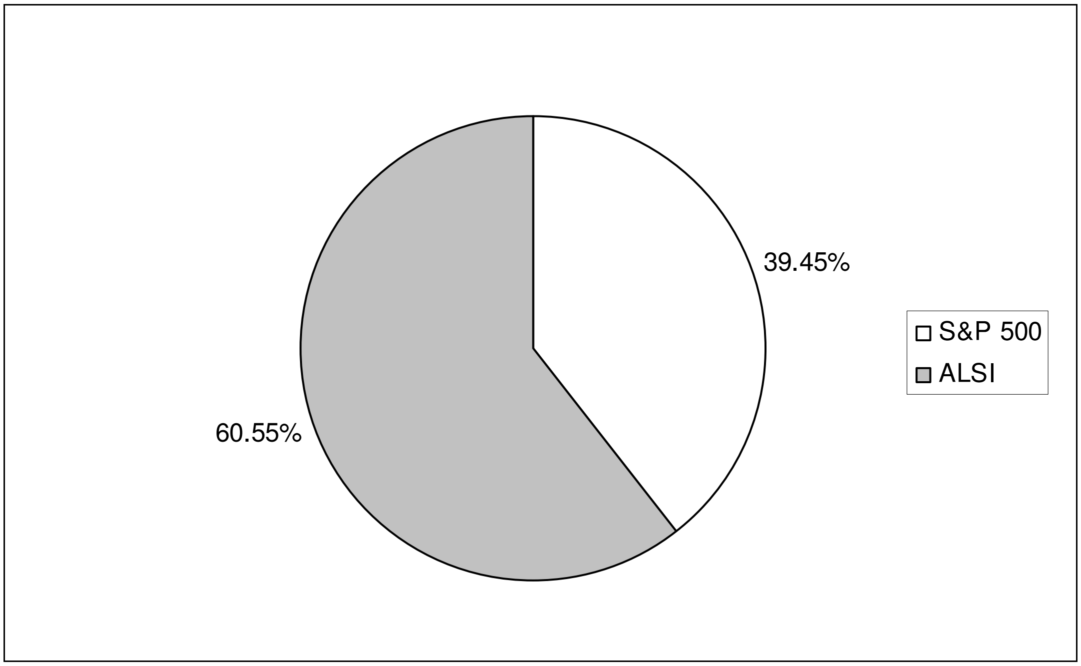
Figure 6.2 **Optimal inter-market allocation (1973 – 1992)**



Source: Derived using Table 6.8.

With reference to Figure 6.3, the asset allocation is based on the broad market indices. What is immediately apparent is the disparity in allocations between Figures 6.2 and 6.3, which is the same as the disparity between Tables 6.8 and 6.9. Figure 6.3 is based on the broad market indices that are predominantly large-cap growth assets. The U.S. small-cap assets display high volatility and are not predominant in the index. As a result the index has a lack of exposure to small-cap assets, thereby resulting in a concomitant increase in overall asset allocation to the U.S. market.

Figure 6.3 **Indices inter-market allocation (1973 – 1992)**



Source: Derived using Table 6.9.

6.6 RESAMPLED INTRA-MARKET ASSET ALLOCATIONS (1973 – 1992)

The resampled allocation of assets is based on the resampling of the data inputs required by the mean-variance optimisation algorithm, as set out in the methodology[42].

[42] See Section 2.6.3 (p. 45).

Table 6.10 Resampled South African asset allocations (1973 – 1992)

	Minimum Variance Portfolio	**Middle Portfolio**	Maximum Return Portfolio
MP L Cap Growth	0.00%	**0.00%**	0.00%
MP L Cap Value	3.15%	**2.96%**	0.30%
MP M Cap Growth	12.74%	**0.96%**	0.00%
MP M Cap Value	25.11%	**44.36%**	50.97%
MP S Cap Growth	35.72%	**3.86%**	0.00%
MP S Cap Value	23.28%	**47.86%**	48.73%
Standard Deviation	20.72%	**21.61%**	22.51%
Geometric Return	7.87%	**10.72%**	11.19%
Sharpe Ratio	0.38	**0.50**	0.50

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

Table 6.11 Resampled U.S. asset allocations (1973 – 1992)

	Minimum Variance Portfolio	**Middle Portfolio**	Maximum Return Portfolio
FF L Cap Growth	78.70%	**21.30%**	0.00%
FF L Cap Value	8.73%	**19.71%**	9.22%
FF S Cap Growth	0.21%	**0.00%**	0.00%
FF S Cap Value	12.37%	**58.99%**	90.78%
Standard Deviation	28.85%	**33.01%**	37.18%
Geometric Return	6.42%	**12.06%**	13.88%
Sharpe Ratio	0.22	**0.37**	0.37

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

With reference to Table 6.12 it is noticed that there is a slightly improved allocation of South African assets, leading to enhanced diversification. This is apparent from the allocation of 0.96 percent of assets to South African mid-cap growth category as well as the significant increase in allocation to the South African small-cap growth category of 12.72 percent. The main observation is that the change to the allocation is less significant than was anticipated. This may be period specific with more significant results being delivered in ensuing periods, or the asset allocation weightings using historical data reflect the optimal allocation of assets fairly.

Table 6.12 South African resampled versus actual asset allocations (1973 – 1992)

South African Assets	Resampled Asset Allocations	Actual Asset Allocations
MP L Cap Growth	0.00%	0.00%
MP L Cap Value	2.96%	2.71%
MP M Cap Growth	0.96%	0.00%
MP M Cap Value	44.36%	41.57%
MP S Cap Growth	3.86%	12.72%
MP S Cap Value	47.86%	43.00%

Source: Derived using Tables 6.5 and 6.10.

With reference to Table 6.13 it is noticed that there is a slightly improved allocation of U.S. assets, leading to enhanced diversification. This is apparent by the increased allocation to U.S. large-cap assets, and the reduction of the more volatile U.S. small-cap assets, to 58.99 percent. The main observation is that the change to the allocation is less significant than was anticipated. This may be period specific with more significant results being delivered in ensuing periods, or that the asset allocation weightings using historical data reflect the optimal allocation of assets fairly.

Table 6.13 U.S. resampled versus actual asset allocations (1973 – 1992)

U.S. Assets	Resampled Asset Allocations	Actual Asset Allocations
FF L Cap Growth	21.30%	19.44%
FF L Cap Value	19.71%	17.34%
FF S Cap Growth	0.00%	0.00%
FF S Cap Value	58.99%	63.21%

Source: Derived using Tables 6.6 and 6.11.

6.7 RESAMPLED INTER-MARKET ASSET ALLOCATIONS (1973 – 1992)

Following the determination of resampled asset allocations for the respective markets the final step is to derive the allocation per market. This process is achieved by using the middle portfolio data in Tables 6.10 and 6.11, the derived inter-year rates of return in Annexure 8 and retro-actively calculating the resampled optimal portfolios' inter-year rates of return. These inter-year rates of return for the respective markets are then used to derive the resampled cross-correlations between the respective markets, as well as the resampled standard deviation and resampled geometric rates of return for the period 1973 – 1992, as reflected in Annexure 9. This data is then used for the mean-variance optimisation algorithm to derive the split of assets per market by way of the efficient frontier, as reflected in Annexure 10.

Table 6.14 Resampled inter-market asset allocations (1973 – 1992)

	Minimum Variance Portfolio	Middle Portfolio	Maximum Return Portfolio
United States Market	**21.43%**	35.71%	41.78%
South African Market	**78.57%**	64.29%	58.22%
Standard Deviation	**20.20%**	20.82%	21.43%
Geometric Return	**11.21%**	11.40%	11.42%
Sharpe Ratio	**0.55**	0.55	0.53

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992.

With reference to Table 6.15 it is noted that there is a minimal adjustment in the allocation of assets between markets as a result of the adjustments to the intra-market asset allocations.

Table 6.15 Inter-market resampled versus actual allocations (1973 – 1992)

Inter-Market Allocations	Resampled Asset Allocations	Actual Asset Allocations
United States Market	21.43%	19.92%
South African Market	78.57%	80.08%

Source: Derived using Tables 6.8 and 6.14.

6.8 MARKETS MEAN REVERSION

In order for forward looking portfolios to be designed around historical data inputs there needs to be significant evidence of mean reversion amongst the asset classes. This is tested for in accordance with the methodology[43], with the results set out in Tables 6.16 and 6.17.

Tables 6.16 and 6.17 are daunting in their presentation. The statistical breakdown is by asset class, with the first line of data being the correlation coefficients and the second line the coefficients of determination. The objective is to interpret the levels of predictability, using the coefficients of determination.

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[43] See Section 2.9 (p. 50).

Table 6.16 **South African equity serial-correlation (R) and coefficient of determination (R^2) statistics (1973 – 1992)***

Rolling Time Periods (Years)									
1	2	3	4	5	6	7	8	9	10
MP L Cap Growth									
12.75	51.44	69.85	64.64	63.07	57.86	56.20	58.40	68.25	67.57
1.62	26.46	48.79	41.79	39.77	33.48	31.58	34.11	46.58	45.66
MP L Cap Value									
-22.70	23.81	42.80	30.40	27.75	13.27	-5.96	4.25	51.20	19.63
5.16	5.67	18.32	9.24	7.70	1.76	0.35	0.18	26.21	3.85
MP M Cap Growth									
7.59	50.64	72.69	68.60	75.70	80.92	75.47	80.96	85.94	77.77
0.58	25.64	52.84	47.06	57.31	65.49	56.96	65.54	73.85	60.48
MP M Cap Value									
7.50	48.51	61.79	66.97	71.23	73.74	78.98	67.68	78.64	65.49
0.56	23.53	38.18	44.85	50.73	54.37	62.38	45.80	61.84	42.90
MP S Cap Growth									
26.78	57.19	70.41	76.40	71.46	75.12	76.21	79.22	76.69	79.26
7.17	32.71	49.57	58.37	51.06	56.43	58.08	62.76	58.81	62.83
MP S Cap Value									
12.01	54.57	66.45	74.47	65.34	65.73	67.49	79.27	63.89	68.25
1.44	29.77	44.16	55.46	42.69	43.21	45.54	62.84	40.82	46.59
*Figures are presented as percentages (%).									

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

Table 6.17 **U.S. equity serial-correlation (R) and coefficient of determination (R^2) outcomes (1973 – 1992)***

Rolling Time Periods (Years)									
1	2	3	4	5	6	7	8	9	10
FF L Cap Growth									
9.65	33.06	59.38	72.64	72.49	75.01	74.34	77.69	87.12	75.97
0.93	10.93	35.26	52.77	52.55	56.26	55.26	60.36	75.90	57.72
FF L Cap Value									
11.60	32.77	60.95	69.53	67.65	70.81	69.01	61.93	55.77	36.68
1.35	10.74	37.14	48.35	45.77	50.15	47.62	38.35	31.10	13.46
FF S Cap Growth									
0.26	24.63	48.63	59.78	62.67	65.28	63.49	66.60	64.61	57.54
0.00	6.06	23.65	35.74	39.28	42.62	40.31	44.35	41.74	33.11
FF S Cap Value									
18.01	37.38	62.86	70.02	68.32	74.76	76.19	66.19	60.30	54.83
3.24	13.97	39.52	49.03	46.68	55.89	58.05	43.82	36.36	30.06
*Figures are presented as percentages (%).									

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

The same process applied to the plethora of asset classes is applied to the broad market indices in order to view the overall mean reversion characteristics thereof. The outcome of this process is represented in Table 6.18.

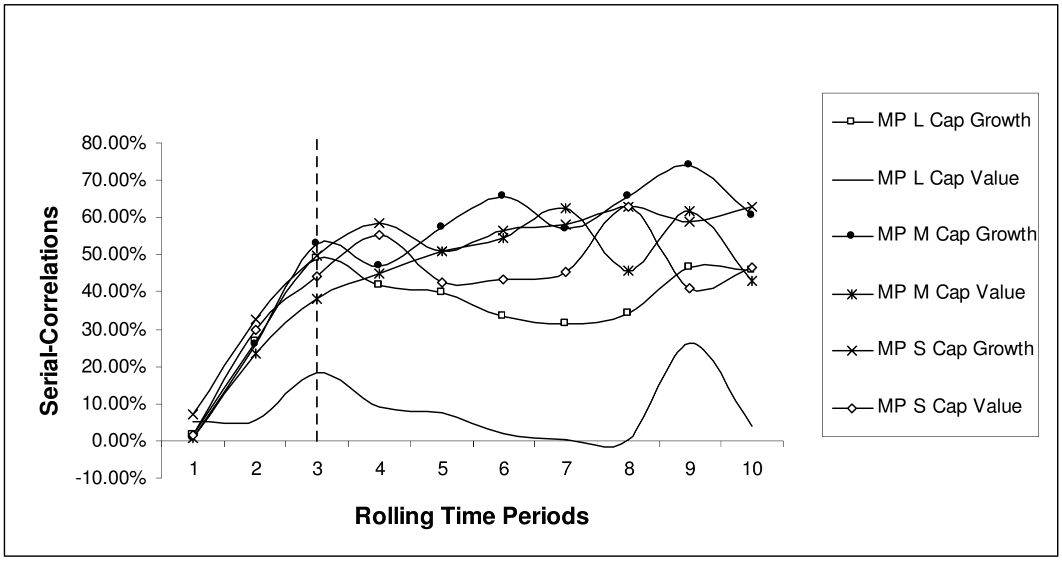
Table 6.18 **Serial-correlation (R) and coefficient of determination (R^2) outcomes for the broad market indices (1973 – 1992)***

Rolling Time Periods (Years)									
1	2	3	4	5	6	7	8	9	10
ALSI									
8.56	47.06	65.53	62.06	61.44	61.67	63.29	54.86	70.72	63.57
0.73	22.14	42.94	38.52	37.75	38.03	40.06	30.10	50.02	40.41
S&P 500									
-6.87	25.87	57.52	67.81	71.21	75.95	83.42	78.70	85.71	83.06
0.47	6.69	33.09	45.98	50.70	57.68	69.59	61.93	73.47	68.99
*Figures are presented as percentages (%).									

Source: Derived in accordance with established methodology using data for the period 1973 – 1992.

The analysis of the South African asset class serial-correlation calculations, as reflected in Table 6.16, and set out graphically in Figure 6.4, reveals that there is significant, broad based predictability in returns from the three year mark, with the MP M Cap Growth attaining a high 85.94 percent correlation coefficient.

Figure 6.4 South African asset classes mean reversion

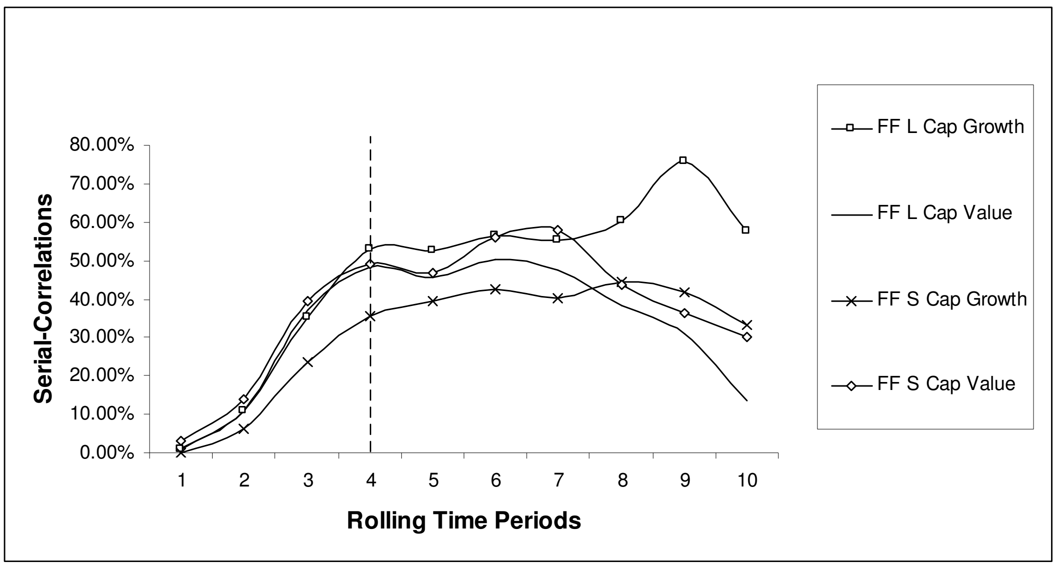


Source: Derived using Table 6.16.

In an environment that may presume market efficiency, namely that there is no possibility of knowing which way the market is heading, this is a strong indication that there is a significant level of predictability. With reference to Table 6.18, the broad South African market index displayed a serial-correlation coefficient, for the one year period, of a mere 8.56 percent, indicating a level of determination of only 0.73 percent, which implies stochastic behaviour. This level of correlation rises to 65.53 percent at the three year level, clearly indicating trend-like behaviour the greater the time period under review. The high levels of correlation imply that there is significant mean reversion in order to convert stochastic or even cyclical behaviour, to trend-like behaviour.

With reference to Table 6.16, an analysis of the individual South African asset classes reveals characteristics analogous to that of the broad market, indicated above, with the exception of large-cap value assets, as indicated in Figure 6.4. The South African large-cap value assets show a negative correlation coefficient of -22.70 percent at the one year period. Although this may signal mean reversion (cyclical behaviour) the levels of correlation remain inordinately low through different time periods. This characteristic equates to low levels of determination, and therefore implies that any investment in this asset class is inherently risky as a result of the lack of adherence to past characteristics and the low levels of predictability.

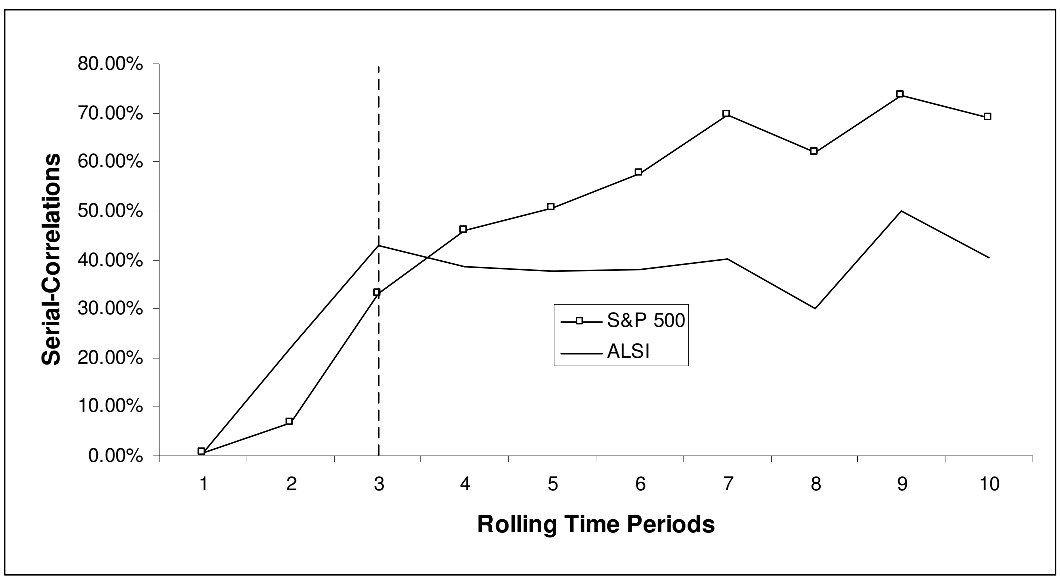
Figure 6.5 U.S. asset classes mean reversion



Source: Derived using Table 6.17.

The analysis of the U.S. asset class serial-correlation coefficients, as reflected in Table 6.17, and set out graphically in Figure 6.5, reveals that there is significant, broad based predictability in returns from the four year mark. The FF L Cap Growth asset class attained a high 87.12 percent correlation coefficient, which indicates that as much as 75.90 percent of the future period return can be predicted using past performance. Given that the U.S. broad based market index serial correlation coefficients, for the one year period, are a mere -6.87 percent, indicating a level of determination of only 0.47 percent, which implies stochastic behaviour, the markets notably manifest trend-like behaviour the greater the time period under review. The high levels of correlation imply that there is significant mean reversion in order to convert stochastic or even cyclical behaviour, to trend-like behaviour.

Figure 6.6 Broad market indices mean reversion



Source: Derived using data from Table 6.18.

Notably, with reference to Figure 6.6, the predictability levels for the U.S. market, as measured by the determination coefficient, seem to continue rising relative to the South African, the greater the time period. This phenomenon will be discussed in the ensuing chapter.

Finally, with reference to Figures 6.4 and 6.5, it is noted that the levels of predictability for the individual assets classes seems to decline somewhat, the longer the time period under review. This may have implications for long term investors, and will be discussed in the ensuing chapter.

6.9 ACTUAL INTRA-MARKET ASSET ALLOCATIONS (1974 – 2002)

This process will be calculated in accordance with Section 2.5 (p. 41) of the methodology. The outcomes are listed in Tables 6.19 – 6.22.

Table 6.19 Actual South African asset allocations (1974 – 1997)

	1974-1993	1975-1994	1976-1995	1977-1996	1978-1997	**1973-1992**
MP L Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	**0.00%**
MP L Cap Value	1.00%	3.69%	5.66%	8.73%	0.00%	**2.71%**
MP M Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	**0.00%**
MP M Cap Value	33.00%	0.00%	5.00%	0.00%	0.00%	**41.57%**
MP S Cap Growth	18.73%	26.12%	22.15%	19.69%	27.79%	**12.72%**
MP S Cap Value	47.27%	70.20%	67.20%	71.58%	72.21%	**43.00%**
Standard Deviation	23.71%	23.10%	23.16%	22.38%	22.88%	**21.36%**
Geometric Return	12.28%	15.38%	15.69%	16.98%	16.14%	**10.13%**
Sharpe Ratio	0.52	0.67	0.68	0.76	0.71	**0.47**

Source: Derived in accordance with established methodology using data for the period 1973 – 1997.

Table 6.20 Actual South African asset allocations (1979 – 2002)

	1979-1998	1980-1999	1981-2000	1982-2001	1983-2002	**1973-1992**
MP L Cap Growth	0.00%	0.00	0.00%	0.00%	0.00%	**0.00%**
MP L Cap Value	0.00%	0.00	8.25%	19.73%	5.81%	**2.71%**
MP M Cap Growth	0.00%	0.00	0.00%	0.00%	0.00%	**0.00%**
MP M Cap Value	5.25%	30.48	0.00%	0.00%	0.00%	**41.57%**
MP S Cap Growth	28.85%	25.10	28.70%	26.94%	29.55%	**12.72%**
MP S Cap Value	65.90%	44.41%	63.06%	53.32%	64.64%	**43.00%**
Standard Deviation	24.61%	19.86%	21.08%	20.92%	21.23%	**21.36%**
Geometric Return	13.15%	11.88%	9.18%	10.44%	10.13%	**10.13%**
Sharpe Ratio	0.53	0.60	0.44	0.50	0.48%	**0.47**

Source: Derived in accordance with established methodology using data for the period 1979 – 2002.

Table 6.21 Actual U.S. asset allocations (1974 – 1997)

	1974-1993	1975-1994	1976-1995	1977-1996	1978-1997	**1973-1992**
FF L Cap Growth	24.09%	24.23%	22.79%	22.05%	24.39%	**19.44%**
FF L Cap Value	0.00%	0.00%	0.00%	0.00%	0.00%	**17.34%**
FF S Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	**0.00%**
FF S Cap Value	75.91%	75.77%	77.21%	77.95%	75.61%	**63.21%**
Standard Deviation	29.79%	27.66%	24.79%	24.80%	24.60%	**33.24%**
Geometric Return	16.53%	18.46%	16.45%	16.67%	18.01%	**11.82%**
Sharpe Ratio	0.55	0.67	0.66	0.67	0.73	**0.36**

Source: Derived in accordance with established methodology using data for the period 1973 – 1997.

Table 6.22 Actual U.S. asset allocations (1979 – 2002)

	1979-1998	1980-1999	1981-2000	1982-2001	1983-2002	**1973-1992**
FF L Cap Growth	31.24%	55.59%	29.15%	32.80%	39.45%	**19.44%**
FF L Cap Value	0.00%	0.00%	32.84%	0.00%	0.00%	**17.34%**
FF S Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	**0.00%**
FF S Cap Value	68.76%	44.41%	38.00%	67.20%	60.55%	**63.21%**
Standard Deviation	23.31%	20.28%	23.02%	24.10%	28.27%	**33.24%**
Geometric Return	18.20%	18.29%	18.58%	19.57%	13.60%	**11.82%**
Sharpe Ratio	0.78	0.90	0.81	0.81	0.48	**0.36**

Source: Derived in accordance with established methodology using data for the period 1979 – 2002.

With reference to Tables 6.19 – 6.22, upon analysing the South African and U.S. asset classes it is immediately apparent how much ensuing 20 year period historical asset allocations deviate from the original asset allocations (1973 – 1992), as indicated in bold.

This finding suggests that the use of historical data, from a particular period, may be inappropriate for a future period, and may hold dire risks for the investor. These findings indicate the necessity for an adjustment process which will allow for a dynamic adjustment to asset allocations.

