GRADE 8 AND 9 TOWNSHIP LEARNERS' RESPONSE TO A PROBLEM-BASED MATHEMATICS EXTENSION PROGRAMME

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Declaration

I, Monique Duvenhage, declare that the thesis hereby submitted for the M.Ed. degree at the University of the Free State is my own, independent work and has not previously been submitted by me at another university/faculty. I furthermore cede copyright of the thesis in favour of the University of the Free State.

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Dedication

This thesis is dedicated to my two loving children,

Gustav Dawid and Inge Lise.

"May God's blessing keep you always
May your wishes all come true
May you always do for others
And let others do for you
May you build a ladder to the stars
And climb on every rung

May you grow up to be righteous
May you grow up to be true
May you always know the truth
And see the lights surrounding you
May you always be courageous
Stand upright and be strong

May your hands always be busy
May your feet always be swift
May you have a strong foundation
When the winds of changes shift
May your heart always be joyful
May your song always be sung
And may you stay
Forever young"

Joan Baez

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Summary

A need to nurture learners so that they are ready for the demands of the future suggests the need for a re-evaluation of traditional teaching approaches that tend to be teacher-centred and promote rote learning and routine drill procedures. In contrast, learner-centred problem-based teaching approaches are aimed at enabling learners to think critically, regulate their own learning, shift the boundaries of their own capabilities and be more receptive to meaningful and long-lasting learning. However, skilful guidance and scaffolding from the teacher is a key determining factor for the success of such an approach. It is, therefore, unsurprising that, in South African schools, particularly in the townships, where teachers' mathematics pedagogical and content knowledge is known to be low, problem-based teaching is rarely practiced. Further, the cognitively taxing and time-consuming problem-based teaching approach seems incompatible with training for high-stakes examinations, which is prevalent in this context. Little is known about the way learners in this context respond to problem-based extension programmes.

To contribute to filling this gap in the literature, I conducted this research using a case study design informed by the pragmatic paradigm and the framework for integrated methodology. A mixed methods research design comprising qualitative and quantitative data and using inductive analysis to produce rich, detailed accounts of learners' responses, was used. This was done to investigate how 27 Grades 8 and 9 learners responded cognitively and affectively to a problem-based mathematics extension programme that lasted two hours a day for five days. The learners were selected by applying non-probability purposive sampling. Triangulation and peer examination were used to strengthen validity and reliability. Data analysis was inductive, iterative and pragmatic, and guided by the research questions. Content analysis was performed using existing codes from Carlson and Bloom's (2005) multidimensional problem-solving framework to code the various data sources using NVivo software. This was followed by thematic analysis, in which themes were derived inductively from the coded data in answer to the research questions. This was done in the form of five assertions.

I assert that the learners employed resources, many of which were incorrect, and displayed a diverse repertoire of heuristics, but generally needed to be prompted. Furthermore, across the intervention period, the learners responded to the teacher’s modelling of monitoring by increasingly posing Why and How questions but appeared unable to apply these questions to direct engagement in iterative problem-solving. Additionally, the learners engaged in the first three phases of the problem-solving process, but showed no engagement in the checking phase, nor were mathematical intimacy and integrity evident, although strong affective responses throughout the programme were visible.

This study is not representative of Grades 8 and 9 South African learners at township schools; therefore, subsequent studies are needed to explore the extent to which the study could be generalisable to the broader population and more typical contexts. The significance of this study is that it demonstrates that it is possible to conduct problem-based extension programmes effectively to engage at least a sub-population of South African township learners both cognitively and affectively. A rich description of this programme is provided to serve as a model for the creation of similar programmes.

Key terms: *problem-based teaching, mathematics education, multidimensional problem-solving framework, township education*

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List of abbreviations and acronyms

CAPS	Curriculum Assessment and Policy Statement
FraIM	Framework for Integrated Methodologies
IPM	Information processing model
MKT	Mathematics Knowledge for Teaching
MPS framework	Multidimensional problem-solving framework
OBE	Outcomes-based education
PCK	Pedagogical Content Knowledge
PBL	Problem-based learning
STEM	Science, technology, engineering, and mathematics
ZPD	Zone of proximal development

CHAPTER 1: BACKGROUND AND RATIONALE

"Education is what remains
after one has forgotten what
one has learned in school"

Albert Einstein

1.1 INTRODUCTION

My view on teaching resonates with that of Hobden (2005, p. 308), who maintains that "there is a need to change what is taught and learnt, and how it is taught and learnt". Traditional approaches rarely lead to mastery of a topic or skill (Savery, 2006), therefore, to change and improve on traditional teaching and learning approaches, it is necessary to encourage relevant, meaningful, and long-lasting learning (Orton & Frobisher, 1996; Ridlon, 2009; Savery, 2006). Research shows that learner-centred approaches, as opposed to teacher-centred approaches, like problem-based learning (PBL), are more effective in promoting meaningful learning (Boaler, 2013; Voskoglou, 2008; Walker, Leary, Hmelo-Silver, & Ertmer, 2015). Furthermore, meaningful learning incorporates everyday experiences, which add value to what is learnt (Hobden, 2000), and nurture skills required to make a meaningful contribution to preparing learners for the modern world (Brown-Martin, 2017).

South Africa's democratic government is committed to ensuring that all children are provided with basic education, to meet the Millennium Development Goals (United Nations, 2013), and to prepare South African learners for the demands of the future (Reddy et al., 2016). Despite these commitments and endeavours, the quality of South African education is still in a dire state, especially for mathematics, where performance, according to in both national and international benchmarking tests, is extremely poor (Baller, Dutta, & Lanvin, 2016; Gurney-Read, 2016; Reddy *et al.*, 2016). In attempts to address this situation, national policies that focus on learner-centred and problem-based instructional practices, especially in mathematics, have been initiated (Department of Basic Education, 2011a, 2011b). South Africa is not the only country that has proposed such policies in response to concerns about and the need to develop mathematics problem-solving abilities in its learners (see, for example, National Research Council, 2000). This change in policies, or curriculum reform, stipulates a greater focus on what may be considered to be minimally guided approaches, such as problem-based, discovery, inquiry, experiential, and constructivist learning (Bruner, 1961; Jonassen, 1991; Schoenfeld, 1992), with the aim of encouraging learner-centredness in education.

Since implementing constructivist approaches in a minimally guided manner causes extreme cognitive load, which is not conducive to learning for novices (Kirschner, Sweller, & Clark, 2006), a carefully scaffolded approach that will support engagement in deep learning, coupled with skilful prompting, is necessary for approaches such as PBL to be successful (Hmelo-Silver, Duncan, & Chinn, 2007). It seems reasonable to assume that it would be possible to create learning experiences that would enable learners from less privileged contexts, such as South African townships, to enjoy these benefits too. However, there is a dearth of literature available on this topic, with most of what is available referring mainly to implementation difficulties and teacher misconceptions regarding problem-based and inquiry-focussed learning processes (Motala, Dieltiens, & Sayed, 2009).

The context of township schools has a direct effect on the resilience of their learners (Mampane & Bouwer, 2011). Furthermore, impoverished pedagogy, which is mostly evident in township schools, very rarely promotes higher order thinking or quality learning (Hugo, Bertram, Green, & Naidoo, 2008; Hoadley, 2018). Learners in these contexts are, therefore, seldom exposed to problem-based instructional strategies.

1.2 RESEARCH OBJECTIVE AND RESEARCH QUESTIONS

In this research, the main objective was to investigate how South African township learners respond to a carefully scaffolded, problem-based instructional strategy. This topic was investigated so as to shed light on whether it is reasonable to expect learners who likely have limited skill levels (Spaull, 2013; Spaull & Kotze, 2015; Van den Berg, Spaull, Wills, & Kotze, 2016), and who may be used to having low expectations imposed on them (Hobden & Hobden, 2019; Hugo et al., 2008), to undergo the cognitively taxing process of problem-solving. This knowledge will enhance our understanding of the applicability of the benefits of such an instructional strategy to contexts other than Western classrooms, in which they are typically studied.

The research question that guided the collection and analysis of data is the following:

> *How do Grade 8 and 9 township learners respond to a problem-based mathematics extension programme?*

With this question in mind, the study unfolded in accordance with the following subsidiary questions:

a) How do learners engage in the cognitive components of problem-solving?

b) What are learners' affective responses to the programme?

1.3 THEORETICAL REFERENTS

Various terms and concepts are central to this study, and framed the conceptualisation of this programme, the research questions, and the data collection. Section 1.3.1 summarises these concepts, which will be discussed in more detail throughout this dissertation.

1.3.1 Theoretical framework

Carlson and Bloom's (2005) multidimensional problem-solving (MPS) framework was used as the conceptual framework for this study. Carlson and Bloom drew on a large body of literature to conceptualise the MPS framework; these sources ranged from early work on problem-solving (Pólya, 1957), which merely focussed on describing the problem-solving process, to more recent work that identifies the attributes of the problem-solver that contribute to problem-solving success (DeFranco, 1996; Schoenfeld, 1992; Geiger & Galbraith, 1998; Carlson, 1990a; all cited in Carlson and Bloom, 2005). Supplementary studies reveal the influence that various affective domains (e.g. beliefs, attitudes, and emotions) have on the problem-solving process. The integration of all the problem-solving domains resulted in the creation of the comprehensive MPS framework (refer to Figure 3.6 in Section 3.4).

This framework consists of four phases (first column of the MPS in Figure 3.6, Section 3.4), derived from a combination of the work of Pólya, and Garofalo and Lester (as cited in Carlson & Bloom, 2005). These four phases are viewed as consistently occurring in the problem-solving process, and are orientation, planning, executing, and checking. Furthermore, general behaviours (e.g. sense-making, organising) that occur during each phase were identified by Carlson and Bloom (2005) during their close observations of 12 mathematicians participating in problem-solving. The framework also characterises various problem-solving attributes (resources, affect, heuristics, and monitoring), taken from Schoenfeld (1992), and describes their roles and significance during each of the four problem-solving phases. Some key concepts will be briefly discussed below.

Resources relating to problem-solving are described as “formal and informal knowledge” about the specific content domain, which includes facts, algorithmic procedures, routine procedures, and definitions relating to a specific topic (Carlson & Bloom, 2005, p. 48). The term *monitoring* refers to the problem-solver’s metacognitive reflection regarding the effectiveness of the problem-solving process (Schoenfeld, 1992, p. 355). The *heuristics* dimension of the framework describes specific procedures and approaches regarding the problem-solving activity, e.g. constructing a diagram or attempting a parallel problem (Carlson & Bloom, 2005). *Affect* encompasses the beliefs, attitudes, and emotions of the problem-solver, which have a powerful effect on the behaviour of the problem-solver (Schoenfeld, 1992).

1.3.2 Pedagogical approach

The pedagogical approach followed during the mathematics extension programme that formed part of this research study can be described using Stott's (2008) ladder approach instructional model (see Figure 1.1). This model emerged from her research into the promotion of critical thinking. I consider this instructional model to be consistent with the recommendations for teaching practice that arise in relation to the theoretical discussions presented in Chapters 2 and 3, as summarised below. The essence of this ladder approach should serve as reference when reading all the chapters in this dissertation.

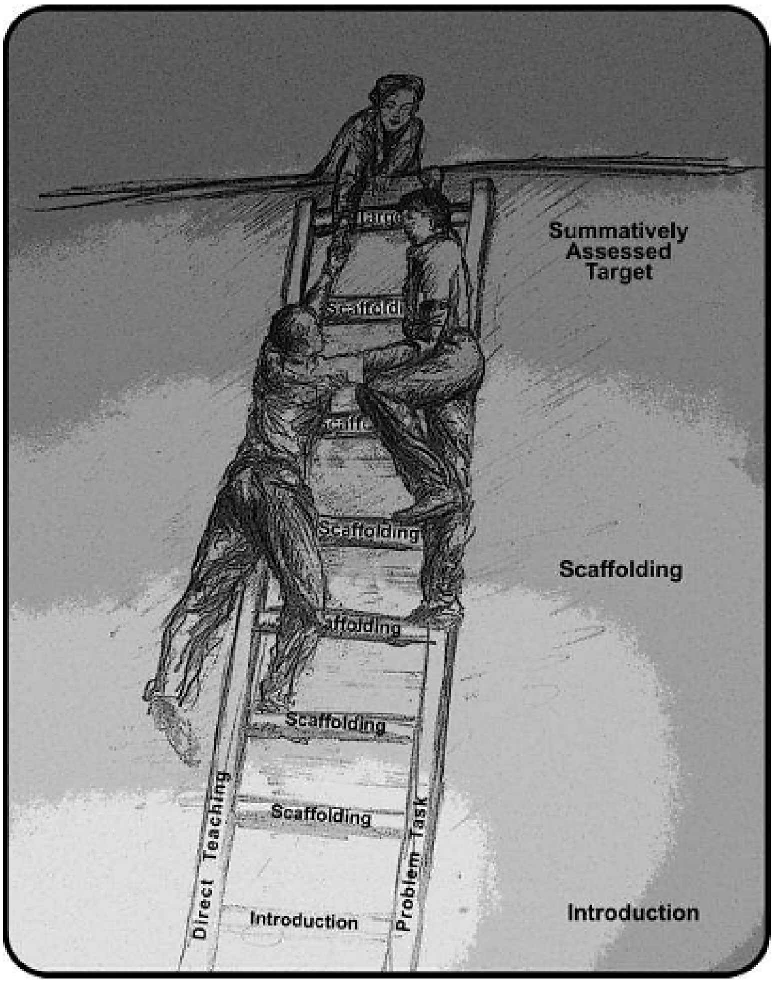

Figure 1.1: The ladder approach

Source: Stott (2008, p. 220)

In reference to this ladder metaphor (refer to Figure 1.1), the two sides of the ladder represent the two main components of the model: (1) direct teaching by the teacher, and (2) individual learner engagement in the problem-solving task. These components run parallel throughout this study's mathematics extension programmes' problem-solving process. The amount of guidance, or engagement in the two opposite sides of the ladder, shifts according to (1) the phase of the problem-solving where learners find themselves, and (2) the amount of scaffolding needed from the teacher, which differs according to the cognitive capabilities of each individual learner. This shift in engagement, according to Stott (2008), with the opposite sides of the ladder relates to the discussion in Chapter 2, which states that the teacher's pedagogical approach moves across the instructivist-constructivist-continuum in response to the needs of learners. Although the shift in classroom instruction alternates between either direct teaching by the teacher, and the learner's engagement in the problem-solving task, the influence of both of these main components must be present throughout the learning sequence.

The two sides of the ladder are held together by the rungs. The rungs represent the scaffolding tools that connect the two components. As the learner improves in his/her critical thinking and problem-solving skills, with assistance, he/she metaphorically climbs up the ladder. Throughout the climbing process, assistance, by both the teacher, who pulls from the top (see Figure 1.1), and peers, who generally assist on a similar thinking level, is required for problem-solving success.

1.4 DESCRIPTION OF THE PROGRAMME

The holiday extension programme that was central to the research study and which is reported on in this dissertation, took place in July 2016 and involved 27 township-based Grade 8 and 9 learners, who participated in a mathematics, science and English programme for five days, six hours per day. The learners were bussed in to the University of the Free State from schools in Motheo District (Botshabelo and Bloemfontein township schools) every day to attend the holiday programme. They received refreshments every day, and returned home with homework, to help them prepare for the following day's work.

The context used throughout the programme was engagement in an investigation into factors that affect the maximum height and range of a water rocket. The role of the mathematics section of the programme was to enable learners to take appropriate measurements to calculate the rocket's height during flight. During the first three days of the programme, learners engaged in activities to prepare them to use trigonometry to measure and calculate the maximum height of the water rocket from observing its flight. On the fourth day, the water rocket was launched, and data were collected and analysed to complete the mathematical calculations. On the fifth day, the learners wrote reports on the entire investigation process and presented the entire week's work to their peers.

I assisted the learners to construct their own mathematics knowledge, through problem-based strategies, which focussed on guided scaffolding and skilful prompting (Anderson, 2002). The essence of the mathematics programme revolved around basic trigonometric ratios to which, according to the Curriculum and Assessment Policy Statement (CAPS) (Department of Basic Education, 2011a, 2011b) for mathematics, learners are introduced only in Grade 10. The programme will be described in greater detail in Chapter 5.

1.5 RESEARCH APPROACH

1.5.1 Research aim

In this research, I aimed to investigate the response of Grades 8 and 9 township learners to the problem-based mathematics extension programme briefly described in Section 1.4. The goal of the investigation was to add to the paucity of literature regarding problem-based mathematics approaches in the South African context, particularly those involving learners from poor communities. My aim in doing this was to identify and encourage the use of relevant, implementable strategies for this context.

1.5.2 Research design, data collection and sampling

A case study design, informed by the pragmatic paradigm and the framework for integrated methodology, was used (Plowright, 2011).The focus of this case study was how Grade 8–9 learners respond while being actively engaged in mathematics problem-solving activities in the described extension programme. This paradigm and framework are flexible and responsive to the complexities of real classroom practice.

Non-probability purposive sampling was applied to select learners for this study (Plowright, 2011, pp. 42-43), since this method addressed the needs of this particular study best. The criteria of sampling were that (1) the learners attended Grades 8 or 9 in township schools near Bloemfontein, (2) the learners had voluntarily produced a project for the Expo for Young Scientists competition over the past three months, from which interest in mathematics and science was assumed, and (3) the learners wanted to participate in the programme during a week of their winter holidays.

A number of qualitative methods, which will be elaborated on further in Chapter 4, such as "observer as participant", "full observer", "written questions answered face-to-face", and "interpretational use of artefacts" were used (Plowright, 2011, pp. 66, 67, 78, 95). Being a "participant observer" (Plowright, 2011, p. 67) allowed me, as the researcher, to answer the research questions with deeper understanding, since I was intimately involved in the process of investigation. A variety of data sources, namely pre- and post-tests, audio- and video- recorded lessons, written work and audio-recorded focus group discussions, were used to enhance validity and reliability, as discussed below.

1.5.3 Data analysis

Data analysis was inductive, iterative and pragmatic, and guided by the research questions.

Content analysis was performed – I used existing codes from Carlson and Bloom's (2005) MPS framework to code the various data sources, using NVivo software. I used these existing codes to form an idea of the relative prevalence of each of the components of this framework.

During *thematic analysis,* I used the coded data to inductively derive themes, which assisted me to answer the research questions by presenting five assertions in answer to the research questions, as presented in Chapter 6.

1.5.4 Validity and reliability

Triangulation and peer examination were used to strengthen internal validity and reliability of this study. Three sets of data sources were coded by the researcher and a peer. These six sets (three different sources, coded separately by two people) were compared by running a coding comparison query in the software program NVivo, from which a Cohen's Kappa coefficient and percentage agreement results was obtained. Rich descriptions of the programme and the learners' responses to the programme will be given in Chapters 5 and 6, to enable readers to judge the validity of the claims to knowledge that the study makes, and to form what Stake (1994) refers to as naturalised generalisations. Naturalised generalisation refers to the reader being able to extract elements from the case that are relevant to their particular circumstances.

1.5.5 Limitations

Since non-probability purposive sampling was used, the sample is not representative of all Grades 8 and 9 learners from South African townships. The learners in the sample tended to be relatively strong academically, and internally motivated. In addition, the programme was presented in an environment that had ample resources and support, which is very different to the classroom contexts that these learners are usually exposed to. However, the aim was to study township learners' responses to an exemplary PBL programme, rather than to investigate whether PBL is viable in typical, impoverished contexts. Further research could explore the extent to which this particular study could be generalisable to the broader population and more typical township contexts.

1.6 ETHICAL STANDARDS

Sensitivity to ethical standards and the research process was clearly communicated to all learners before they engaged with the programme, with emphasis on the voluntary nature of their participation in the study and the confidentiality of their responses (Strydom, 2005; Mertens 2010).

1.7 THESIS OVERVIEW

This chapter (Introduction) provided an overview of the research study. In Chapter 2 (Literature review), I will elaborate on the literature and empirical findings relevant to this study. In Chapter 3 (Theoretical framework), I will continue discussing the theoretical framework that frames this study. In Chapter 4 (Research design and methodology) I will discuss the research design and methodology of this study, by elaborating on the data corpus, and the validity and reliability of the data. Chapter 5 (Description of programme) will provide a rich description of the extension programme, and Chapter 6 (Interpretation and analysis) will contain analytical and interpretative discussions on the cognitive and affective responses of the township learners to the programme. In Chapter 7 (Summary and implications for research and practice) I will conclude with a summary of the findings and provide suggestions regarding the limitations and implications of this study.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter will start with a discussion of the need for problem-based learning approaches in education. Then, I will sketch an overview of the context of South African township schools, by explaining the quintile system, the bimodal educational system that is prevalent in South Africa, and the type of pedagogy predominant in these contexts. I will elaborate on the South African curriculum by summarising the history of the different curricula that South Africa has seen, and then continue to discuss the dire situation of mathematics education in South Africa. The role that problem-based, constructivist and instructivist approaches play in the current curriculum will be addressed subsequently. I will conclude the chapter with empirical references to similar studies through a discussion on the gap in literature that this study hopes to address.

2.2 WHY THE NEED FOR PROBLEM-BASED LEARNING?

Effective teaching approaches are needed to promote meaningful and long-lasting learning (Orton & Frobisher, 1996; Ridlon, 2009; Savery, 2006). Teaching approaches that are effective are usually aimed at enabling the learner to master specific skills and knowledge to accomplish learning goals (Tambara, 2015). Traditionally, effective approaches to mathematics have tended to be teacher-centred, and often require rote learning and routine drill procedures (Voskoglou, 2008; Walker et al., 2015; Boaler, 2013). Savery (2006) maintains that traditional approaches rarely lead to in-depth mastery of a topic or skill, and that learner-centred approaches, such as PBL, are more effective for promoting meaningful learning.

Educationists and learning theorists who favour problem-based approaches are in agreement that teaching methods that are used in classrooms should foster skills that encourage learners to think critically (Lai, 2011), regulate their own learning (Loyens, Magda & Rikers, 2008), and shift the boundaries of their own capabilities (Boaler, 2013). Furthermore, effective teaching should entail the teacher providing the learners with quality opportunities to engage in practical experience, critical thinking, exploration, and real-world PBL (Dweck, 2015; Haylock, 2010; Ingavarson & Rowe, 2008).

PBL addresses the desperate need to nurture learners so that they are ready for the demands of the future, specifically the demands of the fourth industrial revolution. The fourth industrial revolution, or industry 4.0, refers to a "change of the technological, economic, and social systems in industry" (Dombrowski & Wagner, 2014, p. 1). It refers to, among other aspects, artificial intelligence, genetic editing, automation, mobile supercomputing, and intelligent robots. This revolution offers incredible possibilities and solutions and is expected to create opportunities for jobs that have not been invented yet.

Exciting as this revolution might seem, the threat it poses to illiterate people is extreme. 'Illiterate' does not refer to people who cannot read or write, but rather, to people who cannot "learn, unlearn, and relearn" (Alvin Toffler, cited by Brown-Martin, 2014, p. 1). The skill to *learn, unlearn, and relearn* requires a growth mindset (Dweck, 2006; Boaler, 2013), and can be linked to the core skills that are listed in a detailed report of a survey that the World Economic Forum conducted in 2015. The aim of this report was to investigate the core skills needed for industry 4.0. The survey found that 36% of all employment opportunities across all industries require complex problem-solving as one of their core skills (World Economic Forum, 2016, p. 22). The list of core skills identified is given in Table 2.1.

Table 2.1: Change in demand for core work-related skills, 2015-2020, all industries

Skill	Scale of skills demand in 2020
Cognitive abilities	15%
Systems skills	17%
Complex problem solving	36%
Content skills	10%
Process skills	18%
Social skills	19%
Resource management skills	13%

Skill	Scale of skills demand in 2020
Technical skills	12%
Physical abilities	4%

Source: World Economic Forum (2016, p. 22)

Education, specifically the role the teacher fulfils, is at the heart of preparing current and future generations to thrive in industry 4.0. Unfortunately, it is almost impossible to embrace industry 4.0 in the majority of South African schools, since the digital revolution (3IR) has not been incorporated completely yet (Roberts, 2015). Furthermore, the majority of South African schools have not integrated technology into their teaching and learning approaches, due to, among other limitations, limited funding (Centre for Excellence in Financial Services, 2017). It is hoped that integrating different curriculum subjects and presenting learners with real-world problems that relate to their everyday experiences, will nurture the skills required to make a meaningful contribution to preparing learners for industry 4.0 (Brown-Martin, 2017).

Even though South African township schools present unfavourable contexts for education change, human rights issues dictate that educationists should attempt to improve the chances of future success of learners in such contexts (Schweisfurth, 2013) Learners need to be guided, through problem-based approaches, to find value in what they learn at school, which should not be isolated from their everyday experiences (Hobden, 2000).

2.3 CONTEXT OF SOUTH AFRICAN TOWNSHIP SCHOOLS

In the following section I define townships, elaborate on the quintile system of South Africa, and distinguish between township and urban schools – a distinction which is relevant to understanding South Africa's bimodal education system. This section will conclude by describing the type of education that is dominant in the poverty-stricken areas of South Africa.

2.3.1 Defining township schools

According to Donaldson (2014:108), townships are defined as "areas that were designated under apartheid legislation for exclusive occupation by people classified as blacks, coloureds, and Indians. Townships have a unique and distinct history, which has had a direct impact on the socioeconomic status of these areas and how people perceive and operate within them."

Before South Africa's political transformation of 1994, the education system was racially segregated regarding financial support, curriculum, and education department (Molefe & Brodie, 2010; Ramnarain & Schuster, 2014). These separations discriminated against black learners, specifically regarding education. Furthermore, only 20% of the teachers who were assigned to teach maths or science at the so-called 'black schools' had a suitable qualification (Ramnarain & Schuster, 2014, citing Murphy).

2.3.2 Defining townships within the South African quintile system

The quintile system was introduced in 1998, as part of the National Norms and Standards policy, to assist learners with paying school fees. Government schools in South Africa are divided into five categories, based on a school's catchment area's unemployment and illiteracy rates, as well as the socio-economic status of the learners' parents. Quintile 1 schools serve the socioeconomically most disadvantaged communities, and quintile 5 schools serve the richest communities. Table 2.2 presents data that was issued in the *Government Gazette* on 28 April 2017 by Angelina M. Motshekga, the minister of Basic Education, and shows the distribution of South Africa's nine provinces' quintile percentages.

Table 2.2: National poverty distribution for 2017

National Poverty Distribution Table						
	Quintiles					
%	1 poorest	2	3	4	5	Total
EC	27.3	24.7	19.6	17	11.4	100%
FS	20.5	20.9	22.4	20.8	15.4	100%
GP	14.1	14.7	17.9	21.9	31.4	100%
KZN	22.1	23.2	20.2	18.7	15.8	100%
LP	28.2	24.6	24.2	14.9	8	100%
MP	23.1	24.1	21.5	17.7	13.5	100%
NC	21.5	19.3	20.7	21.4	17.1	100%
NW	25.6	22.3	20.8	17.6	13.7	100%
WC	8.6	13.3	18.4	28	31.7	100%
SA	20	20	20	20	20	100%

Source: Department of Basic Education (2017, p. 5)

Quintile 1, 2, and 3 schools are usually found in South Africa's township and rural residential areas, which are mostly characterised by being underdeveloped racially segregated urban areas, where violence, crime, and poverty are typical (Mampane & Bouwer, 2011). These low-quintile schools, or township or rural schools, might not have basic facilities, are often underfunded, ill-resourced, and have overcrowded classrooms (Mampane & Bouwer, 2011). Teachers at these schools are generally poorly qualified, or completely unqualified in maths and science-related subjects (Fish, Allie, Pelaez, & Anderson, 2017).

The schools that fall in quintiles 4 and 5 are usually found in and around urban areas. Urban schools are schools that are near large city and or town centres. These schools are better equipped, the classes have a small teacher-to-student ratio, and teachers are generally better qualified. Learners who attend these schools have good English language skills, have access to the internet and learning resources, and have middle-class socio-economic backgrounds (Fish et al., 2017).

According to Spaull (2013), South Africa has a bimodal education system, which means that learners in quintile 5 (and some quintile 4) schools, which equates to 25% of the population, can compete in the international realm, while the remaining 75% of learners, who attend quintile 1, 2, and 3 schools, often fare extremely badly in international benchmarking tests (Stott, 2018, citing Spaull & Kotze and Pretorius & Spaull). Section 2.3.2 will elaborate on the bimodal education system.

2.3.3 Bimodal education system

The bimodal distribution in the South African education system and the performance of learners are consistent throughout numerous independent surveys and benchmarking tests (Spaull, 2013). The majority of all South African learners who attend quintile 1, 2, and 3 schools perform, on average, very poorly on independent, national and international tests, in comparison to the minority from more functional schools (quintile 4 and 5), who perform much better (Fleisch, 2008). This consistent performance distribution is illustrated in the following three graphs, where the population is divided into 1) four wealth quartiles (Figure 2.1 – Spaull (2013) on SACMEQ III), 2) school language (Figure 2.2 – PIRLS), and 3) former department (Figure 2.3 – NSES) (Spaull, 2013, pp. 4, 5).