6.10 ACTUAL INTER-MARKET ASSET ALLOCATIONS (1974 – 2002)

Following the determination of asset allocations for the respective markets the final step is to derive the allocation per market. This process is achieved first by using the data in Tables 6.19 – 6.22, the inter-year rates of return in Annexures 11, 14, 17, 20, 23, 26, 29, 32, 35 and 38, and retro-actively calculating the optimal portfolios' inter-year rates of return. These inter-year rates of return for the respective markets are then used to derive the cross-correlations between the respective markets, as well as the standard deviation and geometric rates of return for the rolling 20 year periods between 1974 – 2002, as reflected in Annexures 12, 15, 18, 21, 24, 27, 30, 33, 36 and 39. This data is then used for the mean-variance optimisation algorithm to derive the split of assets per market by way of the efficient frontiers, as reflected in Annexures 13, 16, 19, 22, 25, 28, 31, 34, 37 and 40. The results thereof are shown in Tables 6.23 and 6.24.

Table 6.23 Actual inter-market asset allocations (1974 – 1997)

	1974-1993	1975-1994	1976-1995	1977-1996	1978-1997	**1973-1992**
United States Market	31.44%	36.83%	42.75%	38.41%	42.04%	**19.92%**
South African Market	68.56%	63.17%	57.25%	61.59%	57.96%	**80.08%**
Standard Deviation	21.34%	19.17%	18.60%	19.52%	18.96%	**20.20%**
Geometric Return	14.45%	17.52%	16.93%	17.63%	17.87%	**11.18%**
Sharpe Ratio	0.68	0.91	0.91	0.90	0.94	**0.55**

Source: Derived using data from the multiple asset classes for the period 1973 – 1997.

Table 6.24 Actual inter-market asset allocations (1979 – 2002)

	1979-1998	1980-1999	1981-2000	1982-2001	1983-2002	**1973-1992**
United States Market	48.63%	41.78%	39.68%	34.67%	24.22%	**19.92%**
South African Market	51.37%	58.22%	60.32%	65.33%	75.78%	**80.08%**
Standard Deviation	20.02%	16.22%	17.91%	18.58%	19.78%	**20.20%**
Geometric Return	16.57%	15.33%	13.83%	14.73%	12.18%	**11.18%**
Sharpe Ratio	0.83	0.95	0.77	0.79	0.62	**0.55**

Source: Derived using data from the multiple asset classes for the period 1979 – 2002.

With reference to Section 6.9 (p. 185) and the instability of the intra-market asset allocations it is expected that the inter-market asset allocations will manifest similar characteristics. The data analyses, set out in Tables 6.23 and Tables 6.24, confirm this and further support the notion that there is a necessity for an adjustment process which will allow for a dynamic adjustment to asset allocations.

6.11 RESAMPLED VERSUS ACTUAL EFFICIENT FRONTIER PORTFOLIO RETURNS

Using the established resampled asset allocations as at 1992, as set out in Tables 6.10, 6.11 and 6.14, the inter-year (1993 – 2002) arithmetic rates of return are derived as per Annexures 41, 43, 45, 47, 49, 51, 53, 55, 57 and 59, which will be used as a comparison to the actual rates of return, reflected in Annexures 11, 14, 17, 20, 23, 26, 29, 32, 35 and 38, using the correlation coefficient equation, and the coefficient of determination. The results are set out in Tables 6.25, 6.26 and 6.27.

Table 6.25 South African equities (1993 – 2002)

Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1993	99.60%	99.20%
1994	97.54%	**95.14%**
1995	98.38%	**96.79%**
1996	98.06%	**96.15%**
1997	97.18%	**94.43%**
1998	97.88%	**95.80%**
1999	98.84%	**97.69%**
2000	96.09%	**92.33%**
2001	95.99%	**92.13%**
2002	95.78%	**91.73%**

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

Table 6.26 U.S. equities (1993 – 2002)

Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1993	99.78%	99.57%
1994	99.76%	99.52%
1995	99.62%	99.25%
1996	99.59%	99.19%
1997	99.70%	99.40%
1998	99.78%	99.55%
1999	97.44%	**94.94%**
2000	99.23%	**98.46%**
2001	99.74%	99.48%
2002	99.70%	99.40%

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

Table 6.27 Inter-market equities (1993 – 2002)

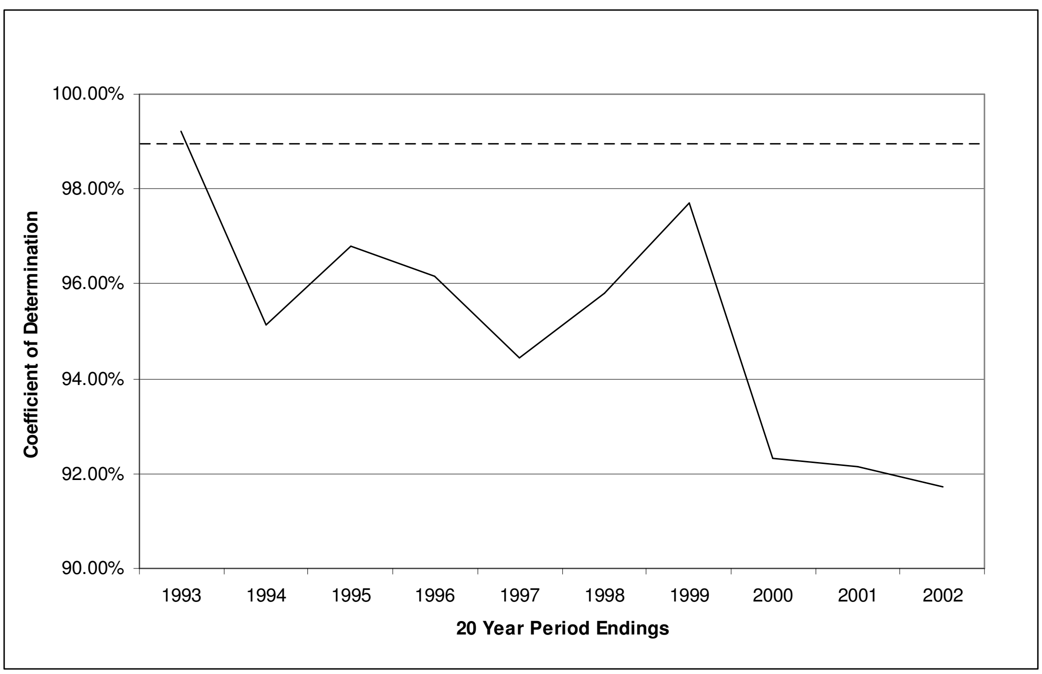
Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1993	96.85%	93.79%
1994	89.91%	80.84%
1995	95.16%	90.56%
1996	90.77%	82.39%
1997	86.18%	74.27%
1998	81.66%	66.69%
1999	87.12%	75.90%
2000	87.76%	77.01%
2001	90.60%	82.09%
2002	95.97%	92.11%

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

An effective, predetermined asset allocation should manifest itself retroactively by displaying a coefficient of determination (R^2) of one when measured relative to the actual optimal asset allocation. Therefore the effectiveness of any asset allocation is measured by comparing the annual returns, over a specified period, of the resampled asset allocations relative to the returns, over the same specified period, of the actual optimal asset allocations. In this way it is possible to measure the amount of return determined by the resampled asset allocation.

With reference to Table 6.25, as indicated in bold, and Figure 6.7 it is noted that the South African assets show an increasing tendency to capture less of the return than is desired by the methodology[44], namely 99 percent, when the original resampled asset allocation is left unadjusted. Any variable less than 99 percent may indicate the need to redetermine the asset allocations. The decreasing levels of determination are understandable given the instability within the asset allocations, as highlighted in Sections 6.9 (p. 186) and 6.10 (p. 190). In addition, given the frequency of variables below 99 percent this may indicate that the original asset allocations were not in line with historical means, thereby further indicating the need for redetermination of the asset allocations.

Figure 6.7 South African equities (1993 – 2002)



Source: Derived using data from Table 6.25.

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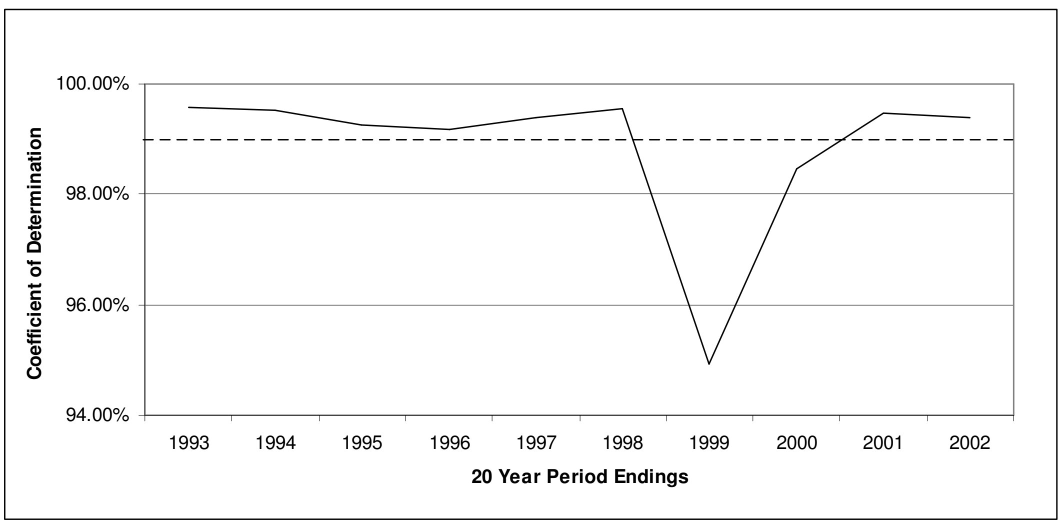
[44] See Section 2.7 e) (p. 49).

With reference to Table 6.26, as indicated in bold, and Figure 6.8 it is noted that the U.S. asset classes show a slight tendency to capture less of the return than is desired by the methodology, namely 99 percent, and then a reversion to above the 99 percent level. Given the infrequency of variables below 99 percent, this may indicate that the original asset allocations were in line with historical means, which may indicate that the U.S. asset classes are more stable relative to their historical averages or the selection of asset classes serendipitously happened to resemble the historical averages.

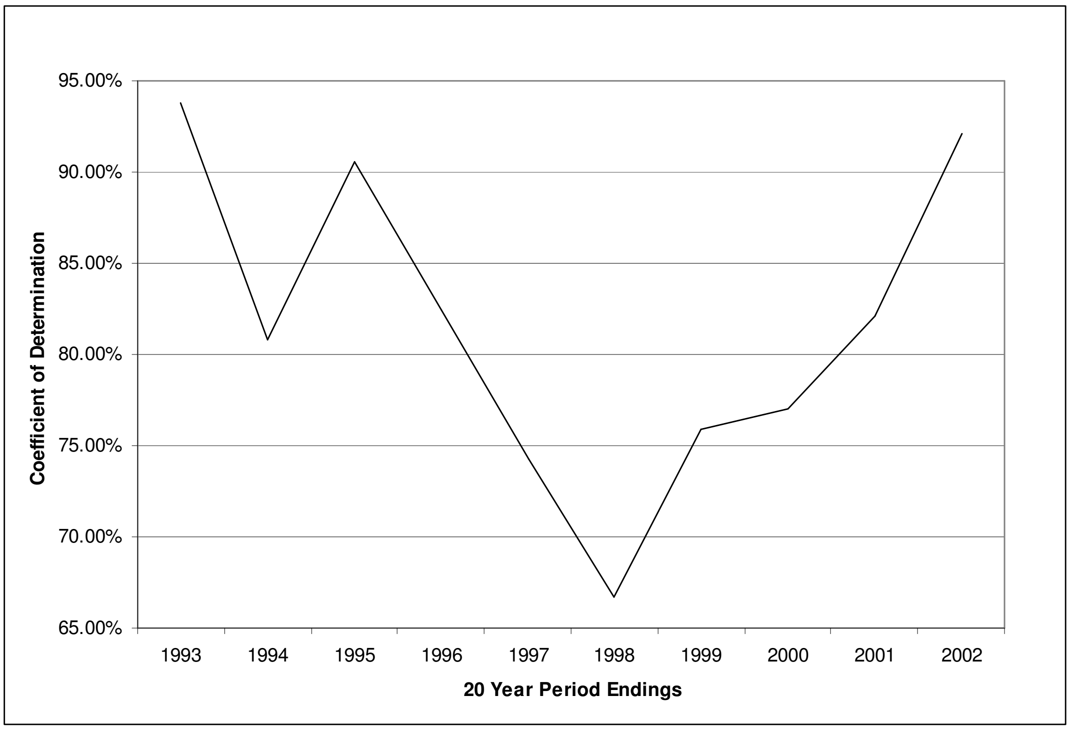
Figure 6.8 U.S. equities (1993 – 2002)



Source: Derived using data from Table 6.26.

With reference to Table 6.27 and Figure 6.9, given the fact that the intra-market asset allocations appear inherently unstable, and given that the inter-market asset allocations are determined by the intra-market allocations, it is not surprising that the levels of determination at the inter-market level are prone to significant changes from period to period. The implication of this is that asset class instability has the tendency to maximise the movement away from an optimal allocation at the inter-market level which may result in a significant difference in portfolio outcomes relative to the optimal solution. Asset class instability is therefore an imperative that requires periodic attention.

Figure 6.9 Inter-market equities (1993 – 2002)



Source: Derived using data from Table 6.27.

6.12 PERIODIC REVIEW OF RESAMPLED ASSET ALLOCATIONS

In accordance with the methodology[45] where the coefficient of determination is below 99 percent, as indicated in bold in Tables 6.25 and 6.26, the resampled

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[45] See Section 2.7 f) (p. 49).

asset allocations need to be redetermined, using the same methodology[46] as applied before. The results thereof, for the different periods, are reflected in Tables 6.28 and 6.29.

Table 6.28 Redetermined resampled South African asset allocations

	1975-1994	1980-1999	1981-2000	1982-2001	**1973-1992**
MP L Cap Growth	0.00%	0.00%	0.00%	0.00%	**0.00%**
MP L Cap Value	3.96%	0.27%	8.10%	18.51%	**2.96%**
MP M Cap Growth	0.00%	0.00%	0.00%	0.00%	**0.96%**
MP M Cap Value	9.33%	32.64%	8.35%	2.62%	**44.36%**
MP S Cap Growth	23.44%	28.62%	28.74%	27.87%	**3.86%**
MP S Cap Value	63.28%	38.47%	54.81%	51.01%	**47.86%**

Source: Derived in accordance with established methodology using data for the period 1975 – 2002.

[46] See Section 2.6.3 (p. 45).

Table 6.29 Redetermined resampled U.S. asset allocations

	1980-1999	1981-2000	1982-2001	**1973-1992**
FF L Cap Growth	51.24%	31.54%	30.99%	**21.30%**
FF L Cap Value	20.98%	39.22%	12.87%	**19.71%**
FF S Cap Growth	0.00%	0.00%	0.00%	**0.00%**
FF S Cap Value	27.78%	29.24%	56.14%	**58.99%**

Source: Derived in accordance with established methodology using data for the period 1980 – 2002.

The redetermined asset allocations are then used to determine the redetermined resampled inter-year returns, which in turn are compared to the actual inter-year returns. The results of the comparison are shown in Tables 6.30, 6.31 and 6.32.

Table 6.30 Redetermined South African equities (1993 – 2002)

Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1993	99.60%	99.20%
1994	97.54%	**95.14%**
1995	99.97%	99.95%
1996	99.86%	99.71%
1997	99.69%	99.39%
1998	99.87%	99.75%
1999	99.12%	**98.25%**
2000	98.06%	**96.16%**
2001	99.35%	**98.69%**
2002	98.63%	**97.29%**

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

Table 6.31 Redetermined U.S. equities (1993 – 2002)

Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1993	99.78%	99.57%
1994	99.76%	99.52%
1995	99.62%	99.25%
1996	99.59%	99.19%
1997	99.70%	99.40%
1998	99.78%	99.55%
1999	97.44%	**94.94%**
2000	98.72%	**97.45%**
2001	96.38%	**92.88%**
2002	99.92%	99.84%

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

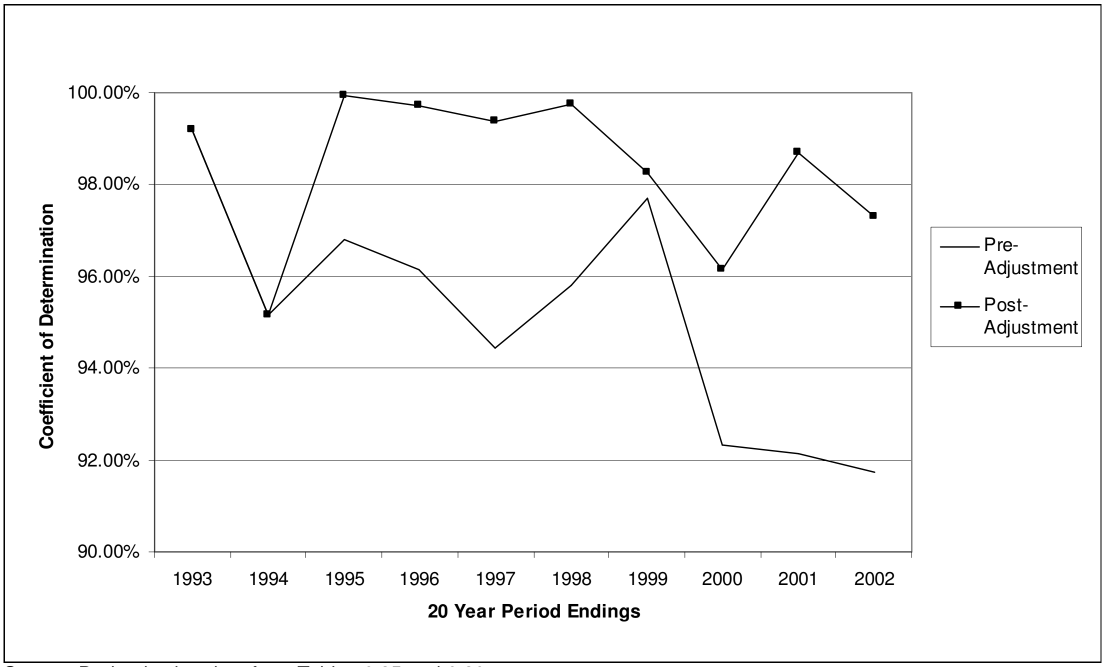
Table 6.32 Redetermined inter-market equities (1993 – 2002)

Year	Correlation Coefficient (R)	Determination Coefficient (R^2)
1974 - 1993	96.85%	93.79%
1975 - 1994	89.91%	80.84%
1976 - 1995	98.84%	97.69%
1977 - 1996	99.76%	99.53%
1978 - 1997	98.89%	97.79%
1979 - 1998	96.48%	93.08%
1980 - 1999	96.43%	92.98%
1981 - 2000	93.21%	86.89%
1982 - 2001	98.15%	96.34%
1983 - 2002	94.99%	90.23%

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

By redetermining the intra-market asset allocations, mutatis mutandis, with reference to the South African asset classes, it is apparent that the post-redetermined coefficients of determination are significantly better than the pre-redetermined figures. This is graphically represented in Figure 6.10. Although there remain occasions that the post-redetermined figures are below the desired 99 percent level, as indicated in bold in Tables 6.30 and 6.31, indicating that the process is not absolute; it is apparent that the results are significantly enhanced.

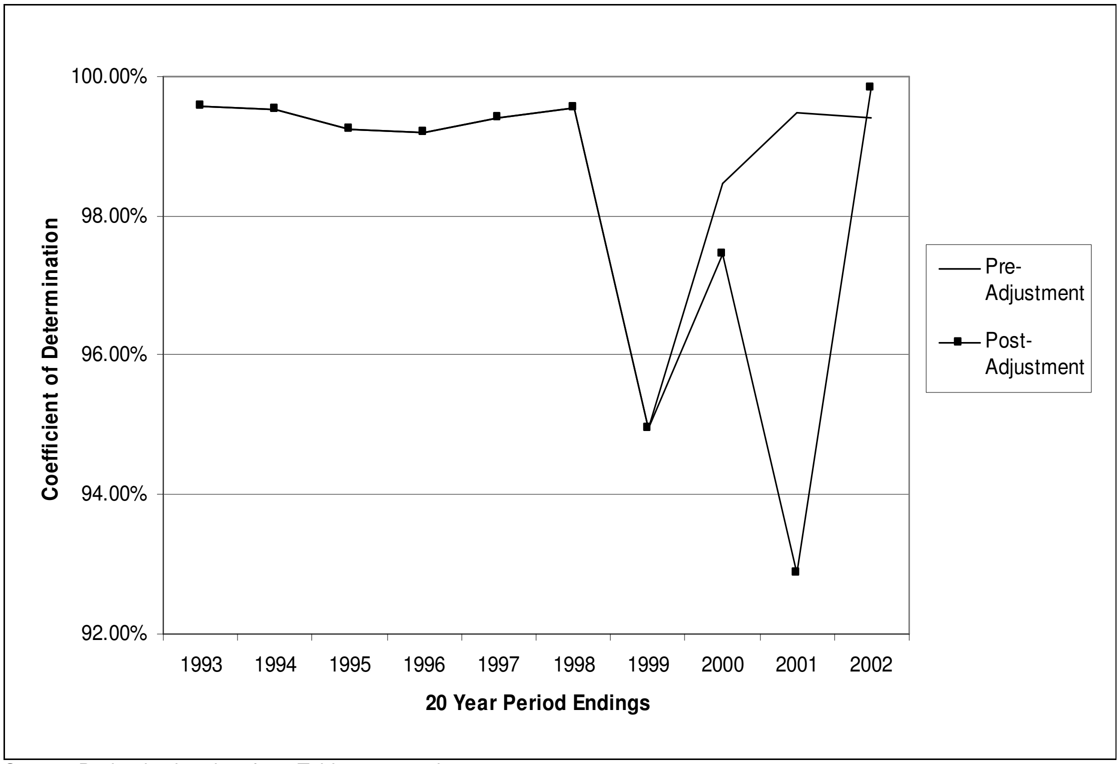
Figure 6.10 South African equities post redetermination (1993 – 2002)



Source: Derived using data from Tables 6.25 and 6.30.

By redetermining the intra-market asset allocations, mutatis mutandis, with reference to the U.S. asset classes, it is apparent that the post-redetermined coefficients of determination initially are significantly worse than the pre-redetermined figures; however by 2002 the adjustment process has corrected the asset allocations. This is graphically represented in Figure 6.11. This indicates that where there is a fundamental move away from historical asset allocations, the adjustment process will eventually correct the allocations. However during periods of extreme volatility, the correction process is subject to a delayed effect which could result in the misallocation of assets until some form of stability prevails. Stability in this respect does not refer to lower levels of volatility, as represented by standard deviation since volatility is inherent in equity markets, but stability rather refers to the performance rankings of asset classes in relation to their long-term positions.

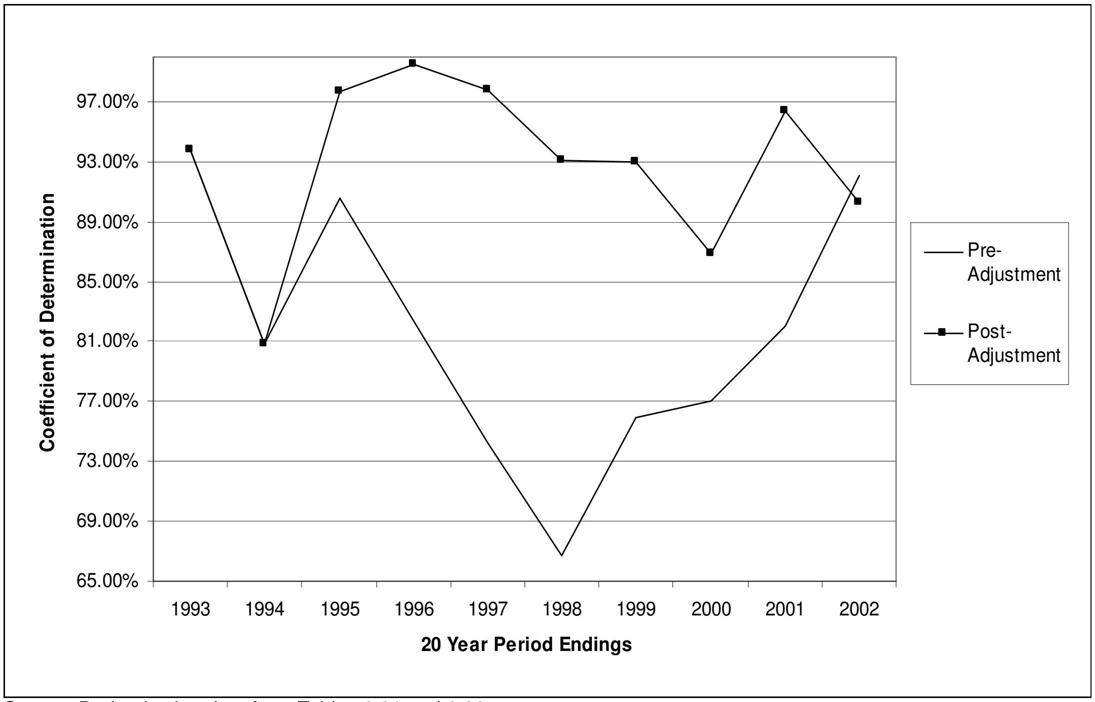
Figure 6.11 U.S. equities post-redetermination (1993 – 2002)



Source: Derived using data from Tables 6.26 and 6.31.

In light of the intra-market findings and the complexity of adjusting asset allocations over time it is interesting to note that post-redetermined inter-market coefficients of determination are significantly better than the pre-redetermined figures, as reflected graphically in Figure 6.12.

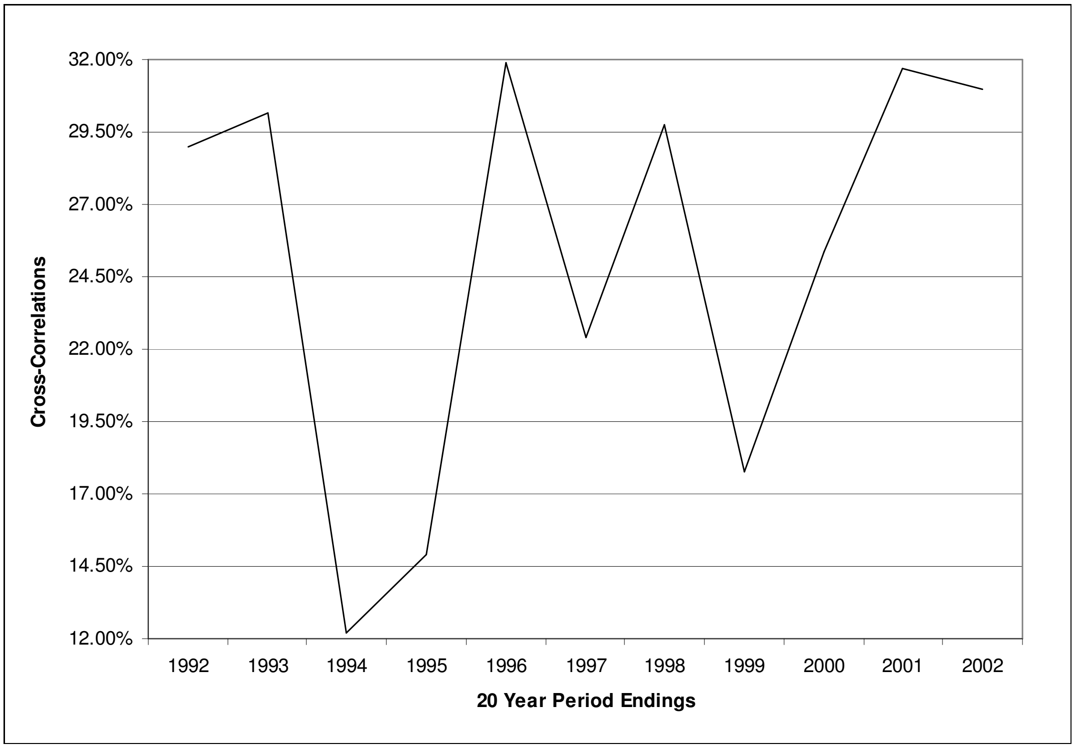
Figure 6.12 Inter-market equities post-redetermination (1993 – 2002)



Source: Derived using data from Tables 6.21 and 6.26.

There still remains the possibility that all markets may move away from their historical asset allocations simultaneously, however with cross-correlations not being that high for rolling 20 year periods, as indicated in Figure 6.13, the probability of this occurring appears slight.

Figure 6.13 Inter-market cross-correlations (1992 – 2002)



Source: Derived using data for the period 1973 - 2002.

6.13 RESAMPLED DIVERSIFICATION EFFECTIVENESS

Section 6.6 (p. 173), pertaining to both markets, made the observation that 'change to the allocation is less significant than was anticipated', and it was suggested that a more optimal allocation may arise during ensuing periods. The implication was that the resampled asset allocation did not provide additional diversification benefits and was optimal relative to the actual allocation. This did not provide support for the hypothesis that mean-variance optimisation results in an effective allocation of assets, as well as effective investment diversification by constructing more optimally diversified portfolios.

By comparing the redetermined resampled and actual asset allocations for ensuing periods the hypothesis could be examined further. The results of this are set out in Tables 6.33 and 6.34.

Table 6.33 South African equities (1992 – 2002)

	1975 – 1994		1980 – 1999		1981 - 2000		1982 – 2001	
	RRes	Actual	RRes	Actual	RRes	Actual	RRes	Actual
MP L Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
MP L Cap Value	3.96%	3.69%	**0.27%**	**0.00%**	8.10%	8.25%	18.51%	19.73%
MP M Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
MP M Cap Value	**9.33%**	**0.00%**	32.64%	30.48%	**8.35%**	**0.00%**	**2.62%**	**0.00%**
MP S Cap Growth	23.44%	26.12%	28.62%	25.10%	28.74%	28.70%	27.87%	26.94%
MP S Cap Value	63.28%	70.20%	38.47%	44.41%	54.81%	63.06%	51.01%	53.32%

Source: Derived using data from Tables 6.19, 6.20 and 6.28.

It is noted in both markets, as indicated in bold, that there is a significant improvement in investment diversification through a more intuitive allocation of assets. The implication is that previous resampled asset allocations may have been optimally allocated. Furthermore, the results seem to lend support to the hypothesis that resampled allocations lead to effective asset allocations.

Table 6.34 **U.S. equities (1992 – 2002)**

	1980 – 1999		1981 - 2000		1982 – 2001	
	RRes	Actual	RRes	Actual	RRes	Actual
FF L Cap Growth	51.24%	55.59%	31.54%	29.15%	30.99%	32.80%
FF L Cap Value	**20.98%**	**0.00%**	39.22%	32.84%	**12.87%**	**0.00%**
FF S Cap Growth	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
FF S Cap Value	27.78%	44.41%	29.24%	38.00%	56.14%	67.20%

Source: Derived using data from Tables 6.22 and 6.29.

6.14 RISK, REWARD AND DIVERSIFICATION

With reference to Table 6.35 it is apparent that the mean-variance model does indeed produce a premium in the form of risk reduction and enhanced returns, through the use of diversification relative to investments that are 100 percent invested in a particular market. Additionally, it is noted that the redetermined resampled portfolio produces the same premium outcome relative to the actual portfolio.

With reference to the redetermined resampled portfolio, it is noted that the Sharpe ratio is 0.67 compared to the actual outcome of 0.62. This seems to indicate that the resampled portfolio delivers an additional premium. Mathematically, of course, this is impossible since there is only one efficient portfolio that is perfect in hindsight. The fact that the redetermined resampled portfolio appears to produce a premium is a function of its slightly different asset allocation, and may be period specific.

Regarding the market specific portfolios it is noted that the resampled portfolio is significantly superior on a risk-adjusted basis, with a Sharpe ratio of 0.67 compared to the South African portfolio at 0.46 and the U.S. portfolio at 0.49.

Table 6.35 Risk, reward and diversification comparison (1983 – 2002)

	Actual	Redetermined		Resampled
United States Market	24.22%	100%	0%	22.02%
South African Market	75.78%	0%	100%	77.98%
Standard Deviation	19.78%	31.57%	20.87%	18.08%
Geometric Return	12.18%	15.60%	9.67%	12.20%
Sharpe Ratio	0.62	0.49	0.46	0.67

Source: Derived using data from Table 6.24 and Annexure 62.

6.15 INVESTMENT PORTFOLIO DETERMINATION AND ANALYSES

In accordance with the methodology[47] the geometric returns for the primary, secondary and control portfolios are derived and set out in Table 6.36.

[47] See Sections 2.4.8 (p. 28) and 2.4.9 (p. 30).

Table 6.36 Investment portfolio returns (1993 – 2002)

Portfolio	Unrebalanced			Rebalanced		
	Real Return	Standard Deviation	Sharpe Ratio	Real Return	Standard Deviation	Sharpe Ratio
Redetermined Resampled	14.57%	23.56%	0.62	14.03%	24.28%	0.58
Resampled	12.08%	27.20%	0.44	13.64%	26.08%	0.52
Constrained	8.95%	22.11%	0.40	10.10%	20.87%	0.48
Equal Weighted	7.24%	22.20%	0.33	8.89%	19.72%	0.45
Market Index	7.24%	22.23%	0.33	8.96%	20.61%	0.43
Rates of return are geometric, and net of inflation. Portfolios are ranked according to rebalanced Sharpe ratios.						

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

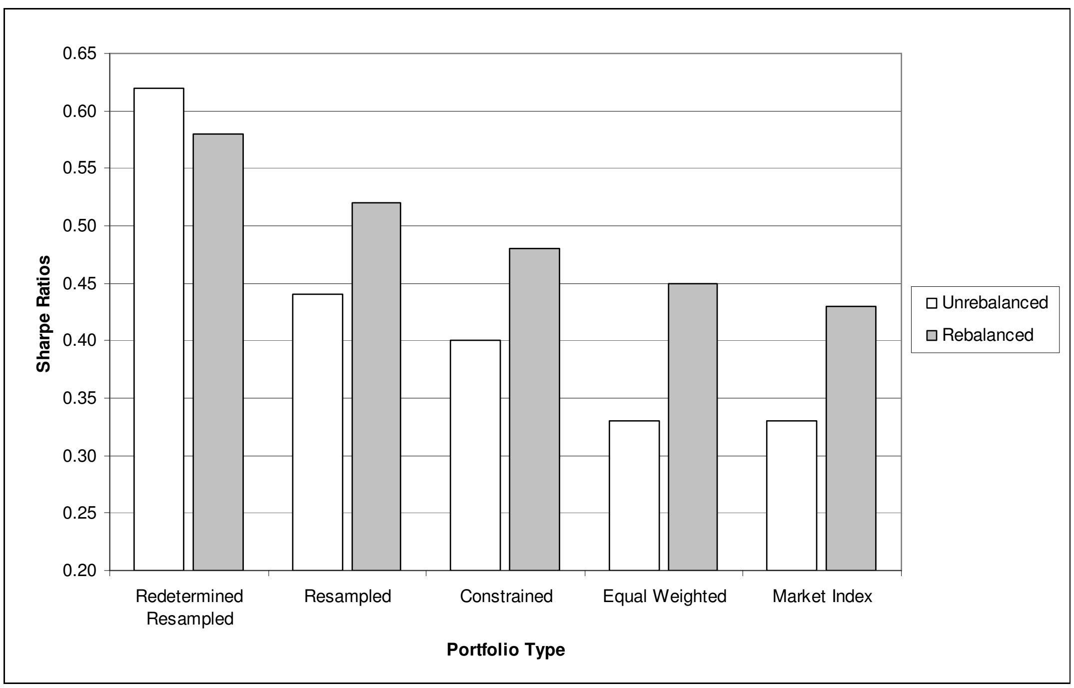
The analyses of the portfolios reveal a number of findings, and are graphically depicted in Figure 6.14.

Firstly, the redetermined resampled portfolio is significantly better than the remaining portfolios, with the remaining performances being resampled, constrained, equal-weighted and market index respectively. This performance outcome applies to both rebalanced and unrebalanced portfolios alike.

Secondly, when the rebalanced portfolios are compared to the unrebalanced portfolios, the rebalanced portfolios are significantly better in all but one instance, namely the redetermined resampled portfolio where the unrebalanced portfolio is superior. Interestingly, although the redetermined resampled unrebalanced portfolio was supreme, the redetermined resampled rebalanced portfolio was significantly better than any of the alternative portfolios.

This finding shows that, although a rebalanced portfolio is likely to yield a superior outcome, this is not assured. In this instance the impact on the rebalanced redetermined resampled portfolio was a result of the fact that the U.S. redetermined resampled intra-market asset allocation showed significant divergence from the actual asset allocation.

Figure 6.14 Investment portfolio Sharpe ratios (1993 – 2002)



Source: Derived using data from Table 6.36.

6.16 VALUE AVERAGED PORTFOLIOS

Using the rebalanced data, as summarised in Table 6.36, and applying the appropriate methodology[48], the value averaged portfolio internal rates of return as well as terminal values are calculated and compared to those that are not value averaged. These findings are summarised in Table 6.37.

Table 6.37 Investment portfolio internal rates of return (1993 – 2002)

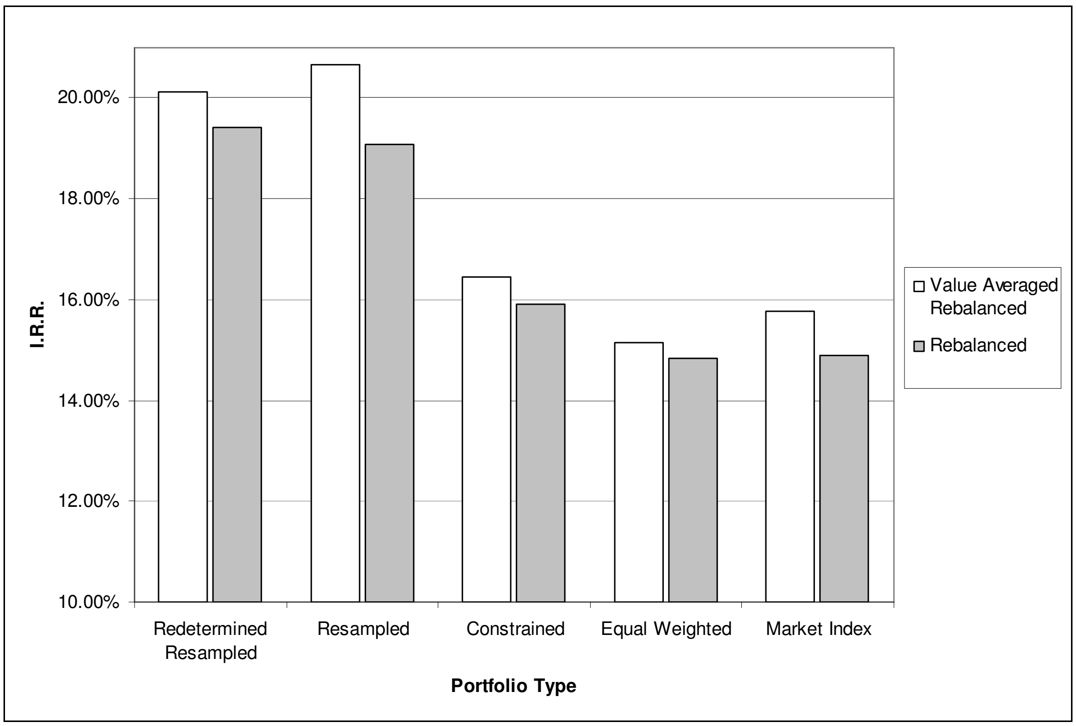
Portfolio	Value Averaged Rebalanced		Rebalanced	
	IRR	VA Terminal Value	IRR	Terminal Value
Redetermined Resampled	20.12%	R8,106,729.38	19.43%	R7,050,560.87
Resampled	20.66%	R7,640,608.56	19.08%	R6,829,864.02
Constrained	16.46%	R6,202,469.24	15.92%	R5,078,294.68
Equal Weighted	15.16%	R6,214,040.32	14.84%	R4,581,025.08
Market Index	15.76%	R5,863,749.80	14.90%	R4,605,962.10
Portfolios are rebalanced and value averaged. Value Averaged terminal values are calculated using rolling historical real rates of return.				

Source: Derived using data from the multiple asset classes for the period 1993 – 2002.

[48] See Section 2.10 (p. 51).

The outcome of the application of the passive formula strategy value averaging is graphically set out in Figure 6.15, where it is apparent that the combination of value averaging and rebalancing yields an internal rate of return superior to the portfolios that stochastically fluctuate.

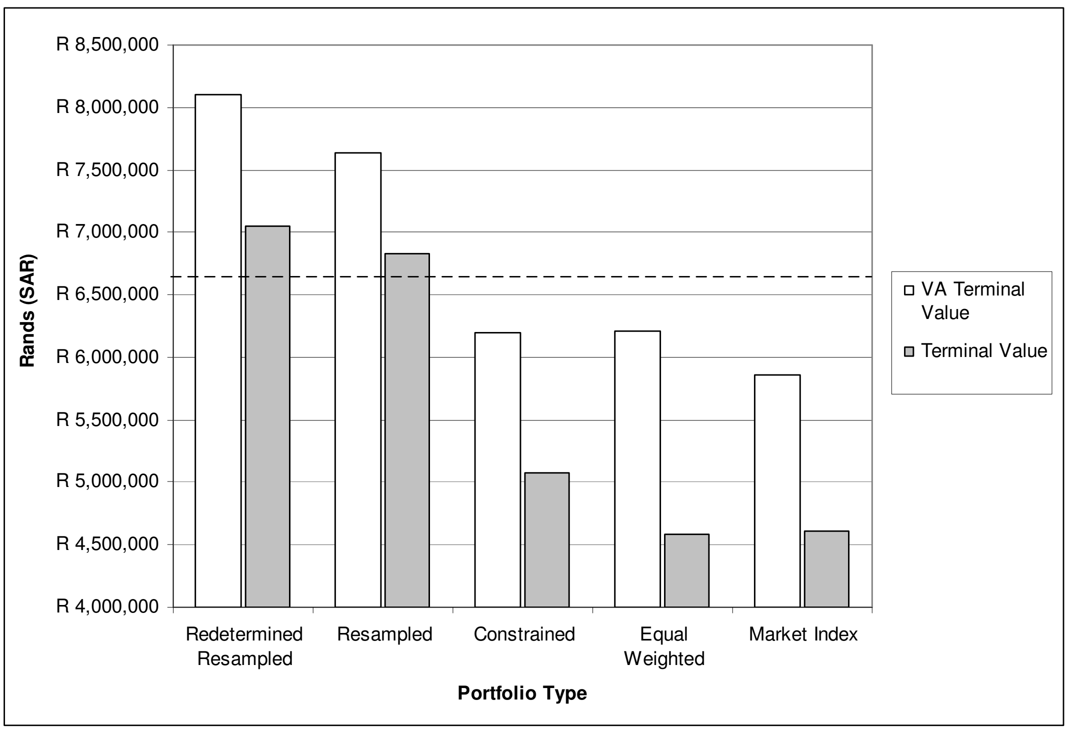
Figure 6.15 Investment portfolio internal rates of return (1993 – 2002)



Source: Derived using data from Table 6.37.

In addition it is observed in Figure 6.16 that the process of resampling and adjusting for the shift in asset allocations, combined with value averaging has a tendency to produce significantly higher terminal values than alternative portfolios. Interestingly it is noted that the value averaged terminal values of the redetermined resampled and resampled portfolios are significantly higher than the terminal value of R6,648,471.31, this being the value pertaining to a portfolio derived from the actual allocations for the period 1983 – 2002, and where an investment was made for the period 1993 – 2002 (10 years) only (as indicated by the dotted line, and having fluctuated stochastically). This tends to highlight the possibility that the process of value averaging adds significantly to the terminal values by allowing the investor to sell equities when a market is overvalued relative to the long-term rates of return, and to acquire significantly more equities when a market is undervalued relative to the long-term rates of return.

Figure 6.16 Rebalanced portfolio terminal values (1993 – 2002)



Source: Derived using data from Table 6.37.

6.17 CONCLUSION

Using the data analyses and findings established in Chapter 6, an appropriate procedure would be to discuss these, and to arrive at certain recommendations. This, therefore is the purpose of the ensuing chapter, and allows for the free interpretation of such findings.

CHAPTER 7

DISCUSSION AND RECOMMENDATIONS

7.1 INTRODUCTION

This chapter will use the detailed data analyses and findings in Chapters 6 and 7, to develop a critical interpretation of the results, and their implications, as well as to provide recommendations for further research and/or the optimal use of the research findings.

7.2 INTERPRETATION OF THE FINDINGS

The primary purpose of the study is to seek an understanding of the following:

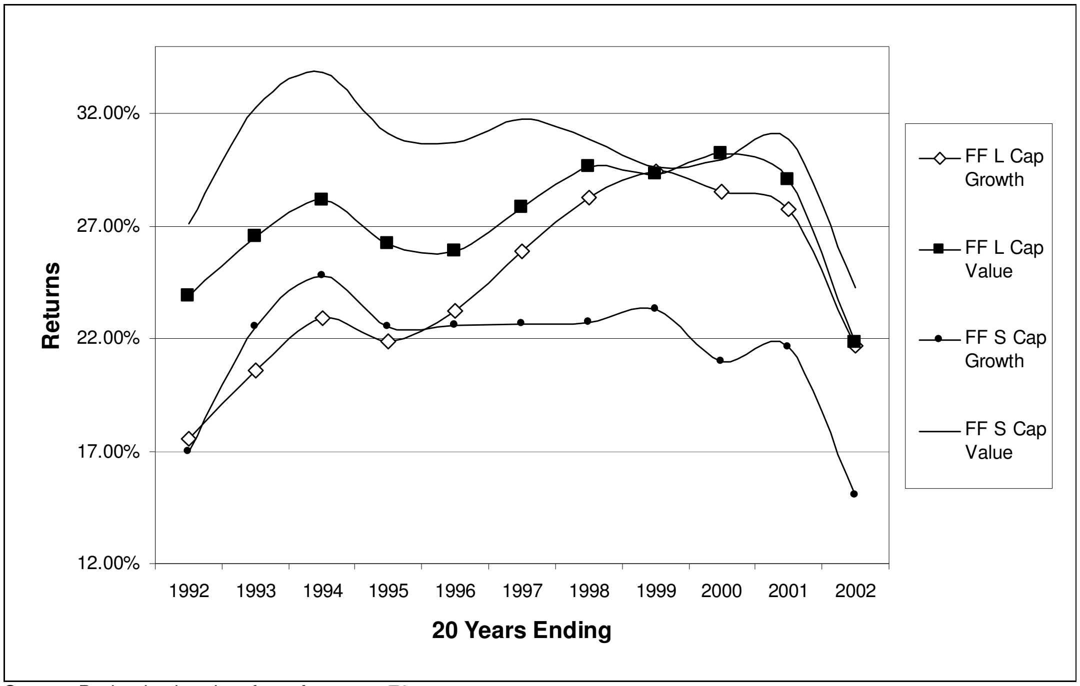
a) whether mean-variance optimisation could be used to build optimal, forward-looking investment portfolios, using passive investment instruments; and

b) whether ongoing structured portfolio management using value averaging and rebalancing techniques, could result in investors capturing more of the potential returns, leading to enhanced returns relative to past performances.

Therefore it is imperative to present a systematic discussion of the findings in order to reveal whether the research has made progress towards the achievement of these primary purposes.

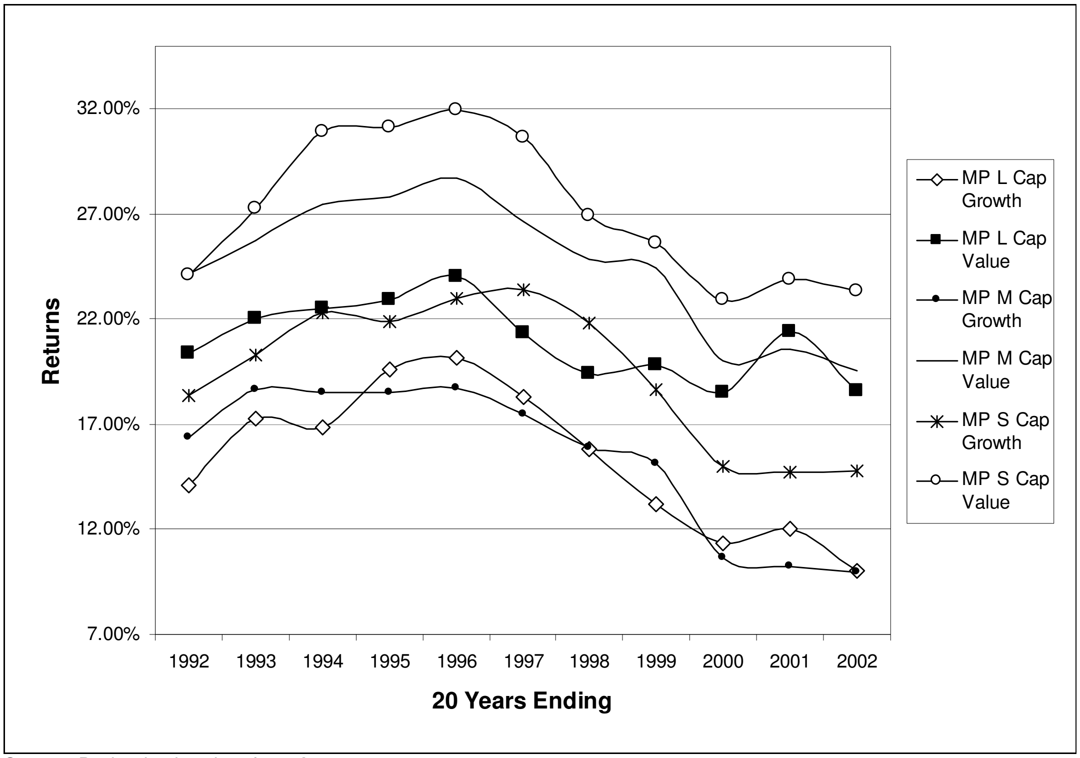
At the outset, with reference to Figures 7.1 and 7.2, it is noticed that South African asset classes display similar characteristics to those of the U.S. asset classes as far as style investing is concerned, namely that value investing seems to deliver an investment premium relative to growth investing. In fact it seems that the South African value assets are distinctly superior to the growth assets, particularly regarding the MP S Cap Value and the MP M Cap value assets. This phenomenon is as distinct in the U.S. market, except that the FF L Cap Growth assets seem to display a strong performance at times. This may have been ephemeral. Therefore the evidence seems to suggest that any portfolio should have a weighting in favour of value assets, particularly in light of the evidence that value investing, at the 20 year level, is predominant and manifests prevailing characteristics.