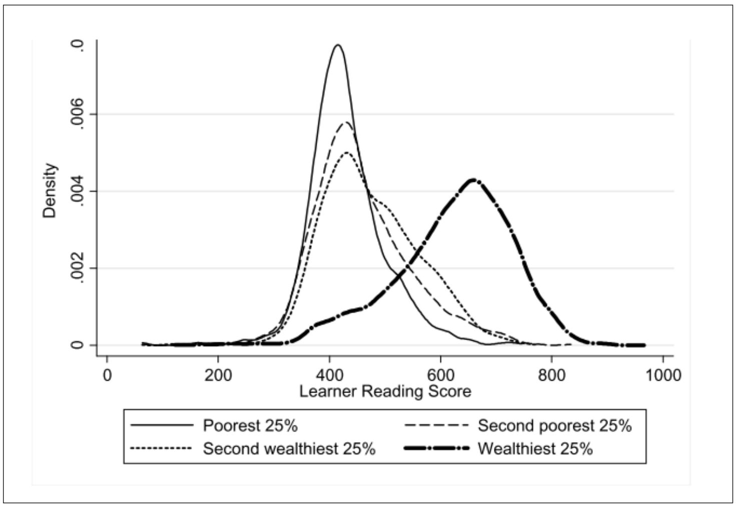

Figure 2.1: Distribution of Grade 6 reading performance by school wealth quartile

Source: Spaull's (2013, p. 4) calculations for the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) III of 2007

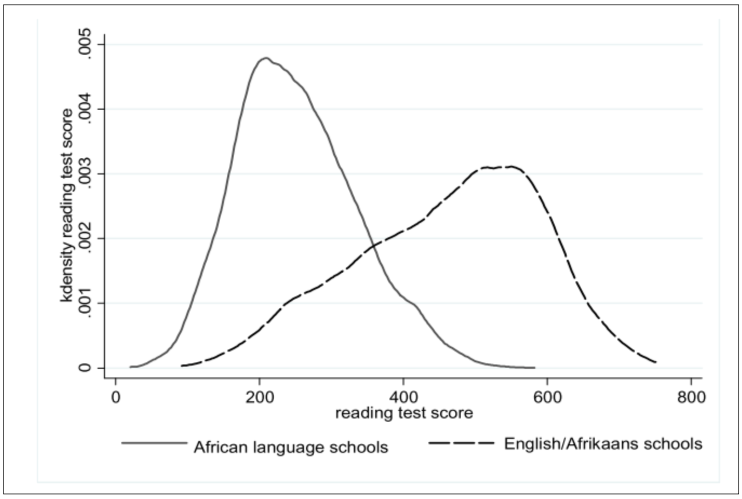

Figure 2.2: Distribution of Grade 5 literacy achievement by language of school

Source: Spaull (2013, p. 5) citing Shepherd (2011) on the 2006 Progress in International Reading Literacy Study (PIRLS) data.

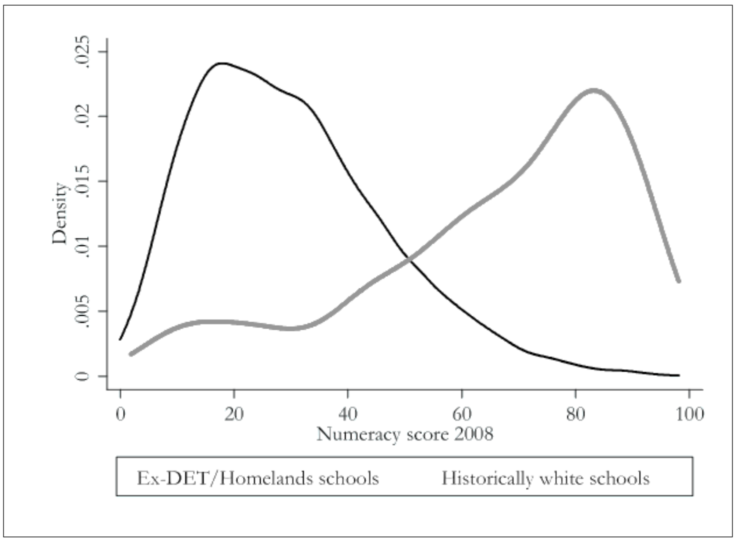

Figure 2.3: Distribution of Grade 4 numeracy achievement by historical education department

Source: Spaull (2013, p. 5) citing Taylor (2011) on the 2007/8/9 National School Effectiveness Study (NSES).

From the graphs in Figures 2.1–2.3, it is clear that a dual system permeates South Africa's education system, since two normal curves are prevalent in three aspects of learner performance – those relating to reading performance, literacy achievement, and numeracy achievement. A dualistic system, or bimodality, does not only occur within learner performance – it is also evident in other aspects that have a direct influence on learner performance (Ramnarain & Schuster, 2014; Spaull, 2013; Mampane & Bouwer, 2011). Table 2.3 categorises performance, textbooks, school factors, and home background as representative of the four school wealth quartiles. It can be seen in Table 2.3 that learners in wealth quartile 4 schools are more likely to have access to conditions and resources that are conducive to learning, e.g. quality teachers with minimal absenteeism, educated parents, access to textbooks and computers, than poorer learners. It is, therefore, unsurprising that there is a considerable variation in learner achievement variables, such as reading and mathematics achievement, of quartiles 4 and 1–3 learners.

Table 2.3: Distribution of various schooling statistics across school wealth distribution quartiles (Grade 6 – SACMEQ III)

Category	Variable	School Wealth Quartiles 1	2	3	4	Total
Performance	Reading score	430.5	457.8	474.0	623.7	**494.9**
	Mathematics score	450.9	467.1	470.7	593.8	**494.8**
	Proportion functionally illiterate[4]	43.3%	33.3%	25.6%	4.1%	**27.3%**
	Proportion functionally innumerate	56.9%	48.6%	44.8%	8.4%	**40.2%**
	Reading teacher reading score	731.8	738.9	732.9	827.0	**757.7**
	Maths teacher mathematics score	719.6	729.1	751.7	863.5	**763.6**
Textbooks	Has own reading textbook	34.4%	42.3%	38.2%	66.1%	**45.0%**
	Has own mathematics textbook	27.6%	35.8%	32.3%	50.9%	**36.4%**
School factors	Gets homework "Most days of the week"	49.9%	52.1%	46.1%	75.8%	**56.1%**
	Self-reported teacher absenteeism (days)	24.2	22.7	20.1	11.6	**19.7**
	Repeated at least 2 grades	10.9%	9.3%	10.3%	1.8%	**8.1%**
	Pupil-Teacher-Ratio	36.3	34.8	35.5	30.5	**34.3**
	School in urban area	5.5%	21.4%	31.2%	73.3%	**31.9%**
	Student very old (14y+)	23.7%	20.1%	14.0%	2.0%	**15.3%**
Home background	Speaks English at home 'Always'	5.6%	7.4%	9.2%	39.5%	**15.3%**
	Student has used a PC before	11.8%	39.9%	51.4%	94.9%	**47.8%**
	More than 10 books at home	17.3%	23.0%	30.8%	67.2%	**34.1%**
	At least one parent has matric	29.9%	40.6%	49.3%	77.2%	**48.5%**
	At least one parent has a degree	4.7%	7.8%	10.7%	28.7%	**12.8%**

4: By this definition, a functionally illiterate learner cannot read a short and simple text and extract meaning, while a functionally innumerate learner cannot translate graphical information into fractions or interpret everyday measurements into units.

Source: Spaull (2013, p. 7)

Spaull (2013) uses these large-scale education statistics and analyses to determine if current curriculum, teaching practices, and interventions are effective. In addition to the efficacy of educational practices, these statistics report on the prevalence of two distinctly different educational systems.

From the discussion above, it is clear that there are two quite different education systems in South Africa: one is well functioning, and the other is in a dire state, and each needs different resources, support, and pedagogical and methodological approaches. Learners in township schools (quintiles 1–3 or wealth quartile 1 and 2, and sometimes 3) respond differently to interventions and pedagogical approaches, compared to more fortunate learners in functioning schools (quintiles 4–5, and wealth quartile 4). These responses by learners in township schools need to be uniquely orchestrated if pedagogies are to be applicable and effective in these poverty-stricken areas (Van der Berg et al., 2011).

2.3.4 Pedagogy in poverty

Hugo (2020, p. 118) describes the pedagogy that most township schoolteachers use as a pedagogy that is "stripped of any power and any effect", due to the nature of this poor, chant-like, instructivistic pedagogy that is common in poor communities. An example of a transcript, taken from Ursula Hoadley's book, *Pedagogy in Poverty*, shows typical dialogue between teachers and learners in these schools (Hugo citing Hoadley, 2020, pp. 118–119):

> *Teacher: Listen, on page 63, how a tree lives and grows. It says that . . . what does it say people? How a tree lives and grows. What does it say?*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: What does it say?*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: What does it say?*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: Umthi uphula njani ukhube nyani. Say it in Xhosa.*
>
> *Learners: Umthi uphula njani ukhube nyani.*
>
> *Teacher: Say it again in English, how a tree lives and grows.*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: There are those who are not talking. I don't hear you. How a tree lives and grows.*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: I don't hear some [of you], how a tree lives and grows.*
>
> *Learners: How a tree lives and grows.*
>
> *Teacher: I don't hear you. How a tree lives and grows.*
>
> *(Grade 3 classroom, Khayelitsha, South Africa, 2003)*

The dialogue example above represents a common scenario in township schools, where learners are positioned in a "passive reactive role" during lessons (Hoadley & Muller, 2019,

p. 111). The low level of cognitive engagement by learners, which is commonly found in township classrooms, is unfavourable for conceptualisation and deeper learning (Schoenfeld, 2016). Hoadley (2018, p. 1) describes this type of pedagogy, as follows:

> *no distinctions or judgements are made of individual learners and their activity and initiative and spontaneity on the part of the students is closed down. Pedagogy is a form of marking time, a rite of rote.*

Learners in township schools are mostly subjected to poor pedagogies, as described in the extract above, and are almost never exposed to learner-centred approaches, such as PBL (Stott, 2018; Hugo, 2020). Teachers who implement these pedagogies do not necessarily know what excellence in teaching looks like, since it is likely that the majority of teachers who teach in township schools only have reference to their own impoverished schooling, their current teaching contexts might not be conducive to learning, and they might be surrounded by colleagues who use similar pedagogical approaches (Hobden & Hobden, 2019). Hoadley and Muller (2019), in their contribution to the book *South African Schooling: The Enigma of Inequality*, concede that a successful pedagogy that is appropriate for the majority of South African schooling contexts has not yet been identified, although they emphasise that teachers' cognitive horizons need to be shifted to pedagogies that promote meaningful learning in poorer contexts.

2.4 CURRICULUM IN SOUTH AFRICA

The South African curriculum has undergone significant changes over the past 25 years. Education change was encouraged by the drastic changes that the political realm underwent. The next section will elaborate on the South African curriculum from past to present, the stance of mathematics education in South Africa, and, finally, the balance between constructivist and instructivist approaches in the township context.

2.4.1 Past to present

After 1994, the Department of Education implemented three initiatives to stimulate reform of the national curriculum. These initiatives included abolishing the apartheid curriculum, implementing continuous assessment in all subjects and schools, and implementing outcomes-based education (OBE) (Jansen, 1998). Curriculum 2005 served as the vehicle through which OBE was implemented (Cross et al., 2002) in 1998, and it was, in turn, revised to form the National Curriculum Statement, and later a Revised National Curriculum Statement. The current South African curriculum, the CAPS, which was implemented in 2012, replaced the Revised National Curriculum Statement.

The OBE curriculum entailed a constructivist approach to teaching (Stott, 2018), with an emphasis on mastering outcomes in each learning topic; this approach replaced the emphasis on content covered over a set time (Jansen, 1998). These outcomes were explicit in directing assessments towards stipulated goals. Exit outcomes, such as higher-order competencies, which includes PBL and critical thinking, replaced subject-matter mastery (Armstrong, 1999) as the main focus. All involved in the teaching community of South Africa were obliged to attend workshops, so that they were ready for the first phase of implementation. These workshops caused confusion and an aversion to OBE and its pedagogies (Stott, 2018, citing Chrisholm).

Among the points of criticism relating to OBE, as reported by Rooth (2005, citing Jansen 1999), was that pedagogical changes were too drastic for the majority of South African schools, it increased teachers' administrative workload, leaving little time for lesson preparation, assessments remained basically the same, even though the OBE approach was completely different to previous curricula, and important curriculum content was trivialised. Furthermore, OBE was difficult to implement in schools where resources were scarce, and teachers underqualified (Taylor & Vinjevold, 1999). However, some educationalists argue that OBE had many positive aspects, such as encouraging teachers to start thinking creatively, learners being more engaged in lessons, and equity being addressed (Rooth, 2005).

Curriculum 2005 was guided by the principles of OBE, with the focus on a more progressive pedagogy (Cross et al., 2002). Curriculum 2005 enforced a clean break from the education system of the apartheid era. Reactions to Curriculum 2005 ranged from acceptance to condemnation. The envisioned results of Curriculum 2005 were not achieved, and a review committee proposed the introduction of the National Curriculum Statement, which was implemented in 2002.

The National Curriculum Statement's main focus was on teaching and learning outcomes, instead of subject-specific content. However, because learners performed very poorly in national and international benchmarking tests, another, revised curriculum was introduced in 2012, namely the CAPS (Mnguni, 2019).

A shift in pedagogical approaches occurred from the National Curriculum Statement, which proposed that teachers take a learner-centred approach, to the CAPS, which is highly prescriptive and content-heavy, and encourages a teacher-centred pedagogy (Hoadley & Muller, 2019). The writers of the CAPS document clearly envisaged this teacher-centred pedagogy resulting in meaningful learning, as evidenced by their appeal to learners to take "an active and critical approach to learning, rather than rote and uncritical learning of given truths" (Department of Basic Education, 2011a, p. 4), and it was expected to be implemented by all teachers.

Over the course of about 25 years, South Africa has experienced four drastic changes in curriculum, but does not seem to be able to address the challenges that the majority of South African schools face (Hoadley, 2018), although there are signs of modest progress having been made, when viewed over time (Van der Berg & Gustafsson, 2019). The performance of South Africa's learners, and the quality and effectiveness of the South African educational system, is still regarded as being in crisis (Hoadley & Muller, 2019).

2.4.2 Mathematics education in South Africa

South Africa's democratic government has, since 1994, committed itself to ensuring that all children are provided with basic education, in order to achieve the Millennium Development Goals (United Nations, 2013). Twelve years after these Millennium Development Goals were set, South Africa, according to the World Economic Forum, was ranked 139th out of 139 countries for mathematics and science education quality, and 137th out of 139 countries with regard to education system quality (Baller et al., 2016). The Trends in Mathematics and Science Studies (TIMSS) results confirm this bleak state of mathematics education in South Africa, by ranking South Africa at the bottom of the list (Gurney-Read, 2016). The belief among researchers regarding the dire state of mathematics education in South Africa was strengthened by the fact that, since 1995, South Africa has performed poorly in both national and international assessments of mathematics achievement (Spaull & Kotze, 2015).

To address the issue of improved mathematics learning, both national and international policies have been initiated which focus on learner-centred and problem-based instructional practices (Department of Education, 2007; National Research Council, 2000). These approaches have various labels, including problem-based, discovery, inquiry, experiential and constructivist learning (Bruner, 1961; Jonassen, 1991; Creswell, 2007; Schoenfeld, 2016). Kirschner et al. (2006, p. 75) categorise all these types as minimally guided approaches and argue that such instruction is detrimental to effective learning, since minimal guidance discounts the limitations imposed by the small capacity of working memory in human cognitive architecture. They maintain that the "advantage of guidance begins to recede only when learners have sufficiently high prior knowledge to provide internal guidance".

It is unlikely that authors, such as Kirschner, would consider most South African mathematics learners as having this minimum amount of prior knowledge. This is hardly surprising, given the fact that most South African schools are situated in high-poverty areas (townships) with limited resources, low expectations posed on learners, and relatively poor teaching quality (Mampane & Bouwer, 2011). The heavy content focus of the current CAPS curriculum (Department of Basic Education, 2011a) serves as an additional cognitive obstacle to the successful implementation of problem-based teaching strategies (which will be further elaborated on in the next chapter). On the other hand, Hmelo-Silver et al. (2007) argue that PBL need not be minimally guided instructional practice but can also be a carefully scaffolded approach (refer to Chapter 3 for more detail) that focusses on deeper learning.

As mentioned in the previous section, in spite of South Arica's curriculum reform to focus on more learner-centred approaches (Umalusi, 2014), the culture of teacher-centeredness is still very strong in township schools (Tambara, 2015). This culture makes the application of problem-based pedagogies almost impossible.

2.4.3 Balance between constructivist and instructivist approaches

The constructivism-instructivism debate divides approaches to teaching and learning into two distinctly different camps, especially concerning approaches to PBL. To elaborate further on my own view concerning instructional practice, it is, first, necessary to define the relevant terms.

Constructivism is based on the philosophy of pragmatism (Dewey, 1916) and has characteristics of knowledge growth and self-regulated, active and lifelong learning, and contradicts behaviourist approaches, where learning is regulated externally through direct instruction (Schwartz et al., 2009; Duffy & Jonassen, 1992). Proponents of constructivism reject the view that information is absorbed throughout, and assert, instead, that knowledge is constructed (Von Glasersfeld, 1998). Constructivists argue that, for learners to enhance the depth and retention period of their learning, teacher feedback should be limited and cognitive load (which is discussed further in Chapter 3) should be enhanced (Stott, 2018, citing Wise & O'Neill). Constructivist approaches refer to learning taking place within authentic contexts through, for example, PBL (Savery, 2006).

Instructivism emphasises repetition, memorisation, absorption of information, and drill exercises to embed knowledge (Porcaro, 2011). Instructivists promote the provision of guidance and continuous feedback to reduce the cognitive demand placed on the learner (Stott, 2008, citing Klahr). Teacher-centred approaches are central to the instructivist pedagogy, in contrast to constructivists' learner-centred approaches (Onyesolu et al., 2013).

To reduce all learning theories to either constructivist or instructivist categories is simplistic, as each theory is complex. Porcaro (2011) deems it helpful to visualise the two theories on opposite poles of the continuum of educational practice. Depending on the learning context and required outcomes, the focus of instruction should move accordingly across this continuum (Schweisfurth, 2013).

Ramnarain and Schuster (2014) refer to Ausubel's theoretical framework for learning and instruction to view learning in relation to instructional approaches. The vertical axis of Figure 2.4 represents the *continuum* of instruction, where reception learning refers to direct instruction, which is favoured by instructivists, and discovery learning refers to inquiry instruction, which is favoured by constructivists. The nature of learning, ranging from rote to meaningful learning, is represented on the horizontal axis. PBL pedagogy is aimed at promoting meaningful learning (which will be discussed further in Chapter 3) (Malan, Ndlovu, & Engelbrecht, 2014), which falls in quadrants I and IV, and, consequently, rote learning falls in quadrants II and III. Thus, Ausubel's representation (in Figure 2.4) stands in agreement with Porcaro (2011) and Ramnarain and Schuster (2014), who argue that meaningful learning can take place in both instructivist and constructivist approaches. Furthermore, discovery learning can be effective if it is orchestrated well, and the quality of feedback and assistance given by the teacher is a strong determinant of success (Hmelo-Silver et al., 2007). However, discovery learning (discussed further in Chapter 3) can also have a negative effect on the learner if too little guidance is provided in an unstructured learning environment (Kirschner et al., 2006; Ramnarain & Schuster, 2014).

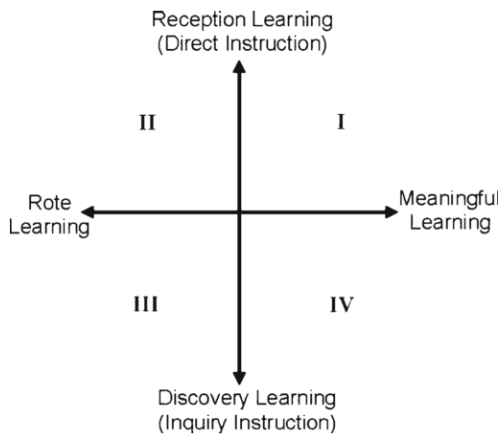

Figure 2.4: Ausubel's representation of types of instruction and nature of learning

Source: Ramnarain and Schuster (2014, p. 632)

Stott (2018) suggests that instructivist pedagogical approaches are more appropriate for the majority of South African low-quintile schools, because of the prodigious obstacles that have to be overcome in these contexts. That said, Stott (2018) does not completely disregard the appropriateness of a constructivist approach in township schools, if there is enough time to reach the learning outcomes, if the language proficiency and prior knowledge of the learners are sufficient, and if the teacher serves to be an invaluable human resource throughout the learning process.

Therefore, problem-based approaches should not be an either-or choice between constructivism and instructivism, but rather a combination of the two approaches. The combination should be adapted according to the needs of learners in the unique township school context, to assist them to obtain a sense of structure and direction.

2.5 GAP IN THE LITERATURE

A review of the literature shows that, although research into PBL for mathematics has been conducted in South African schools (Nieuwoudt, 2015), very little such research has been performed in township schools (Naroth & Luneta, 2015; Nel & Luneta, 2017; Chirinda & Barmby, 2018). The literature I could find regarding PBL in these township contexts suggests that, although some meaningful learning patterns can be identified throughout the mathematical PBL process (Malan et al., 2014), the negative impact of external factors and learners' contexts (Moloi, 2013), coupled with overcrowded and under-resourced classrooms (Chirinda & Barmby, 2018), have on the efficacy of PBL, cannot be ignored.

Stott (2019) conducted a study on the natural science section of the same programme that the current study investigated. From her findings, she maintains that, although the applicability of an inquiry-focussed approach in township schools appears to be an impractical ideal, a carefully conceptualised holiday inquiry programme proved to be effective for promoting learning among higher-achieving learners. This study extends Stott's earlier work (2019) among higher-achieving township learners attending a holiday extension programme, into the domains of problem-based learning and mathematics learning. She researched these learners' perceptions of mathematics problem-based learning at such a holiday programme. This holiday programme was conceptualised and conducted using the ladder approach teaching strategy developed by Stott (2008) as an outcome of action research she had conducted on her own practise teaching for Grade 10–12 physical sciences in a rural, but highly functional, South African school. Stott proposes the ladder approach as an effective teaching strategy for promoting critical thinking, including problem-based learning, but reminds us that the context of her study was not typical of South African schools, which may limit generalisation. One of the aims of this study was to investigate the suitability of the ladder approach for use in the context in which it was applied here, namely in a week-long holiday extension programme for Grades 8 and 9 township learners who showed an interest in science and mathematics.

2.6 CONCLUSION

In this chapter, I presented arguments relating to the need for problem-based learning, especially in the context of township schools. I elaborated on the context of South African schools, and I explained the quintile system and the bimodal system, which permeate South Arica's education. The emphasis was on the predominant pedagogical approaches in poverty-stricken circumstances, and I provided an example of a dialogue that is often evident in township schools. I discussed curriculum reform, and the current state of mathematics education in South Africa. I discussed the balance required between constructivist and instructivist approaches, particularly in impoverished contexts. I ended with a discussion of existing literature related to the promotion of problem-based learning in the South African context and pointed out the gap that this study intended to address. In Chapter 3, I will unpack the theoretical framework on which this study was conceptualised.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1 INTRODUCTION

In this chapter, I aim to situate this study within the existing understanding of PBL, by providing a literature-based theoretical framework, which was used to approach data analysis and interpretation. I will begin with analogies relating to the way people process information, and I will define PBL. This definition will serve as the basis for my arguments about the necessity of scaffolding when minimally guided approaches to learning are used. I will then move on to discuss the MPS framework, which is central to this study, and served as the theoretical framework for interpreting the observations made.

3.2 HOW DO PEOPLE PROCESS INFORMATION?

My understanding of how people process information is based on Richard Atkinson and Richard Shiffrin's widely accepted information processing model (IPM), which was published in 1968 (Gagné, Yekovich, & Yekovich, 1993; Atkinson & Shiffrin, 2016), and on John Sweller's cognitive load theory (Mayer, 1988; Sweller, 1988; Paas, Renkl, Sweller, Paas, & Renkl , 2003; Mayer, 2009). These two models are often applied together in literature, since cognitive load theory builds on the theory of the IPM. Furthermore, they are relevant to issues regarding minimal guidance, cognitive overload, and scaffolding in relation to learning approaches (Kirschner et al., 2006; Hmelo-Silver et al., 2007), all of which are particularly relevant when higher-order thinking skills are targeted (Stott, 2019). These two models are discussed in Sections 3.2.1 and 3.2.2.

3.2.1 Information processing model

3.2.1.1 **Analogies to the information processing model**

An analogy to describe the IPM likens the functioning of the human cognitive architecture to a computer (Turple, 2016). Information enters the human brain, likened to a computer, through the senses (likened to a mouse or keyboard), then, this information is processed in our working memory (the analogous equivalent being the processor or RAM), where it is saved and later recalled from the section in our brains responsible for long-term memory (likened to the hard drive). Finally, an output is delivered (analogously through the monitor) in response to the initial stimulus that had been received. This basic explanation contradicts behaviouristic notions, where each response is caused by a stimulus, and no processing is required (Skinner, 1953). Refer to Figure 3.1 for a schematic representation of the computer analogy.

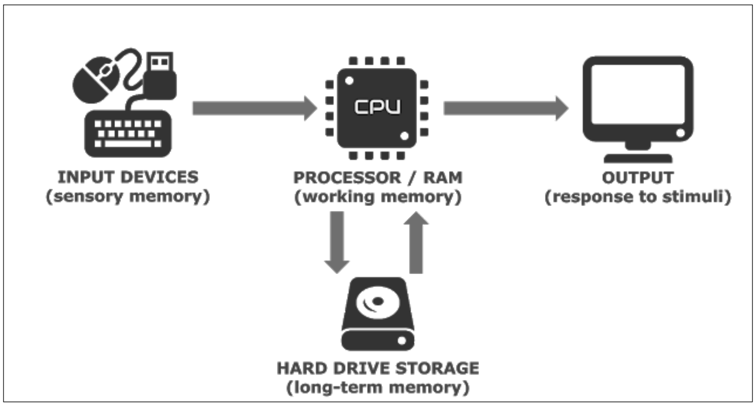

Figure 3.1: IPM computer analogy

Source: Turple (2016)

To describe the different phases of the IPM, I refer to a class situation. When a learner needs to give an explanation in response to a question asked by the teacher, a learner's sensory memory may discard irrelevant information (such as birds chirping in the trees, or other learners searching for books in their suitcases) and keep the focus on the question that the teacher asked. The information from the learner's sensory memory is passed through to their working memory, where it is either processed or rejected. The learner giving the answer to the teacher's question might be an example of an output action. If the learner is able to undergo knowledge chunking through possessing a well-organised knowledge structure within their long-term memory, and due to having routinised skills through drill exercises and practice, then a successful output is probable, since chunking and routinisation reduce cognitive load in working memory (Kirschner, 2002).

3.2.1.2 **Theory behind the information processing model**

There are many information processing theories, namely Craik and Lockhart's (1972) three levels of information processing, Gibson's (1979) bottom-up theory, Rumelhart et al.'s (1986) parallel-distributed processing model, the Baddeley and Hitch model of working memory (Baddeley, 2001) and Driscoll's (2001) cognitive information processing model, that have their roots in the traditional concepts of Atkinson and Shiffrin's IPM (Atkinson & Shiffrin, 2016). Despite disagreements on many levels, these different theories agree that 1) limited capacity exists in working memory, 2) there is some kind of control system in sensory memory that deals with stimuli, and that 3) interaction takes place between stored and new information (Lutz & Huitt, 2003; Turple, 2016). The key element of the IPM is that learning and memory are viewed as "discontinuous and multi-staged" (Lutz & Huitt, 2003, p. 2). The general model for information processing theory identifies three stages of memory: sensory memory, short-term or working memory, and long-term memory (Atkinson & Shiffrin, 2016). For a schematic representation of the IPM, refer to Figure 3.2.

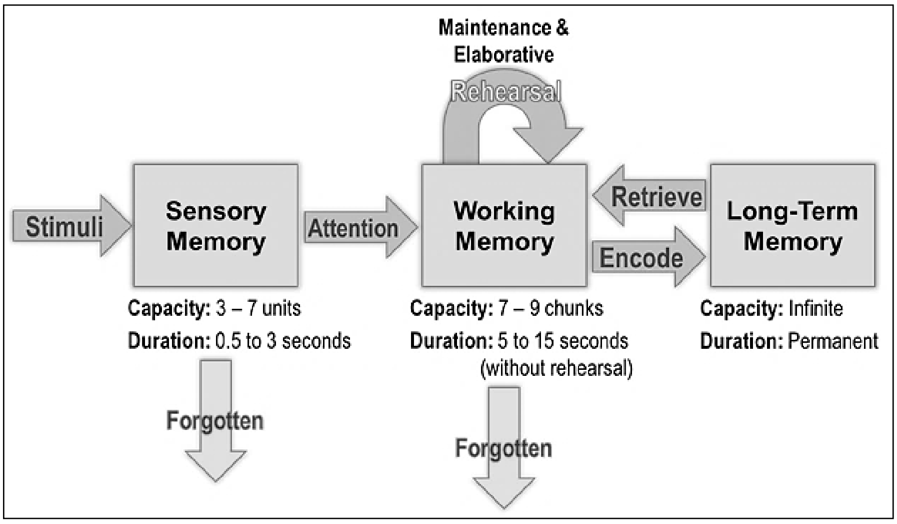

Figure 3.2: Information processing model.

Source: Based on Mayer (1988, p. 15)

Sensory memory serves to gather information through the senses. This information is altered by means of receptor cell activity, into a form that the brain can process. This information (or memories) only lasts for a short time, approximately three seconds, and is usually in the unconscious state. The sensory memory filters all information, and only focuses on the necessary aspects. This stage of the IPM serves to catch the attention and progress the information to working memory (Lutz & Huitt, 2003).

Working or short-term memory consists of three components (Baddeley, 2001): the executive control system, which is responsible for information selection, processing methods, and transfer of information to the long-term memory, and the auditory-loop and visual-spatial check pad, which is responsible for processing auditory and visual information, respectively. Sensory information transmitted to working memory lasts for 15–20 seconds; working memory has a limited capacity of 5–9 chunks of information. A chunk is a collection of similar pieces of information which have been grouped, and by chunking information one can free up space in the working memory (Young, Ten Cate, O'Sullivan, & Irby, 2016). According to the IPM, the limited capacity of the working memory is the limiting factor for learning by novices (Sweller, 1988; Van Merriënboer, Kirschner, & Ketser, 2003), since novice learners cannot chunk as much information as experts (Ockelford, 2002). The quality of processing that takes place in the working memory is affected by the level of cognitive load that a person experiences (Mayer, 1996).