Figure 7.1 **U.S. assets classes 20 year rolling time period returns**



Source: Derived using data from Annexure 70.

Figure 7.2 **South African asset classes 20 year rolling time period returns**



Source: Derived using data from Annexure 71.

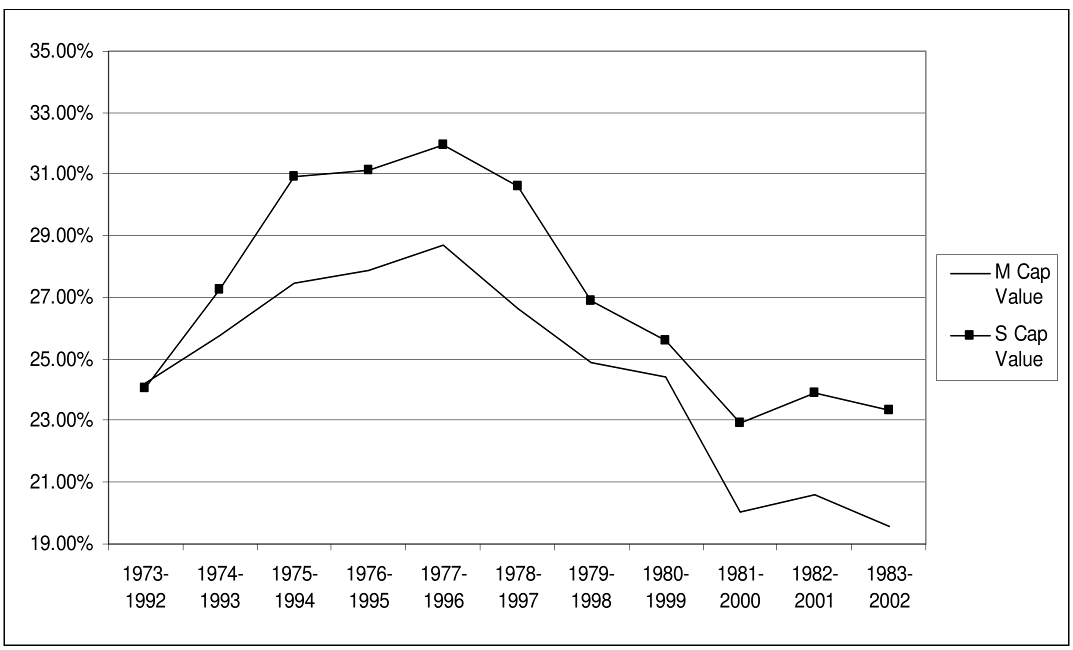
More profound however are the differences amongst the sectors based on size. It has been a widely held belief that large-cap assets are less volatile than the smaller cap asset classes, and this is in fact manifested in the U.S. markets, but is not the case in South Africa. In fact the large-cap value assets also seem to manifest little predictability over time. This finding would seem to suggest that the large-cap assets, within the South African environment are not a viable asset class, being unpredictable and excessively risky for the level of return.

Interestingly, with reference to Tables 6.33 (p. 208) and 6.34 (p. 209), where varying asset allocations from different time periods are compared, it is noted that certain asset classes remain excluded in every period, indicating that it may be prudent to ignore such asset classes from inclusion in any future portfolio. These asset classes include South African large-cap growth, South African mid-cap growth and U.S. small-cap growth assets. The fact that growth shares, by definition, have larger capitalisations than value shares means that the South African large-cap index, the ALSI 40, will predominantly consist of South African large-cap growth assets. The great significance of the finding is that South African active managers were unable to outperform the ALSI index, net of costs, for the period 1976 – 2001 (Pawley, 2002 p. 92), where the ALSI 40 comprises 76.38 percent of the ALSI index, and has a correlation coefficient of 97.06 percent (Pawley, 2002, p. 79). The implications are profound, since the evidence seems to suggest that the active managers have been unable to outperform an asset class that is a relatively inferior performer across multiple time periods. In fact, even if active managers had displayed the ability to outperform the broad market index, with its heavy weighting in South African large-cap growth assets, in light of the findings, this would not constitute superior value creation relative to what could have been achieved, and suggests that active investors revisit their asset allocation models, or their investment philosophies.

With reference to Table 6.33 (p. 208), it is noted that the South African mid-cap value asset allocation tends to receive a low allocation to the portfolio in contrast to the 1973 – 1992 historical allocation of 41.57 percent, as shown in Table 6.12 (p. 176). This anomaly is what can be described as optimiser instability. Mean-variance optimisation tends to select asset classes with the lowest cross-correlation. In this regard, firstly it is observed that there is an insignificant difference in the rates of return for the rolling 20 year periods 1973 – 2002 as set out in Figure 7.3.

Secondly, it is noted that the South African small-cap value assets displayed somewhat lower cross-correlations for the period 1973 – 1992, with the exception of the small-cap growth assets, as evidenced by Table 7.1. When the cross-correlations are analysed for the ensuing rolling 20 year periods it is observed that the small-cap value assets displayed lower cross-correlations throughout, except for the period 1980 - 1999 and 1981 – 2000.

Figure 7.3 Rolling 20 year returns (1973 – 2002)



Source: Derived using data from Annexure 61.

With regards the 1980 – 1999 period the allocation to the mid-cap value assets amount to 30.48 percent. Therefore it is prudent to deduce that the mid-cap value assets are receiving little inclusion in the portfolios due to the high levels of cross-correlation relative to an alternative asset class, namely the small-cap value assets.

Table 7.1 Selected asset class cross-correlations (1973 – 1992)

	MP M Cap Value	MP S Cap Value
MP L Cap Growth	72.16	65.00
MP L Cap Value	77.47	57.92
MP M Cap Growth	75.66	67.79
MP S Cap Growth	73.77	83.22

Source: Derived using data from Annexure 1.

Insofar as market mean reversion is concerned it is noted that markets display a high level of predictability beyond three years. Considering that average investment holding periods are less than two years, as indicated in Figure 7.4, this has implications.

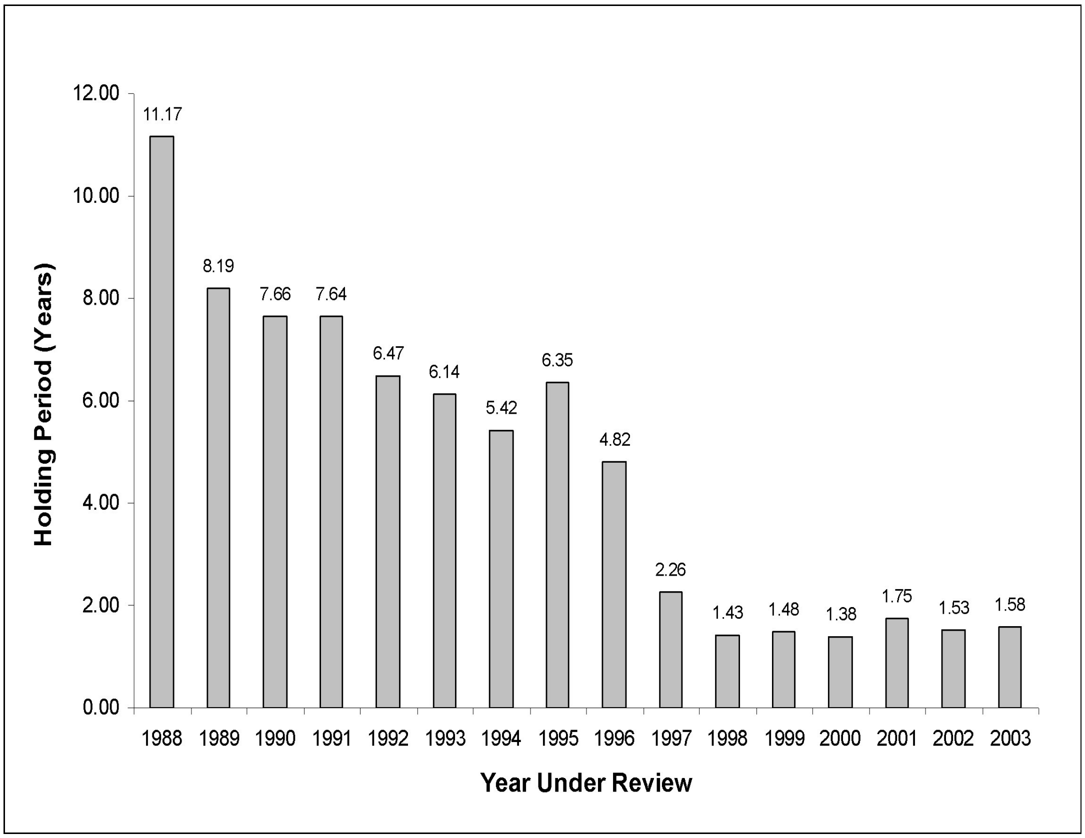
What is of interest is that both the U.S. asset classes and the South African asset classes have a tendency for predictability levels to manifest an inverse relationship with time. Superficially this may seem to hold dire consequences for the long term investor that has rigidly rebalanced to a set asset allocation. In the context of the research however, this is not an imperative since the asset allocation process is treated as dynamic, meaning that the mean reversion aspects of a fixed asset allocation may not be of relevance, although it is acknowledged that this may require further study.

Furthermore, with reference to Figure 6.5 (p. 185), it seems apparent that the predictability levels of the U.S. market continue to rise when, with reference to Figure 6.4 (p. 184), the individual assets that largely constitute the market, seem to manifest declining predictability levels. Logically this indicates that the portfolio predictability levels are determined by the relative weightings of the assets within the portfolio, and by association the predictability levels. In this regard it is noted that the FF L Cap Growth asset class, which by definition would constitute the majority of the S&P 500 index, displays the same characteristic of continually rising predictability levels.

With reference to Figure 6.5 (p. 185), it seems that the predictability levels of the South African market remain approximately constant when, with reference to Figure 6.3 (p. 183), the individual assets that largely constitute the market, seem to manifest declining predictability levels. Again, this indicates that the portfolio predictability levels are determined by the relative weightings of the assets within the portfolio, and by association the predictability levels. In this regard it is noted that the MP L Cap Growth asset class, which by definition would constitute the majority of the ALSI index, displays the same characteristic of approximately constant predictability levels.

Investors seemingly will be making asset allocations, in search of returns that manifest themselves over longer periods of time. The caveat is that, without the investment in time, there is the high probability that investors will not realise the long-term mean reversion benefits, and may find themselves inappropriately invested during most time periods.

Figure 7.4 South African investor holding periods (1988 – 2003)



Source: Pawley, 2002, p. 78 and updated using data from the Association of Collective Investments.

This is evident from the lower predictability results for smaller time periods, as reflected in Tables 6.16 (p. 180) and 6.17 (p. 181). These findings lend support to the hypothesis that market efficiency and market predictability can co-exist. The opposing hypotheses merely apply to different investment time horizons.

With the high levels of volatility prevalent amongst the asset allocations from one time period to the next, as indicated in Tables 6.19 – 6.22 (pp. 186 – 189), it is logical to deduce that it is imprudent to use historical data inputs that are established in a period that may not manifest any similar characteristics to ensuing periods, without providing for the adjustment thereof. In this instance it appears, with reference to Figures 6.10 (p. 205) and 6.11 (p. 206) that a process of adjustment does provide for enhanced return outcomes relative to the actual returns; however, the process is not without its problems. It is noted that when markets are in a state of flux relative to historical data, as was the case in the U.S. market, culminating in the tripling thereof in five years (Shiller, 2000, p. 4) by the year 2000, followed by the subsequent reversal, the adjustment process seems unable to provide the appropriate asset allocation. This has implications worthy of analysis.

In the first instance, it can be deduced, based on the apparent success of the adjustment process, that as soon as the extraordinary market behaviour ceases, appropriate asset allocations will re-emerge.

In the second instance, the state of market flux may become more permanent, in which case the adjustment process could be ineffective as an asset allocator, and would manifest itself in the form of low levels of long-term predictability, as measured using the coefficient of determination. Based on the high predictability findings this seems improbable.

In the third instance, should globalisation become more prevalent as evidenced by foreign direct investment flows, resulting in the increase in inter-market return correlations, a state of market flux within a dominant market, namely the U.S., would result in similar patterns in alternative markets, thereby reducing the risk reduction benefits of market diversification. This may not appear to be the case in the foreseeable future. Inter-market correlations for the U.S. market relative to South Africa, although higher for shorter time periods, are not displaying any tendency to strengthen their correlation for the 20 year time periods. Correlation coefficients remain within a range of between 12 and 30 percent. Insofar as foreign direct investment is concerned the South African market is insignificant, and does not appear on the top 25 destinations for investment (AT Kearney, 2002, p. 36). The most attractive markets remain China, U.S. and United Kingdom. Until the South African market attracts significant increases in foreign direct investment, the investor can conclude that any form of market flux is likely to be offset by market diversification.

In relation to the resampled data inputs relative to the actual historical data inputs, by and large, resampled data does produce more optimal asset allocations as shown in Tables 6.33 (p. 208) and 6.34 (p. 209). The implications, as mentioned earlier, are that historical data inputs may not be appropriate since they are determined in a particular time period, and only represent one of an infinite number of possible asset allocations. Resampled data inputs seem to provide an intuitive allocation of assets, thereby providing for enhanced investment diversification, and in turn reduced portfolio risk, as measured by standard deviation. Insofar as a retro-active returns comparison is concerned, with reference to Table 6.35 (p. 210). This is not to say that a resampled portfolio will always be superior to the actual outcome since this is impossible. The only reason the resampled portfolio appeared superior was that the allocations were somewhat different to the actual final portfolio. In this regard when the actual portfolio returns were compared to the redetermined resampled portfolio, the coefficient of determination was 90.23 percent, as indicated in Table 6.32 (p. 203). This suggests that the outperformance of the actual portfolio was a result of the 9.77 percent[49] of performance not determined by the actual allocation, namely the outperformance was a result of the asset allocation differential.

The crux of the above discussion is whether the portfolios derived through mean-variance optimisation lead to returns that are superior to the control portfolios. In this respect it is observed that the redetermined resampled portfolio is superior to the alternative portfolios, with the unrebalanced redetermined resampled portfolio being the top performer. It is noted that all the rebalanced portfolios are superior with the one exception. One can posit that this is a result of the fact that the redetermined long-term asset allocation and returns are not similar to their historical averages. With markets showing a

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[49] 100% - 90.23% (Table 8.26) = 9.77%

tendency to mean revert, it is postulated that it is unlikely that the disparity between asset allocations and returns and historical averages will persist for extended periods. Furthermore it is too risky to allow an investment to proceed without rebalancing due to the risk of excessive exposure to an asset allocation established using a historical perspective, as well as excessive exposure to higher growth assets over time, thereby minimising the benefits of diversification.

Based on the premise that markets have a tendency to mean revert, and it seems that the evidence bears this out, the value averaging approach to portfolio management clearly produces a superior outcome, as measured by the internal rate of return, as represented in Table 6.37 (p. 213) and Figure 6.14 (p. 214). The implications of this finding are more far reaching than may be immediately apparent. During rapidly advancing markets the predetermined value would be readily achieved. Should this state of affairs continue for a protracted period of time, this would allow for a cash surplus in excess of the value desired. This cash surplus in turn would attract a level of return which has not been factored in to the internal rates of return. This implies that the levels of return afforded by value averaging are understated at best. Moreover, by withdrawing surplus funds the initial investment is reduced, thereby reducing the investor's exposure to the market during excessive advances. Conversely, during excessive declines, unless the decline is protracted, equity purchases are made out of the surplus funds thereby enhancing the buy-low sell-high phenomenon.

Finally, given the imperative to diversify, and in this regard diversification includes across markets and currencies, it is worth noting that the research presented a process of enabling the combination of multiple asset classes within multiple markets. This may not at first seem apparent, or significant.

Any combination of foreign assets as discussed in the literature always makes use of broad market indices, or indices derived by combining broad market indices (Michaud, 1998, p. 15, Gibson, 2000, p. 154 – 156, Malkiel, 1999, p. 213 – 217, Bernstein, 2000a, p. 46 – 53 and others), as set out in Figure 6.2 (p. 173), and therefore are always presented as a single asset class representing a particular market. Although this is useful in demonstrating the benefits of diversification across markets, the practical benefits are muted especially in light of the research on asset classes, specifically size and style investing. Given that the South African and the U.S. markets manifest return premiums as a result of the size/style phenomenon, a methodology that did not take this into account would not have had any practical significance. In this regard a methodology[50] was constructed that solved for multiple asset classes at the intra-market level first, and then solved for the integration of markets.

Utilising data from Tables 6.1 (p. 160) and 6.2 (p. 161) and solving for the inter-market without solving for the intra-markets first a significant outcome occurs, as indicated in Table 7.2.

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[50] See Section 2.5 (p. 41).

Table 7.2 Combined inter-market asset allocation (1973 – 1992)

	Minimum Variance Portfolio	Middle Portfolio
MP M Cap Growth	20.17%	0.00%
MP M Cap Value	35.46%	46.91%
MP S Cap Value	20.25%	0.00%
FF L Cap Value	24.12%	0.00%
FF S Cap Value	0.00%	53.09%
Standard Deviation	19.25%	23.66%
Geometric Mean	10.41%	13.93%
Sharpe Ratio	0.54	0.59

Source: Derived using data from Tables 6.1 and 6.2, and Annexure 73.

A crucial aspect of the research is that without solving for the individual markets first the portfolio is exposed to excessive market related risk through inadequate intra-market diversification.

Multiple asset classes provide improved diversification and return benefits within the individual markets, and then provide enhanced benefits due to the integration of the markets. This can be observed by referring to Figure 6.1 (p. 172) where it is apparent that the U.S. market exposure is significantly reduced relative to Figure 6.2 (p. 173).

Interestingly it seems that although currency volatility has a significant impact on the final value of foreign investments the primary reason for foreign exposure should be the low correlations relative to domestic equities. In this regard, and specifically South Africa, it is prudent to make reference to the ensuing quote.

" … it was mooted in the press that all South Africans should have at least 20 - 30 percent … invested offshore. Current recommendations are closer to 70 – 80 percent" (Kruger, De Kock and Roper, 2001, p. 1).

Kruger, De Kock and Roper (2001) highlight the typical approach to portfolio construction, as displayed by investors. Investors fail to build portfolios based on the benefits of diversification, but rather build portfolios as pyramids of assets (Statman, 2000, p. 3), thereby overlooking the cross-correlations of the asset classes, and instead focus on the wealth generating abilities of the assets based on the most recent past performance, in this instance the rapid depreciation of the South African currency.

The imperative is that diversification, as highlighted throughout the research, is not based on absolute wealth creation per se, but critically includes the cross-correlations between the competing asset classes. This phenomenon would be overlooked without the use of the mean-variance model principles.

7.3 RELATIONSHIP BETWEEN THE HYPOTHESES AND THE FINDINGS

It is a research imperative to compare the findings to the established hypotheses in order to ascertain whether the research is in a position to make a contribution in line with what was proposed. In this regard, a discussion is appropriate and will be presented in the form of points numbered to correspond with the relevant hypotheses[51].

a) The issue of whether mean-variance optimisation, using resampled data, led to performance improvements was researched. The results appear below.

   i) In regards to effective asset class selection it is noted, with reference to Tables 6.33 (p. 208) and 6.34 (p. 209), that the weaker asset classes are reduced in significance in a portfolio, with this seeming to hold from period-to-period. The significance of this is that the probability of enhanced returns is somewhat improved by maintaining a relatively low allocation to poor performing asset classes. Therefore it seems that the research was effective in selecting the appropriate asset classes. This hypothesis is therefore accepted.

   ii) With reference to effective asset allocation relative to actual portfolios it is noted in Section 6.13 (p. 207), and Tables 6.33 (p. 208) and 6.34

[51] See section 1.6 (p. 10).

(p. 209) that mean-variance optimisation does indeed produce an enhanced portfolio in terms of asset class diversification and weighting, however only when the approach to determining asset allocations is treated as a dynamic process. In this regard Sections 6.9 (p. 186) and 6.10 (p. 190) draw attention to the dynamic nature of asset allocations, thereby supporting the theory that periodic resampling is required. In this regard the hypothesis is partially accepted since the effectiveness of the allocation of assets hinges on the effectiveness of the rebalancing process that tests the original asset allocation for optimality.

iii) Given the complexity of combining multiple asset classes within a market, and in turn combining global markets it is noted, with reference to Sections 6.5 (p. 168) and 6.7 (p. 177), that the mean-variance optimiser provides a definitive asset allocation between markets. Interestingly in this regard it is noted by selecting the appropriate asset allocations amongst competing asset classes, that the foreign market exposure is significantly lower relative to a broad market index approach, thereby reducing excessive exposure to the foreign market, which has a risk reduction effect due to the exchange rate factor.

Table 7.3 Inter-market effectiveness test (1983 – 2002)

	Inter-Market	SA Intra-Market
Real Geometric Mean	11.66%	10.43%
Standard Deviation	19.67%	24.28%
Sharpe Ratio	0.59	0.43

Source: Derived using data from Annexures 60 and 62.

With regards to the effectiveness of global diversification relating to returns, with reference to Table 7.3 it is apparent that inter-market diversification leads to return optimisation relative to a portfolio wholly invested in the domestic market. In this regard the hypothesis is accepted.

iv) The effective management of risk is measured by comparing the Sharpe ratios of the rebalanced redetermined resampled portfolio to a rebalanced broad market index portfolio. With reference to Figure 6.13 (p. 212) it is noted that the rebalanced redetermined resampled portfolio has a Sharpe ratio significantly higher than the rebalanced broad market index portfolio. In this regard the broad market index portfolio had the lowest risk-adjusted returns relative to the alternative portfolios. Therefore, from a risk perspective, mean-variance optimisation does produce risk reduction benefits. This hypothesis is accepted.

b) Given the broad benefits emanating from the use of a mean-variance optimiser, using resampled data, these benefits would be minimised without a strategy to manage the portfolio for sustainable superior performance.

i) With regards to portfolio rebalancing the research findings[52] established that rebalancing a portfolio is indeed a superior technique, as indicated in Figure 6.13 (p. 212). A caveat though is that rebalancing does not produce superior outcomes 100 percent of the time. However based on the balance of probabilities, rebalancing seems to be an optimal technique, based on risk-adjusted returns alone. The technique applied was a hybrid of calendar rebalancing, namely that the portfolio was reviewed annually, and contingent rebalancing, i.e. that the asset allocations were redetermined when the desired coefficient of determination was below the predetermined threshold. This hypothesis is partially accepted since the rebalancing process does not always produce superior results to the unrebalanced alternative.

ii) Concerning passive formula strategies, the technique applied was the concept of value averaging. Figure 6.14 (p. 214) indicates that the internal rate of return was superior across all measured portfolios where value averaging was applicable. Therefore the hypothesis is accepted.

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[52] See Section 6.15 (p. 210).

c) Given the findings that actual asset allocations display a tendency to be extremely dynamic, as set out in Tables 6.19 – 6.22 (pp. 186 – 189). This tendency results in inappropriate asset allocations with regards to asset class diversification. Given also that the resampled asset allocations, as set out in Tables 6.33 and 6.34 (pp. 208 – 209), display a marked enhancement in terms of asset class diversification, and thirdly that the resampled asset allocations are derived using stochastic modelling, it can be deduced that stochastic data input determination is a more effective technique of deriving data inputs, for use in a mean-variance optimiser, than the deterministic linear extrapolation of historical data. In this regard the hypothesis is accepted.

d) With reference to Figure 6.13 (p. 212), and Tables 6.33 and 6.34 (pp. 208 – 209), the findings indicate that the resampled portfolios display over-weighting in value and small-cap equities. The performance, as measured on a risk-adjusted basis, indicates that style and size investing is demonstrably better than pursuing a standard broad market investment approach. In this regard the hypothesis is accepted.

In summary, the research effectively set about conducting quantitative experiments in order to support or refute the hypotheses.

7.4 FURTHER CONTRIBUTIONS TO THE BODY OF KNOWLEDGE

An important research imperative is to make an original contribution to the existing body of knowledge. In this regard a definition of the requirement, original, is in order. There is the literal interpretation which includes 'initial, first, never having existed or occurred before' (Locke, Spirduso and Silverman, 2000, p. 58), moreover in research the word original importantly includes all studies that have deliberately been used to test the accuracy of results or the viability of conclusions made in previous studies.