Long-term memory has unlimited capacity, in contrast to the limitations of the working memory. Possessing well-structured and interconnected knowledge structures in long-term memory enables chunking, and, subsequently, efficient use of the working memory space. The efficacy of long-term memory depends on how well information is organised, which is affected by the encoding and retrieval processes (Baddeley, 2001; Lutz & Huitt, 2003). The amount and quality of information retrieved from long-term memory relies on the manner in which information was stored in the first place (Bransford, Sherwood, Hasselbring, Kinzer, & Williams, 2009). Information can only be transferred from working memory to long-term memory if links are made between long-term memory and the working memory (Raaijmakers & Shiffrin, 2003). The limited capacity of working memory, relating to the ability to categorise, move, and store information in knowledge structures (or schemas) in long-term memory, is central to the cognitive load theory, which will be discussed in Section 3.2.2.

Finally, the sequential IPM model, though influential in the field of information processing, is criticised for reducing the complexities of the human cognitive architecture to being too linear, and for making unjustified assumptions, such as "consciousness which operates at the highest theoretical level", about the processing aspects of consciousness (Hardcastle, 1995:105). Nevertheless, the manner in which IPM holistically describes the processing of information and the role that short-term memory, working memory, and long-term memory play, coupled with cognitive load theory, are considered appropriate for forming a framework for interpreting learners' responses to a PBL intervention referred to in this study.

3.2.2 Cognitive load theory

Cognitive load theory is based on the concept that a person has, at a given time, a limited amount of figurative space available in their working memory, and when they undertake a thought process or learn something new, those thoughts and ideas take up a certain amount of figurative space in their working memory (Kirschner, Sweller, Kirschner, & Zambrano, 2018). According to Miller (1955), in his classic paper, *The Magical Number Seven, Plus or Minus Two Some Limits on Our Capacity for Processing Information,* this limited amount of space that is available is known to only accommodate seven plus or minus two units of information in working memory.

Therefore, the essence of cognitive load theory is connecting the limited capacity of working memory to an infinite capacity of long-term memory (Baddeley, as cited in Kirschner, 2002). The capacity of working memory can be increased through schema automation and creation, by imposing tolerable levels of cognitive load (Kirschner, 2002), which are useful for managing demanding tasks during the problem-based learning process (Sweller, 1988). About twenty years after the cognitive load theory was introduced, the concept of *chunking*, also referred to as schema construction, was coined by William G. Chase and Herbert A. Simon, who describe it as a technique for isolating and grouping information in long-term memory, to reduce the cognitive load in working memory (Chase & Simon, 1973). These chunks can then be retrieved later, when needed, by working memory (Chi, Glaser, & Rees as cited in Sweller, 1988).

Schema automation occurs after extensive practice has been applied to a certain procedure, until it can be carried out fluidly and precisely with minimal conscious effort, and, therefore, the cognitive activity required to perform the procedure is less strenuous, since this cognitive activity occupies less space in working memory (Kotovsky, Hays, & Simon, 1985). Even though the total number of these elements (chunks) active in working memory is limited (Miller, 1955), the complexity and size of each element is limitless (Sweller, 1988).

Furthermore, cognitive load is defined as “a multidimensional construct that represents the load that performing a particular task imposes on the cognitive system of a learner” (Paas & Van Merriënboer, 1994, p. 353). There are three types of cognitive load that impact greatly on working memory, where the volume of working-memory space occupied during instruction equates to the sum of these three types. These three types – intrinsic load, extraneous load, and germane load – are discussed below (Sweller, 1988; Sweller, Merriënboer, & Paas, 1998; Kirschner, 2002).

Intrinsic cognitive load refers to a certain level of difficulty associated with each instruction. Even though the level of difficulty cannot be altered by the instructor, the information can be divided into separate subschemas and taught separately.

Extraneous cognitive load is extrinsic information (which is not relevant) to the instruction given. Instructional designers can reduce this load by excluding information that is not directly relevant to the targeted learning outcome.

Germane cognitive load is experienced through the learners’ sense-making activities, such as their use of cognitive strategies to construct, process, and automate information to improve existing schemas. Germane load is to be promoted, since it is associated with deep learning (Van Merriënboer & Sweller, 2005).

Figure 3.3 on the next page illustrates the concept of cognitive load in early learners who are performing a difficult or unknown task, a medical handoff.

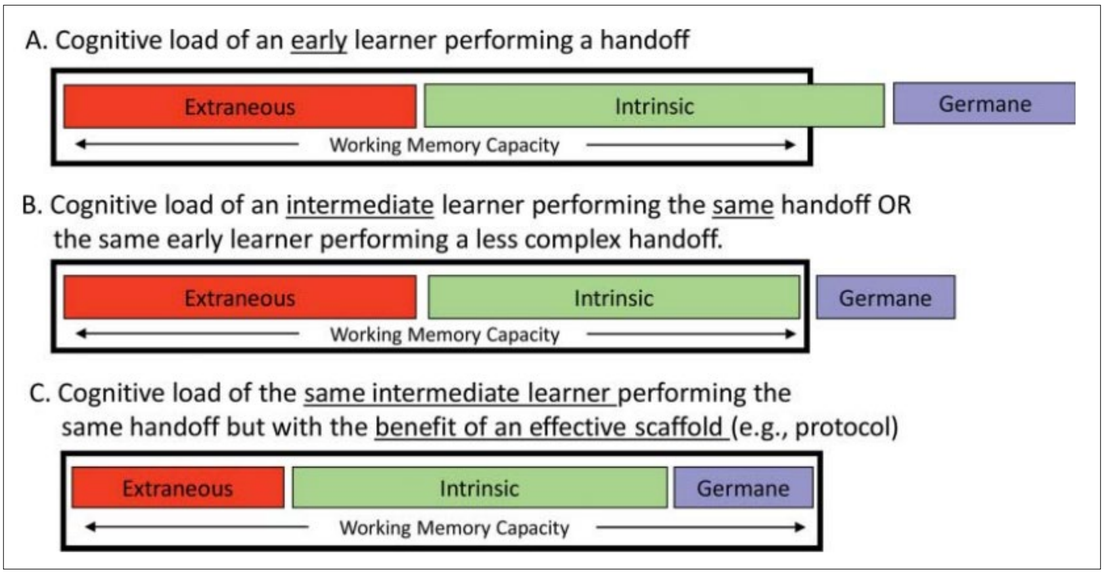

Figure 3.3: Interaction of cognitive load and working memory capacity during a difficult task

Source: Young et al. (2016)

When the instructional design is inappropriate or the information provided too complex, extraneous and intrinsic cognitive load is increased, which leaves little or no room for germane cognitive load (Van Merriënboer et al., 2003). Cognitive load theory was designed to “provide guidelines intended to assist in the presentation of information in a manner that encourages learner activities that optimize intellectual performance” (Sweller et al., 1998, p.251), therefore, teachers can use cognitive load theory to help them understand the value of scaffolding and removal of extraneous material from their instructional design. In terms of cognitive load theory, these techniques are intended to reduce intrinsic and extraneous cognitive load and, so, help learners to optimise the space available in their working memories for germane cognitive load.

The difference between experts and novices is reliant on the quality and effectiveness of a person’s retrieval of stored knowledge (or schemas) from long-term memory (Kirschner, 2002). The ability to chunk information extensively is a characteristic of experts (Chase & Simon, 1973). Furthermore, experts, in contrast to novices, can retrieve organised information from long-term memory seamlessly, which causes the limitations of working memory to disappear (Kirschner, 2002). This explains why the limitation of working memory size only applies to novices (Kirschner, 2009).

Since working memory is so limited, well designed instructions are necessary to assist learners in processing complex instructions effectively. This applies, in particular, to novice learners who experience the limitations associated with cognitive load, due to their limited ability to chunk information. In the context of a mathematics problem-based learning programme, learners who have had little prior exposure to higher-order thinking or problem-based programmes are considered to be novices.

3.3 MATHEMATICS KNOWLEDGE FOR TEACHING

In Shulman's (1986) seminal work he concludes that besides content knowledge and curricular knowledge, teachers need a third kind of knowledge which he calls 'pedagogical content knowledge' (PCK). He suggests that PCK "goes beyond knowledge of the subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman, 1986, p. 9). Building on Schulman's concept of PCK, Mathematics Knowledge for Teaching (MKT) was coined by Ball and colleagues (Ball & Bass, 2000; Ball, Lubienski & Mewborn, 2001), suggesting that for teachers to teach mathematics well, they need to be able to unpack or decompress the mathematics they know and have learned, so as to be able to make it accessible to learners.

Teachers who possess advanced mathematics knowledge for teaching can anticipate how learners will approach a mathematics problem. Advanced teachers can design the most appropriate instructional strategies and scaffolding approaches for the specific mathematics problem and context (Hill & Ball, 2009; Tambara, 2015).

An exploratory study by Hill et al. (2008) suggests that teachers with excellent mathematics knowledge exhibit a strong link between mathematics knowledge for teaching and high quality mathematics instruction, and were able to achieve better results in PBL, compared to teachers with less mathematics knowledge and lower mathematics knowledge for teaching. Tambara (2015, pp. 49-50) summarised the study of Hill and Ball (2009), and deduced that teachers with high mathematics knowledge for teaching were able to (Tambara, 2015, pp. 49–50),

- *"avoid mathematical errors and missteps;*

- *deploy their mathematical knowledge to support more rigorous explanations and reasoning, and to better analyse and make use of learner mathematical ideas than would otherwise have been possible;*
- *create rich mathematical environments for their learners;*
- *be critical of their mandated curriculum, and to like to invest considerable time in identifying and synthesising activities from supplemental resource material;*
- *provide high-quality mathematical lessons;*
- *provide high-skill responses to learners; and*
- *choose examples wisely to ensure equitable opportunities for learning."*

This list of competencies associated with having high mathematics knowledge for teaching highlights, (1) the vital importance of mathematics knowledge for teaching in any effective lesson, more so during cognitively challenging approaches, like PBL, and (2) that the teacher's skilful input is fundamental to the success of PBL approaches (Sweller et al., 1998; Van Merriënboer et al., 2003; Schmidt, Loyens, Van Gog, & Paas, 2007).

3.3.1 Problem-based learning

PBL originated as an attempt to improve the efficacy of medical education (Strobel & Van Barneveld, 2009, citing Barrows, 1996). Since its origin, PBL has been defined in a vast number of ways, but the essence of these definitions correlates throughout. This study will be guided by the definition of PBL given by Savery (2006, p. 9) which states that PBL is,

> *an instructional (and curricular) learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem.*

Generally, PBL is based on collaborative, learner-centred teaching approaches through authentic, relevant, and real-life learning tasks (Merrill, 2002; Van Merriënboer et al., 2003; Hmelo-Silver et al., 2007). Learners are cognitively engaged in an environment where the teacher facilitates the learning process. Teachers are only to intervene on a just-in-time basis regarding the provision of relevant content knowledge (Hmelo-Silver et al., 2007; Strobel & Van Barneveld, 2009). Integrating knowledge, skills and attitudes is encouraged by PBL, which enables learners to eventually transfer these newly integrated characteristics to other non-task-specific environments (Strobel & Van Barneveld, 2009; Cantürk-Günhan, Bukova-Güzel, & Özgür, 2012).

The following steps, according to Cantürk-Günhan et al. (2012, p. 148), are common to PBL. These steps served as the basis from which this study was approached:

1. *"The teacher presents the PBL scenario to students along with its evaluation rubric and the steps students should follow to solve the scenario.*
2. *Groups work on the scenario in the presence of the teacher.*
3. *Students do individual research on topics related to the scenario.*
4. *Students share with their group the research they have done, and they establish the problems to solve.*
5. *Students work collaboratively and cooperatively with their team members to propose a solution to the problems they stated.*
6. *Students present their work to their classmates and hand out their report.*
7. *The teacher comments on the similarities and differences among the groups' work and pulls out of their presentations the main concepts involved in the problems and their solutions.*
8. *The teacher evaluates the report."*

The role the teacher plays is of vital importance to the success of PBL. It is not sufficient to only follow the steps mentioned above; the teacher should, in addition, possess a specific body of knowledge to ensure that a deep mathematics understanding is formed (Tambara, 2015). In contrast to PBL, traditional approaches to teaching and learning are understood as teacher-centred instructional practices. Critics of such practices characterise them as being associated with curricula in which the content is compartmentalised, and as resulting in passive and shallow learning (Greening, 1998; Erickson, 1999; Barrows, 2000; Hmelo-Silver & Barrows, 2006; Walker et al., 2015). Although this is a simplistic characterisation (Taber, 2010), it does appear to be a fair description of the pedagogy typical of classrooms in the developing world (Schweisfurth, 2013), including in South African township schools (Hoadley, 2018).

3.3.2 Mathematics problems vs problem-solving

Pólya (1957) directed his early work on problem-solving mainly to describing the process, which was referred to as a linear process with four phases. Approximately a decade later, in the late 1960s, PBL first appeared in the literature as a pedagogical approach (Schoenfeld, 2016; Loyens et al., 2008). This appearance of PBL as a pedagogy soon led to confusion about exactly what problems, problem-solving and PBL did and did not entail (Schoenfeld, 2016). Therefore, I deem it necessary to clearly distinguish between the terms problem-solving and mathematics problems. To define the word problem, Schoenfeld (2016) refers to Webster's (1979, p.1434) dictionary definitions:

> *Definition 1: "In mathematics, anything required to be done, or requiring the doing of something."*
>
> *Definition 2: "A question . . . that is perplexing or difficult."*

The first definition above is consistent with the traditional notion of doing routine procedures in mathematics. Routinised and previously practised mathematics exercises may be referred to as being perplexing, and, therefore, classed as problems, according to Definition 2. However, Schoenfeld (2016) maintains that such a categorisation would be inappropriate, since no matter how difficult a person may perceive a mathematics question to be, if they have previously encountered a similar example, the question cannot be seen as being a problem in the sense of Definition 2.

Initially (1970–1982) problems were defined based on the question characteristics alone, i.e. as being difficult questions. However, more recent studies highlight that whether a question is perceived as being difficult is just as dependent on the characteristics of the solver as the characteristics of the question (Lester, 1994; Carlson & Bloom, 2005). Each learner has unique capabilities and working memory capacity (Kirschner, 2002; Schmidt et al., 2007), which make a particular task mundane for one learner and a problem for another.

Traditionally, learners were taught problem-solving techniques and to use specified algorithms, and they practised these algorithmic techniques in repetitive drill exercises to eventually master the particular technique (Schoenfeld, 2016). Although a shift in instructional focus across the constructivist-instructivist continuum (as discussed in Chapter 2) is necessary (Jonassen, 1991; Ertmet & Newby, 1993), the sole focus of intentionally teaching problem-solving, and approaching problem-solving purely from an instructivist perspective, can affect the learners' mathematical knowledge and understanding negatively (Schoenfeld, 2013).

A learner's sense of understanding of mathematics is established from their prior experiences of the subject. If learners are expected to engage in problem-solving in the sense of "what you do when you don't know what to do" (Wheatley, 1995: 2), they will form a deep understanding of the particular topic and use mathematics meaningfully in other domains too (Schoenfeld, 2013).

3.3.3 Problem-based learning: A minimally guided approach?

Minimally guided teaching approaches have been referred to as constructivist learning, discovery learning, inquiry learning, experiential learning, and PBL (Kolb & Fry, 1975; Bruner, 1961; Papert, 1980; Schmidt, 1983; Jonassen, 1991; Kirschner et al., 2006). These approaches are pedagogically similar, and hold the general view that learners construct their own knowledge through the process of discovery in a learner-centred environment, where the instructor serves as a facilitator of the learning process (Savery, 2006).

In contrast with the view that PBL has to be minimally guided, Hmelo-Silver et al. (2007, p. 100) argue that an assumption about this approach, that "learners need to explore phenomena and/or problems without any guidance", has repeatedly been confirmed to be unsound (Hmelo-Silver et al., 2007, citing Mayer, 2004). They argue, instead, that PBL should be a well-guided and scaffolded pedagogical approach, bearing in mind that providing too much or too little guidance might threaten the success of PBL (Albanese & Mitchell, 1993). PBL requires teachers to change the instructional guidance they provide at various points in their teaching by operating across the constructivist-instructivist continuum, ranging from providing direct instruction, to taking a just-in-time instructional approach, to offering no guidance at all (Schmidt et al., 2007; Schwartz et al., 2009; Strobel & Van Barneveld, 2009).

Clearly, such a complex pedagogy requires the teacher to possess a rich pedagogical repertoire and, in response to classroom situations, to make appropriate decisions about the approach to use from this repertoire. Therefore, the skill of the teacher plays a crucial role in the efficacy of PBL (Cantürk-Günhan et al., 2012).

3.3.4 Instructional scaffolding and problem-based learning

To assist with this cognitively taxing task of PBL without compromising the positive effects of the learning process (Greening, 1998; Van Merriënboer & Sweller, 2005), tailored support should be given according to the learners' needs. Support in the form of instructional scaffolding should be provided throughout the learning process (Sawyer, 2006; Hmelo-Silver et al., 2007).

During instructional scaffolding, the teacher adjusts the level of support provided according to the learner's level of cognitive potential (refer to Figure 3.4); when the learner is highly skilled, little or no assistance is needed during an easy task, and when the same task is presented to a learner with very few skills or prior knowledge, more assistance is needed (Wilson & Devereux, 2014; Beed, Hawkins, & Roller, 1991).

The theory of instructional scaffolding is based on Lev Vygotsky's theory of the zone of proximal development (ZPD) (Vygotsky, 1978), which refers to the field between what a learner is capable of accomplishing on their own, and what they can achieve with the help of a knowledgeable peer or teacher, which would not have been possible without assistance (Wilson & Devereux, 2014). Refer to Figure 3.4 for a chart representation of ZPD.

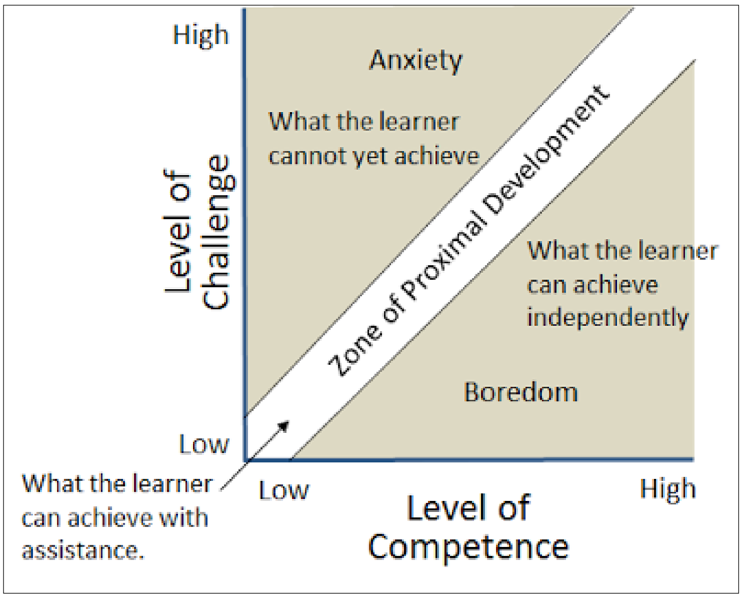

Figure 3.4: Zone of proximal development

Source: Redd (2014)

It is noteworthy that boredom creates equal levels of frustration and anxiety, which emphasises that tasks should be designed for learners to stay in their ZPD (Redd, 2014; Wilson & Devereux, 2014; Murphy, Scantlebury, & Milne, 2015). When instructional scaffolding is implemented effectively, learners operate within their ZPD, which (1) has a positive impact on the task at hand (Wilson & Devereux, 2014), and (2) influences future cognitive development, by building capacity in their working memory to eventually complete similar tasks unassisted, through transfer to their long-term memory (Landry, Miller-Loncar, & Swank, 2002). Learner independence is the ultimate goal of scaffolding.

Both "high challenge" and "high support", according to Mariani (1997), are needed for effective instructional scaffolding (refer to Figure 3.5). This model of scaffolding links to Figure 3.4, which also refers to the necessity of high support for difficult tasks and, conversely, that low challenge-low support is pointless, whereas high challenge-low support, or low challenge-high support, pushes learners out of their ZPD.

High challenge **Low support** >>Frustration >>Short cuts	**High challenge** **High support** >>Engagement >>Transformation
Low challenge **Low support** >>Pointlessness >>Boredom	**Low challenge** **High support** >>Busy work >>Dumbing down

Figure 3.5: The "high challenge, high support" scaffolding model

Source: Wilson and Devereux (2014, p. 93)

The framework above (Figure 3.5), if implemented well, enables the teacher to contribute to improving the learning taking place, through scaffolding that offers rich potential for meaningful learning in constructivist approaches, such as PBL (Wilson & Devereux, 2014). In addition, a specific teaching strategy, such as the ladder approach (Stott, 2008) that was explained in Chapter 2, can serve as an efficient tool to combine the theoretical aspects discussed in this chapter. The ladder approach was used throughout the intervention of the study (refer to Chapter 5).

3.4 MULTIDIMENSIONAL PROBLEM-SOLVING FRAMEWORK

Embedded in the existing body of problem-solving literature, is Carlson and Bloom's (2005) MPS framework, which they created after studying the responses of 12 mathematicians undertaking a problem-solving process. This framework provides a thorough classification of phases, attributes that relate to each phase, and cycles that occur in the problem-solving process (refer to Figure 3.6). The MPS framework also identifies various behaviours that problem-solvers exhibit during this process.

Phase • Behaviour	Resources	Heuristics	Affect	Monitoring
Orienting • Sense making • Organising • Constructing	Mathematical concepts, facts and algorithms were accessed when attempting to make sense of the problem. The solver also scanned her/his knowledge base to categorise the problem.	The solver often drew pictures, labelled unknown and classified the problem.	Motivation to make sense of the problem was influenced by their string curiosity and high interest. High confidence was consistently exhibited, as was strong mathematical integrity.	Self-talk and reflective behaviours helped to keep their minds engaged. The solvers were observed asking, "What does this mean?", "How should ii represent this?', "What does that look like?".
Planning • Conjecturing • Imagining • Evaluating	Conceptual knowledge and facts were accessed to construct conjectures and make informed decisions about strategies and approaches.	Specific computational heuristics an geometric relationships were accessed and considered when determining a solution approach.	Beliefs about the methods of mathematics and one's abilities influenced the conjectures and decisions. Signs of intimacy, anxiety, and frustration were also displayed.	Solvers reflected on the effectiveness of their strategies and plans. They frequently asked themselves questions such as, "Will this take me where I want to go?", "How efficient will Approach X be?".
Executing • Computing • Constructing	Conceptual knowledge, facts and algorithms were accessed when executing. Without conceptual knowledge, monitoring of constructions was misguided.	Fluency with a wide repertoire of heuristics, algorithm, and computational approaches were needed for the efficient execution of a solution.	Intima y with the problem, integrity in constructions, frustration, joy, defence mechanisms and concern for aesthetic solutions emerged in the context of constructing and computing.	Conceptual understanding and numerical intuitions were employed to reflect on the sensibility of the solution progress and products when constructing solution statements.
Checking • Verifying • Decision making	Resources, including well-connected conceptual knowledge informed the solver as to the reasonableness of the solution attained.	Computational and algorithmic shortcuts were used to verify the correctness of the answers and to ascertain the reasonableness of the computations.	As with the other phases, many affective behaviours were displayed. It is at this phase that frustration sometimes overwhelmed the solver.	Reflections on the efficiency, correctness and aesthetic quality of the solution provided useful feedback to the solver.

Figure 3.6: The multidimensional problem-solving framework

Source: Carlson and Bloom (2005, p. 67)

In the table in Figure 3.6, the first column denotes the four *problem-solving phases* (orienting, planning, executing, checking), with examples of general behaviour (i.e. verifying, decision-making, etc.) being listed below each phase name. In columns 2, 3, 4, and 5, the key roles of the specific attribute (resources, heuristics, affect, monitoring) are described in relation to the problem-solving phases indicated in the first column. The problem-solving phases and attributes are discussed in the next sections.

3.4.1 Phases of the problem-solving process

George Pólya is renowned for his work on conceptualising the problem-solving process and its nature in relation to mathematics. In building on Pólya's work, Lester and Garofalo (1989) maintain that a shift between problem-solving phases is only possible when metacognitive decisions lead to cognitive action. Carlson and Bloom (2005) labelled the four phases as (i) orienting, (ii) planning, (iii) executing, and (iv) checking, which are each defined below.

During the first phase, *orienting*, behaviours of understanding the problem, and constructing and organising the information received, are identified (Carlson & Bloom, 2005; Schoenfeld, 2016).

The *planning* phase refers to constructing conjectures, identifying goals and patterns, imagining the route to follow in order to reach the goal, and evaluating the plans made, through accessing one's conceptual knowledge and heuristics (Carlson & Bloom, 2005; Pólya, 1957).

Throughout the *executing* phase, the problem-solver engages in behaviours that include carrying out calculations based on the conjectures made in the planning phase, through accessing (factual and conceptual) knowledge, writing logical mathematical statements, and implementing strategies planned (Lester & Garofalo, 1989; Carlson & Bloom, 2005).

In the course of the last phase, *checking* the accuracy of the written mathematical statements, conjectures, and calculations (which were done in the executing phase) is verified and justified. If the initial conjecture is correct, the goal (which was set in the planning phase), has been reached. If the calculations and conjectures are incorrect, the checking phase results in a rejection of the solution, which directs the problem-solver back to the planning phase to rework the problem. Carlson and Bloom (2005) argue that the rejection and acceptance of solutions in the checking phase result in the problem-solving process being cyclical, and not linear, as initially argued by Pólya (1957).

3.4.2 Attributes of problem-solving

Early studies in problem-solving involved mainly descriptions of the process, whilst more recent studies emphasise the problem-solving behaviours and attributes that are conducive to problem-solving success (Wilson, Fernandez, Hadaway, 1993; Carlson & Bloom, 2005; Voskoglou, 2008; Tambara, 2015). These problem-solving behaviours are determined by the person's problem-solving attributes, which involve resources, heuristics, monitoring, and affect (Stott, 2002; Carlson & Bloom, 2005; Schoenfeld, 2016).

Resources, defined by Carlson and Bloom (2005, p. 50), are "the conceptual understandings, knowledge, facts, and procedures used during problem solving". These resources can be chunked (or clustered) together to maximise capacity in working memory (Sweller et al., 1998; Baddeley, 2001; Young et al., 2016), so that stored knowledge (or long-term memory) can be utilised (Stott, 2002). When resources in long-term memory are used, learning for understanding occurs, which causes intrinsic motivation (Stevenson & Palmer, 1994), since new knowledge is integrated into cognitive schemas. This intrinsic motivation can positively impact the belief system of the problem-solver (Stott, 2002). Furthermore, problem-solving success strongly relates to the role control plays in attaining resources (Vinner, 1997); Schoenfeld (2016, p. 23) maintains that "[i]t's not just what you know; it's how, when, and whether you use it", which means that, even though a problem-solver might possess the resources required to solve a problem, they do not always access those resources.

Heuristics is described as specific strategies and procedures which the problem-solver follows, and methods they use whilst grappling with a problem (Carlson & Bloom, 2005; Tambara, 2015). These specific procedures and/or methods are used to reduce the cognitive load experienced by the learner (Kirschner, 2002), and require conscious thought (Schoenfeld, 2016), in contrast to automated strategies which are classified as resources transferred from long-term memory (Sweller et al., 1998).

Monitoring, or control, refers to the selection and implementation of approaches, resources, and strategies to be utilised during the problem-solving process (Tambara, 2015), and that influence the solution path. There are three categories into which control can be divided (Carlson & Bloom, 2005):

1. Initial cognitive engagement, where understanding the problem and creating goals are the focus,
2. Cognitive engagement during problem-solving refers to the linking of resources from long-term memory with working memory to connect existing knowledge with new information, to construct logical statements,
3. Metacognitive behaviours that denote reflecting on the effectiveness and efficiency of one's thought process and chosen problem-solving strategy, as well as self-regulatory behaviours during the problem-solving process.

Effective employment of the monitoring (or control) attribute to the problem-solving process is an integral part of the success of PBL (Vinner, 1997; Schoenfeld, 2013; Young et al., 2016), and requires good judgement regarding the utilisation of accessible resources and which strategies to apply to which situations (Schoenfeld, 2013). Furthermore, metacognitive skill is an essential component of a problem-solver's ability to control the learning process (Garner, 1988; Lester & Garofalo, 1989) – when problem-solvers can plan, regulate, and evaluate their own problem-solving approaches, they are self-motivated and fulfilled by the task at hand (Stott, 2002; Van Merriënboer & Sweller, 2005). Stott's (2002) views around metacognition and the successful employment of control during a problem-solving task are embedded in the self-efficacy theory, which relates to people's self-belief regarding their own capabilities to complete a demanding task, which is discussed further in the next paragraph.

Affective aspects, like beliefs, mindsets, attitudes, and emotions impact greatly on the behaviour of the problem-solver (Lester & Garofalo, 1989; Dweck, 2006; Schoenfeld, 2016). Even though a person's beliefs, which are defined as deep-rooted convictions, such as "learning mathematics is mostly memorisation" or "doing mathematics requires lots of practice in following rules" (Schoenfeld, 2016, p. 10), are less evident than emotions (frustration, excitement, anxiety, etc.), they play a key role during problem-solving.

Schoenfeld (2016) argues that a person's belief system determines their perception of a mathematical task and how to approach this task, which may contribute to the success or failure of problem-solving. Schoenfeld (2016), therefore, states that problem-solving success is not purely a result of what a person knows. In line with the aforementioned statements on beliefs, Dweck (2006) coined the terms "growth" and "fixed mindsets", after decades of researching numerous differently aged people. Dweck and other researchers, such as Boaler (2013), found that a person's mindset has a strong effect on their beliefs regarding their ability to complete a difficult task. Learners can develop a growth mindset when they believe that intellect can be improved and that, with effort and exercise, one's brain can grow. Learners with growth mindsets learn more efficiently, exhibit a longing for resilience, problem-solving and challenge, and use failures as learning experiences. People with fixed mindsets, in contrast, foster the notion that a person is either intelligent or not, and when failure sets in, such people lack the determination to persevere, and they give up (Boaler, 2010; Dweck, 2015). The desire of learners to produce only correct mathematics work, due to the prevalence of fixed mindset beliefs, is not conducive to learning, since such a mindset is not conducive to cognitive development (Boaler, 2013). Furthermore, it is extremely unlikely that learners will be able to produce only correct mathematics work if tasks are pitched within learners' ZPD, since operating within one's ZPD is characterised by a degree of failure, which indicates the need for help from a more skilled peer or instructor (Wilson & Devereux, 2014).

3.5 CONCLUSION

In this chapter, I outlined the theoretical framework within which I operated as I conducted this study. A constructivist view of learning was taken, informed by Vygotsky's theory of the ZPD, the cognitive load theory, as well as the IPM. The latter two theories emphasise the limitations of working memory capacity. I continued to discuss mathematical knowledge for teaching, by emphasising the necessity thereof for the effective implementation of problem-based learning approaches. I described PBL by elaborating on general, misguided use of the terms mathematics problems and problem-solving, and followed it by a discussion of the debate whether PBL is necessarily a minimally guided approach, or not. With reference to the latter, I concluded with the view I take regarding the minimally guided approach, namely that instructional scaffolding, incorporating the ladder approach, is vital for the PBL approach. Finally, I described the MPS framework, by elaborating on the phases of the problem-solving process and the attributes that are evident during the problem-solving process. In the next chapter, I will discuss the methodology and research approach taken for this study.

CHAPTER 4: RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

In Chapter 4, I will describe and motivate a case study design that was informed by a pragmatic paradigm and the framework for integrated methodology (Plowright, 2011). This methodology was used to determine how Grades 8–9 learners responded to a problem-based mathematics extension programme. This framework and paradigm are flexible and responsive to the complexities and idiosyncrasies of real classroom practice. Furthermore, by working within the framework for integrated methodology (FraIM), a pragmatic methodology invited the researcher to conduct research that has the aim of "informing decisions and activities that impact on the world or that solve problems" (Plowright, 2011, p. 185).

The research question that guided the collection and analysis of data is:

> *How do grade 8 and 9 township learners respond to a problem-based mathematics extension programme?*

With the abovementioned question in mind, the study unfolded in accordance with the following subsidiary questions:

a) How do learners engage in the cognitive components of problem-solving?

b) What are learners' affective responses to the programme?

I will start this chapter by describing the study, and the fit for research design to research question, after which I will elaborate on the collection and analysis of data. The internal and external validity and reliability will then be discussed in detail, followed by limitations to the study and concluding remarks.

4.2 DESCRIBING THE SAMPLE

A descriptive case study design, as opposed to an explanatory- or exploratory case study design, was used to present a complete description of the phenomenon within the particular context (Merriam, 2009). The holiday extension programme that was central to the research took place in July 2016 and involved 27 Grades 8 and 9 learners from township schools who participated in a five-day mathematics, science and English programme for six hours a day. The learners were selected by applying non-probability purposive sampling (Plowright, 2011, p. 42–43), since this method was considered to be the best for addressing the needs of this particular study. The criteria for sampling included that the learners were in either Grade 8 or 9 in a township school near Bloemfontein, and that each learner had voluntarily entered the Expo for Young Scientists competition, which served as an indication that the chosen participants showed an interest in science, and, therefore, it was assumed, also mathematics, and were willing to do more than the bare minimum, and subsequently participate actively in an intense holiday extension programme during the winter holidays. A detailed description of the programme will be given in Chapter 5.

4.3 FIT OF RESEARCH DESIGN TO RESEARCH QUESTION

The aim of this study was to determine the manner in which, if at all, township learners respond to problem-solving teaching and learning. Plowright's FraIM model, which served as a template which guided me through the research process, also encouraged a more responsive, flexible and open-minded approach to research based on answering the research questions of this study. What is arguably most different about FraIM to other, more traditional, models is that FraIM does not "dictate that you hold a particular philosophical position prior to beginning the research" (Plowright, p. 7). A mixed methods research design, comprising qualitative and quantitative data, and inductive analysis to produce rich, detailed accounts for the learners' responses (Tambara, 2015), was appropriate for achieving the aim of this study. By using a mixed methods design, qualitative and quantitative data were combined and integrated to form an improved understanding of the research problem. The strengths of each method complemented the other to assist in gaining a holistic understanding of the study (Creswell, 2003, 2007; Castro, Kellison, Boyd, & Kopak , 2010).

A variety of qualitative methods were used, such as "observer as participant", during which some interaction between the observer and participant occurs; "full observer", where the researcher has minimal contact with participants; "written questions answered face-to-face", using written pre- and post-tests; and "interpretational use of artefacts", which involves the researcher interpreting learning events and experiences (Plowright, 2011, pp.

66, 67, 78, 95). Being a participant observer (Plowright, 2011, p. 67) allowed me, as the researcher, to answer the research questions with deeper understanding, since I was intimately involved in the process. However, to ensure that these participant observer data, or full observer data, are not biased, other sources of data (e.g. artefacts in the form of learners' written work) were also included.

Mixed methods can be categorised into three types: exploratory, explanatory, and triangulation (McMillan & Schumacher, 2006, p. 28). A triangulation design was used in this study, since the strengths of both qualitative and quantitative methods complement each other to add to the reliability and validity of the data. Triangulation will be elaborated on when the reliability of the study is discussed, in Section 4.6.3.

4.4 DATA COLLECTION PROCEDURES

A variety of data were collected over five consecutive days. Each learner received a workbook designed according to the principles of carefully scaffolded guided inquiry and problem-solving (Hmelo-Silver et al., 2007). Learners' answers in these workbooks were analysed to identify the interaction of problem-solving attributes with the general behaviours exhibited during the four problem-solving phases, to determine the extent to which learners engaged in and responded to problem solving. Every session was video-recorded, and detailed field notes were taken during the course of the extension programme. The opinions of the learners regarding the holiday extension programme were captured with audio-recorded focus group discussions at the end of the week. The group of learners was divided into three smaller groups and focus group discussions were guided by the researcher and her colleagues. A focus group discussion guide was conceptualised before each discussion (refer to Addendum C). Pre- and post-tests (Addendum D) were written by the learners at the start and end of the holiday extension programme respectively. These tests were designed to determine the learners' knowledge base (Schoenfeld, 2016) and to analyse their individual understanding of the concepts that had been taught throughout the week. The tests included questions on basic mathematics concepts, which form part of their current school curriculum, as well as trigonometric concepts that had not been taught to them. Table 4.1 summarises the data corpus, by listing the day of collection and quantity of each data type gathered.

Table 4.1: Relationship between quantity, day of collection, and type of data collected during the holiday extension programme

Type of data collected	Quantity of data collected				
	Day 1	Day 2	Day 3	Day 4	Day 5
Video recordings of lessons	41 min	91 min	110 min	220 min	0
Audio recordings of lessons	2 min	2 min	12 min	0	0
Audio recordings of three different focus groups' discussions (learners).	0	0	0	0	101 min
Audio recordings of individual interviews during lessons (conducted by research assistants)	2 min	16 min	43 min	0	0
Photos of lessons	0	9	5	13	9
Field notes: Maths researcher on maths lessons (written document completed after maths lessons)	1	1	1	1	1
Field notes: Science researcher on maths lessons (written document completed during maths lesson)	1	1	1	1	0
Pre-test	26*	0	0	0	0
Post-test	0	0	0	0	26*
Learner booklets	0	0	0	0	25**

**One learner was absent during the pre-test and one during the post-test (each time a different person).*
*** Two learner booklets were unavailable for collection and analysed subsequently.*

4.5 DATA ANALYSIS

Data analysis was inductive, iterative and pragmatic, and guided by the research questions. Data from the video and audio recordings, and the observer's field notes, are nominal data (data not placed in a certain order or rank) with a low degree of structure, which were enlarged to ordinal data (ordinal data stands in relation to each other in a ranked fashion) by adding information to order or rank the original data (Plowright, 2011, p. 128). Data from the pre- and post-tests, and written work are ratio/interval data with a high degree of structure, which was reduced to ordinal data (Plowright, 2011, p. 129) through the process of data reduction, which grouped the data; new categories of these groups were created.

The video and audio recordings were transcribed and coded using a problem-solving coding taxonomy (refer to Addendum A), which is embedded in the literature relating to the four problem-solving phases and problem-solving attributes (Carlson & Bloom, 2005, p. 51). The MPS framework (Figure 3.1) (Carlson & Bloom, 2005, p. 66) provides a detailed characterisation of phases and cycles that occur consistently in the problem-solving process. This framework was used to categorise the transcribed codes to develop a holistic idea of the ways in which the learners responded to the problem-based holiday extension programme, as guided by the research questions. The data were coded using the software package NVivo. After coding, data were transformed, which combined effective organisation of nonnumerical, unstructured data through indexing, searching, and theorising. Using NVivo software for data coding and analysis increased the reliability of the research process by ensuring transparency and allowing for clear peer examination, as described in Section 4.6. Furthermore, using this software enhanced the rigor, efficiency and effectiveness of the data analysis process.

4.6 VALIDITY AND RELIABILITY

In qualitative research, the researcher serves as the "primary instrument for data collection and analysis" (Merriam, 2009, p. 15), which causes the risk of this human instrument being biased. Therefore, validity and reliability, which will be discussed in this section, are two factors about which a researcher should be concerned throughout the study, since a research study design that meets the standards of these factors increases credibility. These factors should not be seen as independent qualities (Marczyk, Dematteo, & Festinger, 2005), because a measurement is deemed invalid if it is unreliable (Loyal, 2016).

4.6.1 Internal validity

Internal validity refers to whether the findings of a study are legitimate and correspond with reality (Mertens, 1998). The way the data was recorded, collected, and analysed influences the internal validity directly. According to Merriam (2009), triangulation, member checks, peer examination, and stating the researcher's assumptions and biases are strategies that the researcher can apply to increase the internal validity. Triangulation and peer examination were used in this study to strengthen internal validity. Three different data type files (audio file, video file, and field notes) were coded and examined, using the coding taxonomy based on Carlson and Bloom's (2005) multidimensional problem-solving framework (refer to Addendum A for the complete list of codes), by both the researcher and a peer, after which discrepancies and shortfalls were discussed and adjustments were made to the coding taxonomy and approach. To strengthen the internal validity further, after coding all sources of data, the researcher re-evaluated all coded evidence and compared the codes to make sure that different data sources did not refer to the same piece of evidence, and that the accumulated number of codes counted evidence of, e.g. 5 codes, which all referred to a single occurrence (refer to Table 4.2).

Table 4.2: Number of coded attribute evidence per problem-solving phase

		Attributes of problem-solving							
		Resources		Heuristics		Affect		Monitoring	
		Coded evidence	True evidence	Coded evidence	True evidence	Coded evidence	True evidence	Coded evidence	True evidence
Phases of problem-solving	**Orienting**	92	71	38	17	108	76	42	21
	Planning	67	61	11	5	79	59	19	6
	Executing	59	22	8	2	44	23	5	1
	Checking	0	0	0	0	0	0	0	0
	Total	218	154	57	24	231	158	66	28

By recoding the data, as shown in Table 4.2, there was a clear distinction made between coded evidence and true evidence, where coded evidence represents the total number of codes that represent a certain occurrence, and true evidence represents the amalgamated number of occurrences. For example, if a particular occurrence of learners showing evidence of resources in the planning phase was evident, and this particular occurrence was coded three times in field notes, video recordings and learner books (coded evidence), it was re-evaluated to count as only one occurrence (and not three) of evidence (true evidence).

4.6.2 External validity

External validity refers to the possibility of the results of a study being generalised to other cases (Creswell, 2003). This case study is too small for the aim of generalisation; however, the aim was not to generalise, but to understand "how people make sense of their world and the experiences they have in the world" (Merriam, 2009, p. 13). Fuzzy generalisations allow predictions of possible similar occurrences to be made (Bassey, cited in Stott, 2002). Furthermore, "rich descriptions" (Merriam, 2009, p. 16) of a case enable readers to form their own naturalistic generalisations, since understanding is socially constructed (Stake, 1994).

4.6.3 Reliability

Reliability is the degree to which a study produces consistent end correct results (Tambara, 2015); a study is deemed reliable if it can be reproduced under a similar methodology (Golafshani, 2003). To reproduce a study is not easy in the social sciences as it is in the pure sciences, since dealing with the human factor contributes to the complexity of the study (Merriam, 2009). For example, this study would not have necessarily yielded similar results if the teacher had lacked the essential problem-solving knowledge and mathematical knowledge for teaching. Dependability and consistency serve as the key focus areas of reliability in the social sciences. The underlying question in social sciences research is "not whether the results of one study are the same as the results of a second or third study, but whether the results of the study are consistent with the data collected" (Merriam, 1995, p. 56). The strategies I used to improve reliability were (1) triangulation, using more than one method of data collection; (2) peer examination, by a researcher verifying the data results; and (3) ensuring an audit trail, by providing a clear description of the data collection instruments, data analysis and interpretation (see Chapter 5), so that readers can audit the complete process (Merriam, 1995).

In this study, triangulation and peer examination were used to ensure reliability. As mentioned in the Section 4.6.1, three sets of data were coded by the researcher and a peer.

These six sets (three different sources, coded by two people) were compared by running a coding comparison query in the software program NVivo, which produced a Cohen’s Kappa coefficient and percentage agreement result. Refer to Addendum B for an example of how the difference sources were coded in the NVivo program, of the compared codes of one of the data sets (Field notes of 27/06/16), as well as a screenshot of a visual comparison of different codes at a specific part of the field note document.

The Cohen’s Kappa coefficient verifies the occurrence of the themes that appeared in the study. It is a statistical inter-rater reliability measure to determine the level of agreement between two peers (or raters). Figure 4.1 outlines the Cohen's Kappa concordance index.

Kappa coefficient	
$k < 0.00$	no concordance
$0.00 \leq k \leq 0.20$	very mild concordance
$0.21 \leq k \leq 0.40$	mild concordance
$0.41 \leq k \leq 0.60$	moderate concordance
$0.61 \leq k \leq 0.80$	substantial concordance
$0.81 \leq k \leq 1.00$	concordance almost perfect

Figure 4.1: Cohen's Kappa concordance index

Source: Decito, Chueire, and Bonvicine (2013, p. 179)

The inter-rater agreement results (averages of all code agreements) for the three different data sources which were coded by the researcher and a peer are displayed in the Table 4.3 (refer to Addendum B, Table 2, for the complete list of inter-rater reliability of codes). These results show substantial concordance and strong agreement between the coded data of the two raters, suggesting a high degree of reliability in the NVivo coding process.

Table 4.3: Inter-rater agreement results

Data source	Cohen's Kappa	Agree-ment (%)	A and B (%)	Not A and Not B (%)	Dis-agree-ment (%)	A and Not B (%)	B and Not A (%)
Field notes of Day 1	0.6760	96.2556	2.5439	93.7119	3.7444	3.1661	0.5786
Video recording of Day 3	0.7100	96.3249	2.6836	93.6415	3.6751	3.0891	0.5864
Audio recording of focus group Day 5	0.7416	96.5478	2.7341	93.8139	3.4522	2.9080	0.5445

4.7 LIMITATIONS AND ETHICS IN RESEARCH ACTIVITIES

This sample is not representative of Grades 8 and 9 South African learners at township schools, since these learners tended to be relatively strong and motivated academically, compared to the average township learner (McMillan & Schumacher, 2006). Furthermore, the intervention was not conducted in a township context, and it involved expertise and outstanding human resources that are not usually available in township classrooms. However, given the paucity of research in this area and the difficulty of attaining success, it was considered appropriate to conduct this study in the manner most likely to succeed. Subsequent studies could explore the extent to which the study could be generalisable (McMillan & Schumacher, 2006) to the broader population and more typical contexts.

The researcher's sensitivity to ethical standards and the research process were clearly communicated to each individual before learners started with the holiday extension programme, completed any of the documentation (questionnaire or workbook with reflections), or participated in the focus group discussions, where the voluntary aspect of participation and confidentiality of responses were highlighted (Mertens, 2005; Strydom, 2005). Attendance at the intervention and inclusion in the research sample were both voluntary, and independent of each other. All intervention participants gave informed assent and all their parents gave informed consent to be included in the research sample.

4.8 CONCLUSION

This chapter started by restating the research questions that guided the collection and analysis of data, followed by a description of the sample and the fit for the research design to the research questions. Next, an outline of the data collection procedures and types of data collected, and a description of the research instruments were given, with emphasis on the internal and external validity of the research. Subsequently, I described how Cohen's Kappa coefficient was used as a statistical inter-rater reliability measure to determine the reliability of the data. Lastly, the limitations of the study were elaborated on. The results and findings of the NVivo analysis will be elucidated in Chapter 5.

CHAPTER 5: DESCRIPTION OF PROGRAMME

5.1 INTRODUCTION

The aim of this chapter is to provide a rich description of the problem-based mathematics extension programme that resulted in this research. First, the context of the programme will be presented by a discussion on the background of the researcher, followed by the philosophy of the programme. A short summary of the learners' school context will then be given, having been discussed in detail in Section 2.3. A description of the intervention programme will be outlined and followed by a rich description of each day of the programme. These descriptions will add to the understanding of how the data was interpreted and analysed, which will be reported in Chapter 6.

5.2 CONTEXT OF THE PROGRAMME

5.2.1 Background of researcher

At the time of the research, I worked on mentorship programmes, and I mentored mathematics teachers in township schools. This mentorship programme's focus was to impact disadvantaged South African youth through education, and to produce a positive ripple effect in the surrounding communities.

The group of mentors working on these mentorship programmes acts as agents of change in disadvantaged communities, by being leaders of school change (Van der Walt, 2016). The aims of these school mentorship programmes included assisting in developing social capital in poor communities (Taylor, 2020). These programmes have a credible history of bringing about change in school management and the quality of teaching in classrooms, and for opening opportunities for gifted learners in township schools (Van der Walt, 2016; Jacobs, 2018).

Furthermore, the dire situation of STEM (science, technology, engineering, and mathematics) education in the majority of South African schools (Reddy et al., 2016) prompted the University of the Free State to work towards establishing an innovative, independent, STEM Academy on its South Campus, aimed at changing the way teachers are trained and the way learners learn to make greater use of PBL that requires higher-order thinking. I was part of the team tasked with researching the feasibility of using PBL in STEM education for township learners.

My beliefs regarding schooling in South Africa challenge the current landscape of teaching and learning in South African schools. I agree with Hobden (2005, p. 308) that “there is a need to change what is taught and learnt, and how it is taught and learnt”. I am, furthermore, in agreement with Jo Boaler (2013), who states that learners must be encouraged to imagine, to think, to explore, to collaborate, to investigate, to think critically, and to create meaning, and that teachers should be facilitators in a learner-centred and problem-based pedagogical approach.

My background of studying mathematics, applied mathematics and chemistry, coupled with my teaching experience in an inquiry-focussed private school, serves as the basis from which I approached my mathematics mentoring, as well as my view on teaching as a whole. Additionally, one of my mentoring colleagues, a deeper-learning expert, continuously challenged my mentoring approaches, to encourage me to improve on the impact I had on the limiting environment of South African township schools.

Lastly, being part of both initiatives mentioned above, working as a mentor who mentors mathematics teachers in township schools, and being a member of the team tasked to establish a STEM academy, I interacted with learners who showed an interest in science and mathematics. The aforementioned team ignited the idea to run a pilot programme to evaluate the feasibility of a STEM approach in poorer contexts of South Africa, and to evaluate the feasibility of PBL with township learners. Consequently, the conceptualisation of a week-long science, English for science, and mathematics programme was initiated.

5.2.2 Philosophy of the programme

During the conceptualisation of this programme I, in accordance with my view on teaching, used a combination of scaffolding techniques (Van Merriënboer et al., 2003; Hmelo-Silver et al., 2007; Wilson & Devereux, 2014) and a variation of teaching methodologies from across the constructivist-instructivist continuum (Porcaro, 2011; Stott, 2018). I made sure I was sensitive to learners’ cognitive load (Beed et al., 1991; Van Merriënboer et al., 2003), their range of abilities, and external factors, such as hunger, which might impact negatively on their learning experiences (Mampane & Bouwer, 2011). Skilful questioning, a scaffolding technique, was employed when responding to questions or situations where learners felt they were stuck and wanted to give up – this usually occurs when a learner's cognitive load is high (Van Merriënboer et al., 2003). During my prompts and questioning, in accordance with the scaffolding approach, I focussed on posing carefully scaffolded questions to skilfully guide each learner to succeed as they worked within their ZPD (Vygotsky, 1978; Murphy et al., 2015; Wilson & Devereux, 2014). I also decided to use statements that encourage growth mindsets (Boaler, 2013), by responding to possibly incorrect or correct learner answers with encouragement and prompts to undertake deeper thought. Examples include, "you figured out what to do really fast, how did you do it?", "I remembered a time at the beginning of the week when that felt so difficult, now you can do it by yourself", "Even though this was hard, you gave it your best try", and "You cannot do it yet, but you will get it when you practise more" (Video recording; Field notes, Days 1, 2, 3, 5). Additionally, two Grade 12 learners volunteered to serve as teaching assistants throughout the duration of the programme, thereby increasing the individual attention we were collectively able to give the learners.

One of the three essential features of scaffolding is to determine the learners' current level of knowledge, and then to pitch new content just above that level (Beed et al., 1991). Even though I had a certain programme planned, which determined the pitch of instruction at the pre-empted level of learner knowledge and competence, I sometimes had to spontaneously adjust my teaching speed and content according to the leaners' needs, which also forced me to make changes to lesson time frames. Throughout the programme, I worked to encourage each learner to participate in conversations, and to have the confidence to make mistakes, since this assists learners to fully engage in the learning process (Hobden & Hobden, 2019).

When I conceptualised the workbooks, tests, and lesson plans, I was very conscious of the requirement that my instructional design should consider the anticipated limited capacity of the learners' working memory (Van Merriënboer & Sweller, 2005). Consequently, I ensured that I excluded unnecessary details from the workbooks and presented lessons in a varied and learner-centred manner. I avoided all concepts that did not contribute to the learning goal, as such unwanted concepts would only increase the extraneous and intrinsic cognitive load. I encouraged cognitive linking in my prompts, with comments such as "remember when you tried to use Pythagoras in an obtuse triangle" (Audio-recording, Day 2). These approaches were taken with the aim of encouraging the learners to engage in sense-making, i.e. to engage in generative cognitive processing, which offers germane cognitive load (Kirschner, 2002), through minimising extraneous load and managing intrinsic load.

During the tea and lunch breaks I played music, danced with the learners, and engaged in social conversations. I learnt all the participants' names, which I hoped made them feel valued and heard. This appeared effective as, during the focus group discussion on the final day, one of the learners remarked, "it made me feel happy when you learnt our names" (Audio recording, Day 5). I also introduced learners to new knowledge through YouTube videos, music and creative demonstrations; all of which they would not have experienced in their own schools.

I am aware that this extension programme's classroom environment and context are in stark contrast to classrooms in township schools, but the aim was not to recreate their learning environment to see if the learners could respond to new pedagogies within these unfortunate circumstances. Rather, the aim was to investigate if learners with limited prior exposure to problem-solving approaches and limited mathematical conceptual understanding, were able to respond to new learning and teaching strategies.

5.2.3 Context of learners' schools

As discussed in more detail in Section 2.3, township schools such as those that these learners attended, rarely promote deep learning (Hugo et al., 2008; Hobden & Hobden, 2019). The teaching approaches of this pilot programme week and the teaching approaches that the learners are used to are incomparable. Therefore, I approached the intervention and research with an awareness that even modest learner responses to the programme would be noteworthy; I also designed the programme to create opportunities for the learners to achieve well beyond the very limited expectations normally imposed on them.

5.3 THE OUTLINE OF THE PROGRAMME

The conceptualisation of this weeklong problem-based mathematics extension programme was a group effort, which called for input from numerous team members. The aim, conceptualisation and detailed planning were brainstormed with two STEM-focussed research colleagues, an Expo specialist, a natural science subject specialist, an English subject specialist and me, to conduct the mathematics research. Four assistants were tasked with providing additional individual attention to the learners throughout the week. An additional person was tasked with administrative duties, such as organising all the catering, arranging the buses and transport for the learners, and printing all the programme material. One teacher at each school was assigned to act as the liaison between the university and the respective schools. Table 5.1 provides a summary of the team that assisted in the implementation the holiday programmes.

Table 5.1: Task team for the weeklong problem-based mathematics extension programme

Team	Description
Four subject specialists	One mathematics teacher One physical science teacher One English teacher (fluent in the learners' mother tongue) One Expo teacher
Four assistants	Two Grade 12 learners, both with exceptional mathematics, English, and physical science proficiency, from other high-quintile schools assisted the learners with specific Expo-related or subject-related questions Two teachers from the learners' schools attended the sessions to assist and possibly gain from the problem-solving which was modelled
One administrative official	One administrative person to organise buses, stationery, catering, printing, venue bookings, and Wi-Fi access at the university
Two additional conceptual planning team	Two additional external specialised researchers to assist in conceptualising the week-long programme
Contact person at each school	One staff member at each of the representing schools was approached to assist as liaison between the organisers and the school

The learners were bussed in to the University of the Free State from schools in the Motheo District (Botshabelo and Bloemfontein township schools) each day; the weeklong holiday programme involved six hours of contact time per day, with roughly two hours allocated to each of the three subjects offered: mathematics, science, and English for science. Learners received refreshments, and were given homework on Days 1–4, to help them prepare for the following day's work. The work engaged in during each of the three disciplines was embedded in the context of investigating factors affecting the maximum height and range of a water rocket. The role of the mathematics section of the programme was to enable the learners to take appropriate measurements from which they could calculate the rocket's maximum height during flight.

Problem-based strategies, based on guided inquiry and carefully prepared scaffolding, to help learners construct their own knowledge (Anderson, 2002), were used for teaching a specific mathematics topic, trigonometry. Not only was this appropriate for the context, i.e. for the learners to determine the maximum height of the water-rocket, but it was also an appropriate topic for this research, since I was confident that the learners were not already familiar with this topic, because, according to the CAPS (Department of Basic Education, 2011a, 2011b) for mathematics, learners are only introduced to trigonometry in Grade 10. The aforementioned ensured that the trigonometric concepts the learners were taught were completely foreign and authentic to them. Another reason why trigonometry adds to the problem-solving nature of learning is because it requires learners to link triangle drawings with numerical relationships and requires them to manipulate symbols included in those relationships (Weber, 2008), which learners tend to find extremely difficult (Kamber & Takaci, 2018).

To address the taxing cognitive tasks discussed in the previous chapter, I referred to Stott's (2008) conceptualised pedagogical approach. The following steps of the ladder approach proposed by Stott (2008, pp. 219-220) describe the implementation of this weeklong mathematics extension programme for guiding learners to solve the problem of finding the maximum height of the water rocket:

1. *Initial problem engagement*: the teacher demonstrates a scale rocket launch and guides the learners to realise the scope of the problem and aim of the week, which is finding the maximum height of the water rocket. ("This aims to develop learning anticipation" (Stott, 2008, p. 219))
2. *Direct instruction*: Guidance is given to assist learners to link with their prior knowledge. The teacher, through constant provision of scaffolded guidance and

skilful prompts, encourages sense-making by the learners. (“This aims at developing procedural knowledge and an understanding of the relevant fundamental knowledge, as well as encouraging reflection of this in terms of the problem.” (Stott, 2008, p. 21))

3. *Scaffolding worksheet engagement*:
    a. Learners individually engage in answering the related questions in the Learner Workbook (refer to Addendum F). (“Critical thinking during guided self-reflection is aimed for here.” (Stott, 2008, p. 219))
    b. Learners compare their diagrams and calculations with those of the other members of their group. They engage in critical group discussions and choose the best answer or sketch to represent their group’s work to the rest of the class. (”Critical thinking through critical discourse is aimed for here.” (Stott, 2008, p. 219))
    c. The teacher engages in class discussions and encourages further critical discussions, to skilfully guide the learners to conclude which answers are correct, and also to reflect on why their answers might be incorrect. Then, the learners revert to their own individual work to do corrections and reflect on their own answers. (“In this way the scaffolding worksheet acts as a formative assessment tool.” (Stott, 2008, p. 219))
4. Repetition of Steps 2 and 3, until all relevant concepts are covered, and learners are empowered to confidently move to the next section of the problem or Learner Workbook.
5. *Problem task solution*:
    a. Learners answer the relevant problem individually, guided by the scaffolded Learner Workbook questions. (“This requires learners to use their conceptual understanding and procedural knowledge in new and creative ways to solve the subsuming problem and doing so requires critical thought.” (Stott, 2008, p. 220))
    b. Learners present their calculations, verbally, guided by their individual written work, to their group. The group members critique each other, modify their answers, and produce an answer that represents their groups answer

best. ("Critical thinking through discourse is aimed at here." (Stott, 2008, p. 220))

c. The teacher assesses and responds accordingly to the different groups' answers.

The ladder approach was used throughout the five days of the programme. During the first three days of the programme (refer to Table 5.2 for the programme and times of the week), learners engaged in activities to prepare them to use trigonometry to calculate the height of the water rocket. On the fourth day, the water rocket was launched, and data collected and analysed to complete the mathematical calculations. On the fifth day, the learners wrote a report of the entire investigation process and presented the entire week's work to their peers. The data that is used in this dissertation focuses only on the mathematics teaching and learning of this holiday extension programme.

Table 5.2: The outline of the programme for days 1–5

Day	Science	English	Mathematics	Extension (home)
Day 1 Monday Introduction to the maximum height problem	8:30–9:30 Introductory video 11:30–13:00 Pre–test Plan investigation	9:30–11:00 Vocabulary relating to rockets Read article about rockets and the science involved	13:30–15:00 Pre-test Practical demonstration and investigation Linking to prior knowledge Start thinking about height calculations	Science: Go through glossary and explain 'What is happening: Terms and concepts' in own words/other language/pictures Mathematics: Prior knowledge sums
Day 2 Tuesday Investigating Trigonometric ratios	10:30–12:00 Terms and concepts	12:00–13:00; 13:30–15:00 Find key words Make mind maps	8:30–10:00 Triangles Investigating trig ratios Reflection	Science: Read relevant texts: Energy, Newton's third law Maths: Two extension sums

Day	Science	English	Mathematics	Extension (home)
Day 3 Wednesday Top secret information	8:30–10:00 Raw and processed data table preparation Data interpretation Hypothesis	13:30–15:00 Plan report	10:30–12:00 Measuring angles Practical activity on angle of inclination (stationary and moving object) Trig ratios/ identities application	Science: Design the highest-flying rocket possible, with reasons Maths: two sums and prep for launch
Day 4 Thursday Launch day	12:00–13:00 Evaluate data Compare hypotheses to observations	13:30–15:00 Write report	11:00–12:00 Capture data Calculating unknown height (of vertical launch) Line graph construct	Finish report Maths: Finish graph and think about essay
Day 5 Friday Reflecting and presenting	8:30–9:30 Explain observations Improve report Post–test	–	9:30–10:30 Post-test Essay on findings and accuracy of data	–

Source: Modified from Stott (2019, p. 160)

5.3.1 Day 1: Introduction to the maximum height problem

The aim of Day 1 was to challenge the learners with the problem of finding the maximum height that the water rocket would reach. They needed to be guided to realise that their knowledge of the Pythagorean theorem would not be sufficient to do the calculations.