Given that the aforementioned discussions have highlighted many of the contributions that were revealed during the research process, and given the complexity of the study, many of the original contributions may not be immediately apparent and therefore, in some instances, require reiteration.

7.4.1 The data resampling process

Previous research by Jobson and Korkie (1980) stipulated using 500 trials during the simulation process. The number of trials seems to have been heuristically determined, with no explicit explanation being provided therefore. Michaud (1998) replicated this process without explanation, and the literature in regards to the optimal number of trials was silent. The methodology[53] set out the process of determining resampled data inputs using a stochastic simulator.

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[53] See Section 2.6.3 a) (p. 46).

Inherent in the process is an explanation for the use of 500 trials, which is unique to the study, and an imperative for the effective use of a stochastic simulator.

7.4.2 The resampled efficient portfolio

The methodology[54] indicates that a derivative of the Michaud (1998, p. 37) process was applied for the purposes of resampling. In this regard Michaud (1998) suggests that the resampled inputs derived using the stochastic simulator should be equal to the number of trials, namely 500. This process seems heuristic with no explanation for this. The methodology[55] provided clarity by supplying a scientific explanation. The research revealed that there is no need for the dogmatic adherence to the generation of 500 simulated sets of data inputs, and in turn the generation of 500 resampled efficient frontiers. This requirement can be met by utilising the confidence interval measure as set out in Section 2.6.2 (p. 44) and included as part of the process in c) of Section 2.6.3 (p. 46). The use of such a confidence interval measure reduces the need for lengthy computations that do not provide significantly greater levels of accuracy.

[54] See Section 2.6.3 (p. 45).

[55] See Section 2.6.2 (p. 44).

7.4.3 South African asset class determination

Given that the prevailing literature suggests that style and size equity asset classes produce a return premium over and above alternative equity asset classes, and given that the South African market has no such indices[56] or passive investment products, as set out in Section 2.4.10 (p. 31), and, finally, given that the research was premised on the superiority of passive investment products, a major contribution to the body of knowledge was the creation of such asset classes. These proxy asset classes formed a fundamental part of the research, and the superiority of the entire investment process relative to a broad market index approach, as indicated in Chapter 6, would not have been realised otherwise. An imperative for the successful implementation of the findings of this study will, henceforth, be based on the continuation of the established asset classes.

7.4.4 The periodic portfolio rebalancing process

Sections 5.2 (p. 133) and 5.6 (p. 154) reviewed and discussed the prevailing views on portfolio rebalancing. In this regard the study took the position that portfolio rebalancing was a dynamic process. Although Michaud (1998) highlighted a procedure that may reduce the need to trade, the procedure was

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[56] It is acknowledged that the JSE Securities Exchange has, as at 2004, released indices based on growth and value characteristics. However these indices are not further differentiated by size which is a significant aspect of the research. In light of this the South African market is not considered to have appropriate indices.

premised on the original resampled efficient portfolio, which was derived using data inputs that may have become obsolete as a result of being period specific, which may result in a reduction in how optimal the asset allocations remain, henceforth. Since the literature, specifically Jahnke (1997, p. 111) viewed rebalancing as a dynamic process the study sought a methodology that would take this characteristic into account. In this regard Section 2.7 (p. 48) of the methodology highlights an approach based on the Brinson, Hood and Beebower (1986, p. 136) logic, namely a portfolio that has similar assets, and asset weightings, to the broad market should manifest a high coefficient of determination to the market. This methodology, in regards to rebalancing, is unique to the study.

7.4.5 Investment time horizons

Although the literature makes reference to investment time horizons in the context of the subject under review, there is a clear lack of focus on investment time horizons specifically. In this regard the research synthesised the various references to time horizon as set out Section 5.4 (p. 142). The research therefore set forth the position that an optimal holding period would be 10 years, and this position would incorporate the mean reversion of markets, risk reduction through time diversification, normalisation of returns divergence and size and style return premium time horizons. This position adds to the current positions on investment time horizons, however with the unique aspect of incorporating the many theories that have been integrated within the study.

7.4.6 The integration of previous research

Given the volume of existing academic research in focused areas of an otherwise common topic, namely wealth maximisation, and in the context of a Business Management study, it seemed imperative to integrate the many findings in such a manner so as to produce a packaged solution that would lead to an opportunity for the application thereof. In this regard the imperatives of asset allocation, portfolio construction, portfolio rebalancing and value averaging were examined, using only passive investment instruments, and where necessary the required adjustments, additions or replications to the existing hypotheses were made. This integration and contextualisation process, using passive investment instruments, is unique to this study.

To summarise, the reiteration of contributions to the body of knowledge are by no means exhaustive, but merely highlight areas that may be overlooked, and should be read in conjunction with the research findings and discussions highlighted in Sections 7.2 (p. 217), 7.3 (p. 233) and elsewhere, throughout the study.

7.5 LINKING THE FINDINGS AND PREVIOUS RESEARCH

7.5.1 Asset class characteristics

Insofar as the U.S. asset classes are concerned, with reference to Table 6.2 (p. 160), they display characteristics in line with the literature (Fama and French, 1992, p. 445), namely that small-cap assets and value assets produce a return premium in excess of that produced by the alternative equity asset classes. This phenomenon is attributed to receiving a reward for the excessive risk. In this regard small-cap assets have the highest standard deviation, with large-cap assets displaying the least risk, as measured by standard deviation.

With reference to Table 6.1 (p. 159) the South African asset classes display some of the characteristics espoused in the literature, namely that small-cap assets and value assets produce a return premium in excess of alternative asset classes, and this fact supports the literature on emerging markets (Fama and French, 1997, p. 15).

Interestingly, the South African assets have standard deviation characteristics which are contrary to the literature. In this regard the large-cap assets have the highest standard deviation, which suggests that the hypothesis that large-cap assets are the least risky asset class does not apply across markets.

7.5.2 Resampled versus actual intra-market asset allocations

With reference to Table 6.33 (p. 208), it is noted that the redetermined resampled asset allocations display a remarkable level of enhanced asset diversification relative to the actual asset allocations for the periods under review. Michaud (1998, p. 62) postulated that one of the many benefits of resampling data inputs is that it will lead to intuitively better asset allocations, in other words the asset allocations will be more optimally diversified. The findings continue to lend support to this hypothesis.

7.5.3 Mean reversion

With reference to Tables 6.16 (p. 180) and 6.17 (p. 181) it is noted that the individual broad based markets display high levels of predictability from three and four years respectively. These levels of predictability infer mean reversion, since the levels of predictability for smaller time periods are small. These findings support the literature in this regard where it was postulated that markets show high levels of predictability at the three to seven year marks (Thaler, 1992, p. 153).

7.5.4 Contingent rebalancing

The research selected a combination of both calendar and contingent rebalancing approaches based on the literature, namely that a portfolio should be analysed periodically to ensure asset allocations remain optimal, and, should the asset allocation be deemed to be less than optimal, only then should rebalancing occur. In this regard the researcher developed an approach to rebalancing that yielded superior asset allocations as substantiated in Tables 6.25 (p. 193), 6.26 (p. 194), 6.30 (p. 201) and 6.31 (p. 202).

7.5.5 The rebalancing premium

The findings as evidenced by Figure 6.13 (p. 212) strongly support the hypothesis established by Bernstein and Wilkinson (1997, p. 13), namely that dissimilar rates of return between the asset classes may lead to the unrebalanced portfolio providing a superior outcome. Where historical differentials between asset classes remain similar, the rebalanced portfolio will yield a superior outcome. The findings revealed that all the rebalanced portfolios yielded superior returns relative to the respective unrebalanced portfolios, with the exception of the redetermined resampled portfolio. In this instance the asset allocations were redetermined when deemed necessary. This adjustment process could have resulted in the movement away from historical data asset allocations thereby leading to the unrebalanced result.

7.5.6 The value averaging premium

With reference to Figure 6.14 (p. 214) it seems evident that the internal rates of return are higher for a value averaged portfolio relative to a stochastically advancing and declining portfolio. These results were evident across multiple portfolio types, suggesting that mean reversion does lend support to the process. This outcome supports the hypothesis established by Edleson (1993, p. 39).

7.5.7 Mean-variance model risk-reward-diversification benefits

With reference to Table 6.35 (p. 210) it is noted that the benefits in accordance with the mean-variance model, namely risk reduction and return enhancement through the use of diversification, are realised. The findings therefore support the Markowitz (1959) hypothesis.

7.5.8 Investment holding periods

With reference to Figure 7.4 (p. 225), there is an apparent tendency towards reduced investment holding periods. Given the benefits deriving from extended holding periods, as espoused in the literature, this reality of South African investors seems to be at odds with what is suggested as being optimal.

7.6 RECOMMENDATIONS

The recommendations have three dimensions, as set out below.

a) The first are recommendations to the broad investor public.

b) The second dimension would be recommendations to the investment product providers.

c) The third dimension would be recommendations to the financial advisory community.

7.6.1 The broad investor public

In the first dimension it seems prudent to pursue an investment strategy which includes diversification across optimal asset classes, as well as across currencies and markets. In this regard it is imperative that the correct asset classes are identified. The research suggests that the optimal asset classes are significantly in favour of size and style, and in this regard specifically lean towards small-cap value assets in particular, and thereafter the alternate value asset classes such as mid-cap and large-cap value. This is not to say that growth assets do not play a role. They do, and that role is to provide optimal portfolio diversification. The crux is that a significant portion of any portfolio should consist of a value and size orientation.

Of course much of the battle, with regards to enhancing investor returns, would be won if investors embraced the concept of selecting passive investment products in line with the small-cap and value orientations instead of dogmatically pursuing active investor strategies, and in so doing continue to pursue market timing and stock picking strategies. In this regard it must be emphasised that passive investing is an inordinately difficult strategy to beat, and it would serve any investor well to become acquainted with the strategy.

Not all investors would be able to include market diversification, or at the levels required, either due to the lack of investment capital or due to exchange control limitations. In the first instance, it remains an imperative that diversification should take place within the home market. One of the elegant findings of the research is that the resampled mean-variance optimisation process first solves for the individual markets, therefore, should an investor not be able to diversify across global markets, the asset allocation remains as per the intra-market asset allocation. In the second instance, should the investor not be able to make maximum use of the determined foreign market allocation, this can be applied at a lower allocation. In this regard the investor would simply allocate to the foreign market at the reduced allocation, and in turn would allocate within the foreign market in accordance with the required asset class weightings.

Since the benefits as unveiled in the research are premised on a long-term investment horizon, it is recommended that investors significantly increase their investment holding periods which should, at a minimum, be in excess of 10 years in order to enjoy the benefits.

Insofar as the management of the portfolio is concerned it seems prudent to suggest that the portfolio is periodically rebalanced, thereby maintaining the portfolio’s standard deviation characteristics and in turn adjusting for the dynamic nature of asset allocations. Coupled to this is the suggestion that the technique of value averaging be applied which will mechanically direct investor behaviour in the face of market uncertainty, ensuring the sale of equities during an overheated market, and the acquisition of equities during periods of lacklustre performance.

7.6.2 The investment product providers

In the second dimension, an appeal needs to be made to the investment product providers to develop and offer an exchange traded fund and/or a collective investment fund based on the resampled mean-variance optimisation process. In this way the private investor would not have to acquire the necessary skills to benefit from the research, the products would be passive in nature, and all the investor would be required to do to mitigate the levels of risk, based on the investor's risk profile, would be to add a cash component to the portfolio.

7.6.3 The financial advisory community

Finally, it may be prudent to suggest that financial advisors become au fait with the techniques and strategies as espoused in order to be better able to promote the benefits thereof, and to be in a position to educate their clients. In this regard, given some of the opening statements regarding the retirement savings imperative, it is noted that many retirement fund trustees may require the services of their financial advisors, and knowledge of the research project may indeed enhance the returns to be experienced by such a retirement fund.

7.7 SUGGESTIONS FOR ADDITIONAL RESEARCH

A remarkable anomaly relative to the literature was in respect of the South African asset class characteristics. In this regard further research would be suggested to establish whether South African large-cap assets manifest excessive volatility as a result of the lack of diversification, or the over concentration in a particular sector. The outcome of this research may indicate that the over concentration of investment products within the large-cap sector may be hindering the results experienced by active investors.

Given the fact that current literature refers to global diversification in the broad sense, namely the combination of broad market indices, as adjusted for currency, there is the aspect of combining multiple assets within multiple markets that was accomplished within this body of research. In this regard more extensive research, across multiple markets, would be appropriate to highlight the many benefits accruing from the combination of multiple assets within multiple markets, relative to the methodologies espoused in the literature, and additionally to indicate any caveats that may arise.

Given the dearth of literature pertaining to the aspect of appropriate investment holding periods, it seems prudent to suggest that further research should be undertaken to establish what is an appropriate holding period, and why investors are displaying the tendency to reduce their holding periods. The outcome of the research could provide insights for an investor.

The research utilised stochastic simulation modelling to manage the data inputs for a mean-variance optimiser, specifically the mean and standard deviation. Given that the mean-variance optimiser requires a third variable, the cross-correlation between the assets, and given that this correlation between assets provides the windfall gain resulting from diversification, it seems an imperative to seek a method of managing future correlation coefficients for maximum results, instead of merely using historical data, as adjusted.

Finally, the research was premised on the view that investors should invest for the long term[57], and the research solved for the allocation between risky equity assets. There is scope to solve for risky, and less risky assets within a portfolio, where the investors have broadly divergent investment time horizons without having to deviate away from the asset allocations derived within the research.

7.8 DISCUSSION ON THE BROADER IMPLICATIONS

To merely expect the individual investor to adopt, and benefit from the research is rather limited in scope. Given the findings and recommendations there would

[57] See Section 2.13 (p. 52).

be broader implications, as there are currently[58], and these require postulating upon.

7.8.1 New product development

Assuming the development of new products in the form of passive exchange traded funds.

a) The cost structure is minimal relative to current collective investment funds.

b) The exchange traded funds can be classified as collective investments.

c) As a collective investment the assets can form part of a pension funds overall asset allocation, as regulated by Regulation 28 of the Pension Funds Act.

d) As a passive exchange traded fund, with no active management, low transaction fees, probability of higher performance as established during the research and longer holding periods due to the nature of pension funds, pension fund members can expect significantly higher benefits on retirement from their funds.

[58] See Section 1.3 (p. 4).

7.8.2 Investment industry implications

Large portions of retirement funds are managed by the investment industry. The management responsibility garners significant fees. It would be envisaged that a significant reduction in investment management companies would occur, should Section 7.8.1 gain mainstream acceptance, and for this reason it would be an unrealistic expectation. A more realistic expectation, albeit slight, would be the development of collective investment products with a higher fee structure. Unfortunately the research is premised on a long-term investment horizon, which would require a significant paradigm shift by the investment management companies, and for this reason it would not be expected that the products would succeed.

7.8.3 The State

One of the government's imperatives is to ensure the well-being of its people. Unfortunately capitalism is not well suited to assuring the optimal allocation of resources when there is no profit incentive. For this reason government may be well advised to structure a low-cost pension product that only allows for the inclusion of passive exchange traded products in line with the research. Such a product could be offered by government, under the same regulations that govern retirement annuities with the exception individuals or pension schemes whose membership has median earnings below a predetermined threshold, must be compelled to participate in the government product. Those individuals or pension funds above the median threshold, that are of the belief that their efforts will yield higher returns may continue to invest in alternative products.

7.8.4 General discussion

Of course improved investment returns, at least to levels above inflation, would encourage further investment. This in itself would go a long way to improving the welfare of individuals.

No amount of research will convince many individuals that passive investment management is the strategy of choice. Therefore should the research become mainstream there is no risk of abnormalities forming within the equity markets, since the active investors will continue to seek out undervalued assets, and this process will keep the markets efficient.

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ANNEXURE 1

Intra/inter multiple asset class cross-correlation matrix										
1973-1992	FF L Cap Growth	FF L Cap Value	FF S Cap Growth	FF S Cap Value	MP L Cap Growth	MP L Cap Value	MP M Cap Growth	MP M Cap Value	MP S Cap Growth	MP S Cap Value
FF L Cap Growth	1.0000	0.7505	0.8318	0.6512	0.2574	0.4522	0.3848	0.3642	0.5882	0.4213
FF L Cap Value	0.7505	1.0000	0.7098	0.8519	0.0524	0.1907	0.1508	0.0626	0.3565	0.2137
FF S Cap Growth	0.8318	0.7098	1.0000	0.8378	0.3239	0.3788	0.5147	0.4368	0.6729	0.5004
FF S Cap Value	0.6512	0.8519	0.8378	1.0000	0.1551	0.1132	0.2579	0.106	0.4469	0.2987
MP L Cap Growth	0.2574	0.0524	0.3239	0.1551	1.0000	0.7632	0.873	0.7216	0.7998	0.65
MP L Cap Value	0.4522	0.1907	0.3788	0.1132	0.7632	1.0000	0.8226	0.7747	0.6134	0.5792
MP M Cap Growth	0.3848	0.1508	0.5147	0.2579	0.873	0.8226	1.0000	0.7566	0.8193	0.6779
MP M Cap Value	0.3642	0.0626	0.4368	0.106	0.7216	0.7747	0.7566	1.0000	0.7377	0.8019
MP S Cap Growth	0.5882	0.3565	0.6729	0.4469	0.7998	0.6134	0.8193	0.7377	1.0000	0.8322
MP S Cap Value	0.4213	0.2137	0.5004	0.2987	0.65	0.5792	0.6779	0.8019	0.8322	1.0000

Source: Derived using data from the multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data I/Cross Correlations.xls/Actual Classes).

ANNEXURE 2

Actual multiple asset class middle portfolio returns 1973 - 1992		
Year	US Optimum Portfolio	SA Optimum Portfolio
1973	-31.81%	20.44%
1974	-19.34%	-0.51%
1975	91.13%	19.22%
1976	47.98%	-4.01%
1977	13.70%	25.24%
1978	15.80%	34.45%
1979	24.87%	91.00%
1980	10.62%	47.84%
1981	58.43%	8.65%
1982	36.85%	30.01%
1983	59.91%	36.10%
1984	81.47%	3.90%
1985	60.44%	23.60%
1986	-1.61%	36.54%
1987	-13.23%	16.41%
1988	52.64%	17.43%
1989	31.86%	43.40%
1990	-16.66%	9.06%
1991	49.09%	36.61%
1992	42.65%	7.71%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data I/Asset Allocations.xls/Actual Cross-Correlations).

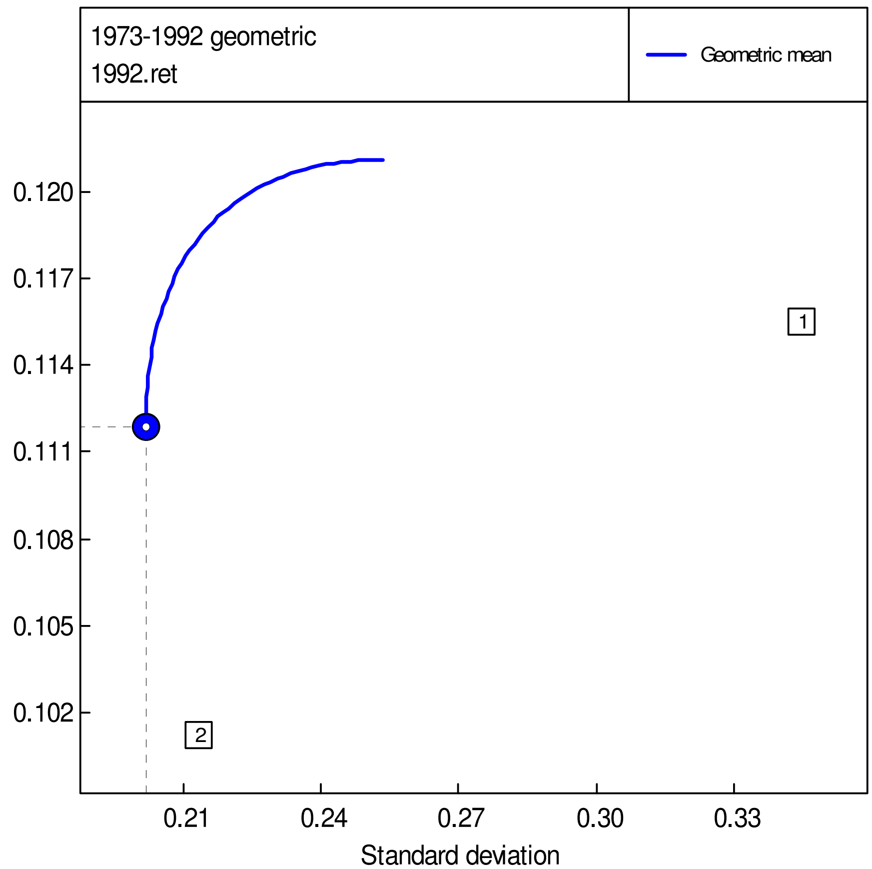
ANNEXURE 3

Summarised actual multiple asset class middle portfolio returns (1973 – 1992)		
	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	25.02%	23.59%
Standard Deviation	34.48%	21.36%
Real Geometric Mean	11.55%	10.12%
Sharpe Ratio	0.33	0.47
Cross-Correlation	0.29	
Real Geometric Mean adjusted for inflation at 13.47%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data I/Asset Allocations.xls/Actual Cross-Correlations).

ANNEXURE 4

Actual inter-market efficient frontier 1973 – 1992



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived, using MVO Plus software and data from the multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1992.ret).

ANNEXURE 5

Actual market returns 1973 - 1992		
Year	S&P 500	ALSI
1973	26.83%	6.31%
1974	24.45%	6.40%
1975	72.64%	-4.03%
1976	23.84%	-3.25%
1977	-7.18%	29.39%
1978	6.56%	36.24%
1979	12.45%	91.30%
1980	18.95%	41.43%
1981	34.18%	0.01%
1982	23.61%	34.00%
1983	41.93%	14.21%
1984	78.90%	9.40%
1985	61.34%	40.99%
1986	1.48%	54.80%
1987	-6.00%	-3.40%
1988	42.25%	14.26%
1989	41.78%	54.93%
1990	-2.60%	-4.85%
1991	40.22%	30.66%
1992	20.37%	-1.87%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the market indices for the period 1973 – 1992 (Archive Reference: Thesis Data I/Market Indices.xls/Sheet3).

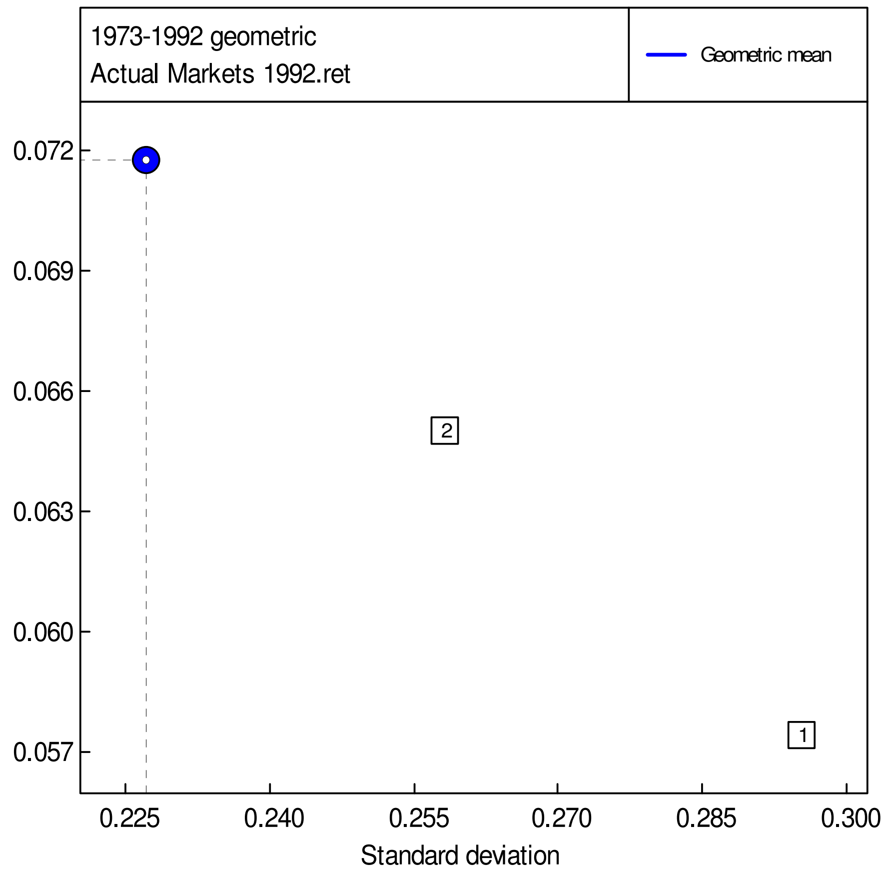
ANNEXURE 6

Summarised actual market returns 1973 - 1992		
Summary	S&P 500	ALSI
Geometric Mean	19.21%	19.98%
Standard Deviation	29.53%	25.83%
Real Geometric Mean	5.74%	6.50%
Sharpe Ratio	0.19	0.25
Cross-Correlation	0.37	
Real Geometric Mean adjusted for inflation at 13.47%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the market indices for the period 1973 – 1992 (Archive Reference: Thesis Data I/Market Indices.xls/Sheet3).

ANNEXURE 7

Actual inter-market indices efficient frontier 1973 – 1992



1. S&P 500 2. ALSI

Source: Derived using MVO Plus software and data from the market indices for the period 1973 – 1992 (Archive Reference: Simulations II/EF US-SA/Actual Markets 1992.ret).

ANNEXURE 8

Resampled multiple asset class middle portfolio returns 1973 - 1992		
Year	US Optimum Portfolio	SA Optimum Portfolio
1973	-31.21%	23.22%
1974	-19.75%	0.10%
1975	90.37%	19.23%
1976	46.97%	-4.98%
1977	12.54%	26.53%
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data I/Asset Allocations.xls/Resampled Cross-Correlations).

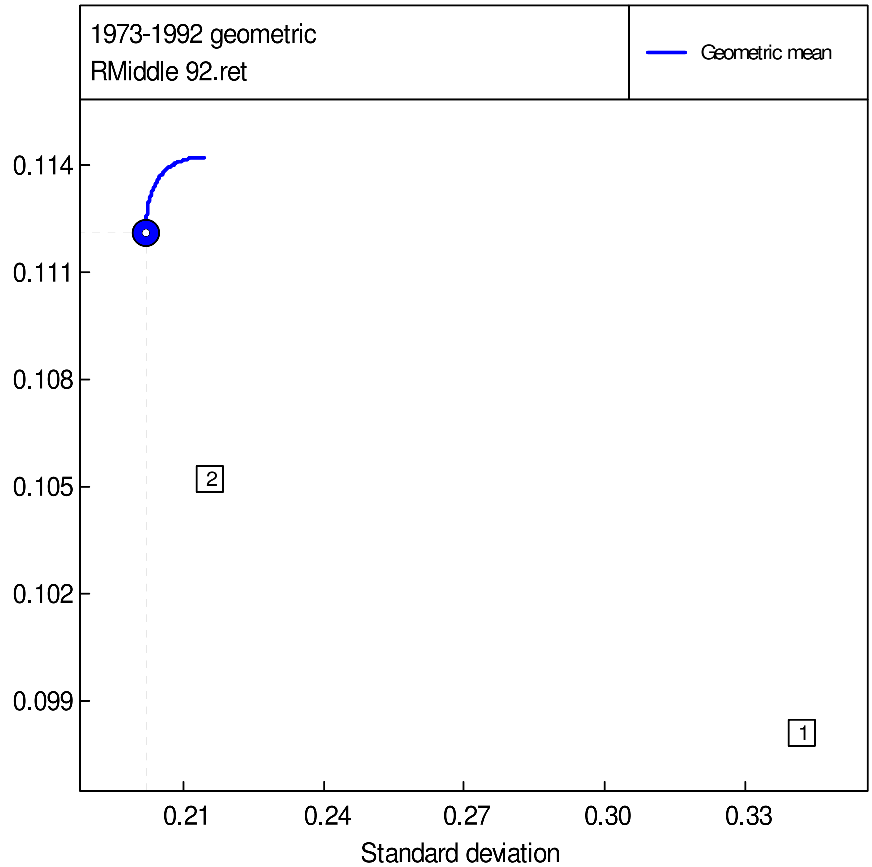
ANNEXURE 9

Summarised resampled multiple asset class middle portfolio returns 1973 - 1992		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	23.28%	23.99%
Standard Deviation	34.21%	21.56%
Real Geometric Mean	9.81%	10.52%
Sharpe Ratio	0.29	0.49
Cross-Correlation	0.27	
Real Geometric Mean adjusted for inflation at 13.47%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data I/Asset Allocations.xls/Resampled Cross-Correlations).

ANNEXURE 10

Resampled inter-market efficient frontier 1973 – 1992



1. US Middle Portfolio   2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Ref US-SA/RMiddle 92.ret).

ANNEXURE 11

Actual multiple asset class middle portfolio returns 1974 - 1993		
Year	US Optimum Portfolio	SA Optimum Portfolio
1974	-18.55%	-1.96%
1975	91.26%	20.72%
1976	48.07%	-3.60%
1977	16.11%	23.67%
1978	18.28%	34.86%
1979	26.64%	90.80%
1980	12.18%	47.22%
1981	57.60%	10.98%
1982	38.24%	27.89%
1983	62.27%	37.59%
1984	77.91%	2.54%
1985	60.03%	24.08%
1986	-2.84%	37.31%
1987	-13.27%	17.22%
1988	52.57%	16.98%
1989	29.97%	41.39%
1990	-17.29%	8.31%
1991	51.82%	35.00%
1992	43.77%	8.87%
1993	33.94%	72.79%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1974 – 1993 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1993).

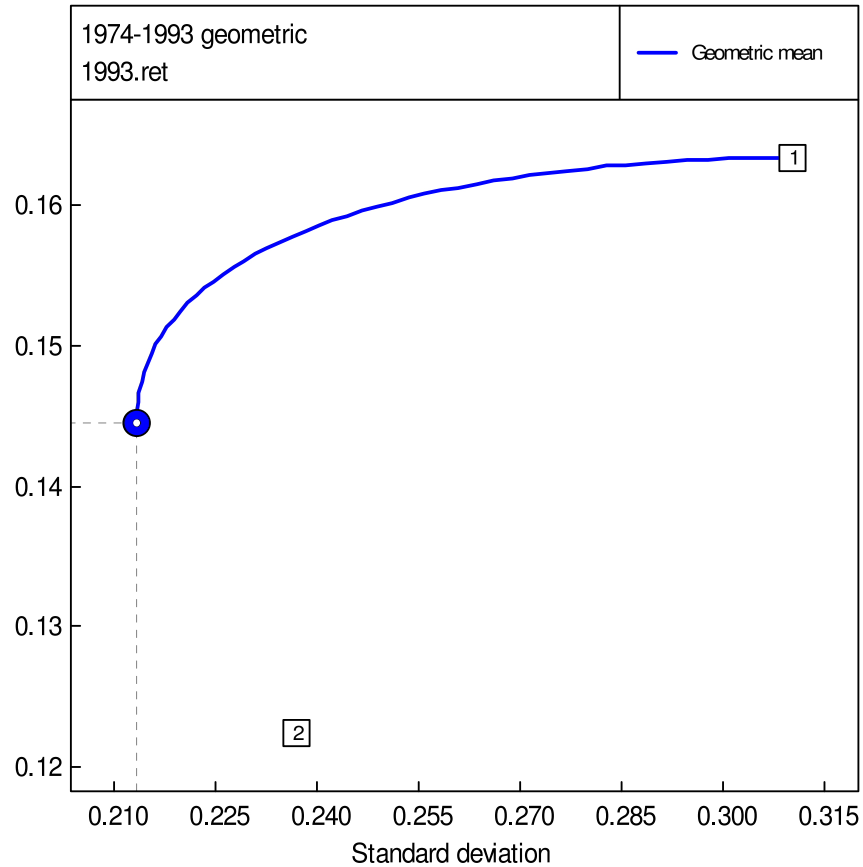
ANNEXURE 12

Summarised actual multiple asset class middle portfolio returns 1974 – 1993		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	29.82%	25.72%
Standard Deviation	31.04%	23.71%
Real Geometric Mean	16.34%	12.24%
Sharpe Ratio	0.53	0.52
Cross-Correlation	0.30	
Real Geometric Mean adjusted for inflation at 13.48%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1974 – 1993 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1993).

ANNEXURE 13

Actual inter-market efficient frontier 1974 – 1993



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1974 – 1993 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1993.ret).

ANNEXURE 14

Actual multiple asset class middle portfolio returns 1975 - 1994		
Year	US Optimum Portfolio	SA Optimum Portfolio
1975	91.22%	25.15%
1976	48.02%	-4.02%
1977	16.07%	20.86%
1978	18.26%	36.27%
1979	26.62%	88.78%
1980	12.20%	43.88%
1981	57.55%	17.44%
1982	38.21%	23.79%
1983	62.22%	43.49%
1984	77.90%	-1.92%
1985	60.04%	26.17%
1986	-2.84%	42.75%
1987	-13.26%	20.06%
1988	52.54%	18.87%
1989	30.00%	37.94%
1990	-17.26%	6.03%
1991	51.83%	29.86%
1992	43.73%	9.16%
1993	33.90%	72.43%
1994	6.72%	52.92%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1975 – 1994 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1994).

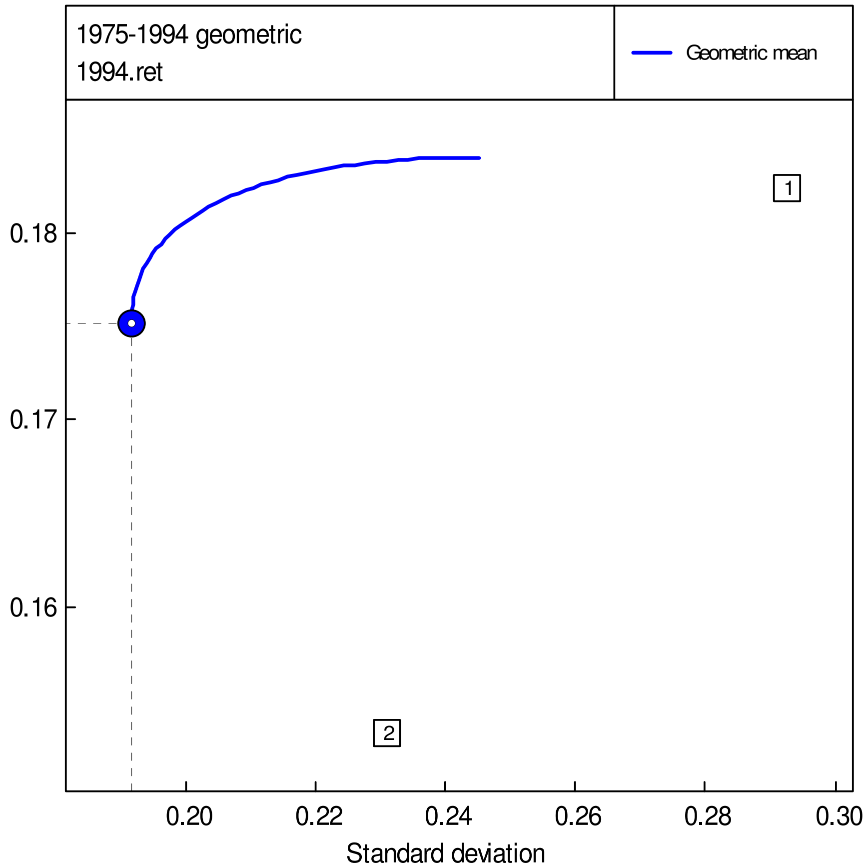
ANNEXURE 15

Summarised actual multiple asset class middle portfolio returns 1975 - 1994		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	31.58%	28.66%
Standard Deviation	29.26%	23.10%
Real Geometric Mean	18.24%	15.32%
Sharpe Ratio	0.62	0.66
Cross-Correlation	0.12	
Real Geometric Mean adjusted for inflation at 13.35%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1975 – 1994 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1994).

ANNEXURE 16

Actual inter-market efficient frontier 1975 – 1994



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1975 – 1994 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1994.ret).

ANNEXURE 17

Actual multiple asset class middle portfolio returns 1976 - 1995		
Year	US Optimum Portfolio	SA Optimum Portfolio
1976	48.60%	-4.31%
1977	16.55%	21.99%
1978	18.47%	35.92%
1979	26.89%	88.70%
1980	12.06%	44.16%
1981	58.06%	15.79%
1982	38.52%	25.27%
1983	62.76%	42.31%
1984	78.10%	-1.02%
1985	59.98%	26.17%
1986	-2.85%	42.45%
1987	-13.44%	19.36%
1988	52.85%	19.25%
1989	29.67%	39.51%
1990	-17.61%	6.41%
1991	51.79%	30.95%
1992	44.19%	8.18%
1993	34.31%	73.61%
1994	6.74%	52.21%
1995	34.07%	25.45%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1976 – 1995 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1995).

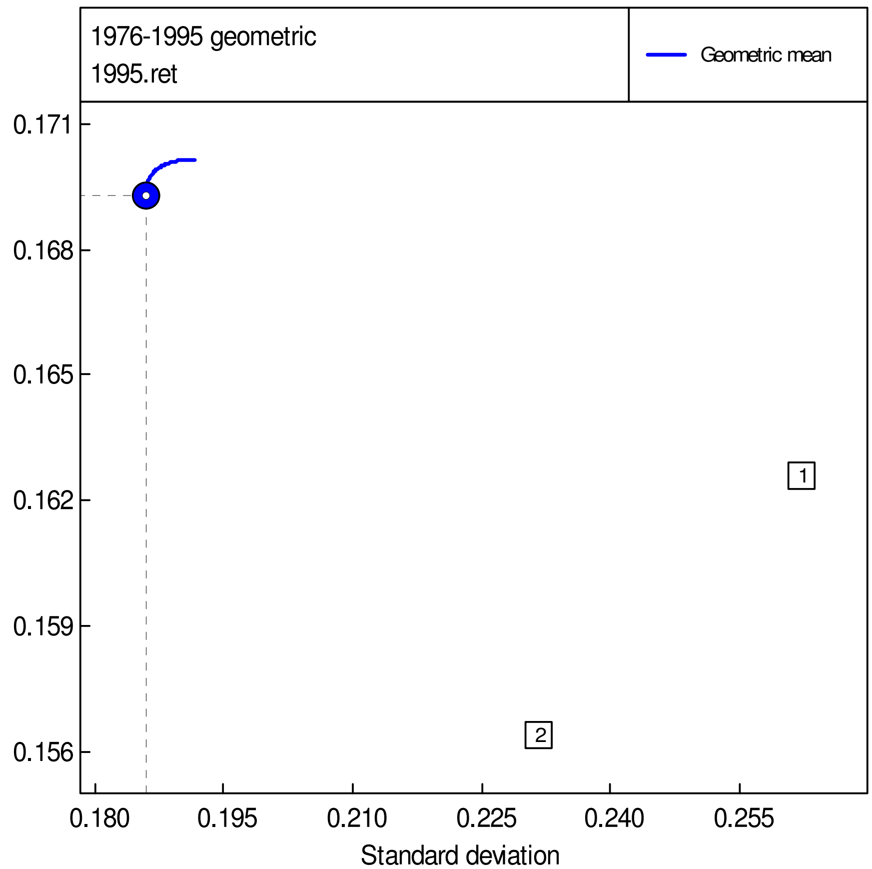
ANNEXURE 18

Summarised actual multiple asset class middle portfolio returns 1976 - 1995		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	29.40%	28.78%
Standard Deviation	26.22%	23.16%
Real Geometric Mean	16.26%	15.64%
Sharpe Ratio	0.62	0.68
Cross-Correlation	0.15	
Real Geometric Mean adjusted for inflation at 13.14%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1976 – 1995 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1995).

ANNEXURE 19

Actual inter-market efficient frontier 1976 – 1995



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1976 – 1995 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1995.ret).

ANNEXURE 20

Actual multiple asset class middle portfolio returns 1977 - 1996		
Year	US Optimum Portfolio	SA Optimum Portfolio
1977	16.80%	22.39%
1978	18.58%	36.00%
1979	27.03%	87.97%
1980	12.00%	43.38%
1981	58.33%	16.02%
1982	38.68%	25.66%
1983	63.05%	42.98%
1984	78.20%	-1.45%
1985	59.95%	26.76%
1986	-2.85%	43.90%
1987	-13.53%	19.54%
1988	53.01%	20.37%
1989	29.50%	40.16%
1990	-17.79%	6.15%
1991	51.77%	30.50%
1992	44.43%	7.14%
1993	34.52%	75.06%
1994	6.74%	52.69%
1995	34.00%	25.86%
1996	48.11%	9.96%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1977 – 1996 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1996).

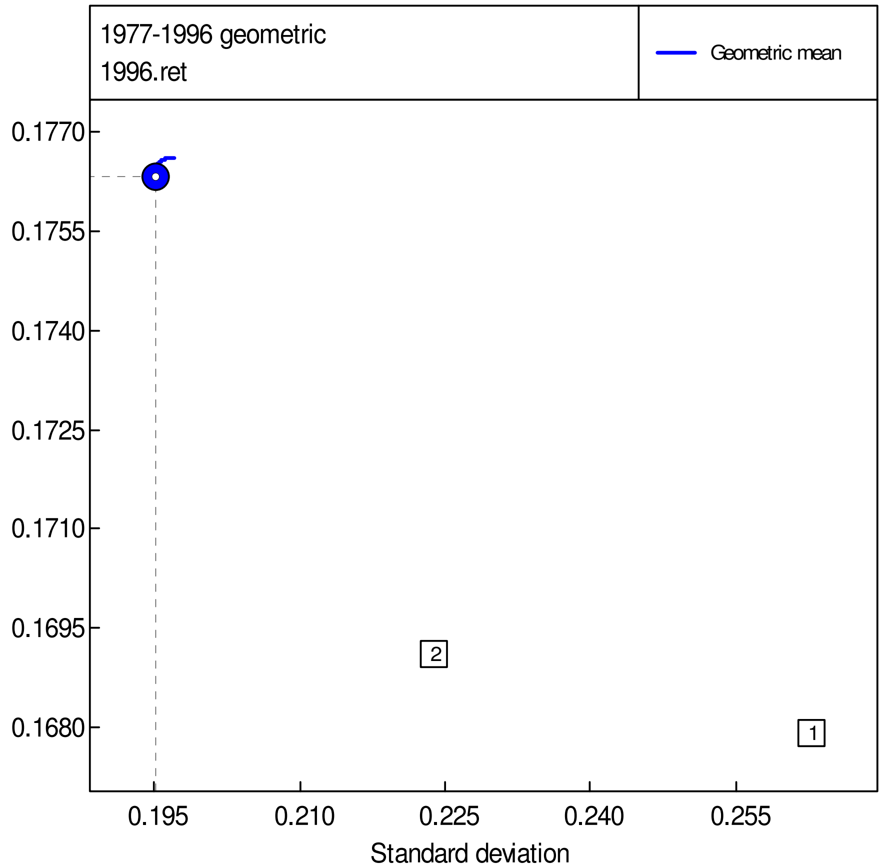
ANNEXURE 21

Summarised actual multiple asset class middle portfolio returns 1977 - 1996		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	29.75%	29.87%
Standard Deviation	26.28%	22.39%
Real Geometric Mean	16.79%	16.91%
Sharpe Ratio	0.64	0.76
Cross-Correlation	0.32	
Real Geometric Mean adjusted for inflation at 12.95%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1977 – 1996 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1996).

ANNEXURE 22

Actual inter-market efficient frontier 1977 – 1996



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1977 – 1996 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1996.ret).

ANNEXURE 23

Actual multiple asset class middle portfolio returns 1978 - 1997		
Year	US Optimum Portfolio	SA Optimum Portfolio
1978	18.24%	36.72%
1979	26.59%	89.51%
1980	12.21%	44.20%
1981	57.49%	18.54%
1982	38.18%	22.90%
1983	62.16%	44.69%
1984	77.87%	-2.36%
1985	60.04%	25.08%
1986	-2.84%	42.19%
1987	-13.24%	20.95%
1988	52.50%	18.43%
1989	30.04%	36.47%
1990	-17.22%	6.25%
1991	51.83%	29.28%
1992	43.68%	10.30%
1993	33.86%	72.61%
1994	6.72%	54.11%
1995	34.21%	26.60%
1996	48.09%	11.03%
1997	43.98%	6.11%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1978 – 1997 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1997).

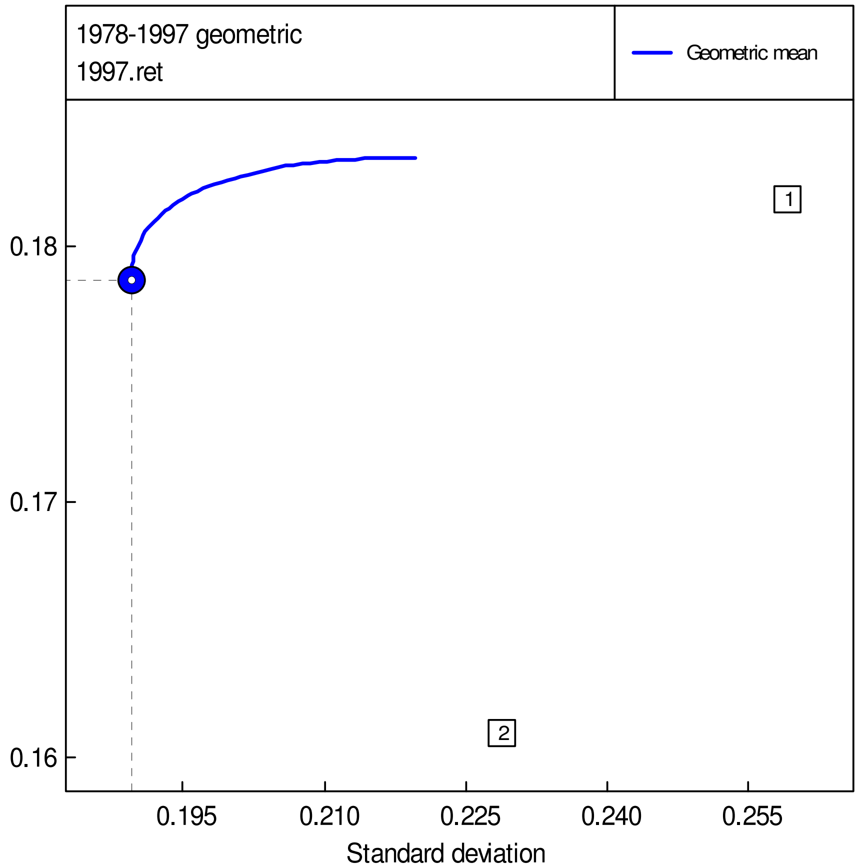
ANNEXURE 24

Summarised actual multiple asset class middle portfolio returns 1978 - 1997		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	31.02%	28.92%
Standard Deviation	25.91%	22.88%
Real Geometric Mean	18.19%	16.09%
Sharpe Ratio	0.70	0.70
Cross-Correlation	0.22	
Real Geometric Mean adjusted for inflation at 12.82%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1978 – 1997 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1997).

ANNEXURE 25

Actual inter-market efficient frontier 1978 – 1997



1. US Middle Portfolio

2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1978 – 1997 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1997.ret).

ANNEXURE 26

Actual multiple asset class middle portfolio returns 1979 - 1998		
Year	US Optimum Portfolio	SA Optimum Portfolio
1979	25.28%	89.62%
1980	12.84%	44.73%
1981	55.04%	17.33%
1982	36.72%	23.28%
1983	59.56%	42.94%
1984	76.92%	-1.52%
1985	60.31%	25.40%
1986	-2.80%	41.16%
1987	-12.39%	19.98%
1988	51.03%	17.65%
1989	31.63%	37.06%
1990	-15.55%	6.35%
1991	52.01%	30.25%
1992	41.48%	10.39%
1993	31.93%	70.95%
1994	6.66%	52.59%
1995	34.81%	25.58%
1996	48.03%	11.55%
1997	43.45%	6.01%
1998	21.48%	-14.21%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1979 – 1998 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1998).

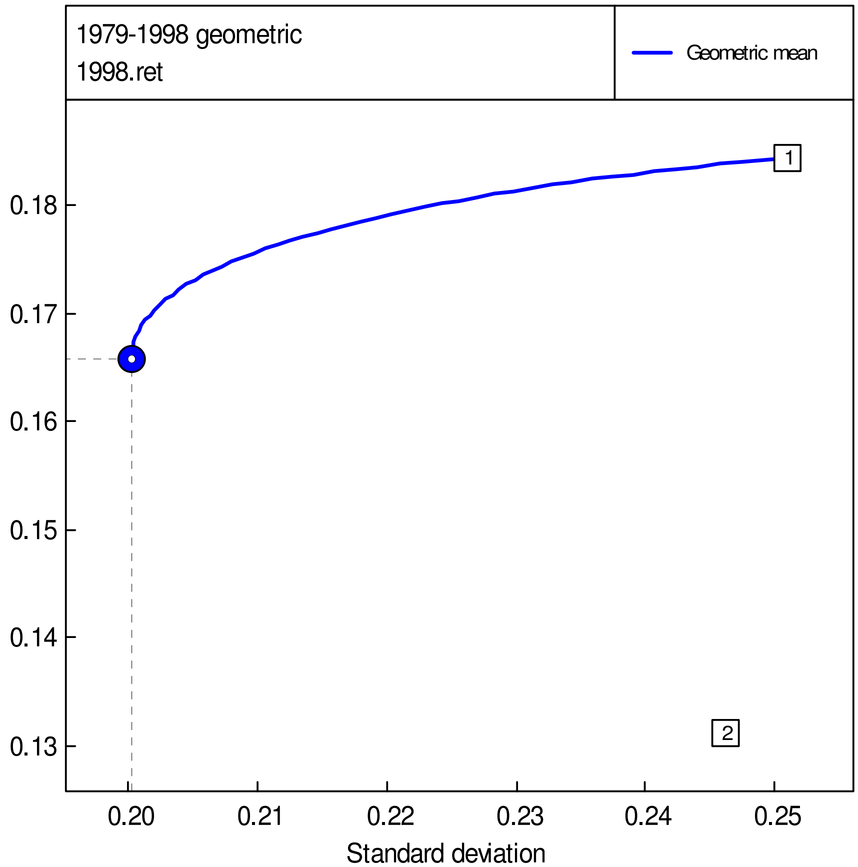
ANNEXURE 27

Summarised actual multiple asset class middle portfolio returns 1979 - 1998		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	31.05%	25.72%
Standard Deviation	25.10%	24.62%
Real Geometric Mean	18.44%	13.11%
Sharpe Ratio	0.73	0.53
Cross-Correlation	0.30	
Real Geometric Mean adjusted for inflation at 12.60%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1979 – 1998 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1998).