The mathematics concepts and terminology I emphasised throughout Day 1 were basic triangle terminology, right-angled triangles and their characteristics, the fundamentals of the Pythagorean theorem, similar triangles, and terminology relating to the launch (range, angle of inclination, launch pad, maximum height, stationary, and observer).

After an enthusiastic welcome, a video of a NASA rocket launch was shown to ignite excitement and anticipation amongst the learners for the week ahead. Then, during the first contact session on Day 1, I introduced the concept of launching a rocket with a small-scale demonstration, using basic stationery. Throughout my demonstration I focussed on emphasising important mathematical terminology (described in the sections to follow) and repeating mathematical concepts that are central to the success of reaching the main goal of calculating the maximum height that a water rocket could reach during the practical launch on Day 4. After my scaled practical demonstration, the learners sketched a 2D diagram of what they had observed, after which they compared their individual sketches within their groups and answered guided questions (refer to Addendum F, Learner Workbook pp. 2–3). Each group decided on the sketch that represents their combined understandings best and presented it on an A0-sized piece of paper. This process enabled me to identify several misconceptions, which I addressed by guiding the class through a discussion of four different representative sketches, which I drew on the chalkboard. I guided the learners, using questioning techniques, to the point where they indicated which of the four options was the best sketch to represent the scaled demonstration. I then directed the discussion to possibilities for calculating the maximum height of the rocket, where, as I anticipated, they reverted to the theorem of Pythagoras. I prompted a discussion to guide the learners to realise that Pythagoras would not be sufficient for calculating the maximum height (refer to the second dialogue in Section 6.2.1.2).

I continued to link the current launch demonstration with the learners' prior knowledge of similar triangles, since similarity would be a key concept in Day 2's investigation. I had included practise problems in the part of the workbook used on Day 1, where learners were required to use Pythagoras to calculate the side lengths of different triangles. Although they would not need Pythagoras for calculating the maximum height of the rocket, I deemed it important for learners to have a sound knowledge of the Pythagorean theorem, which I hoped to develop through requiring them to complete the routine sums given in Addendum F, Learner Workbook p. 5. I did this because I believed that, if the learners did not fully comprehend why they could not use the Pythagorean theorem to solve the problem of determining the maximum height of the water rocket, they would not be able to realise the need for learning a new concept.

At the end of Day 1, I left the learners with two questions: 1) Can a ratio be written as a fraction? I asked this because misconceptions regarding ratios and fractions are known to be prevalent (Ramadianti, Priatna, & Kusnandi, 2019), and this would serve as a conceptual link between Day 1's work and Day 2's work, and 2) How will you calculate the maximum height if you cannot use the theorem of Pythagoras?

5.3.2 Day 2: Investigating trigonometric ratios

The aim of Day 2 was to prompt learners to realise that the relationship between the ratio of similar triangles' various side lengths are the same if the reference angles in all similar right-angled triangles are the same. This contact session was probably the learners' first introduction to trigonometry. My aim was to prompt the learners to find patterns, relationships and similarities without any explicit mention of trigonometry.

Learners revisited, after I prompted them, the terminology of the previous day, by adding appropriate labels to the blank diagram on p. 7 of the Learner Workbook (Addendum F). I made sure that all learners were comfortable with the relevant terminology to be used, since I wanted to prevent unfamiliarity with basic concepts, which would add to learners' cognitive load, since they would need as much mental space as possible to attend to the calculations and relations addressed in the rest of the lesson.

Learners completed the investigation (refer to Addendum F, Learner Workbook p. 8), after which I engaged in guided discussions to assist learners to identify the link between, specifically, the ratio of the opposite side to the adjacent side of right-angled triangles (0,6) with relation to the reference angle of 30^{o}. Learners were then asked to complete the next section in the Learner Workbook (p. 9), which required them to make the link between the previous and the new exercise; and identify that the new question (2.4) consisted of a right-angled triangle with a reference angle of 30^{o}, similar to the investigation sums.

I ended Day 2's programme by asking the learners to reflect on what they had accomplished in only two days. I emphasised how difficult these concepts were and how well they had managed to grasp the new mathematics concepts. At the end of Day 1, it appeared that all learners had been convinced that the only way to find the maximum height of the rocket is to try and find the lengths of the hypotenuse and the range, so that they could calculate the third side (height of the rocket). By the end of Day 2, however, it appeared that many of the learners had realised that knowing the size of a reference angle and only one side length of the same triangle is sufficient information to calculate the height of a right-angled triangle.

5.3.3 Day 3: Top secret information

The aim of Day 3 was (1) to link the concept of ratios and reference angles (learnt in Day 2) with the new concept of a varying angle of inclination, (2) to assist the learners to understand basic trigonometric ratios, (3) to assist learners to use calculators to calculate trigonometric ratios, and (4) to discuss the endeavours of Day 4 (launch day).

On Day 3, I encouraged reflective engagement by the learners, by only engaging in How and When questions. The learners were prompted, through class discussions and completion of sums similar to those of Day 2, to identify that the new mathematics concept that would be covered on Day 3 would be different from the previous day's work, in the sense that the reference angle (angle of inclination) would not be a constant 30^{o} angle anymore. I set up a practical demonstration of a water bottle on a ledge (representing the stationary position of the water rocket), a book as the launchpad, and a box at the position of the observer. I instructed the learners to sketch what they saw, and from their sketches determine the maximum height of the water bottle. The learners were provided with a large wooden inclinometer and protractors. This instruction triggered excitement and increased anticipation of which group would be able to calculate the correct answers first. I did not tell the learners how to use the inclinometer, which encouraged many group conversations and practical trial-and-error behaviour by the groups (which will be discussed further in Chapter 6). Learners were able to construct the correct sketch, but soon realised that the angle of inclination is not 30^{o}, which caused a barrier for them to continue with the sum. After all learners had reached this point – of not knowing how to continue – I handed them sealed envelopes with "top secret information" inside (refer to Addendum F). The enclosed sheet introduced and provided guidance for using the basic trigonometric ratios they needed to be able to work with any sized reference angle. Learners continued to complete practise examples (Addendum F, Learner Workbook p. 14), where the angle of inclination differed in each sum, to enhance their confidence in calculating similar sums on launch day (Day 4).

I ended Day 3 by encouraging learners to engage in thorough planning for launch day, (1) by having clarity about how to measure the angle of inclination of moving objects correctly, (2) by identifying the role that each learner in the group would fulfil, to be optimally efficient in taking measurements, and (3) by revising all important mathematics concepts they had learnt that week.

5.3.4 Day 4: Launch day

The aim of Day 4 was to gather data, correctly, to work efficiently as a team, to do precise measurements and calculations, and to report on all findings in a scientific manner.

Before the learners were directed to the launch area, all groups were given the chance to reflect on their planning of the previous day. Although all groups were able to say what they would be doing when they reached the launch area, it took a bit of effort by the researchers to guide the learners to carry out their plans effectively. It is not clear whether this was because of the divide between planning and practice, or whether there had been a passenger effect, with the leaders having been able to explain what should be done, but struggling with the execution, because it required action by all members of each group, some of whom had not yet internalised the plan.

All groups were responsible for their own data and the researchers or assistants did not help the learners to collect and manage their data. Instead, the learners were expected to work together and help each other if anyone failed to understand something. The learners divided themselves into groups: some launched the rocket, some were positioned at the observer point with the inclinometer, others captured the data and started with rough sketches and calculations, and some evaluated the accuracy of the angle of inclination. A high degree of engagement was evident, although there were five of the learners (19%) whom I observed not cooperating with their teams.

All groups were asked to complete their reporting documents (refer to Addendum F, Learner Workbook pp. 16–20), and to be ready to present their findings of the launch to the rest of the learners on the final day of the programme.

5.3.5 Day 5: Reflecting and presenting

The aim of Day 5 was to give learners the opportunity to present their findings and to reflect on the intervention of the past week, and to emphasise that science and mathematics are enjoyable and engaging subjects.

At the start of Day 5, learners all wrote a post-test in both science and mathematics. Next, learners presented their findings in a scientific manner. This was done in the format used in the Expo for Young Scientists Science Fair. Each learner had a poster board, on which they pasted their investigative report (reported on in greater detail in Stott, 2019), which included their calculations of the maximum height of the rockets launched under various investigated conditions. The learners were randomly paired with one another and required to deliver a verbal presentation of their work. Several adults, including all the teachers involved in the programme, and others invited to view the presentations, walked around the presentations, engaging with the learners. After this, the learners engaged in focus group discussions, after which we ended the week with lunch, took photographs, and engaged in informal social conversations.

5.4 CONCLUSION

In this chapter, I described the context of the intervention programme, the philosophy of the programme, and my background and philosophy of teaching. I presented the detailed programme of the week, followed by a description of the aims and activities of each of the five days. This was a weeklong, problem-based, mathematics-focussed programme, aimed at skilfully guiding learners to inductively employ basic trigonometric ratios in taking measurements and performing calculations to determine the maximum height a water-launched rocket could reach. The focus shifted from Day 1 to Day 2, from revising basic triangle characteristics and the Pythagorean theorem, to investigating the relation between side-length ratios to various, constant 30^{o} reference angle triangles. Then, on Day 3, the focus expanded to include triangles with varying reference angles, and learners' new conceptual knowledge of trigonometric ratios was implemented. On Day 4, the focus continued to expand, by including all concepts learnt the previous three days and applying theory to practice on launch day. The last day's focus was on presenting all findings in a scientific manner and reflecting on the experiences of the past week. In the next chapter, I will analyse and interpret the Grade 8 and 9 township learners' responses to this problem-based programme in accordance with the theoretical framework outlined in Chapter 3 and guided by the study's research questions.

CHAPTER 6: ANALYSIS AND INTERPRETATION OF FINDINGS

6.1 INTRODUCTION

The analysis of data in this study will be formulated to answer the general research question and associated sub-questions listed below, and to present evidence for assertions made in response to these questions. The general research question is:

> *How do grade 8 and 9 township learners respond to a problem-based mathematics extension programme?*

With the abovementioned question in mind, the study unfolded in accordance with the following subsidiary questions, based on the MPS framework (refer to Section 3.4):

a) How do learners engage in the cognitive components of problem-solving?

b) What are learners' affective responses to the programme?

6.2 ENGAGEMENT IN COGNITIVE ATTRIBUTES

In the following section, I will present four assertions based on the data collected. I provide arguments in support of these assertions, by drawing on literature and empirical evidence. The assertions collectively answer the research questions in Section 6.1.

6.2.1 Resources: Assertion 1

> *The learners showed evidence of employment of resources, many instances of which were incorrect, but they generally needed to be prompted to do so.*

6.2.1.1 **Employing resources**

The coded evidence of resources evident over the course of the programme is displayed in Table 6.1, which reports evidence of procedural knowledge, wrong knowledge, conceptual understanding, misconceptions, technology use during calculations, and written materials, when learners referred to textbooks or notes. In mathematics, procedural knowledge relates to the ability to apply the correct algorithms during the problem-solving process (Byrnes & Wasik, 1991), and conceptual knowledge refers to understanding of concepts, principles, and relationships (Hiebert & Lefevre, 1986). Therefore, my understanding of a misconception is that it is an incorrect representation of a mathematics principle or concept. Wrong knowledge and wrong procedures are understood as being evidenced when learners were observed to use algorithms incorrectly or making careless errors. Although learners displayed evidence of using resources (refer to Table 6.1), the most remarkable finding is that learners were comfortable about making mistakes, such that they did not show signs of fear of failing in attempting new ways to solve the problem, which is in contrast to what the literature reports, as will be discussed in the next paragraph.

Table 6.1: Coded evidence of resources

Code	Description of code	Number of codes
RK	Knowledge, facts, and procedures	44
RKN	Wrong knowledge and procedures	52
RC	Conceptual understandings	22
RCN	Misconceptions (negative/opposite of RC)	19
RT	Technology	7
RW	Written materials	10

Source: Terminology of codes from Carlson and Bloom (2005, p. 51)

Table 6.1 indicates that, during the employment of resources, there were, in total, 154 coded pieces of evidence, of which 46% referred to errors, i.e. wrong knowledge (52 codes) and misconceptions (19 codes). The coded evidence for knowledge, wrong knowledge, misconceptions, and conceptual understandings can be broken down further, as displayed in Table 6.2.

Table 6.2: Breakdown of daily coded responses of knowledge and conceptual understandings

	Day 1		Day 2		Day 3		Day 4		Day 5		Total
	Workbooks	Audio & Video Recording	Workbooks	Audio & Video Recording	Workbooks	Audio & Video Recording	Workbooks	Audio & Video Recording	Workbooks	Audio & Video Recording	
Knowledge, facts, and procedures	3	2	4	3	5	6	8	2	11	0	44
Wrong knowledge and procedures	0	8	2	7	9	6	10	4	6	0	52
Conceptual understandings	0	2	1	4	1	5	1	3	2	3	22
Misconceptions (negative/opposite of RC)	1	4	1	3	2	2	0	2	1	3	19

As shown in Table 6.2, no evidence of errors could be found in the learners' workbooks on Day 1, although some errors were evident in the audio and video recordings (8). This finding is consistent with that of Hobden and Hobden (2019), who found that learners from township schools tend to wait for their teacher to give the correct answer, which they copy into their books, to provide immaculate work. Although the book is immaculate, it does not reflect the learner's true sense of understanding and knowledge and does not allow for the vital process of error-making in the learning process (Hobden & Hobden, 2019). There was, however, an increase in coded evidence of errors in workbook data sources of learners over the course of the week, as shown in Table 6.2.

An increase in coded evidence was observed, generally, though the coded evidence of conceptual understanding in workbooks was low throughout the week, since it was difficult to deduce conceptual understanding from the learners' written work – they might have, for example, copied work from a peer. The aforementioned finding is consistent with the findings of Van der Berg, Spaull, Wills, Gustafsson, and Kotzé (2016), who found that learners at township schools usually show very little to no evidence of conceptual understanding and knowledge. The TIMMS results also indicate that mathematics knowledge and understanding of most South African learners are extremely poor (Reddy et al., 2016). A low level of content knowledge and conceptual understanding was evident in this study's findings (Workbooks, audio recordings, video recordings, fieldnotes), which shows that these learners are, indeed, novices, who experienced high cognitive load due to limited working memory (Schmidt et al., 2007).

In the following two paragraphs, examples will be presented from the qualitative data to support Assertion 1. Many misconceptions were evident throughout the programme. The misconception indicated in Figure 6.1 was evident in 17 (65%) of the learners' pre-tests for question 1b. This shows that the majority of the learners could not distinguish between different measurement units. These learners used the side length of the triangle and the angle sizes in one linear equation, which shows that the learners did not realise that addition or subtraction cannot be performed using unlike units.

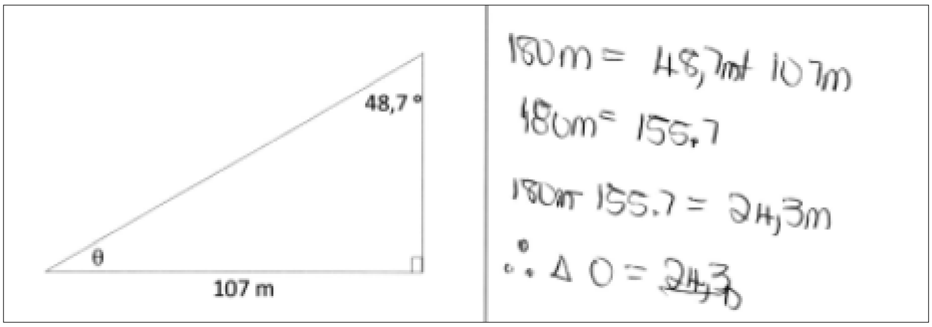

Figure 6.1: Example of the misconception that unlike units can be added

Another misconception, indicated in Figure 6.2, was evident for 11 of the learners (42%), who did not know the mathematical meaning of an equation, in the sense that they viewed the equals sign as a symbol that separates a problem and the answer (Kieran, 1981). The example below indicates that the height, which does not even relate to the question asked, is equal to 48, which is also equal to 107 m, and also equal to 5 136, thus, this learner claims, $h = 48 = 107\ m = 5136 = 5136 \div 107 = 48$.

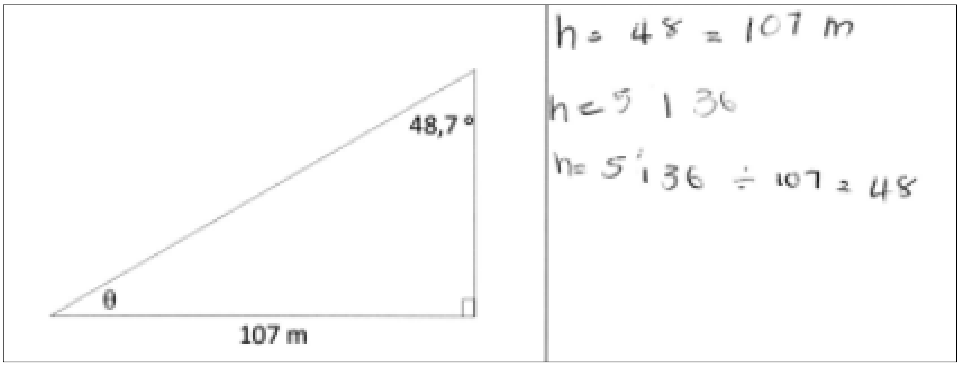

Figure 6.2: Example of the misconception regarding equations

In addition to the two examples illustrated in Figures 6.1 and 6.2, from the pre-test, I detected an increase in misconceptions and wrong knowledge in the written work from Day 1 to Day 4 of the programme (see Table 6.2), which suggests that learners may have started to feel more comfortable about making mistakes in their written work. This finding may suggest that the learners developed an appreciation of the value of mistakes during the problem-solving process from the start to the end of the programme. Even though making mistakes is a crucial part of the learning process, and learners need to have the freedom to make mistakes (Boaler, 2013; Hobden & Hobden, 2019), making mistakes is not encouraged in township schools (Hobden & Hobden, 2019).

6.2.1.2 **Generally needed to be prompted**

Throughout the course of the week, I frequently prompted learners to explain their thinking, by asking Why and How questions (refer also to the dialogues in Section 6.2.1.2), to assist learners with developing conceptual understanding, or to identify the limits of the learners' mathematics knowledge. Learners at township schools generally show evidence of limited mathematics knowledge (Van den Berg et al., 2016), which emphasises the central role a teacher should play in providing scaffolded prompts and guidance to assist learners to engage collaboratively in constructive processing of knowledge (Hmelo-Silver & Barrows, 2006).

On Day 1, I set up a physical demonstration of the water rocket, using general stationery to imitate the launch. Learners had to individually represent what they saw on a sketch in their workbooks (refer to Addendum F, Learner Workbook, p. 2). After walking around in class, I identified four different types of schematic representations given by the learners. I redrew the four sketches on the blackboard, without identifying which learners' sketches each was. I led the class through a discussion, in which we critically analysed each sketch and identified the sketch that represented the demonstration best. The following discussion took place:

Teacher	*Now, in general, everyone displayed the same type of shape. What was it?*
Class	*Triangles.*
Teacher	*Yes, a triangle. Now, why do you think that triangles would help us in finding the height?*

Class	*[silence]*
Teacher	*What do you know about triangles? What theorems can be used when working with triangles?*
Learner A	*Oh! The theorem of Pythagoras.*
Teacher	*Why?*
Learner B	*It will help us find the height.*
Teacher	*Can we only determine the height?*
Learner C	*No.*
Teacher	*Please explain why you disagree.*
Learner C	*You can use it to measure any side.*
Teacher	*Ok, listen to what you are saying. You are saying that the theorem of Pythagoras is used to measure any side. Do you all agree?*
Class	*Yes.*
Teacher	*[frowning and squinting]*
Class	*[silence]*
Teacher	*To measure?*
Learner B	*Oh no! Not to measure; you use it to calculate the side lengths.*

The discussion above illustrates the important role of the teacher in providing guided prompts to skilfully address wrong knowledge and misconceptions, and how learners should express what they mean. The predominant answering method used in township schools is the *chorus method* (Hoadley, 2018), which is exemplified by the class' response of "triangles", or "yes" in the extract above. This method is not conducive to individual reflection or metacognitive engagement (Hugo, 2019). I often reminded the learners to be prepared to justify their answers. I would regularly give prompts, like,

> *Guys, can you see that, as scientists, which you are this week, you need to be careful and choose your words well. You always need to be able to give proof of what you have based your decision on.*

The brief answers of the learners, as exemplified in the dialogue above, are typical of the findings throughout the data: they only justified their answers after prompting by the teacher. At the beginning of the week, there were many sections of teacher-learner dialogue in which the whole class responded to the teacher's questions with silence, possibly since these learners were not used to being expected to answer non-recall questions (Hugo et al., 2008). However, it appeared to me that, towards the end of the week, there were fewer silent responses when I asked questions.

The conversation below illustrates how I guided the learners to realise that the theorem of Pythagoras would not be sufficient to determine the height if one of the three side lengths is known. I endeavoured to regularly model reflection and questioning behaviour, as shown in the first section of the conversation below.

Teacher	*Ok. Now, let us just pause and examine what we have and what we can measure. If you think of the theorem of Pythagoras, at least how many side lengths of the triangle do you need to find the length of the unknown side?*
Class	*Two.*
Teacher	*So, at least two of the tree sides are necessary to work with the theorem of Pythagoras?*
Class	*Yes.*
Teacher	*So, will we be able to measure the range?*
Class	*Yes.*
Teacher	*[encircles the range and draws a tick next to it] Yes. And what about the hypotenuse?*
Class	*Yes.*
Teacher	*Will we be able to measure the hypotenuse with measuring tape?*
Class	*[silence]*
Learner C	*No, that is impossible. We will measure the height.*
Teacher	*Ok, how will you do that?*
Learner C	*Measuring tape.*
Teacher	*And if your rocket goes to the moon?*
Learner C	*[laughs]*
Teacher	*Come on fellow scientists, jump in and help us.*
Learner D	*We cannot measure the height and the hypotenuse. It is impossible.*
Teacher	*Oh goodness me. What now?*
Learner D	*We can only find one of the three sides.*
Teacher	*What does that mean?*
Learner D	*Pythagoras will not work.*
Teacher	*Oh no! Now what now? I think we should just give up.*
Class	*[shouts] No!*
Teacher	*But this is a big problem. You only know how to use Pythagoras, and it cannot help us. We have a huge problem. Or no, actually, you have a big problem. Because at the end of the week you will have to solve this to be able to launch our rocket and I am not giving any answers.*
Class	*[laughs and makes cringing expressions]*
Learner D	*We will have to find a new way.*

Most of the responses in the conversation above came from top achievers. The aim of this discussion was to assist learners to realise that finding the maximum height requires more than using Pythagoras. I had initiated this discussion because I had anticipated that learners would revert to the Pythagorean theorem for solving this question. However, simply conducting this discussion was clearly insufficient for some of the learners to realise the need for additional learning, as suggested by three learners' body language and comments immediately after the discussion above, which suggested that they thought their current knowledge was sufficient to solve the problem (Fieldnotes, Day 1). This is consistent with the reasons proposed by Marais, Van der Westhuizen, and Tillema (2013) for why learners avoid asking for help: (1) not caring about the answer, which refers to a lack of engagement from the learners' side; or (2) ignorance, which refers to a lack of metacognitive awareness that they are in need of assistance. Teachers need to be able to identify these queues and channel the reasoning and thought processes of learners, in order to contribute to meaningful learning.

6.2.2 Heuristics: Assertion 2

The learners displayed a diverse repertoire of heuristics, but generally needed to be prompted.

6.2.2.1 **Variety of heuristics**

As shown in Table 6.3, the learners presented evidence of using a wide variety of heuristics throughout the week. The qualitative descriptions given in Sections 6.2.2 and 6.2.3 provide further support for my claim that learners used a variety of heuristics. The seemingly low number of counts per heuristic in Table 6.3 may be caused by several factors, and it should be borne in mind that (1) the data collection method was not well suited to accessing the learners' cognitive processing, since participant observation, instead of think-aloud protocols, were used, consistent with the pragmatic, intervention-focussed approach taken, furthermore, (2) as explained in Section 4.6.1, counts of evidence of heuristics were consolidated in the analysis process, so that it only reflected once per learner group, and across data sources. It should be pointed out, however, that, since learners largely worked in groups composed of learners with a range of mathematics abilities, it is highly likely that the data that was collected does not represent the activities of all the members of each group.

Table 6.3: Number of true coded instances of evidence of heuristics

Code	Description of code	Number of instances of true evidence
HT	Constructs new statements and ideas	4
HP	Carries out computations	3
HR	Accesses resources	1
HW	Works backwards	1
HO	Observes symmetries and similarities	2
HS	Substitutes numbers	4
HM	Represents situation with a picture, graph, table, or action/movement	5
HC	Relaxes constraints or generalises the problem	1
HD	Subdivides the problem	1
HA	Assimilates parts into whole or adds the subdivided parts to make sense	0
HL	Alters the given problem so that it is easier	1
HE	Looks for a counter example	1

Source: Terminology of codes from Carlson and Bloom (2005, p. 51)

The learners' use of multiple heuristics is in stark contrast to the passivity known to typify South African township learners (Fish et al., 2017; Hoadley, 2018). In fact, the number of heuristics listed in Table 6.3 even contradicts the limited heuristic use found during problem-solving by learners of high quintile schools (Hobden, 2000). However, the learners' use of multiple heuristics is consistent with studies of learners engaging in discovery learning, where ample time, appropriate resources and a supportive environment are involved (Veermans, Van Joolingen & De Jong, 2006). Under these conditions, learners have been shown to display a natural creativity and curiosity (Goldenberg, 2019). Such conditions are not typically present in South African schools, particularly township schools. Given the general low mathematics content knowledge of teachers in township schools (Taylor, 2019; Van der Berg et al., 2016), and the content-heavy curriculum and focus on high-stakes examinations in the South African context in general (Stott & Hobden, 2008), it is unsurprising that low heuristic use is typical amongst South African learners. These conditions in South African schools are not conducive to the time and energy-consuming and knowledge-intensive process required to promote higher-order thinking skills (Stott, 2008), which include problem-solving heuristic use (Singh et al., 2018). Teachers in township schools are known to have very limited expectations of learners, with Hugo et al. (2008) failing to find evidence of question-posing beyond the recall level of Bloom's taxonomy (Anderson & Sosniak, 1994). Although teachers in high quintile schools do tend to expect learners to answer higher-order questions (Hugo et al., 2008), their approach to problem-solving instruction tends to involve modelling a limited variety of heuristics for the learners to copy, rather than prompting learners to generate their own heuristics in an exploratory, problem-solving manner (Hobden, 2005).

6.2.2.2 **Generally needed prompting (guidance provided, and heuristics modelled)**

Modelling and prompting the use of heuristics encourages higher order thinking during the problem-solving process (Snyder & Snyder, 2008). I experienced that, as I did this, initially, the learners displayed behaviour typical of literature descriptions of township learners (for example, that by Hoadley, 2018). However, by the end of the first day, the discussions I observed in two of the four learner groups displayed evidence of heuristic use (Field notes, Audio recording; Day 1). As the week progressed, more instances, and a greater variety, of heuristic use was evident in the data. For 14 of the learners, no evidence of heuristic use could be found in the data. For the remaining 12 learners, heuristic use can be classified along initiation and efficacy dimensions as follows. On the initiation dimension, heuristic use was either guided or spontaneous. On the efficacy dimension, heuristic use can be described as having been fruitful or unfruitful. By guided use of heuristics, I refer to learners mimicking a teacher-modelled heuristic when prompted to do so, as opposed to learners making self-directed heuristic choices, referred to as spontaneous heuristic usage. By fruitful heuristic use, I refer to learners employing heuristics that progressed their solution in a meaningful manner. Although spontaneous heuristic use was observed for nine of the learners, this was never fruitful, whereas guided heuristic use was mostly observed to be fruitful. For three of the learners, fruitful spontaneous use of heuristics was evident by the end of the week. In the sections to follow, I will discuss guided heuristic responses and spontaneous heuristic responses.

a) Guided heuristic response

The following description illustrates learners' use of guided heuristic responses. This occurred on Day 2, after I had shown the learners a demonstration of a scale model of the launch of the water rocket that formed the context of the programme. I asked them how they would calculate the rocket's maximum height. There was general agreement amongst the learners that Pythagoras could be used, revealing the misconception that the Pythagorean theorem can be used to calculate the length of an unknown side given only the length of one of the sides of the triangle.

I then spent 17 minutes modelling, observing symmetries, generalising constraints, sub-dividing the problem, and asking guided questions to assist the learners to alter the given problem, to make it easier. After another 8 minutes of discussions, two learners commented respectively, "oh, this is not so easy", and "Pythagoras will not work" (Video recording, Day 2). This indicated that these learners had realised their initial error. All learners were asked to think about the error and possible ways to solve the problem for homework.

The following day, the learners had to complete the investigation worksheet in their workbooks on page 8 (refer to Addendum F), which guided them to investigate the ratio between different triangles' interior angles and side lengths. This investigation was designed to prompt learners to realise that right-angled triangles of particular reference angles have constant side-length ratios. I had not highlighted this fact before, and since trigonometry, which rests on this premise, is only introduced by the South African curriculum in Grade 10 (Department of Basic Education, 2010), it is extremely unlikely that these Grades 8 and 9 learners were aware of this relationship at this point, nor did I point out to the learners that this investigation was related to the practise problem (refer to Learner Workbook p 9, in Addendum F).

The targeted outcomes of the practise problem included that the learners would be able to 1) identify the similarities between the practise problem and the previous day's investigation, by identifying that all triangles used have a reference angle of 30^o, and that there was only one side length provided for all triangles, and 2) identify symmetries and apply this, to set up an equation to assist in finding the hypotenuse. None of the learners could identify the similarities between the practise problem and the investigation on their own. After three minutes of letting the learners grapple with the practice problem, I asked the following question: "Compare the sums of yesterday and the sum of today; what is exactly the same? Do not answer yet, if you find it, draw a circle around the numbers of shapes or questions which are exactly the same." During the next five minutes learners paged back and forth in their books, 11 learners started writing, and others (15) sat quietly, waiting for me to give the next step. It appeared to me that the top-achieving learners were the only ones with their heads down, writing. I walked around the class and saw that those learners who were writing were either setting up equations, or copying the ratios to the practise problem, and most of the other learners had only circled the 30^o angle and seemed to be stuck – they could not continue. Then, I discussed the reason for identifying similarities, by emphasising the link of similarity and setting up an equation. I showed the learners on the blackboard how to plug values into an equation and instructed them to set up an equation with aspects similar to that of the sums. This still seemed to be too difficult a task for the learners. I realised that the learners still needed to link the 30^o angle with the ratio of 0, 6 ($\frac{6}{10}$). This ratio is the tangent ratio when the reference angle is equal to 30^o. After a class discussion, consensus was reached that an equation should be set up, and this equation would only work for a reference angle of 30^o in both triangles. I guided the learners further with questioning techniques to the final derived equation of $\frac{opposite}{adjacent} = \frac{6}{10}$. This guidance appeared to be sufficient for the learners to complete the calculation and determine the maximum height of the practise problem, since I observed many learners smiling, grabbing their pens, and writing.

Even though learners were able to substitute the values correctly into the equation, the majority (14 learners) could not perform the basic procedure of making the numerator of the fraction on the left-hand side of the equation the subject of the equation. I then spent another 8 minutes revising the concept of making a certain value the subject of the equation, after which learners used this procedural mathematics knowledge to complete the equation. When all learners indicated that they had completed the sum, I did the sum on the board, demonstrating and talking my way through my actions of finding similarities, substituting, and carrying out computations. All the learners seemed to have performed calculations correctly, because, when I asked who had the answer correct, everyone put up their hands. However, when I marked their workbooks, four learners had not corrected their incorrect answers, which indicated that they were either not paying attention, or they were too shy to confess that they could not do it.

The example discussed above describes the way I guided the learners with scaffolding techniques and skilful questioning, in an attempt to serve as an expert collaborator, and as a resource of additional working memory space for the learners to access, thus, reducing the individual cognitive load required for the learners to manage a solution (Kirschner et al., 2018). The description also suggests that the learners were operating in their ZPD (Wilson & Devereux, 2014), since they were unable to make progress without my guidance, but generally responded well to this guidance. I modelled the use of the following heuristics: observe symmetries and similarity, substitute numbers, and carry out computations. At very specific times, I prompted the learners on exactly how to use these heuristics to be able to get to the next step in their calculations. In this example, 22 learners showed evidence of fruitful heuristic use, after being prompted.

b) Spontaneous heuristic response (guidance provided, but no heuristics modelled)

An example of a spontaneous heuristic response occurred on Day 3, during the practical activity (refer to Learner Workbook, p. 12), when learners were given big, manual inclinometers, without being told how to use them. None of the learners knew what an inclinometer was. I gave the instruction: “Each group can get one of these special tools to possibly help you. Discuss in groups and let’s see if you can use it effectively to determine the maximum height” (Video recording, Day 3). Whilst learners were trying to figure out what this new tool was, I placed a water bottle on a ledge, to indicate the position of the water rocket when it has reached its maximum height. I told the learners that, at this instant, I had stopped time to assist them to form a picture of how the maximum height can be calculated. I also told the learners where the launch pad was positioned and reminded them that the observer is situated at the launch pad. This practical activity is a smaller-scale example of what would be happening on Day 4 (launch day), and a bigger-scale example of the first demonstration of a water rocket launch on Day 1, when I had used a highlighter and eraser to represent the rocket and launchpad.

Figure 6.3: Learners of all groups working together to try find the hypotenuse length

I asked the learners to use their inclinometers to assist them to find the maximum height of the water rocket. All the learners wanted to measure the hypotenuse side of the imaginary right-angled triangle formed with the simulated rocket and launch pad serving as apexes, which indicated a misconception – some of the learners reverted back to the idea that Pythagoras should be used to calculate the maximum height, and that they expected a rocket to remain stationary mid-air. To measure the hypotenuse side, learners tried to position all the inclinometers end to end, as shown in Figure 6.3. All the groups spontaneously started working together to do this, and it was evident (Video recordings; Fieldnotes, Day 3) that the learners started questioning each other about whether this method was actually feasible, because one learner asked why they would need an angle of inclination if they could just determine the hypotenuse (Fieldnotes, Day 3).

At this moment, I stepped in and started reflecting on the feasibility of using the inclinometer. I, first, asked them if they would be able to use this method the following day, highlighting the fact that they would not be able to stop time. I asked the following guided questions to indicate their incorrect use of the inclinometer: "Why do you think this tool can be adjusted?", and "Do you think that this tool can maybe also point at the maximum height?" (Fieldnotes; video recording, Day 3). After hearing this, a learner jumped up and said: "it can be used here [standing on the launch pad], it will point", then another learner said, “we can then measure [this] angle, because this is what we need” (Video recording; Fieldnotes, Day 3). This led to the learners trying to work with the inclinometer, as illustrated in Figures 6.3, 6.4 and Figure 6.5.

Figure 6.4: Learners measuring the angle of inclination

Figure 6.5: Learners discussing the unknown height problem

One group (shown in Figure 6.4) correctly measured the range and angle of inclination, and substituted their values into the equation of the previous problem activity (refer to Learner Workbook p. 9), where the ratio was 0,6. The learners did not realise that this would not give the correct answer, because that ratio is only applicable to 30º reference angles. The learners in this group even checked their answers, by working backward, but still did not realise that they did not incorporate the angle of inclination as measured. This showed the spontaneous use of the following heuristics, albeit unfruitful in their efforts: 1) substitution, and 2) works backward.

Another group (shown in Figure 6.5) did physical demonstrations of how the rocket would be launched, and how the observer would open the inclinometer to indicate the point where the rocket would reach the maximum height, using their arms, hands and a variety of stationery. After the learners had spoken to one another within their group, they were observed to run back to their group's table and demonstrate the 3D display of the water rocket, on a 2D surface, for them to collaboratively redraw the triangle on paper. They paged back to the previous sections of their workbooks, and started discussing the previous day's problem activity, in what appeared to be an attempt to find similarities. This group seemed to realise that they could not use the ratio of 0,6, since one of the learners started arguing with another group member, pointing at the 30^{o} angle and the angle of inclination which they had measured as 70^{o}. This group then tried to change the 70^{o} proportionally, to reduce it to 30^{o}, and then started multiplying and dividing with various (incorrect) values (Fieldnote, Day 3). When I asked them what they were doing, they said they wanted to get the angle of inclination to 30^{o} to work with the previous day's equation. This group's final answer was incorrect; thus, they were unfruitful, but showed spontaneous use of the following heuristics: 1) Represents situation with an action, 2) observes symmetries and similarities, and 3) substitutes numbers.

My findings agree with Anggrianto, Churiyah, and Arief (2016), who suggest that using heuristics develops a learner's critical thinking skills. A teacher can promote learners' development of these critical thinking skills through using scaffolding, guided worksheets, and prompting (Stott, 2008). As discussed in Section 2.3.3, this type of prompting is not common in township schools, which is why it was expected that learners would need regular scaffolded guidance, and prompting, since these learners are not used to being expected to engage in higher-order thinking (Stott, 2018). It is likely that these learners do not possess the necessary space in their working memories to be able to solve the problem posed unaided. This is because their novice status and associated inability to chunk information (Kirschner, 2002), would have caused them to experience a high intrinsic load from the problem-solving process. Scaffolding of tasks reduces cognitive load (Belland, Kim, & Hannafin, 2013), and collaboration between peers and the teacher allows pooling of cognitive resources, thus, extending the effective size of working memory (Kirschner et al., 2018).

Those learners who exhibited spontaneous heuristic use were the highest achievers in the sample. This is consistent with cognitive load theory, since academically stronger learners tend to have better organised knowledge structures, and, therefore, experience the limitations of working memory to a lesser degree than weaker learners do (Kirschner et al., 2018).

The following features of the programme appear to be relevant to understanding why the learners were able to display a variety of heuristics, despite this practice being largely foreign in the context of township education: (1) This programme did not focus on test scores and on curriculum coverage; and (2) I incorporated and prompted the use of different heuristics in my teaching. It should be noted, however, that most of the evidence of heuristics involved responses from higher achieving learners within this already higher achieving sample of township learners, which poses a limitation on generalisation of these assertions to the broader population of township learners, or even the population of higher achieving township learners.

6.2.3 Monitoring: Assertion 3

> *Across the intervention period, the learners responded to the teacher's modelling of monitoring by increasingly posing Why and How questions but appeared unable to apply this to direct engagement in iterative problem-solving.*

6.2.3.1 **Teacher modelled monitoring across the programme**

Throughout the course of the week, I focussed on using teaching prompts, such as questioning, by asking How and Why questions, since it is known that using task-appropriate elaboration during the problem-solving process may promote problem-solving success (King, 1991), and may assist in keeping the learners' minds engaged in the problem-solving process (Carlson & Bloom, 2005). Meaningful engagement does not only refer to good communication between the teacher and the student, it also relates to the quality and intensity of learner participation in the problem-solving process. Meaningful engagements are influenced by the type and quality of teacher prompts, including questioning (Khoza & Nyamupangedengu, 2018). Throughout the course of the programme, I aimed to implement the programme in a manner consistent with literature on best practice, namely 1) to skilfully assist the learners' thinking processes, in order to reduce the learners' cognitive load through scaffolding and asking guided questions (as described in Sections 6.2.1 and 6.2.2); 2) to avoid giving the learners direct answers, which would have converted the task from a problem to a routine exercise, which no longer required the learners to operate within their ZPD (see, for example, the first dialogue in Section 6.2.1.2), and 3) to model metacognitive behaviour with the hope of encouraging the learners to mimic my behaviour. Table 6.4 indicates the large number of questions I asked throughout the programme.

Table 6.4: Number of How and Why questions asked by the teacher and learners throughout the course of the week

	Day 1	Day 2	Day 3	Day 4	Day 5
Teacher's count of How and Why questions throughout the week	62	51	55	29	33
Learners' count of How and Why questions throughout the week	2	9	16	4	2

A high count of coded data for teacher questioning was recorded for Days 1–3, since these days were mainly focussed on guiding the learners through difficult problem-solving scenarios related to finding the maximum height of the rocket – extreme effort from my side was needed to make sure the learners stayed actively engaged. The reason for the decrease in recorded teacher questions on Days 4 and 5 was limited data being captured on these days because of limited video and audio recordings being done on these days, since Day 4 was launch day and Day 5 was mainly spent reflecting on the week, and writing pre- and post-tests. This change in data collection pattern is consistent with the pragmatic paradigm that was used (Plowright, 2011) and is, therefore, not considered to be a failing of the research design. However, it is important to interpret the findings consistently with the method used.

6.2.3.2 **Learners increasingly adopted Why and How questioning**

From Table 6.4 it is evident that the learners' use of How and Why questions increased from Day 1 to Day 3, for which the amount of data collected is comparable. My experience of interacting with the learners was that this increase continued to Days 4 and 5, however, as explained in the previous section, this interaction was not captured in the data, given the different focus of the intervention, and, consequently, forms of data collection during these days. During the focus group discussions on Day 5, several incidents were recorded of learners reflecting on their experiences of questioning. However, these incidents were not coded as evidence of asking How and Why questions, since learners did not actually pose such questions in this data. I suggest that learners' increased use of How and Why questions was related to the fact that I had modelled such questions so extensively throughout the week.

The following extract of dialogue between two learners occurred when a more gifted learner explained to one of her peers how she had arrived at the correct answer to an investigation from which they should have concluded their general findings on the relationship between the side length ratios of a triangle and the size of the reference angle (refer to Figure 6.6). The learners should have used these findings as a prompt to complete the succeeding (refer to Figure 6.7) problem-solving exercise.

Learner A: *I think this is right. Because I looked at this sum [pointing to previous table in the investigation as seen in Figure 6.6] and it looked the same as this one [pointing at the current sum's 30^o angle as seen in Figure 6.7]. I saw that [reading from her book – refer to Figure 6.6] even though the sides are not the same, when we round off to one decimal place, they [are] equal, because they all have 90^o angles and their reference angle equals 30^o. They are all similar sides.*

Learner B: *But how? You cannot use it for all sums*

Learner A: *Why not, it works for all 30^o triangles?*

Learner B: *But this is a bigger triangle [referring to the side lengths of question 2.4 in the Learner Workbook)*

Learner A: *If you look at the table, all these answers are the same [pointing to the investigation table on page 8 of the Learner Workbook]. Then I made my equations the same and worked with same ratio of 0,6.*

		Lengths of the sides (in *mm*)			Ratios (Round off to one decmal pace)		
Triangle to measure	Reference Angle	Opposite side (Opp)	Adjacent side (Adj)	Hypotenuse (Hyp)	$\frac{Opp}{Hyp}$	$\frac{Adj}{Hyp}$	$\frac{Opp}{Adj}$
ΔABC	30°	55 ~~40~~ mm	60 mm	70 mm	$\frac{55}{70}$ 0,5	$\frac{60}{70}$ 0,9	55/60 0,6
ΔADE	30°	50 mm	85 mm	98 mm	50/98 0,5	85/98 0,9	50/85 0,6
ΔAFG	30°	65 mm	110 mm	125 mm	65/125 0,5	110/125 0,9	65/110 0,6

What can you conclude? (Discuss in your groups)

Even though the sides are not the same, when we round them off to one decimal place they equal. Because they all have 90° angle and their reference angle equals to 30°. They are all similar; All sides are propotional

Figure 6.6: Learner A's completed investigation (Learner Workbook, Day 2)

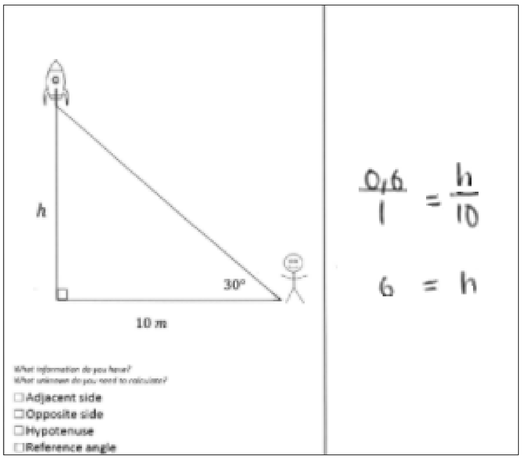

Figure 6.7: Learner A's problem-solving exercise 2.4 (Learner Workbook, Day 3)

The dialogue above is an example of learners engaging in a dialogue in which one learner questions the work of the other. It is noteworthy to state that Learner B did not accept that Learner A was correct. Because Learner A's solution did not make sense to Learner B, he questioned Learner A's calculations. Learner A was so confident about her answer and findings that she immediately responded with reasons why her calculations were correct.

During the focus group discussions on Day 5, learners were divided into three groups, with a maximum of nine learners per group. Various reflection questions regarding their experiences of the programme were asked. I will identify some noteworthy responses.

I asked the learners how they experienced the large amount of questioning that they were introduced to, and what they had learnt from it. The following responses were recorded (Audio recorded focus group discussions, Day 5):

> *When I go back to school, I will continue to ask myself Why, Why, Why with every step of the sum.*
>
> *There is always a reason for an answer, and I need to give it [the reason].*
>
> *Sometimes you write an answer and you do not know what you write.*
>
> *In this week I have learnt to ask questions for everything.*
>
> *I felt comfortable in asking for assistance if I was not sure.*

These responses indicate that the value of questioning oneself during problem-solving seemed to have been conveyed to the learners. One learner acknowledged that I conceptualised all teaching approaches around scaffolding and questioning, by stating that,

> *We were never given an answer. We had to read and think and write and read again and think again.*

I am familiar with the type of teaching and learning scenarios that occur during class time in the township schools, since I spent six years mentoring teachers in such schools. Consistent with descriptions given in academic literature (Hoadley, 2018), I know that teaching in this context is teacher-centred and operates at a low level of engagement from the learners' side. Nevertheless, I wanted to find out how the learners perceived the week's programme, in comparison to what they were used to. Except for one learner who said that "we do difficult work [in our schools]", the learners' responses suggested that their experiences in this programme contrasted with their usual experiences at their schools. One learner responded: "the teacher [at our school] speaks and explains a lot and shows us how to do everything and we only write"; another learner agreed, adding to the previous scenario: "yes, and when we do not understand something at school, the teacher would come and explain, but we would still not understand because he does not explain it clearly or he does not know how to". A third learner, in further agreement with his peers, said

> *Teachers would come into the class and say 'take out your books' and then he writes things on the board, and then erases it all without us copying*

> *everything. He also does not explain what he wrote. Then the lesson is over.*

These three explanations of how the learners experience teaching and learning at their own schools are in agreement with what literature states about the teaching in South African township schools generally, namely that they do not promote higher-order thinking or problem-solving skills (Hugo et al., 2008; Hoadley, 2018; Stott, 2019).

It was suggested by one of the learners that

> *Teachers need to let us think out of the box, and not only learn the textbook. And sometimes the textbook is wrong and then you are right, but you are afraid to tell the teacher, so you leave it (Audio recording, Day 5).*

This quote suggests that, even though the learners experienced this programme as mentally taxing and difficult, they prefer to engage in meaningful learning, in contrast to the rote and boring learning they encounter at their own schools.

6.2.3.3 **Learners appeared unable to engage in iterative problem-solving**

There was no evidence of learners engaging in iterative problem-solving. This is not surprising, since monitoring, which is a type of metacognition, is extremely intensive on working memory – even more so for novices (Kirschner, 2009). Experts, in contrast to these learners, possess well-connected knowledge and rich schemata, which frees up space in working memory (Kirschner et al., 2018). Even though learners were not able to engage in iterative problem-solving, it is remarkable that they were able to start the process of monitoring, checking and questioning. In addition to the aforementioned, the progress made and the responses that some of the learners showed during the contact time of one week, albeit small, is truly noteworthy.

6.2.4 Engagement in cognitive attributes over the problem-solving phases: Assertion 4

> *Learners engaged in the first three phases of the problem-solving process but showed no engagement in the checking phase.*

6.2.4.1 Engagement in the orienting, planning, and executing phases

Following the discussion of problem-solving attributes stated in Assertions 1, 2, and 3, learner responses were evident in three of the four problem-solving phases (orienting, planning, executing, and checking), and across all four of the problem-solving attributes (resources, heuristics, affect, and monitoring) (refer to Table 6.5). However, there was no reliable evidence to suggest that engagement occurred in the checking phase, and furthermore, no indication of a cyclical nature of the problem-solving process.

For problem-solving to be effective for constructing knowledge and developing a deeper understanding, certain problem-solving behaviours and attributes need to be evident in each of the four problem-solving phases (Carlson & Bloom, 2005).

During the orientation phase of the problem-solving process, behaviours of understanding the problem, constructing, and organising the information received are identified (Schoenfeld, 2016). The planning phase refers to the phase when learners construct conjectures and identify goals and patterns through accessing their conceptual knowledge and heuristics (Pólya, 1957; Carlson & Bloom, 2005). During the executing phase, the problem-solver engages in behaviours that include carrying out calculations based on the conjectures made in the planning phase, through accessing (factual and conceptual) knowledge, writing logical mathematical statements, and implementing planned strategies (Lester & Garofalo, 1989; Carlson & Bloom, 2005). During the checking phase, the accuracy of the written mathematical statements, conjectures, and calculations are verified and justified.

Table 6.5: Number of coded attribute evidence per problem solving phase

		Attributes of problem solving							
		Resources		Heuristics		Affect		Monitoring	
		Coded evidence	True evidence	Coded evidence	True evidence	Coded evidence	True evidence	Coded evidence	True evidence
Phases of problem-solving	Orienting	92	71	38	17	108	76	42	21
	Planning	67	61	11	5	79	59	19	6
	Executing	59	22	8	2	44	23	5	1
	Checking	0	0	0	0	0	0	0	0
	Total:	218	154	57	24	231	158	66	28

6.2.4.2 No engagement in the checking phase

True problem-solving does not involve a quick fix. To shift learners' metacognitive behaviours and ways of thinking requires an enormous amount of energy from the teacher's side and requires the consistent expectation that learners engage in the learning process for long periods of time (Stott, 2008). To achieve a shift in metacognitive behaviours or any evidence of higher-order thinking requires constant intellectual challenge from the teacher, coupled with constant scaffolding and support. Although very difficult, especially in the case of novices, evidence of this happening, even in a short programme similar to this research programme (Stott, 2019), can be identified.

No evidence of engagement in the fourth phase of problem-solving, the checking phase, was identified (refer to Table 6.5). According to Carlson and Bloom (2005, p. 70), a learner's ability to, at the right time, access useful mathematical knowledge, is highly dependent on the "richness and connectedness of the individual's conceptual knowledge". In contrast to experts, these learners did not appear to possess the ability to access this useful mathematics knowledge during the metacognitive phase (checking). Since well-connected conceptual knowledge influences all four phases of the problem-solving process, and these learners did not appear to possess these rich conceptual knowledge connections, it is not surprising that learners were unable to engage in the checking phase. Furthermore, for learners to fully engage in all phases of the problem-solving process, they need an enormous reservoir of behaviours, conceptual mathematics knowledge and reasoning patterns, which were not evident during the programme. In addition to the previous point, learners need a great deal of practise and experience in problem-solving approaches to be able to engage in all phases of the problem-solving process (Carlson & Bloom, 2005), which these learners, unfortunately, had not been exposed to.

6.3 AFFECTIVE RESPONSES

6.3.1 Affect: Assertion 5

> *Learners showed strong affective responses throughout the programme, but mathematical intimacy and integrity were not evident.*

6.3.1.1 **Interest in the programme**

Affective responses, as shown in Table 6.6, accounted for the greatest amount of overall coded evidence, compared to the cognitive responses that were coded. This high number of coded affective responses is inconsistent with literature findings in township schools, since learners from township schools rarely show affective responses during lessons, possibly because teacher-centred approaches dominate in these contexts (Hugo et al., 2008; Hoadley, 2018; Stott, 2018). When learners are not actively involved in the learning process, such as in lessons where rote and routine learning are expected, learners could lose interest in the learning process (Wilson & Devereux, 2014). Although routinisation of learning is seen to be effective for producing relatively good marks, learners find lessons that routinise learning boring (Hobden & Hobden, 2019). Low-challenge tasks, usually related to rote and routine learning, cause boredom, which can push learners to operate outside of their ZPD (Wilson & Devereux, 2014). High-challenge tasks, coupled with scaffolded support, can be extremely motivating, and may encourage learners to excel in their learning (Wilson & Devereux, 2014).

Table 6.6: Number of affective responses

Code	Description of code	Number of Codes
AA	*Attitudes*	-
AAE	Enjoyment	12
AAM	Motivation	13
AAI	Interest	8
AB	*Beliefs*	-
ABC	Self-confidence	14
ABE	Pride	9
ABP	Persistence	11
ABM	Multiple attempts are needed in problem solving	3
AE	*Emotions*	-
AEF	Frustration	22
AEA	Anxiety	11
AEC	Confusion	19
AEJ	Joy, pleasure	23
AEI	Impatience, anger	13
AV	*Values/Ethics*	-
AVI	Mathematical intimacy	0
AVG	Mathematical integrity	0

Source: Terminology of codes from Carlson and Bloom (2005, p. 51)

As seen in Table 6.6, general curiosity and *interest* (eight instances of evidence) were mostly displayed during the orientation phase of the problem-solving process, and this was always coupled with *motivation* (13 counts). Motivation appeared to be encouraged when the groups competed against each other to find an answer to my posed questions or challenges.

However, the aspect of time constraints, brought about by competition, may be counter-productive for learners who do not possess as much working memory as the top achievers, which can then encourage a passenger effect and be counterproductive to meaningful learning. Evidence of the occurrence of a passenger effect is suggested in the following two comments by learners during the focus group discussions (Audio recording, Day 5):

> *[when someone copies my work] it makes me feel bad, because that person doesn't want to use his or her own mind to think, they only use ours*
>
> *some kids do not want to do their own work in groups, because they say that the group will say that they are dumb and the others in the group are clever*

The nine records of evidence of *pride* comprised (1) four cases where learners were proud of their work and where their work was, indeed, correct; and (2) five cases where learners showed pride in their solution attempt and for managing to obtain an answer, even though their answers were incorrect.

The three cases coded as *multiple attempts are needed in problem solving*, were all responses from two top-achieving learners (two pieces of evidence coded for one learner, and one coded for another). I initially identified the relevant observations as indications of engagement in iterative monitoring (refer to Assertion 4), but when asked about their multiple attempts, both learners said they redid their sums because of the prompts that I gave, since what they had calculated did not correspond with my prompt. These learners did not seem to question me or try to identify why their answers differed from the prompt I had given, they just assumed they were wrong, and the teacher is always correct (Fieldnotes, Days 1, 3). The learners erased their first attempts before I could take a photograph, even though I had encouraged all the learners to always keep their initial answers.

Interestingly, the high number of negative emotional responses observed, which included *frustration* (22 counts), *anxiety* (11 counts), *confusion* (19 counts), *impatience or anger* (13 counts), were coupled with learners giving up and either waiting for me to assist, or doing something completely different, such as walking around the classroom (Video recording; Fieldnotes, Days 1, 2, 3).

Some pairs of items in Table 6.6 appear to be very similar, and I will describe how I implemented the coding for two such pairs: 1) Joy or pleasure, and Enjoyment; and 2) Motivation, and Persistence. Consistent with literature, I identified evidence of enjoyment as attitudes that are "consistent [and longer-lasting] displays of affects" (Carlson & Bloom, 2005, p. 64). Emotions such as joy or pleasure could last for a shorter time and, to a certain extent, be superficial, and not necessarily portray an attitude of true enjoyment. An example of *joy* would be when the class laughed after I made an amusing comment, and an example of *enjoyment* would be when a certain learner, throughout Day 4 (Fieldnotes, Video recordings, Day 4), asked many questions and regularly stated how much she was enjoying the rocket-launching process.

Another pair of seemingly similar items in Table 6.6, is *persistence* and *multiple attempts are needed in problem solving*. I identified *persistence* as relating to a learner struggling with a given mathematical problem, but the learner not necessarily reverting to different ways of completing the given sum, i.e. they continued along their initial path of approaching the problem to reach an answer. *Multiple attempts are needed in problem solving* was identified when a learner redid a sum completely to try a new approach to solving the problem, and only stopping when, it seemed, they believed they had reached an answer. The latter, if done spontaneously, coupled with evidence of the learner engaging in checking, can be seen as a metacognitive action. As mentioned previously, I did not interpret or code the three points of evidence as being metacognitive, since the learners were responding to a prompt I had given, rather than initiating the checking.

During the focus group discussions of Day 5, some learners reflected on problem-solving and the teacher's scaffolded approaches to learning. A number of learners referred to the practical aspect of learning:

> *It was easy to understand things, because it was not only theory. It was done practically too.*
>
> *Things were not only said once and then left behind. It was repeated in different ways and many times. It made it easy to take notes.*

It was good to have a small highlighter demonstration [on Day 1], then the bigger one on day 3, then the launch day practical [on Day 4]. It made it easy to know what we should do and how we should measure.

It was difficult on day 1 to draw a picture from a practical demonstration. But later it was easy.

One learner highlighted that creativity played a big role in the success of the week:

I feel that we needed to be creative the whole week. I do not think that we could do the sums if we were not able to be creative in thinking

To highlight the necessity of engaging learners affectively in the problem-solving process, so that they find an interest in what they are learning and feel motivated to persevere in the process of problem-solving, I quote a learner in Hobden's (2005, p. 308) study:

I believe school, as it has with all other subjects, has managed to destroy all the interest that is inherent in science— it is, after all, man's attempt at understanding the world around him. But this 'specialness' is lost in boring classroom routine, irrelevant trick questions and such like.

6.3.1.2 **Absence of mathematical intimacy and integrity**

Affective responses are more complex than merely expressing emotions such as enjoyment, anger, or frustration. Affective responses to problem-solving may also involve intimacy, integrity, and meta-effect, which form complex networks of affective pathways, to positively or negatively affect a learner's mathematics problem-solving ability (Carlson & Bloom, 2005; Schoenfeld, 2016). Intimate mathematics experiences (mathematics intimacy) create a deep connection between a learner and his/her mathematics. These types of deep connections have been regarded as behaviours such as a learner cradling his/her work in his/her arms or speaking passionately about the mathematics problems he/she has solved. Furthermore, mathematical intimacy and integrity are directly related, since the absence of integrity creates an obstacle to intimacy, and the absence of intimacy decreases the learner's need for integrity (Carlson & Bloom, 2005).

Negative emotions, as displayed by expressions of frustration, impatience, or anger, may also indicate high levels of mathematical intimacy (Carlson & Bloom, 2005). Although emotional responses like anger and frustration were displayed when, for example, five learners engaged in the checking phase of problem-solving, there is not enough evidence to suggest that these emotions indicated true mathematics intimacy, since emotional engagement in building mathematical meaning (Heyd-Metzuyanim, 2011) could not be determined, due to the nature of the intervention and data collected. In addition, since these learners' honesty relative to their understanding of the mathematics problem was not clear, mathematics integrity could not be coded. The observation that anger and frustration were evident indicates an engaged level of commitment towards a mathematics problem but could not be coded as evidence of mathematical intimacy and/or integrity.