ANNEXURE 28

Actual inter-market efficient frontier 1979 – 1998



1. US Middle Portfolio   2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1979 – 1998 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1998.ret).

ANNEXURE 29

Actual multiple asset class middle portfolio returns 1980 - 1999		
Year	US Optimum Portfolio	SA Optimum Portfolio
1980	15.08%	47.14%
1981	46.32%	11.72%
1982	31.52%	26.47%
1983	50.31%	36.98%
1984	73.53%	2.22%
1985	61.26%	25.02%
1986	-2.65%	37.05%
1987	-9.37%	16.82%
1988	45.77%	15.82%
1989	37.29%	40.37%
1990	-9.62%	7.68%
1991	52.63%	34.60%
1992	33.67%	9.85%
1993	25.05%	69.15%
1994	6.46%	48.24%
1995	36.94%	22.54%
1996	47.84%	12.22%
1997	41.59%	3.72%
1998	33.75%	-9.77%
1999	23.40%	51.36%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1980 – 1999 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1999).

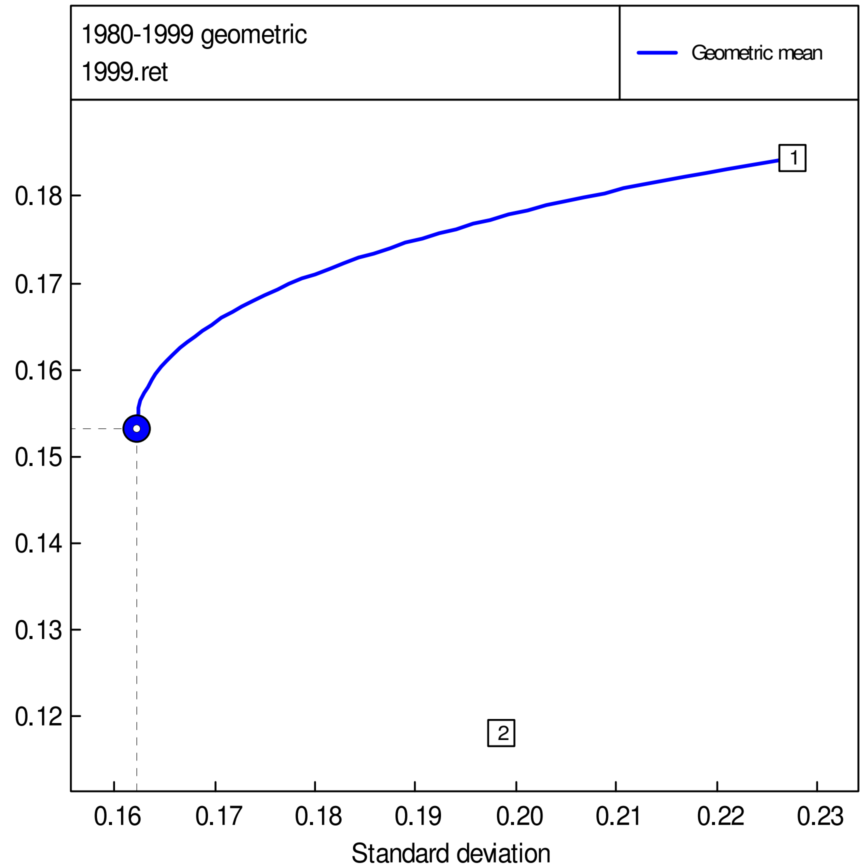
ANNEXURE 30

Summarised actual multiple asset class middle portfolio returns 1980 – 1999		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.63%	23.98%
Standard Deviation	22.76%	19.86%
Real Geometric Mean	18.45%	11.80%
Sharpe Ratio	0.81	0.59
Cross-Correlation	0.18	
Real Geometric Mean adjusted for inflation at 12.18%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1980 – 1999 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 1999).

ANNEXURE 31

Actual inter-market efficient frontier 1980 – 1999



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1980 – 1999 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/1999.ret).

ANNEXURE 32

Actual multiple asset class middle portfolio returns 1981 - 2000		
Year	US Optimum Portfolio	SA Optimum Portfolio
1981	54.21%	15.99%
1982	32.65%	24.18%
1983	52.54%	40.71%
1984	82.72%	-1.23%
1985	61.36%	28.52%
1986	-0.43%	43.01%
1987	-11.48%	18.15%
1988	49.71%	18.39%
1989	36.76%	39.52%
1990	-12.71%	5.30%
1991	47.01%	30.73%
1992	37.20%	8.26%
1993	29.45%	68.97%
1994	3.70%	49.87%
1995	37.61%	24.11%
1996	44.82%	11.83%
1997	40.88%	5.43%
1998	30.62%	-14.42%
1999	16.89%	43.85%
2000	13.22%	-9.50%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1981 – 2000 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2000).

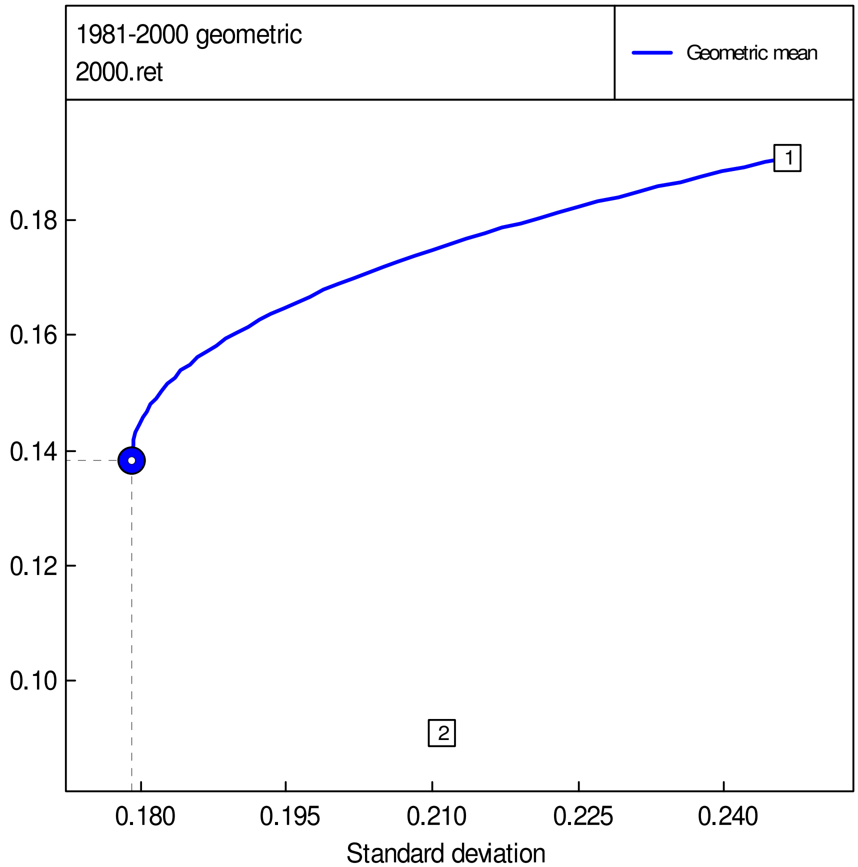
ANNEXURE 33

Summarised actual multiple asset class middle portfolio returns 1981 - 2000		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.85%	20.85%
Standard Deviation	24.65%	21.09%
Real Geometric Mean	19.09%	9.09%
Sharpe Ratio	0.77	0.43
Cross-Correlation	0.25	
Real Geometric Mean adjusted for inflation at 11.76%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1981 – 2000 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2000).

ANNEXURE 34

Actual inter-market efficient frontier 1981 – 2000



1. US Middle Portfolio
2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1981 – 2000 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/2000.ret).

ANNEXURE 35

Actual multiple asset class middle portfolio returns 1982 - 2001		
Year	US Optimum Portfolio	SA Optimum Portfolio
1982	36.39%	26.42%
1983	58.96%	35.99%
1984	76.70%	0.23%
1985	60.37%	32.65%
1986	-2.79%	44.42%
1987	-12.19%	14.76%
1988	50.69%	18.98%
1989	31.99%	43.91%
1990	-15.17%	4.27%
1991	52.05%	32.63%
1992	40.98%	5.08%
1993	31.49%	65.98%
1994	6.65%	44.97%
1995	34.94%	21.25%
1996	48.02%	12.31%
1997	43.33%	3.86%
1998	22.27%	-13.54%
1999	17.33%	47.64%
2000	11.32%	-7.50%
2001	67.30%	32.42%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1982 – 2001 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2001).

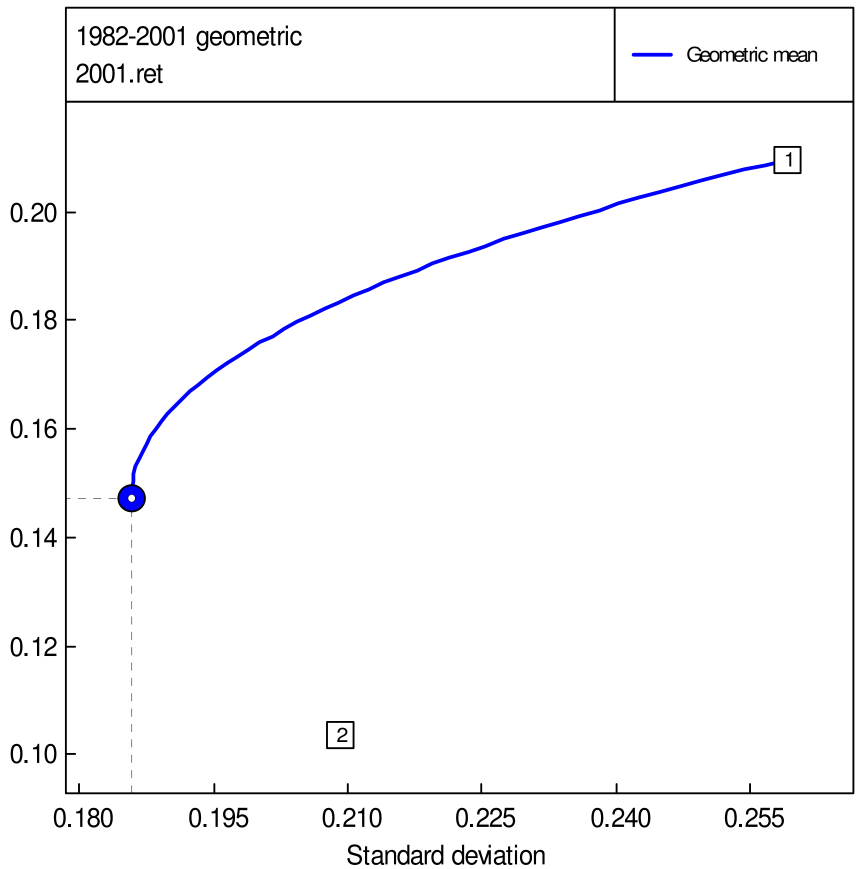
ANNEXURE 36

Summarised actual multiple asset class middle portfolio returns 1982 - 2001		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	32.25%	21.62%
Standard Deviation	25.92%	20.91%
Real Geometric Mean	20.97%	10.35%
Sharpe Ratio	0.81	0.49
Cross-Correlation	0.32	
Real Geometric Mean adjusted for inflation at 11.28%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1982 – 2001 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2001).

ANNEXURE 37

Actual inter-market efficient frontier 1982 – 2001



1. US Middle Portfolio 2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1982 – 2001 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/2001.ret).

ANNEXURE 38

Actual multiple asset class middle portfolio returns 1983 - 2002		
Year	US Optimum Portfolio	SA Optimum Portfolio
1983	56.44%	41.58%
1984	75.78%	-1.53%
1985	60.63%	27.74%
1986	-2.75%	42.66%
1987	-11.37%	18.78%
1988	49.25%	18.15%
1989	33.54%	38.56%
1990	-13.55%	5.47%
1991	52.22%	30.34%
1992	38.85%	8.99%
1993	29.61%	69.26%
1994	6.59%	50.74%
1995	35.53%	24.61%
1996	47.97%	11.83%
1997	42.83%	5.86%
1998	25.62%	-14.50%
1999	19.10%	42.82%
2000	10.53%	-10.00%
2001	62.16%	31.11%
2002	-51.03%	13.96%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the multiple asset classes for the period 1983 – 2002 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2002).

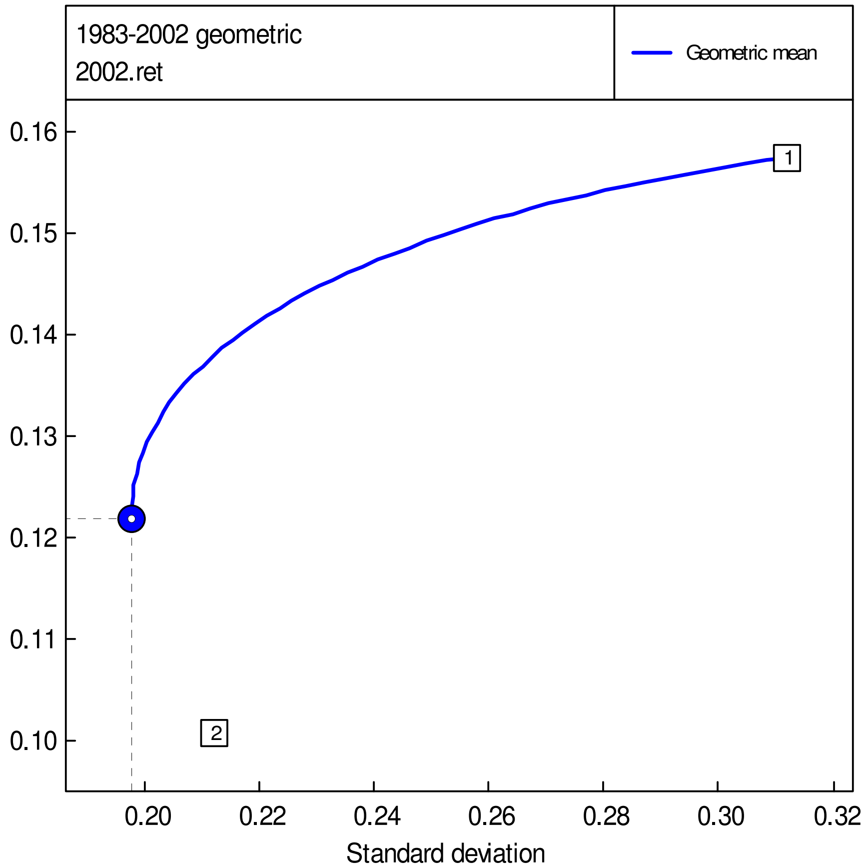
ANNEXURE 39

Summarised actual multiple asset class middle portfolio returns 1983 - 2002		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	26.75%	21.06%
Standard Deviation	31.20%	21.23%
Real Geometric Mean	15.75%	10.06%
Sharpe Ratio	0.50	0.47
Cross-Correlation	0.31	
Real Geometric Mean adjusted for inflation at 11.01%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the multiple asset classes for the period 1983 – 2002 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/EF US-SA Actual 93-02.xls/Cross-Correlation 2002).

ANNEXURE 40

Actual inter-market efficient frontier 1983 – 2002



1. US Middle Portfolio

2. SA Middle Portfolio

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1983 – 2002 (Archive Reference: Thesis Data II/EF US-SA Actual 93-02/2002.ret).

ANNEXURE 41

Resampled multiple asset class middle portfolio returns 1974 – 1993		
Year	US Optimum Portfolio	SA Optimum Portfolio
1974	-19.75%	0.10%
1975	90.37%	19.23%
1976	46.97%	-4.98%
1977	12.54%	26.53%
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1993).

ANNEXURE 42

Summarised resampled multiple asset class middle portfolio returns 1974 – 1993		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	29.02%	26.36%
Standard Deviation	31.09%	24.72%
Real Geometric Mean	15.55%	12.88%
Sharpe Ratio	0.50	0.52
Cross-Correlation	0.26	
Real Geometric Mean adjusted for inflation at 13.48%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1993).

ANNEXURE 43

Resampled multiple asset class middle portfolio returns 1975 – 1994		
Year	US Optimum Portfolio	SA Optimum Portfolio
1975	90.37%	19.23%
1976	46.97%	-4.98%
1977	12.54%	26.53%
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1994).

ANNEXURE 44

Summarised resampled multiple asset class middle portfolio returns 1975 - 1994		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.78%	28.96%
Standard Deviation	29.34%	24.24%
Real Geometric Mean	17.43%	15.62%
Sharpe Ratio	0.59	0.64
Cross-Correlation	0.07	
Real Geometric Mean adjusted for inflation at 13.35%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1994).

ANNEXURE 45

Resampled multiple asset class middle portfolio returns 1976 – 1995		
Year	US Optimum Portfolio	SA Optimum Portfolio
1976	46.97%	-4.98%
1977	12.54%	26.53%
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1995).

ANNEXURE 46

Summarised resampled multiple asset class middle portfolio returns 1976 - 1995		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	28.59%	29.20%
Standard Deviation	26.18%	24.15%
Real Geometric Mean	15.45%	16.06%
Sharpe Ratio	0.59	0.67
Cross-Correlation	0.12	
Real Geometric Mean adjusted for inflation at 13.14%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1995).

ANNEXURE 47

Resampled multiple asset class middle portfolio returns 1977 – 1996		
Year	US Optimum Portfolio	SA Optimum Portfolio
1977	12.54%	26.53%
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1996).

ANNEXURE 48

Summarised resampled multiple asset class middle portfolio returns 1977 - 1996		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	28.87%	30.08%
Standard Deviation	26.15%	23.25%
Real Geometric Mean	15.92%	17.12%
Sharpe Ratio	0.61	0.74
Cross-Correlation	0.29	
Real Geometric Mean adjusted for inflation at 12.95%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1996).

ANNEXURE 49

Resampled multiple asset class middle portfolio returns 1978 – 1997		
Year	US Optimum Portfolio	SA Optimum Portfolio
1978	15.11%	34.78%
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1997).

ANNEXURE 50

Summarised resampled multiple asset class middle portfolio returns 1978 - 1997		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.42%	28.45%
Standard Deviation	25.89%	24.42%
Real Geometric Mean	17.60%	15.62%
Sharpe Ratio	0.68	0.64
Cross-Correlation	0.19	
Real Geometric Mean adjusted for inflation at 12.82%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1997).

ANNEXURE 51

Resampled multiple asset class middle portfolio returns 1979 – 1998		
Year	US Optimum Portfolio	SA Optimum Portfolio
1979	24.16%	91.37%
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
1998	22.59%	-11.26%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1998).

ANNEXURE 52

Summarised resampled multiple asset class middle portfolio returns 1979 - 1998		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	31.00%	25.79%
Standard Deviation	25.68%	26.10%
Real Geometric Mean	18.40%	13.19%
Sharpe Ratio	0.72	0.51
Cross-Correlation	0.26	
Real Geometric Mean adjusted for inflation at 12.60%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1998).

ANNEXURE 53

Resampled multiple asset class middle portfolio returns 1980 – 1999		
Year	US Optimum Portfolio	SA Optimum Portfolio
1980	10.64%	48.06%
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
1998	22.59%	-11.26%
1999	14.59%	65.57%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1999).

ANNEXURE 54

Summarised resampled multiple asset class middle portfolio returns 1980 - 1999		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.49%	24.88%
Standard Deviation	25.94%	23.30%
Real Geometric Mean	18.30%	12.70%
Sharpe Ratio	0.71	0.55
Cross-Correlation	0.12	
Real Geometric Mean adjusted for inflation at 12.18%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 1999).

ANNEXURE 55

Resampled multiple asset class middle portfolio returns 1981 – 2000		
Year	US Optimum Portfolio	SA Optimum Portfolio
1981	57.65%	8.06%
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
1998	22.59%	-11.26%
1999	14.59%	65.57%
2000	13.56%	-15.26%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2000).

ANNEXURE 56

Summarised resampled multiple asset class middle portfolio returns 1981 - 2000		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	30.90%	21.45%
Standard Deviation	25.82%	24.53%
Real Geometric Mean	19.14%	9.69%
Sharpe Ratio	0.74	0.39
Cross-Correlation	0.22	
Real Geometric Mean adjusted for inflation at 11.76%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2000).

ANNEXURE 57

Resampled multiple asset class middle portfolio returns 1982 – 2001		
Year	US Optimum Portfolio	SA Optimum Portfolio
1982	36.14%	31.68%
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
1998	22.59%	-11.26%
1999	14.59%	65.57%
2000	13.56%	-15.26%
2001	64.82%	23.41%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2001).

ANNEXURE 58

Summarised resampled multiple asset class middle portfolio returns 1982 - 2001		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	32.16%	22.26%
Standard Deviation	26.23%	24.26%
Real Geometric Mean	20.88%	10.98%
Sharpe Ratio	0.80	0.45
Cross-Correlation	0.21	
Real Geometric Mean adjusted for inflation at 11.28%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2001).

ANNEXURE 59

Resampled multiple asset class middle portfolio returns 1983 – 2002		
Year	US Optimum Portfolio	SA Optimum Portfolio
1983	58.65%	37.32%
1984	81.61%	4.14%
1985	60.60%	22.06%
1986	-1.43%	36.79%
1987	-12.92%	17.09%
1988	52.11%	18.96%
1989	32.69%	44.38%
1990	-15.96%	9.86%
1991	48.79%	36.78%
1992	41.70%	6.48%
1993	32.89%	79.82%
1994	4.96%	50.51%
1995	35.73%	23.73%
1996	46.17%	8.78%
1997	42.58%	-1.74%
1998	22.59%	-11.26%
1999	14.59%	65.57%
2000	13.56%	-15.26%
2001	64.82%	23.41%
2002	-52.08%	15.12%
US Market figures are adjusted for currency, and reflected as SAR returns.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2002).

ANNEXURE 60

Summarised resampled multiple asset class middle portfolio returns 1983 - 2002		
Summary	US Optimum Portfolio	SA Optimum Portfolio
Geometric Mean	26.63%	21.44%
Standard Deviation	32.37%	24.28%
Real Geometric Mean	15.62%	10.43%
Sharpe Ratio	0.48	0.43
Cross-Correlation	0.21	
Real Geometric Mean adjusted for inflation at 11.01%. Cross-Correlation figures calculated before adjusting for currency.		

Source: Derived using data from the resampled multiple asset classes for the period 1973 – 1992 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 93-02.xls/Cross-Correlation 2002).

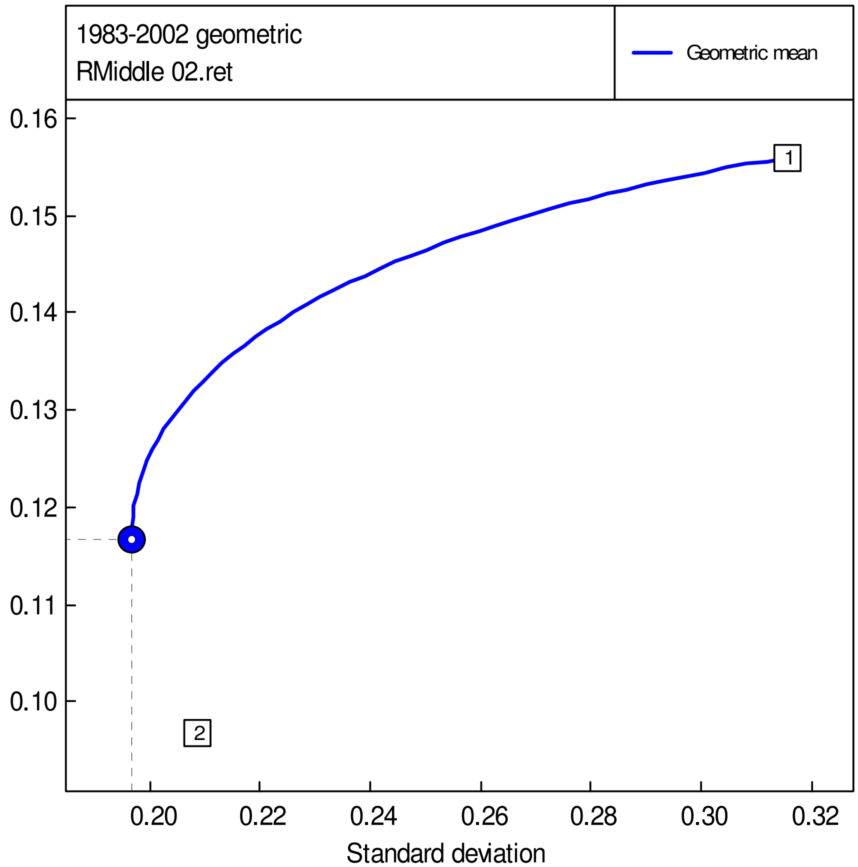
ANNEXURE 61

Rolling 20 year geometric rates of return		
Rolling Time Periods	MP M Cap Value	MP S Cap Value
1973 - 1992	24.20%	24.07%
1974 - 1993	25.77%	27.27%
1975 - 1994	27.49%	30.93%
1976 - 1995	27.85%	31.11%
1977 - 1996	28.73%	31.96%
1978 - 1997	26.66%	30.63%
1979 - 1998	24.87%	26.91%
1980 - 1999	24.44%	25.59%
1981 - 2000	20.01%	22.93%
1982 - 2001	20.59%	23.90%
1983 - 2002	19.55%	23.34%
Average Geometric Rate of Return	**24.56%**	**27.15%**

Source: Derived using data from the actual asset classes for the period 1973 – 2002 (Archive Reference: Thesis Data I/Rolling Mid & Small Returns 20.xls).