6.4 CONCLUSION

In Chapter 6, I stated the following five assertions concerning the responses of the sample of Grades 8 and 9 learners from township schools to the problem-based mathematics extension programme:

- The learners showed evidence of employment of resources, many instances of which were incorrect, but they generally needed to be prompted to do so.
- The learners displayed a diverse repertoire of heuristics, but generally needed to be prompted.
- Across the intervention period, the learners responded to the teacher's modelling of monitoring by increasingly posing Why and How questions but appeared unable to apply this to direct engagement in iterative problem-solving.
- Learners engaged in the first three phases of the problem-solving process but showed no engagement in the checking phase.
- Learners showed strong affective responses throughout the programme, but mathematical intimacy and integrity were not evident.

In this chapter, I presented and supported my interpretation that the Grades 8 and 9 learners from township schools largely positively engaged in the programme, and they responded well cognitively within the understandable limitations of their working memory. Because of those limitations, a teacher's skilful guidance and a scaffolded teaching approach are crucial components of the programme's success. In the final chapter of this dissertation, I will consider possible implications of these findings.

CHAPTER 7: SUMMARY AND IMPLICATIONS FOR RESEARCH AND PRACTICE

7.1 INTRODUCTION

This was a case study informed by a pragmatic paradigm and the framework for integrated methodology, was used (Plowright, 2011). The aim of the study was to investigate the responses of the Grades 8 and 9 learners at township schools to the mathematics problem-solving extension programme they participated in in this study. Carlson and Bloom's (2005) MPS framework was used as the conceptual framework for this study. Data were collected from various sources and coded using NVIVO. A rich description of the data was given in Chapter 5, and this served as basis for analysing the data, presented in Chapter 6. A summary of the knowledge claims I make will be given below. I will continue to elaborate on the possible limitations of this study, and the implications for further research and practice.

The research question that guided this study is:

> *How do Grade 8 and 9 township learners respond to a problem-based mathematics extension programme?*

With the abovementioned question in mind, the study unfolded in accordance with the following subsidiary questions:

a) How did the learners engage in the cognitive components of problem-solving?

b) What were the learners' affective responses to the programme?

The holiday extension programme that was central to the research involved 27 Grades 8 and 9 learners from township schools, who participated in a five-day mathematics, science and English programme for six hours a day. The aim of the mathematics section of the programme was to use problem-based teaching, implemented according to the ladder approach teaching strategy (Stott, 2008) to teach trigonometry, so that the learners could apply this gained knowledge to measure and calculate the maximum height that a water rocket reaches, from observations of its flight.

7.2 SUMMARY OF KNOWLEDGE CLAIMS

On completion of this study, I am convinced that learners from low-quintile schools who show an interest in science and mathematics, and who are committed to a fair amount of self-regulatory work, can respond positively to a problem-based mathematics extension programme. Additionally, I made the following five assertions concerning the responses of the Grades 8 and 9 township learners in the sample to the problem-based mathematics extension programme:

- The learners showed evidence of employment of resources, many instances of which were incorrect, but they generally needed to be prompted to do so.
- The learners displayed a diverse repertoire of heuristics, but generally needed to be prompted.
- Across the intervention period, the learners responded to the teacher's modelling of monitoring by increasingly posing Why and How questions but appeared unable to apply this to direct engagement in iterative problem-solving.
- Learners engaged in the first three phases of the problem-solving process but showed no engagement in the checking phase.
- Learners showed strong affective responses throughout the programme, but mathematical intimacy and integrity were not evident.

The learners' responses to the programme, as laid out in the five assertions above, were remarkable, especially since, (1) it is unlikely that they had had prior experience with true problem-based learning (Hugo et al., 2008; Hobden & Hobden, 2019), and (2) it is very likely that these learners had poor mathematics knowledge, like most learners of South African no-fee paying schools (Reddy, Juan, Isdale, & Fongwa, 2019) and, therefore, had very limited working memory capacity, given their likely extreme novice status (Kirschner, 2009), thereby reducing the likelihood that engagement in problem-solving activity would be productive (Kirschner et al., 2006).

I will now reflect on the appropriateness of the various theoretical tools I used to guide the conceptualisation and implementation of both the mathematics problem-solving programme, and the research relating to this programme.

7.3 APPROPRIATENESS OF THE LADDER APPROACH FOR THIS CONTEXT

Though the ladder approach was, initially, conceptualised for a well-functioning school's Grades 10–12 learners in their everyday science (Stott, 2008), this programme showed that this approach can work in other circumstances too, given the evidence coded throughout the week (refer to Chapter 6). Since my broad planning of the programme, guided by the ladder approach (see Chapter 5, Section 5.3) and the implementation of the approach (as described in Chapter 5), correlated, I consider the ladder approach to have served as an effective planning tool.

Further, the ladder approach guided me in moving across the constructivist-instructivist continuum, in a manner appropriate for these learners' needs, as exemplified in the rich descriptions in Chapter 5.

The ladder approach is geared for solving a problem. The strong affective response referred to in Assertion 5 appears to be associated with the motivating effect that this problem-based approach had, as learners showed great affective responses in their attempts to solve the target problem, i.e. finding the maximum height of the water rocket.

7.4 APPROPRIATENESS OF COGNITIVE LOAD THEORY AND INFORMATION PROCESSING MODEL FOR UNDERSTANDING THE TRENDS OBSERVED

Cognitive load theory and the IPM of learning were useful for directing the planning and interpretation of the PBL approach and for directing the interpretation of the responses observed.

I found that the cognitive load theory and IPM were valuable and useful, particularly for exposing the importance of scaffolding to guide the choices I made in creating the resources, planning my teaching and conducting the lessons. In my opinion, these two theories are cognisant of the limitations imposed by the learners' working memory and guided me to be sensitive to signs that the learners may have been operating outside their ZPR and, therefore, required an appropriate prompt from me. Being aware of these theories guided me to be more observant of learner queues, which, in response, lead me to modify my instructional approach.

With reference to my five assertions, after analysing the data (Chapter 6), all assertions showed positive responses to PBL, which can be interpreted, in terms of cognitive load theory and IPM, as indicating successful implementation of my scaffolded guidance. These assertions also indicate that learners were only able to respond to a certain level of PBL, and these two theories, the cognitive load theory and the IPM, assisted me to understand why these limitations might be present in learners (novices).

I, therefore, experienced cognitive load theory and IMP to be versatile enough to guide all aspects of the creation, execution and evaluation of this problem-based programme. This is particularly remarkable considering that these theories tend to be used by proponents of instructivism (e.g. Kirschner et al., 2006), whereas PBL pedagogy tends to be considered to be more constructivist in nature (Kirschner et al., 2006). Perhaps, as argued by authors such as Hugo (2019), this finding points to the value of drawing on both extremes of the instructivist-constructivist continuum, as will promote the attainment of particular outcomes best, when designing instruction.

7.5 APPROPRIATENESS OF MULTIDIMENSIONAL PROBLEM-SOLVING FRAMEWORK FOR ANALYSING THE DATA

This framework was originally designed by Carlson and Bloom (2005) to study problem-solving behaviours, using think-aloud protocol for experts; nonetheless, it was a very useful heuristic to use to study novices too. The plausibility of this framework, and its relative fit for application in a novice context, makes it quite remarkable. This framework assisted me to code and group the data, which provided more structure to my data analysis process. However, in this study, I was not able to identify metacognitive behaviour.

The fact that novices exhibited responses, in terms of coded evidence, to PBL, might raise questions about the validity and reliability of coding; however, as discussed in Chapter 4, triangulation and peer-reviewed data comparison were used to strengthen the reliability and validity of the data.

7.6 LIMITATIONS OF THE STUDY

A number of limitations were evident in this study, among which the following:

- The claims made about learners' responses to the described teaching and learning programme cannot be generalised to all South African township learners, since the sample was not representative of South African township learners. This is because the learners who participated were chosen because they had voluntarily completed a project for the Expo for Young Scientists competition. It is, therefore, somewhat unsurprising that they showed great interest in the programme and were motivated to participate fully. Therefore, the claims made are confined to township learners who demonstrate interest and perseverance in mathematics and/or science in a similar manner as the learners I included in this study.
- The success of the programme relied greatly on the skills of the teacher. If a teacher's mathematics knowledge for teaching is insufficient, or his/her pedagogical approaches are not in line with true mathematics problem-solving, the responses of the learners might not be equally evident and replicating the programme might not render similar results (Tambara, 2015).
- The teaching context of this programme is not similar to everyday township classroom contexts, which have limited resources and are not always conducive to learning (Mampane & Bouwer, 2011). However, the aim of this study was not to research replication in township school contexts; instead, the focus of this study was to determine if township learners would be able to respond to PBL when conducted in a manner conducive to meaningful engagement occurring.

7.7 IMPLICATIONS OF THE STUDY

The first part of this section will elucidate a metaphor for the pedagogical approach that was used throughout this study, and then, recommendations for further research and practice will be discussed.

7.7.1 Recommendations for practice

In South Africa, there are, and have been, similar programmes piloted and implemented with problem-based approaches (Human, Hofmeyr, Human, Makae, & Van Koersveld, 2010), however, the majority of these programmes, especially those run by the Department of Education, are focussed on curriculum coverage and exam training. These programmes do not provide extension in learning and critical thinking for top-achieving or interested learners. The need for extension, especially in light of our current reality of industry 4.0, is vital if we wish to avoid an even bigger divide in equity regarding the next generation of leaders. If no extension programmes are available, learners at township schools – also those who great potential – will fall even further behind learners at high quintile schools, and can be expected to stay behind (Spaull, 2013).

I wish to call on other tertiary institutions and organisations to implement similar holiday programmes, to ignite hope and develop critically thinking in learners, to counter the impoverished pedagogies and passive learning they are usually exposed to (Hoadley, 2018; Hugo, 2019). I further recommend that teachers should be invited to attend these holiday programmes, where a parallel focus on the teachers' pedagogical development can be incorporated simultaneously.

7.7.2 Recommendations for further research

The following recommendations for further research are made:

- The efficacy of the programme prototype suggested here, conducted by different teachers, in a variety of South African contexts, with a variety of South African learners, could be investigated.
- The efficacy of a teacher professional development initiative aimed at developing teachers' ability to conduct problem-based extension programmes, such as the one described here, could be investigated.
- In the future, related studies that include think-aloud protocols within the data collection process are anticipated to improve the ability to identify individual metacognitive behaviour, which may give more insight into the learners' engagement with the problem-solving process.

7.8 CONCLUSION

In this chapter, I started by giving a summary of the knowledge claims I made in this study, which focussed on the assertions that resulted from the analysis of the data collected in this study. I continued by discussing the limitations of this study, after which I elaborated on the implications and recommendations for future research and practise arising from this study.

Given the national and international drive to develop learners for jobs that have not been invented yet, through equipping the next generation with the necessary skillsets – where problem-solving skills make up the biggest proportion of anticipated future key skills – it is imperative that all learners, not only learners from high-quintile and wealth-quartile schools, are granted the opportunity to be exposed to programmes similar to that described in this study.

Through my mentoring experience in these township schools, I encountered learners with potential far beyond comprehension. Learners who, regardless of their unfathomable circumstances, report to school every day with smiles on their faces and the hope to gain knowledge; learners whose eyes sparkle when a I call out their names and acknowledge their existence; learners who leave the class with a tangible sense of joy after I shared a motivational story; learners who will be our leaders of tomorrow; learners who truly want to make the world a better place. It is towards these learners that we have a responsibility to unlock their incredible potential. A combined effort from all sectors is needed to impact the learners who, due to exposure to impoverished pedagogies, need extension programmes the most. Let our collaborative efforts ignite a hopeful response from the learners' lives we endeavour to touch.

Lastly, I rest my hope for our future generation in Jeremiah 29:11:

> *For I know the plans I have for you, declares the Lord, plans to prosper you, and not to harm you, plans to give you **hope** and a **future**.*

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Addendum A: Coding taxonomy

Coding of problem-solving Phases (Rows of MPS Framework):

OR/OH/OA/OM	Orienting during Resources/Heuristics/Affect/Monitoring
PR/PH/PA/PM	Planning during Resources/Heuristics/Affect/Monitoring
ER/EH/EA/EM	Executing during Resources/Heuristics/Affect/Monitoring
CR/CH/CA/CM	Checking during Resources/Heuristics/Affect/Monitoring

Coding of problem-solving Attributes (Columns of MPS Framework):

1. RESOURCES

RK	Knowledge, facts, and procedures
RKN	Wrong knowledge
RC	Conceptual understandings
RCN	Misconceptions (negative/opposite of RC)
RT	Technology
RW	Written materials

2. HEURISTICS

HT	Constructs new statements and ideas
HP	Carries out computations
HR	Accesses resources
HW	Works back- wards
HO	Observes symmetries
HS	Substitutes numbers
HM	Represents situation with a picture, graph, table, or action/movement
HC	Relaxes constraints or generalises the problem
HD	Sub- divides the problem
HA	Assimilates parts into whole or add the sub-divided parts to make sense
HL	Alters the given problem so that it is easier
HE	Looks for a counter example

3. AFFECT

AA	Attitudes	
	AAE	Enjoyment
	AAM	Motivation
	AAI	Interest
AB	Beliefs	
	ABC	Self-confidence
	ABE	Pride
	ABP	Persistence
	ABM	Multiple attempts are needed in problem solving
AE	Emotions	
	AEF	Frustration
	AEA	Anxiety /

AEC Confusion
AEJ Joy, pleasure
AEI Impatience, anger
AV Values/Ethics
AVI Mathematical intimacy
AVG Mathematical integrity

4. MONITORING

MP *Initial Cognitive Engagement*
MPE Effort is put forth to read and understand the problem
MPO Information is organized
MPG Goals and givens are established and represented
MPS Strategies and tools are devised, considered, and selected
ME *Cognitive Engagement During Problem Solving*
MES Evidence of sense making
MEM Effort is put forth to stay mentally engaged
MEL Effort is put forth to construct logically connected statements
MM *Metacognitive Behaviours During Problem Solving*
MMQ Reflects on the efficiency and effectiveness of cognitive activities
MMM Reflects on the efficiency and effectiveness of the selected methods
MMC Exerts conscious effort to access resources/mathematical knowledge
MMG Generates conjectures
MMV Verifies processes and results
MMR Relates problem to parallel problem
MMP Refines, revises, or abandons plans as a result of solution process
MME Manages emotional responses to the problem-solving situation
MMI Engages in internal dialogue

Addendum B: Examples of coded data using NVivo software

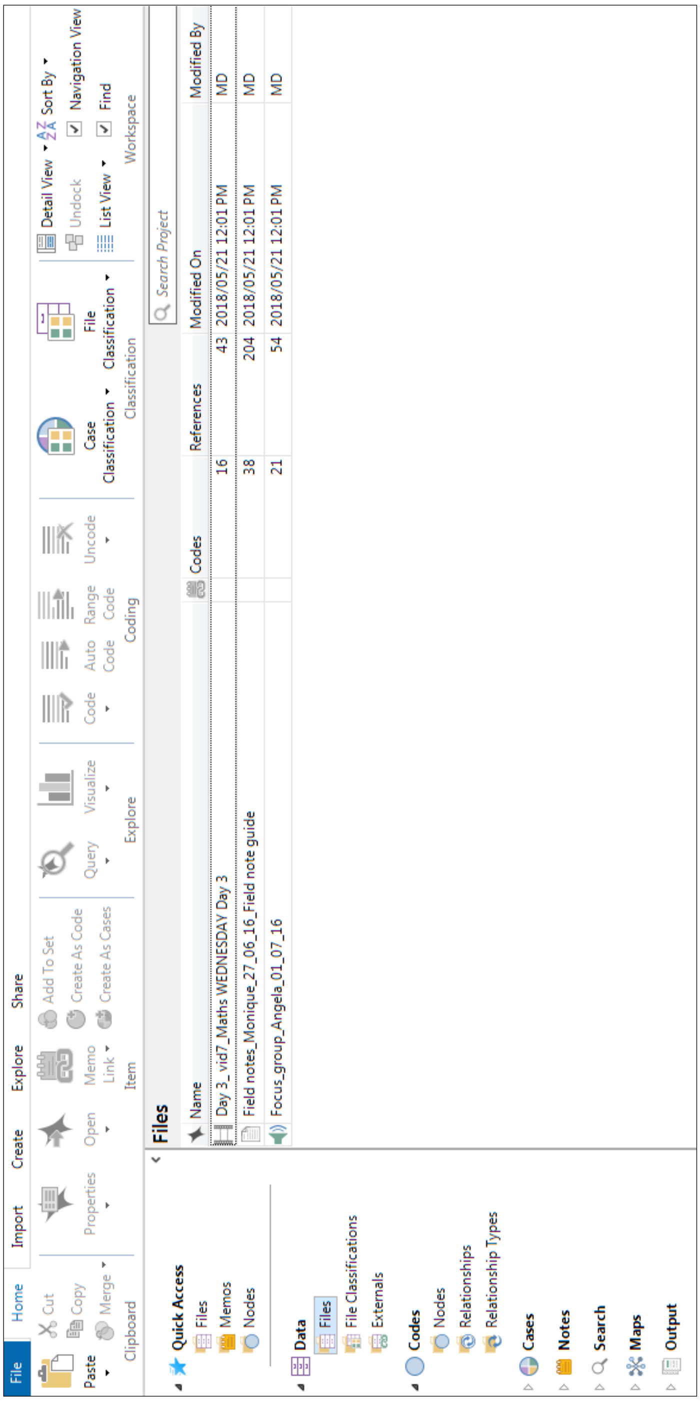


Figure i: Interface of NVivo programme - Summary of three sources that were coded and compared

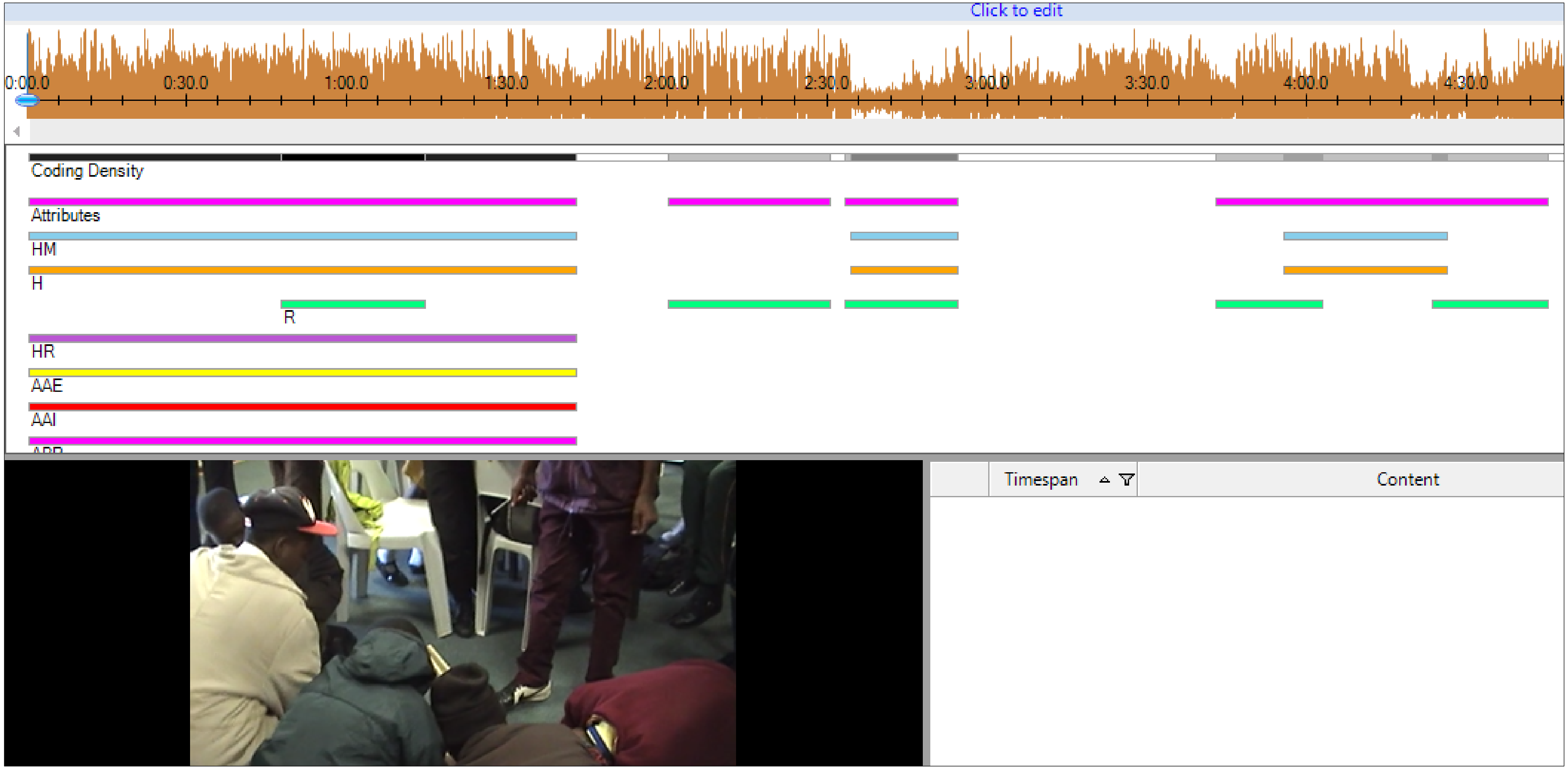


Figure ii: Coded data of Source 1 - Maths Video Day 3

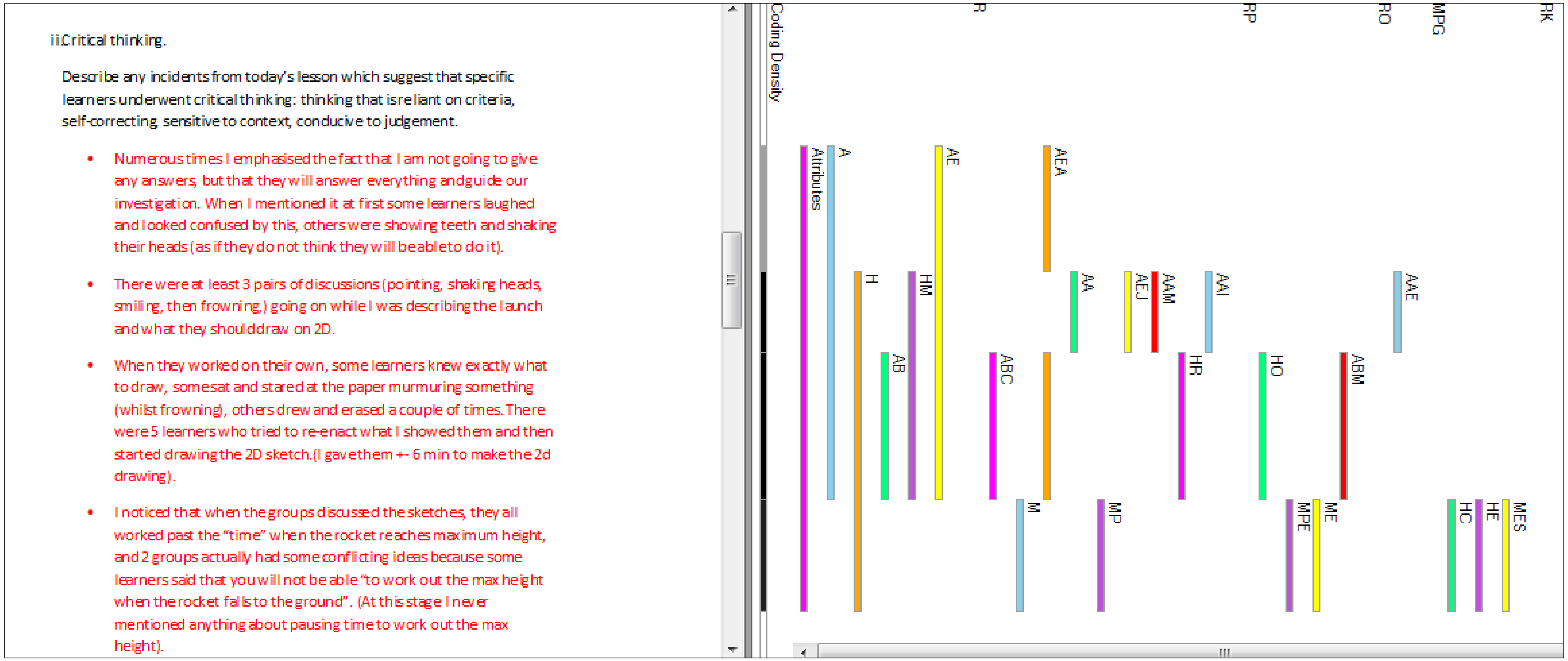


Figure iii: Coded data of Source 2 - Fieldnotes

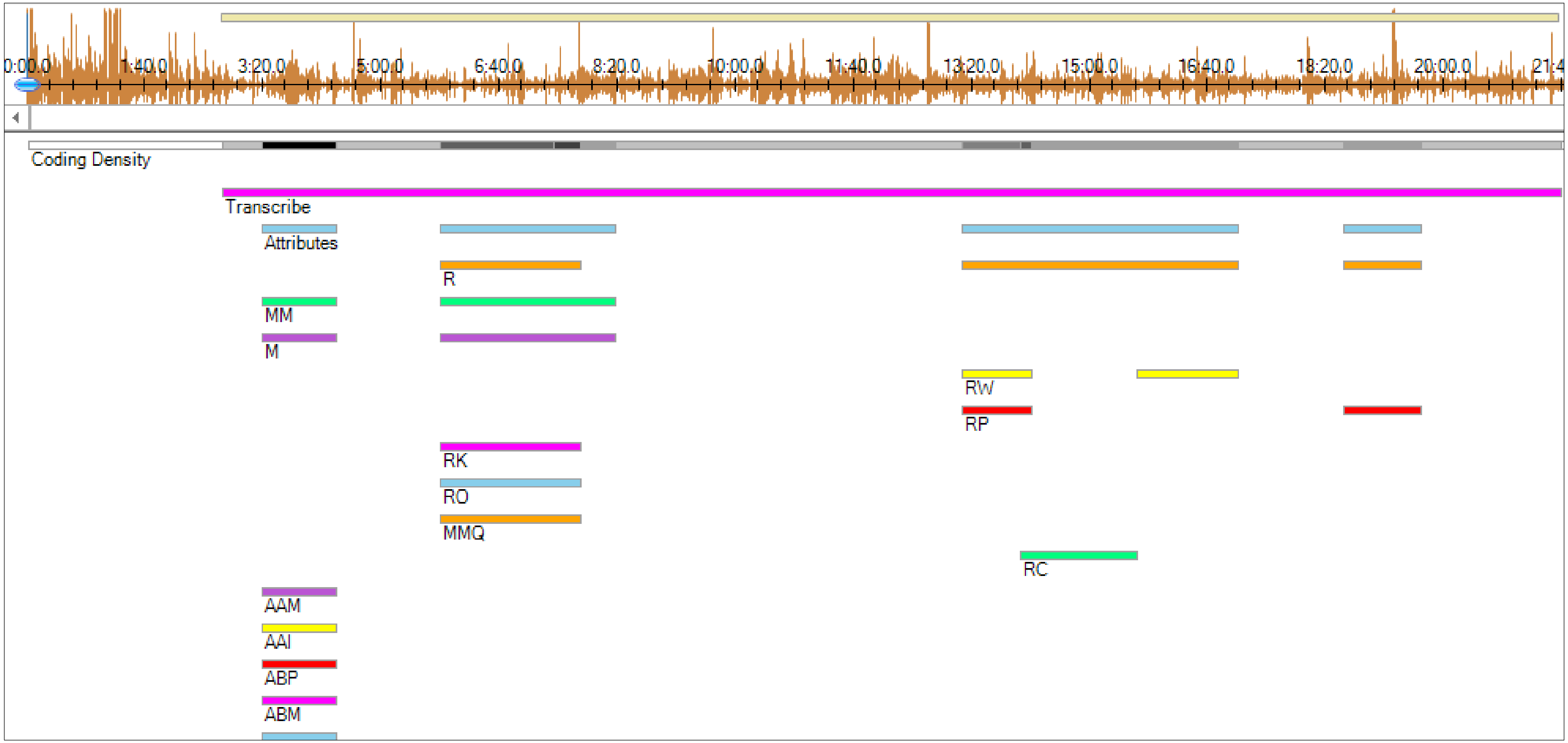


Figure iv: Codes of Source 3 - Audio focus group

Table i: All data coded from the 3 sources

Attributes	Different Attributes of MPS Framework: Resources, Heuristics, Affect, Monitoring	3	85
R	Resources	3	23
RK	Knowledge, facts, and procedures	2	3
RC	Conceptual understandings	2	2
RT	Technology	1	2
RW	Written materials	2	4
RO	Teachers as Resources	2	3
RP	Peers as Resources	3	9
H	Heuristics	2	18
HT	Constructs new statements and ideas	1	1
HP	Carries out computations	1	1
HR	Accesses resources	2	3
HW	Works back- wards	0	0
HO	Observes symmetries	1	2
HS	Substitutes numbers	0	0
HM	Represents situation with a picture, graph, table, or action/movement	2	9
HC	Relaxes constraints or generalises problem	1	1
HD	Sub- divides the problem	0	0
HA	Assimilates parts into whole or add the sub-divided parts to make sense	0	0
HL	Alters the given problem so that it is easier	0	0
HE	Looks for a counter example	1	1
A	Affect	3	32
AA	Attitudes	3	13
AAE	Enjoyment	2	3
AAM	Motivation	2	5
AAI	Interest	3	5
AB	Beliefs	3	11
ABC	Self-confidence	1	5
ABE	Pride	0	0
ABP	Persistence	3	4
ABM	Multiple attempts are needed in problem solving	2	2
AE	Emotions	1	8
AEF	Frustration	0	0
AEA	Anxiety	1	4
AEJ	Joy, pleasure	1	4
AEI	Impatience, anger	0	0
AV	Values/Ethics	0	0
AVI	Mathematical intimacy	0	0
AVG	Mathematical integrity	0	0
M	Monitoring	3	12
MP	Initial Cognitive Engagement	1	4