ANNEXURE 62

Redetermined resampled inter-market efficient frontier 1983 – 2002



1. US Middle Portfolio   2. SA Middle Portfolio

US Middle Portfolio	22.02%	100%	0%
SA Middle Portfolio	77.98%	0%	100%
Standard Deviation	19.67%	31.57%	20.87%
Geometric Mean	11.66%	15.60%	9.67%

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1983 – 2002 (Archive Reference: Thesis Data II/Res US-SA 93-02/Res US-SA 01-02.xls/Cross-Correlation 2002, Thesis Data II/REF US-SA/RMiddle 02_frontier.txt and Thesis Data II/REF US-SA/RMiddle 02_summary.txt).

ANNEXURE 63

Actual ALSI versus proxy ALSI		
	Proxy ALSI	Actual ALSI
1973	1.40%	6.31%
1974	13.04%	6.40%
1975	-9.30%	-4.03%
1976	-6.08%	-3.25%
1977	27.24%	29.39%
1978	29.55%	36.24%
1979	85.87%	91.30%
1980	40.61%	41.43%
1981	-3.72%	0.01%
1982	32.86%	34.00%
1983	10.83%	14.21%
1984	6.81%	9.40%
1985	45.82%	40.99%
1986	52.76%	54.80%
1987	-1.63%	-3.40%
1988	17.29%	14.26%
1989	53.34%	54.93%
1990	-11.16%	-4.85%
1991	30.40%	30.66%
1992	-1.04%	-1.87%
1993	49.79%	53.78%
1994	23.91%	22.52%
1995	9.62%	8.63%
1996	7.40%	9.52%
1997	-6.94%	-4.30%
1998	-7.80%	-9.57%
1999	55.21%	60.47%
2000	-3.25%	-0.01%
2001	21.77%	27.95%
2002	-6.21%	-6.75%
R	0.9912	
R^2	0.9826	
Returns are arithmetic and are inclusive of income.		

Source: Derived using JSE Securities Exchange ALSI and derived ALSI index for the period 1973 – 2002. (Archive Reference: Thesis Data I/Proxy Total Returns.xls).

ANNEXURE 64

Comparison of asset class returns (1973 – 2002)		
	Real Geometric Return* **	Standard Deviation
91 Day Treasury Bills	-4.46%	4.51%
12 Month Fixed Deposit	-4.10%	3.44%
3 Month Negotiable Certificate Of Deposit	-3.61%	4.43%
ABSA Property Index***	-0.82%	9.26%
Short Term Government Bonds	-4.67%	14.97%
JSE Securities Exchange	6.50%	25.23%

* Adjusted for inflation @ 11.46%.
** Adjusted for income taxation, mutatis mutandis @ assumed 40%. No capital gains tax assumed.
*** For the period 1976 – 2002, and adjusted for inflation @ 11.48%

Source: Primary Data SARB Quarterly Bulletin No.'s 111 - 227

ANNEXURE 65

Total unit trusts versus ALSI (1988 – 2002)				
Year	Unit Trusts		ALSI	
1988		R 10,000		R 10,000
1989	38.54%	R 13,854	54.93%	R 15,493
1990	-3.24%	R 13,405	-4.85%	R 14,741
1991	31.77%	R 17,664	30.66%	R 19,260
1992	-3.58%	R 17,032	-1.87%	R 18,901
1993	30.19%	R 22,174	53.78%	R 29,066
1994	15.87%	R 25,693	22.52%	R 35,611
1995	9.18%	R 28,053	8.63%	R 38,683
1996	6.83%	R 29,970	9.52%	R 42,368
1997	-0.89%	R 29,702	-4.30%	R 40,545
1998	-7.00%	R 27,624	-9.57%	R 36,666
1999	21.56%	R 33,579	60.47%	R 58,840
2000	-4.14%	R 32,190	-0.01%	R 58,837
2001	13.60%	R 36,567	27.95%	R 75,281
2002	-7.76%	R 33,731	-6.75%	R 70,198
Geometric Return	8.44%		13.87%	
Arithmetic Return	10.07%		17.22%	
Standard Deviation	15.62%		24.79%	
Sharpe Ratio	0.54		0.56	

Source: Derived using data accessed at the Association of Collective Investments, www.aut.co.za (Archive Reference: Thesis Data I/Active vs Passive.xls).

ANNEXURE 66

Stochastic ALSI portfolio performance (1973 – 2002)				
Year	ALSI Returns	Price	Invested	Investment Value
1972		R10.00	-R 1,000,000	R 1,000,000
1973	6.31%	R10.63	0.00	R 1,063,094
1974	6.40%	R11.31	0.00	R 1,131,132
1975	-4.03%	R10.86	0.00	R 1,085,560
1976	-3.25%	R10.50	0.00	R 1,050,243
1977	29.39%	R13.59	0.00	R 1,358,889
1978	36.24%	R18.51	0.00	R 1,851,388
1979	91.30%	R35.42	0.00	R 3,541,770
1980	41.43%	R50.09	0.00	R 5,009,104
1981	0.01%	R50.10	0.00	R 5,009,666
1982	34.00%	R67.13	0.00	R 6,713,010
1983	14.21%	R76.67	0.00	R 7,667,049
1984	9.40%	R83.88	0.00	R 8,387,874
1985	40.99%	R118.26	0.00	R 11,826,010
1986	54.80%	R183.06	0.00	R 18,306,379
1987	-3.40%	R176.83	0.00	R 17,683,144
1988	14.26%	R202.05	0.00	R 20,204,935
1989	54.93%	R313.04	0.00	R 31,303,800

ANNEXURE 66 CONTINUED

1990	-4.85%	R297.84	0.00	R 29,784,051
1991	30.66%	R389.15	0.00	R 38,914,505
1992	-1.87%	R381.89	0.00	R 38,188,505
1993	53.78%	R587.28	0.00	R 58,727,823
1994	22.52%	R719.51	0.00	R 71,951,011
1995	8.63%	R781.59	0.00	R 78,159,447
1996	9.52%	R856.04	0.00	R 85,603,840
1997	-4.30%	R819.21	0.00	R 81,921,313
1998	-9.57%	R740.84	0.00	R 74,084,172
1999	60.47%	R1188.87	0.00	R 118,886,511
2000	-0.01%	R1188.80	0.00	R 118,880,006
2001	27.95%	R1521.05	0.00	R 152,105,271
2002	-6.75%	R1418.35	0.00	R 141,835,311
Geometric Return		17.96%		
Opening Balance		R 1000,000.00		
Shares Owned		100000		
Terminal Value		R 141,835,311.11		
I.R.R.		**17.33%**		

Source: Derived using ALSI data for the period 1972 – 2002, and Edleson (1993, p. 39) methodology. (Archive Reference: Thesis Data I/Value Averaging.xls/Lump Sum).

ANNEXURE 67

Value averaged ALSI portfolio performance (1973 – 2002)						
Year	ALSI Returns	Price	Invested	Investment Value	Shares Owned	Value Line
1972		R10.00	-R 1,000,000	R 1,000,000	100000.00	R 1,000,000
1973	6.31%	R10.63	-R 116,481	R 1,063,094	110956.85	R 1,179,576
1974	6.40%	R11.31	-R 136,331	R 1,131,132	123009.50	R 1,391,400
1975	-4.03%	R10.86	-R 305,921	R 1,085,560	151190.46	R 1,641,263
1976	-3.25%	R10.50	-R 348,129	R 1,050,243	184337.94	R 1,935,996
1977	29.39%	R13.59	R 221,291	R 1,358,889	168053.18	R 2,283,656
1978	36.24%	R18.51	R 417,569	R 1,851,388	145498.79	R 2,693,747
1979	91.30%	R35.42	R 1,975,751	R 3,541,770	89714.49	R 3,177,481
1980	41.43%	R50.09	R 745,809	R 5,009,104	74825.41	R 3,748,082
1981	0.01%	R50.10	-R 672,647	R 5,009,666	88252.40	R 4,421,150
1982	34.00%	R67.13	R 709,307	R 6,713,010	77686.24	R 5,215,085
1983	14.21%	R76.67	-R 195,350	R 7,667,049	80234.16	R 6,151,593
1984	9.40%	R83.88	-R 526,333	R 8,387,874	86509.10	R 7,256,275
1985	40.99%	R118.26	R 1,671,243	R 11,826,010	72377.18	R 8,559,332
1986	54.80%	R183.06	R 3,153,252	R 18,306,379	55152.29	R 10,096,388
1987	-3.40%	R176.83	-R 2,156,803	R 17,683,144	67349.24	R 11,909,463
1988	14.26%	R202.05	-R 440,253	R 20,204,935	69528.18	R 14,048,123
1989	54.93%	R313.04	R 5,194,124	R 31,303,800	52935.55	R 16,570,838

ANNEXURE 67 CONTINUED

1990	-4.85%	R297.84	-R 3,780,222	R 29,784,051	65627.65	R 19,546,572
1991	30.66%	R389.15	R 2,481,994	R 38,914,505	59249.58	R 23,056,679
1992	-1.87%	R381.89	-R 4,570,591	R 38,188,505	71218.08	R 27,197,119
1993	53.78%	R587.28	R 9,743,742	R 58,727,823	54626.72	R 32,081,085
1994	22.52%	R719.51	R 1,462,381	R 71,951,011	52594.25	R 37,842,097
1995	8.63%	R781.59	-R 3,530,274	R 78,159,447	57111.01	R 44,637,652
1996	9.52%	R856.04	-R 3,764,309	R 85,603,840	61508.37	R 52,653,530
1997	-4.30%	R819.21	-R 11,720,403	R 81,921,313	75815.28	R 62,108,871
1998	-9.57%	R740.84	-R 17,095,049	R 74,084,172	98890.45	R 73,262,170
1999	60.47%	R1188.87	R 31,149,062	R 118,886,511	72689.78	R 86,418,341
2000	-0.01%	R1188.80	-R 15,523,440	R 118,880,006	85747.85	R 101,937,053
2001	27.95%	R1521.05	R 10,184,442	R 152,105,271	79052.20	R 120,242,561
2002	-6.75%	R1418.35	-R 29,711,378	R 141,835,311	100000.00	R 141,835,311
Geometric Return				17.96%		
Opening Balance				R 1000,000.00		
Terminal Value				R 141,835,311.11		
I.R.R.				**20.16%**		

Source: Derived using ALSI data for the period 1972 – 2002, and Edleson (1993, p. 39) methodology. (Archive Reference: Thesis Data I/Value Averaging.xls/Lump Sum).

ANNEXURE 68

ALSI rolling time period returns					
Year	Annual Returns	5 Year Returns	10 Year Returns	15 Year Returns	20 Year Returns
1973	6.31%				
1974	6.40%				
1975	-4.03%				
1976	-3.25%				
1977	29.39%	6.33%			
1978	36.24%	11.73%			
1979	91.30%	25.64%			
1980	41.43%	35.78%			
1981	0.01%	36.68%			
1982	34.00%	37.64%	20.97%		
1983	14.21%	32.87%	21.84%		
1984	9.40%	18.82%	22.18%		
1985	40.99%	18.75%	26.98%		
1986	54.80%	29.59%	33.09%		
1987	-3.40%	21.37%	29.25%	21.11%	
1988	14.26%	21.39%	27.00%	21.69%	
1989	54.93%	30.13%	24.35%	24.78%	
1990	-4.85%	20.29%	19.51%	24.71%	
1991	30.66%	16.28%	22.75%	27.23%	
1992	-1.87%	16.65%	18.99%	24.91%	19.98%
1993	53.78%	23.79%	22.58%	25.92%	22.21%
1994	22.52%	18.11%	23.98%	22.23%	23.08%
1995	8.63%	21.28%	20.79%	20.10%	23.84%
1996	9.52%	17.08%	16.68%	20.83%	24.61%
1997	-4.30%	16.49%	16.57%	18.15%	22.75%
1998	-9.57%	4.76%	13.87%	16.33%	20.26%
1999	60.47%	10.57%	14.28%	19.33%	19.21%
2000	-0.01%	8.75%	14.85%	16.63%	17.16%
2001	27.95%	12.18%	14.60%	15.16%	18.61%
2002	-6.75%	11.60%	14.02%	14.89%	16.48%

Source: Derived using ALSI data for the period 1972 – 2002. (Archive Reference: Thesis Data I/Rolling Returns 5.xls/SA).

ANNEXURE 69

South African bonds rolling time period returns					
Gross	Annual Returns	5 Year Returns	10 Year Returns	15 Year Returns	20 Year Returns
1973	9.69%				
1974	-7.79%				
1975	4.50%				
1976	0.91%				
1977	13.33%	3.86%			
1978	18.25%	5.44%			
1979	16.61%	10.51%			
1980	-10.86%	7.05%			
1981	0.65%	6.99%			
1982	15.39%	7.38%	5.61%		
1983	6.38%	5.13%	5.28%		
1984	-2.44%	1.45%	5.88%		
1985	14.72%	6.70%	6.87%		
1986	46.24%	14.98%	10.91%		
1987	6.12%	13.06%	10.19%	8.04%	
1988	2.07%	12.13%	8.58%	7.52%	
1989	18.29%	16.54%	8.73%	9.32%	
1990	10.99%	15.77%	11.14%	9.76%	
1991	15.85%	10.50%	12.72%	10.77%	
1992	41.35%	17.02%	15.03%	12.42%	10.22%
1993	26.13%	22.08%	17.00%	12.90%	10.99%
1994	-18.27%	13.38%	14.95%	10.26%	10.32%
1995	32.13%	17.40%	16.58%	13.19%	11.62%
1996	3.66%	14.82%	12.64%	13.41%	11.77%
1997	29.39%	12.81%	14.90%	14.28%	12.52%
1998	-3.19%	7.00%	14.29%	13.57%	11.40%
1999	43.03%	19.67%	16.48%	16.50%	12.54%
2000	24.15%	18.19%	17.80%	17.12%	14.42%
2001	21.42%	21.98%	18.35%	15.67%	15.50%
2002	11.87%	18.49%	15.61%	16.08%	15.32%

Source: Derived using Bonds data for the period 1972 – 2002. (Archive Reference: Thesis Data I/Rolling Returns 5.xls/SA).

ANNEXURE 70

U.S. assets classes 20 year rolling time period returns				
Gross	FF L Cap Growth	FF L Cap Value	FF S Cap Growth	FF S Cap Value
1992	17.58%	23.87%	16.98%	27.11%
1993	20.61%	26.56%	22.50%	32.26%
1994	22.95%	28.12%	24.77%	33.84%
1995	21.86%	26.24%	22.53%	31.14%
1996	23.25%	25.88%	22.59%	30.73%
1997	25.88%	27.82%	22.69%	31.79%
1998	28.27%	29.61%	22.71%	30.86%
1999	29.47%	29.28%	23.32%	29.63%
2000	28.55%	30.23%	20.98%	29.93%
2001	27.76%	29.07%	21.61%	30.89%
2002	21.67%	21.85%	15.06%	24.29%
Adjusted for currency, therefore SAR based, and are nominal.				

Source: Derived using data from the actual asset classes for the period 1973 – 2002 (Archive Reference: Thesis Data I/Rolling Returns 5.xls/US (SAR)).

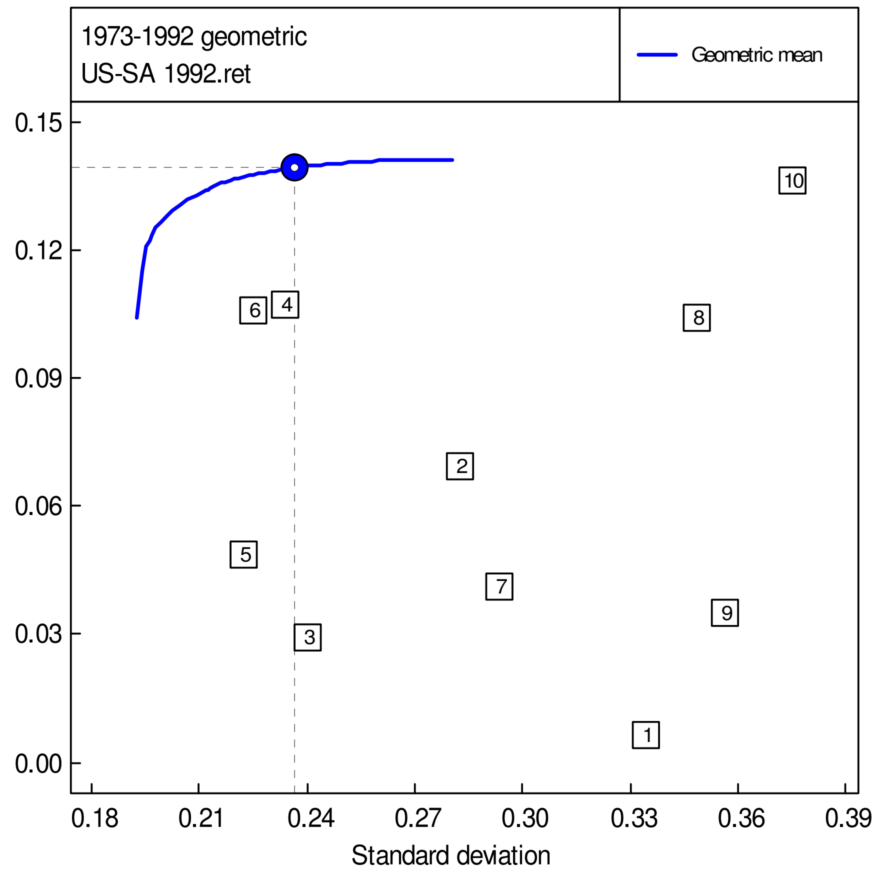
ANNEXURE 71

South African assets classes 20 year rolling time period returns						
Gross	MP L Cap Growth	MP L Cap Value	MP M Cap Growth	MP M Cap Value	MP S Cap Growth	MP S Cap Value
1992	14.10%	20.39%	16.40%	24.20%	18.35%	24.07%
1993	17.28%	22.05%	18.62%	25.77%	20.34%	27.27%
1994	16.85%	22.52%	18.48%	27.49%	22.29%	30.93%
1995	19.64%	22.95%	18.55%	27.85%	21.90%	31.11%
1996	20.15%	24.07%	18.76%	28.73%	22.97%	31.96%
1997	18.31%	21.35%	17.50%	26.66%	23.43%	30.63%
1998	15.80%	19.42%	15.91%	24.87%	21.85%	26.91%
1999	13.20%	19.85%	15.16%	24.44%	18.64%	25.59%
2000	11.35%	18.53%	10.65%	20.01%	15.02%	22.93%
2001	12.03%	21.43%	10.22%	20.59%	14.70%	23.90%
2002	10.05%	18.56%	9.96%	19.55%	14.81%	23.34%
Returns are nominal.						

Source: Derived using data from the actual asset classes for the period 1973 – 2002 (Archive Reference: Thesis Data I/Rolling Returns 5.xls/SA).

ANNEXURE 72

Actual combined inter-market efficient frontier 1973 – 1992



1. MP L Cap Growth 2. MP L Cap Value 3. MP M Cap Growth 4. MP M Cap Value

5. MP S Cap Growth 6. MP S Cap Value 7. FF L Cap Growth 8. FF L Cap Value

9. FF L Cap Growth 10. FF L Cap Value

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1972 – 1992 (Archive Reference: Simulations II/EF SA/US-SA 1992.ret).

ANNEXURE 73

Combined Inter-Market Asset Allocations 1973 - 1992			
	Minimum Variance Portfolio	Middle Portfolio	Maximum Return Portfolio
MP L Cap Growth	0.00%	0.00%	0.00%
MP L Cap Value	0.00%	0.00%	0.00%
MP M Cap Growth	20.17%	0.00%	0.00%
MP M Cap Value	35.46%	46.91%	29.30%
MP S Cap Growth	0.00%	0.00%	0.00%
MP S Cap Value	20.25%	0.00%	0.00%
FF L Cap Growth	0.00%	0.00%	0.00%
FF L Cap Value	24.12%	0.00%	0.00%
FF S Cap Growth	0.00%	0.00%	0.00%
FF S Cap Value	0.00%	53.09%	70.70%
Standard Deviation	19.25%	23.66%	28.07
Geometric Return	10.41%	13.93%	14.13
Sharpe Ratio	0.54	0.59	0.50

Source: Derived using MVO Plus software and data from the multiple asset classes for the period 1972 – 1992 (Archive Reference: Simulations II/EF SA/US-SA 1992_frontier.txt).

ABSTRACT

The research is a quantitative study that formulates an approach to future portfolio asset allocations within the South African domestic equity market, and the diversification of assets across global markets, specifically the U.S.A. The research takes the view that investors are rational, have a long term investment horizon and seek investment wealth maximisation by applying a sustainable investment strategy towards the ongoing management of the portfolio.

Investors experience a significant negative divergence in investment outcomes relative to the potentially achievable result. This negative divergence is a result of the lack of a strategic approach to, and an understanding of asset allocations, and the lack of a sustainable approach to the management of a portfolio. Repetitive sub-optimal investment performance, below the levels of inflation, is an investment disincentive with negative micro and macro implications.

The purpose of the study is therefore to address the issue sub-optimal investment performance through the effective application of a strategy that includes the integration of the mean-variance model through the use of a mean-variance optimiser, using resampled data inputs, the mean reversion of markets, passive investment management, appropriate asset class selection and the ongoing management of a portfolio, using both calendar and contingent rebalancing techniques, and passive formula strategies.

The challenge is accordingly to develop a reliable asset allocation model that accommodates past performance, and which is stable enough to produce optimised forward-looking investment portfolios, which are able to address the issue of optimal asset allocation and selection, within a global context, and which produce optimised investment outcomes, taking cognisance of the fact that the future is unknowable and dynamic.

The research methodology makes a positivist assumption that something exists and can be numerically tested. In this regard various portfolios are constructed, using passive investment instruments, in accordance with mean-variance model principles, using resampled data inputs to minimise the instability of the mean-variance optimiser. This resampling process is fundamental to the research, and incorporates the use of a stochastic simulator. A unique aspect of the research was solving the issue of multiple market integration particularly when the domestic markets are comprised of multiple asset classes. Finally, the resultant resampled efficient portfolios are compared to control portfolios in order to ascertain whether the resampling process indeed offers a return premium.

Due to the dynamic nature of equity markets contingent and calendar rebalancing strategies are applied to the asset allocation in order to maintain an optimal portfolio. This dynamism may necessitate the adjustment of asset allocations. The test for asset allocation optimality takes the form of measuring portfolio outcome correlations to the actual market outcome. Where the portfolio is sub-optimal the asset allocations are redetermined, otherwise the portfolio is merely rebalanced to the original asset allocations.

Regarding the management of the portfolio a value averaged passive formula strategy is applied. This process acknowledges that markets may behave stochastically over the short term, therefore a predetermined value line is derived that the portfolio is to achieve. This value line is based on a long term equity premium plus inflation. Should the portfolio breach the value line on the upside a portion of the investment is liquidated, conversely when the portfolio fails to reach the value line the portfolio is elevated to the value line by means of increasing the investment.

The results of the research manifest unambiguous results in favour of resampled portfolios. In this regard, therefore, data resampling does seem to produce stable portfolio results that are effective at capturing a higher proportion of future returns than a simple market portfolio. Furthermore, the rebalancing process, although not absolutely perfect, does provide a level of adjustment to the asset allocation to ensure optimality. Finally, management of the portfolio through value averaging unambiguously provides an internal rate of return in excess of a portfolio that is allowed to stochastically rise and fall.

In summary, the integration of the identified processes clearly provides a performance premium in excess of alternative approaches, and within a framework that is sustainable from period to period.

KEY TERMS

1. Asset Allocation Determination
2. Mean-Variance Optimisation
3. Resampled Data Inputs
4. Stochastic Simulation Modelling
5. Portfolio Rebalancing
6. Value Averaging
7. Modern Portfolio Theory
8. Passive Investing
9. Resampled Efficient Portfolios
10. Value Investing
11. Size Investing
12. Diversification
13. Efficient Market Hypothesis
14. Mean Reversion