MPE	Effort is put forth to read and understand the problem	1	2
MPO	Information is organised	1	1
MPG	Goals and givens are established and represented	1	1
MPS	Strategies and tools are devised, considered, and selected	0	0
ME	Cognitive engagement during problem solving	2	3
MES	Evidence of sense making	2	2
MEM	Effort is put forth to stay mentally engaged	1	1
MEL	Effort is put forth to construct logically connected statements	0	0
MM	Metacognitive behaviours during problem solving	2	5
MMQ	Reflects on the efficiency and effectiveness of cognitive activities	1	1
MMM	Reflects on the efficiency and effectiveness of the selected methods	0	0
MMC	Exerts conscious effort to access resources/mathematical knowledge	0	0
MMG	Generates conjectures	0	0
MMV	Verifies processes and results	0	0
MMR	Relates problem to parallel problem	1	1
MMP	Refines, revises, or abandons plans as a result of solution process	0	0
MME	Manages emotional responses to the problem-solving situation	1	1
MMI	Engages in internal dialogue	2	2

Table ii: Inter-rater reliability of codes

Code	Cohen's Kappa coefficient	Percentage Agreement of codes by Researcher and Colleague					
		Agreement (%)	A and B (%)	Not A and Not B (%)	Disagreement (%)	A and Not B (%)	B and Not A (%)
Attributes	0.6879	84.29	45.01	39.29	15.71	12.85	2.86
Attributes\A	0.5648	83.42	15.95	67.47	16.58	16.58	0
Attributes\A\AA	0.6482	95.01	5.13	89.88	4.99	4.88	0.12
Attributes\A\AA\AAE	0.5953	97.6	1.85	95.76	2.4	2.28	0.12
Attributes\A\AA\AAI	0.6616	96.61	3.56	93.05	3.39	3.39	0
Attributes\A\AA\AAM	0.8107	98.48	3.41	95.07	1.52	1.52	0
Attributes\A\AB	0.6997	90.51	8.68	81.83	9.49	9.49	0
Attributes\A\AB\ABC	0.652	92.71	8.07	84.65	7.29	7.29	0
Attributes\A\AB\ABE	1	100	0	100	0	0	0
Attributes\A\AB\ABM	0.3047	96.09	0.53	95.56	3.91	3.91	0
Attributes\A\AB\ABP	0.3617	97.27	0.09	97.18	2.73	2.73	0
Attributes\A\AE	0.2955	85.45	2.12	83.33	14.55	14.55	0
Attributes\A\AE\AEA	0.2709	90.41	2.02	88.39	9.59	9.59	0
Attributes\A\AE\AEF	1	100	0	100	0	0	0
Attributes\A\AE\AEI	1	100	0	100	0	0	0
Attributes\A\AE\AEJ	0.1244	92.22	0.11	92.12	7.78	7.78	0
Attributes\A\AV	1	100	0	100	0	0	0
Attributes\A\AV\AVG	1	100	0	100	0	0	0
Attributes\A\AV\AVI	1	100	0	100	0	0	0
Attributes\H	0.5261	82.84	15.14	67.7	17.16	9.58	7.58
Attributes\H\HA	1	100	0	100	0	0	0
Attributes\H\HC	0.8983	96.14	0	96.14	3.86	3.86	0
Attributes\H\HD	0.1297	94.91	0	94.91	5.09	0	5.09
Attributes\H\HE	0.1583	96.14	0	96.14	3.86	3.86	0
Attributes\H\HL	1	100	0	100	0	0	0
Attributes\H\HM	0.5423	89.05	8.16	80.89	10.95	9.83	1.12
Attributes\H\HO	0.4391	93.79	0	93.79	6.21	6.21	0
Attributes\H\HP	0.2873	98.89	0	98.89	1.11	1.11	0
Attributes\H\HR	-0.0398	92.2	0	92.2	7.8	4.44	3.36
Attributes\H\HS	1	100	0	100	0	0	0
Attributes\H\HT	0.2499	95.51	0.8	94.7	4.49	0.3	4.19
Attributes\H\HW	1	100	0	100	0	0	0
Attributes\M	0.4544	91.21	4.3	86.91	8.79	7.85	0.94
Attributes\M\ME	0.3626	96.09	1.17	94.91	3.91	3.91	0
Attributes\M\ME\MEL	1	100	0	100	0	0	0
Attributes\M\ME\MEM	0.9888	99.97	1.17	98.8	0.03	0.03	0
Attributes\M\ME\MES	0.8566	96.11	0	96.11	3.89	3.89	0
Attributes\M\MM	0.9964	99.99	1.87	98.12	0.01	0.01	0
Attributes\M\MM\MMC	1	100	0	100	0	0	0

Attributes\M\MM\MME	1	100	0	100	0	0	0
Attributes\M\MM\MMG	1	100	0	100	0	0	0
Attributes\M\MM\MMI	0.9964	99.99	1.87	98.12	0.01	0.01	0
Attributes\M\MM\MMM	1	100	0	100	0	0	0
Attributes\M\MM\MMP	1	100	0	100	0	0	0
Attributes\M\MM\MMQ	1	100	0	100	0	0	0
Attributes\M\MM\MMR	1	100	0	100	0	0	0
Attributes\M\MM\MMV	1	100	0	100	0	0	0
Attributes\M\MP	0.2941	91.25	1.25	90	8.75	7.81	0.94
Attributes\M\MP\MPE	0.3165	95.14	1.24	93.9	4.86	3.91	0.95
Attributes\M\MP\MPG	0.1147	96.1	0	96.1	3.9	3.9	0
Attributes\M\MP\MPO	0.1087	98.72	0	98.72	1.28	1.28	0
Attributes\M\MP\MPS	1	100	0	100	0	0	0
Attributes\R	0.5439	88.77	8.67	80.1	11.23	8.17	3.06
Attributes\R\RC	1	100	1.07	98.93	0	0	0
Attributes\R\RK	0.9986	99.17	3.86	95.31	0.83	0	0.83
Attributes\R\RO	0.3059	95.95	0.25	95.7	4.05	4.05	0
Attributes\R\RP	0.399	93.56	2.45	91.1	6.44	4.22	2.23
Attributes\R\RT	1	100	0	100	0	0	0
Attributes\R\RW	0.2784	97.52	0.29	97.23	2.48	1.73	0.75
AVERAGE:	0.6760	96.2556	2.5439	93.7119	3.7444	3.1661	0.5786

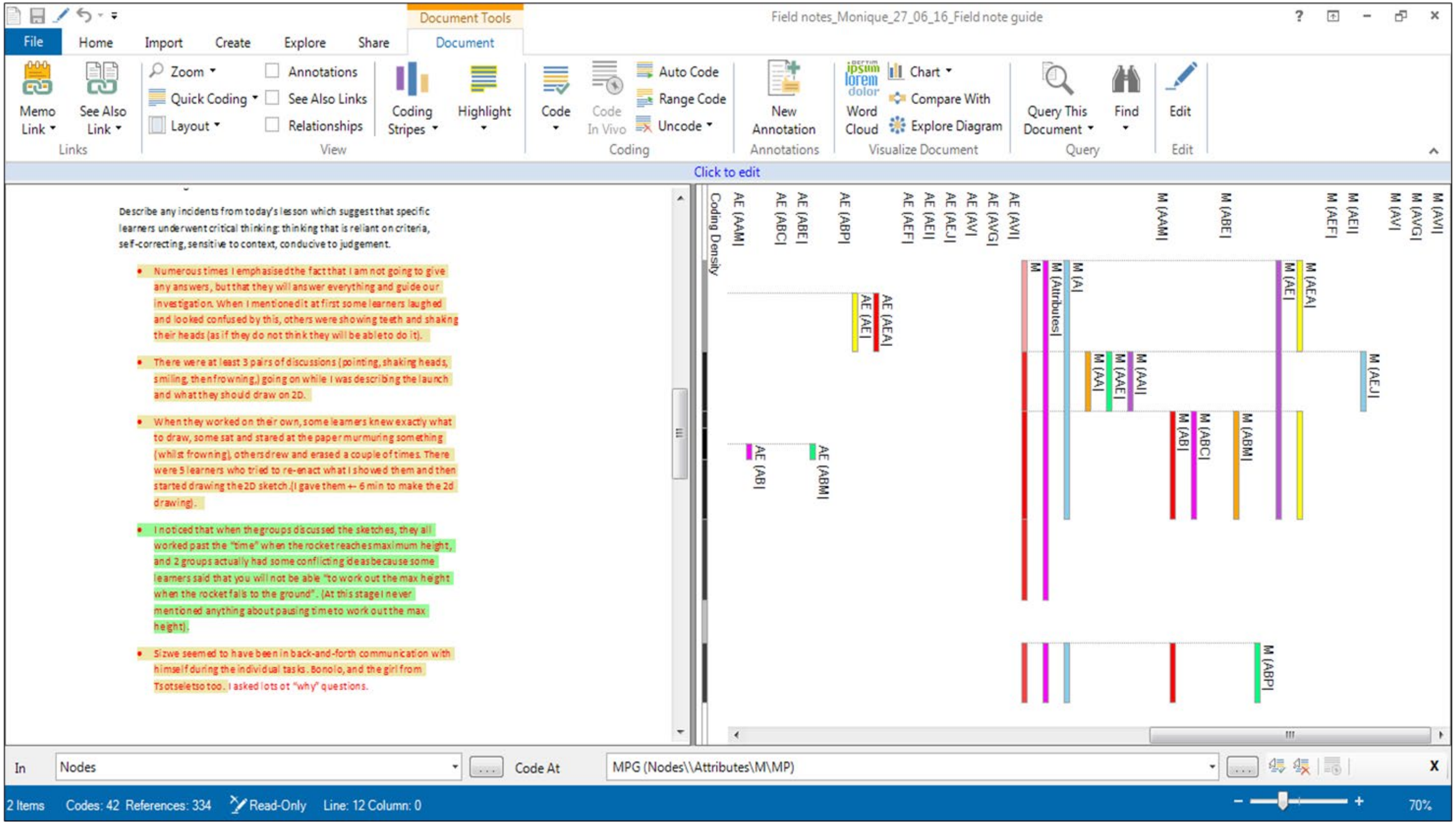


Figure v: Screen shot of the NVivo program, displaying a section of the correlating codes (i.e. ABE, AEF, AAM) between the researcher (M) and her colleague (AE)

Addendum C: Focus group discussion guide

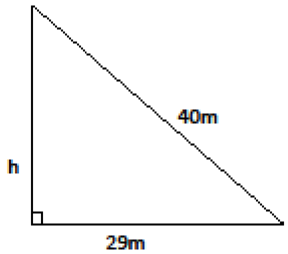
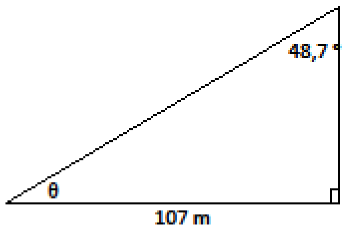
1. Learning goals
    a. Remembering.
    How well do the learners feel they can remember terms, facts and procedures they were taught in this week?
    b. Understanding.
    How well do the learners feel they understand the principles, concepts and skills they were taught in this week?
    c. Self-regulating.
    How much extra effort did the learners put into learning this week? To what extent did they do the extension activities? Were they self-motivated?
    d. Thinking.
        i. Creative thinking.
        Did they think creatively in this week? Did they come up with new ideas they hadn't thought of before? Describe when and what caused this.
        ii. Critical thinking.
        - Did they think critically in this week? Did they question others and expect reasons for claims? Did they give reasons for claims they made? Did they always try to correct themselves and others? Describe when and what caused this.
    e. Communicating.
    - Did they learn how to communicate better in writing and verbally? Describe what helped / hindered them with this.
2. Issues arising.
    - What made the week enjoyable / not enjoyable?
    - What made the learning effective / not effective?
    - How would they suggest we improve a program like this in the future?
3. Learners' perceptions.
    - Did they enjoy / not enjoy the week? Why?
    - Did the week make them like / understand / want to do science and maths more or less?
    - Did the week make them think science and maths are easier or more difficult / nicer or less nice than they used to think?

Addendum D: Pre- and post-test

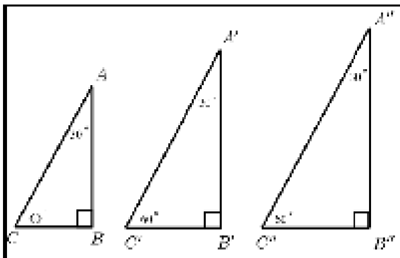
Mathematics Grade 8 (30 min) **NAME:** ______________________

Total:
/25

Question 1 (7 Marks)

Schematic representation. **Calculate the unknown variable:**	**Calculations:** (Round off to the second decimal place. Answers should be in metres.)
	(4)
	(3)

Question 2 (2 Marks)

	Are the tree triangles on the left Similar? If so, give **TWO** reasons for you statement.

Question 3 (3 Marks)

Explain, by means of a sketch and also a definition what "Angle of Inclination means:

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Addendum E: Top secret information

Follow the following clues to help you with your calculations:

1. Use your Calculator to determine the following:

   $Sin\ 30° =$ _______________

   $Cos\ 30° =$ _______________

   $Tan\ 30° =$ _______________

2. Use the Table (2.3.) you completed on Day 2 to complete the following:

   $\frac{Opposite\ side}{Adjacent\ side} =$ __________________

   $\frac{Opposite\ side}{Hypotenuse} =$ __________________

   $\frac{Adjacent\ side}{Hypotenuse} =$ __________________

3. Use Clue #1 and Clue #2 to complete the following:

$Sin\ \theta = \frac{\square}{\square}$	$Cos\ \theta = \frac{\square}{\square}$	$Tan\ \theta = \frac{\square}{\square}$

(Remember that θ represents a variable/angle.)

Addendum F: Learner workbook

UFS Project Week
27 June – 1 July 2016

MATHEMATICS

Name & Surname: ______________________________

Group Name: ______________________________

Contents

Day 1 (Monday, 27 June 2016)

1.1. Up, Up and Away: Practical Demonstration

A practical demonstration will be done. Each member of the group needs to draw rough sketch of his/her observations. Add as much information to your sketch as possible (angle size, height, distances etc.). Try to be as accurate as possible. After a group discussion, a second sketch will be made where the group works together.

First 2D sketch of your observations (done individually):

Questions on your first observation sketch:

1. Compare your sketch to your group members' sketches. List some differences:

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2. Do you have enough information to calculate the maximum height (altitude) that the rocket will reach? (List the information that you have indicated in your sketch)

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3. What extra information do you need to complete your calculations to determine the height?

4. Discuss in your groups any creative possibilities to calculate the height. List your ideas:

5. Refer to the discussions done in your group, and draw a possible sketch that will help you to calculate the maximum height that the rocket will reach:

Second 2D sketch of the group's observations:

6. Name the differences between the First and Second 2D sketches:

7. Is there a Mathematical challenge that you can identify? What is it?

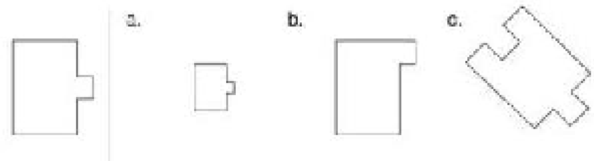
12. Linking with prior knowledge

In Mathematics, every piece of knowledge that you have acquired in the past will serve as building blocks to new knowledge. It is like building a house: You cannot build the roof if the foundations are not set. Before you can solve the complex problems of this week's Rocket Project, you need to understand the basics. Once you can do the basic calculations, your way will be open to take on the challenges that await you. It is your responsibility to make sure you understand the basics.

The following sums need to be completed at home:

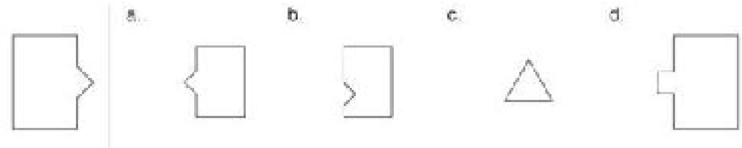
1) Select the shape that is SIMILAR to the shape on the left:

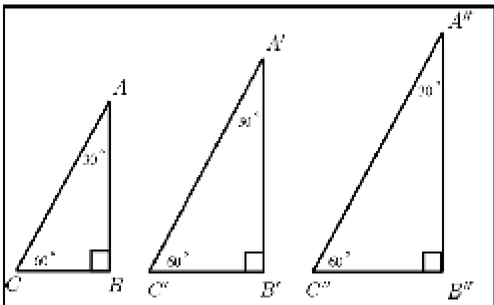
2) Select the shape that is SIMILAR to the shape on the left:



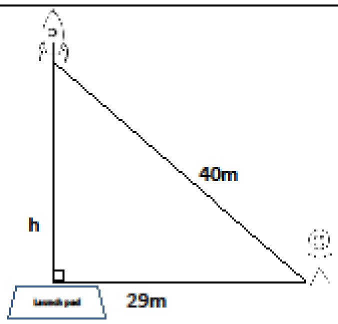
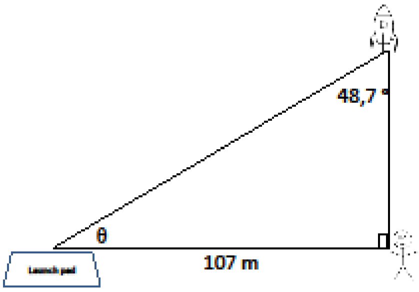
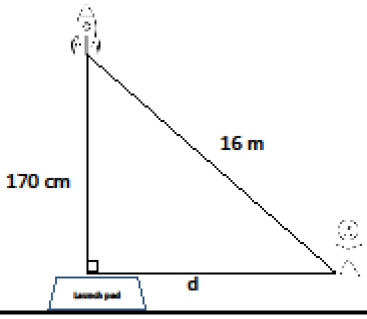
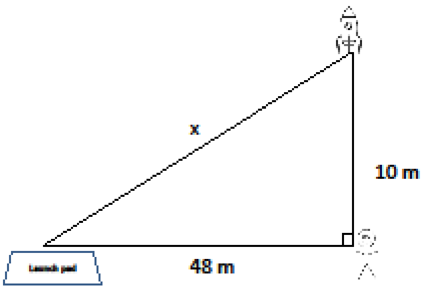
3) Select the shape that is SIMILAR to the shape on the left:

4) Select the shape that is SIMILAR to the shape on the left:





Are the tree triangles on the left Similar? If so, give TWO reasons for you statement.

Schematic representation. **Calculate the unknown variable:**	**Calculations:** (Round off to the second decimal place. Answers should be in metres.)
	
	
	
	

Can Ratios be written as a Fraction? **If you answered "Yes", give an example to explain your answer:**

1.3. A question to think about:

HOW will you calculate the maximum altitude/height that your rocket will reach?

Day 2 (Tuesday, 28 June 2016)

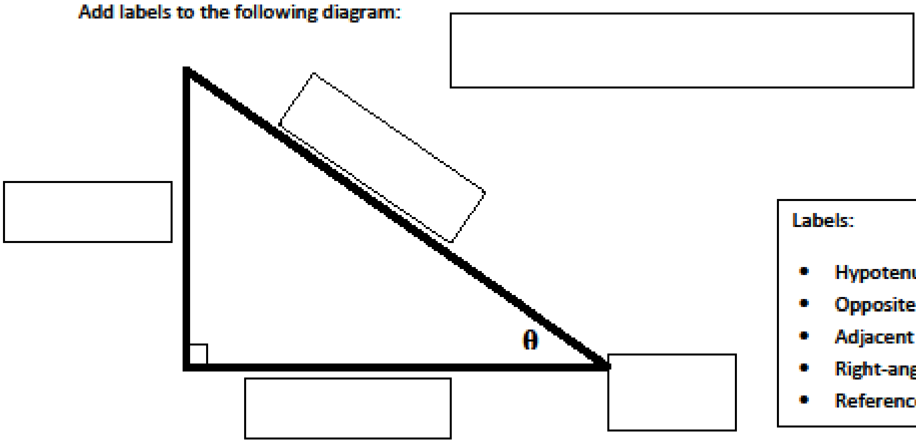
2.1. Did you know

We often use the letter *x* to indicate an unknown variable like the **unknown length of a side** (although we can use any other letter too, such as *h, d, b, z, k* etc).
When we want to indicate the size of an **unknown angle**, we often use the Greek symbol θ (we can also use α , β , γ etc.).
The Greek alphabet:

Α α	alpha	Ν ν	nu
Β β	beta	Ξ ξ	ksi
Γ γ	gamma	Ο ο	omicron
Δ δ	delta	Π π	pi
Ε ε	epsilon	Ρ ρ	rho
Ζ ζ	zeta	Σ σς	sigma
Η η	eta	Τ τ	tau
Θ θ	theta	Υ υ	upsilon
Ι ι	iota	Φ φ	phi
Κ κ	kappa	Χ χ	chi
Λ λ	lambda	Ψ ψ	psi
Μ μ	mu	Ω ω	omega

2.2. Tringles

Add labels to the following diagram:



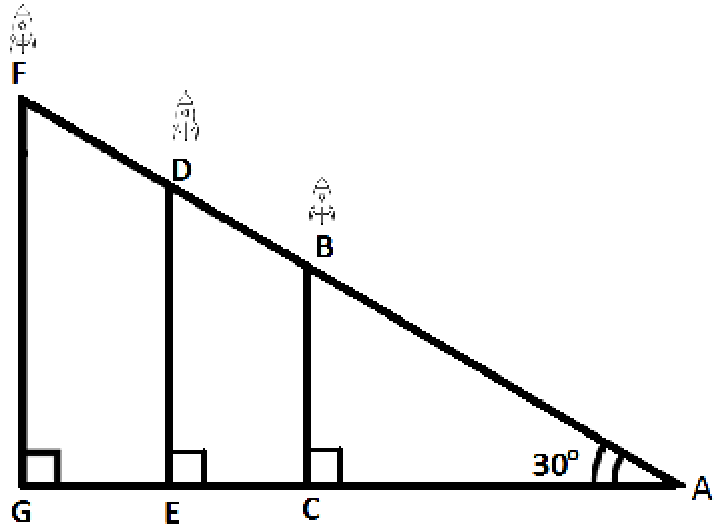
Labels:

- Hypotenuse
- Opposite side
- Adjacent side
- Right-angled triangle
- Reference angle

2.3. Investigation

a) In the diagram below, measure the length of the side opposite the 30° angle, adjacent to the 30° angle, and the hypotenuse. Record your results in the table below.

b) Calculate the Ratios in the table using a calculator. Round all ratios off to one decimal place.



		Lengths of the sides (in mm)			Ratios (Round off to one decmal pace)		
Triangle to measure	Reference Angle	Opposite side (Opp)	Adjacent side (Adj)	Hypotenuse (Hyp)	$\frac{Opp}{Hyp}$	$\frac{Adj}{Hyp}$	$\frac{Opp}{Adj}$
ΔABC	30°						
ΔADE	30°						
ΔAFG	30°						

What can you conclude? (Discuss in your groups)

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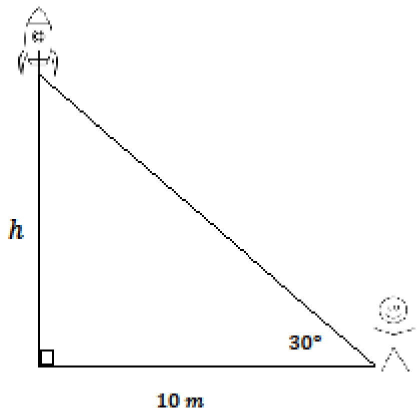
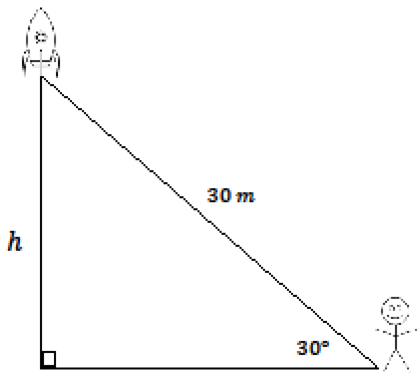
2.4. Work in groups to calculate the unknown height in the diagrams below:

<table>
<tr>
<td>


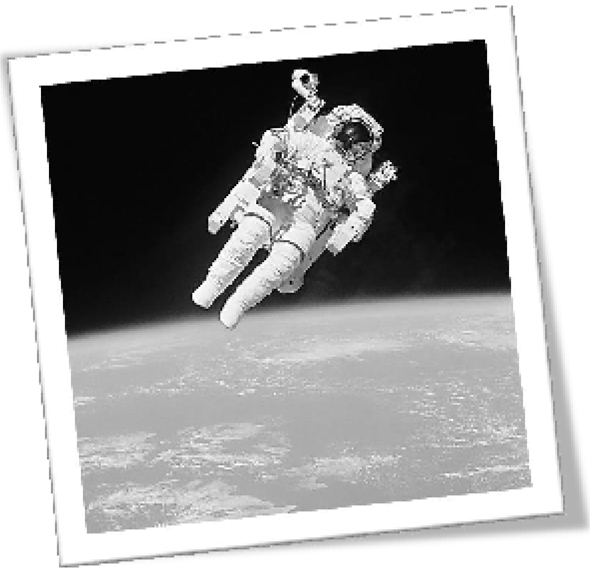
What information do you have?
What unknown do you need to calculate?
☐ Adjacent side
☐ Opposite side
☐ Hypotenuse
☐ Reference angle
</td>
<td></td>
</tr>
<tr>
<td>


What information do you have?
What unknown do you need to calculate?
☐ Adjacent side
☐ Opposite side
☐ Hypotenuse
Reference angle
</td>
<td></td>
</tr>
</table>

2.5. WHAT?! Can I really do it?

What skills have you learnt today? Were you able to figure out a difficult Mathematical problem without the teacher giving you the answers? Will you be able to use this problem-solving skill at school? Are you starting to believe in yourself?

Write a short paragraph, answering the questions asked above.

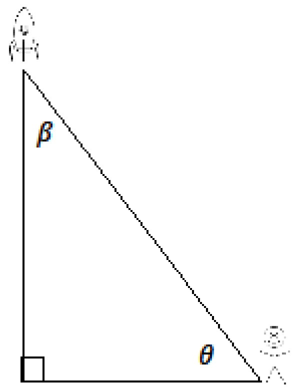


Day 3 (Wednesday, 29 June 2016)

3.1. Basic measurements

a) To accurately measure the following angles, you are going to use a ______________________.

b) In the diagram below, accurately measure the sizes of angles θ, β:

	$\theta =$ $\beta =$

3.2. Measuring the Angle of Inclination

a) Explain, in your own words or by making use of a sketch, what you understand the "Angle of Inclination" is:

Practical Activity

b) There will be an object placed at an unknown height. You need to work with your group to determine the **height** (***h***) (from the ground) of the object, as well as the **angle of inclination** (**θ**), by using the instruments provided. First draw a detailed 2D sketch of your findings, and then do the calculations.

2D Sketch	**Calculations**
What information do you have? *What information do you need to calculate?* ☐ Adjacent side ☐ Opposite side Hypotenuse ☐ Reference angle	

c) Do you think that you need more skills to be able to do the calculation? Give a reason for your answer.

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3.3. Top Secret Information.

Each group received an envelope that contains crucial information that might help you to complete the calculations above.

Stick your copy of this information to the next page and complete the questions.

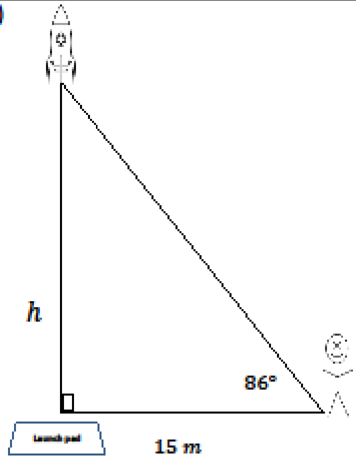
Top secret information:

****Now, use your Top Secret clues to complete question 3.2.b.
Remember to set up an Equation to solve the unknown height.**

3.4. Calculating the Unknown height

Calculate the unknown height of the following diagrams (Complete this at home and check your answers):

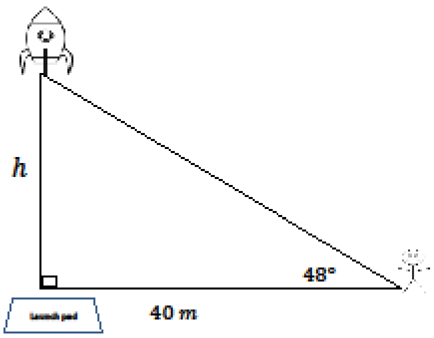
1)



What information do you have?
What unknown do you need to calculate?

- ☐ Adjacent side
- ☐ Opposite side
- ☐ Hypotenuse
- ☐ Reference angle

2)



What information do you have?
What unknown do you need to calculate?

- ☐ Adjacent side
- ☐ Opposite side
- ☐ Hypotenuse
- ☐ Reference angle

Answers: **1) height = 214,5 m** **2) height = 44,4 m**

3.5. Measuring the angle of inclination of moving objects

Will there be a difference in measuring the angle of inclination of your Rocket and the object that was stationary (standing still)? Why?

How will you measure the angle of inclination of a moving object? *(Class discussion, make notes)*

3.6. What to expect on Launch Day (tomorrow, Thursday)

- Each Group will have to record the following:
    1. Distance (in metres) of observer from launch pad
    2. Angle of inclination of **every** launch
- A neat sketch to show all information will be drawn
- The Height will be calculated after the launch, using the recorded data (1, and 2) above and the skills you have acquired this week.
- Data analysis will be done based on recorded data and the Graphic Display of the data.
- (Read through tomorrow's Worksheet to help you prepare.)

<u>REMEMBER</u> that TEAM WORK is the key to success. You will have to work together to collect all data. Some team members might have to launch the rocket, whilst others measure the angle.

Day 4 (Thursday, 30 June 2016)

Work in groups and complete the following Worksheet, based on the Launch. Remember that TEAM WORK is the key to today's success.

LAUNCH DAY

Date of launch: ____________________ Time of first launch: ____________________

Place of launch: ____________________

Group Name: ____________________________

Group Members: ____________________________

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Recorded Data (On Vertical Launches only) – to be done during the launch of the Rocket:

1. Distance of Observer (who measures the angle of inclination) from Launch Pad: ________________

2. Angle of inclination:

Launch number:	Angle of Inclination:
1	
2	
3	
4	
5	
6	

3. 2D Sketch of Observations:

Calculations (to be done <u>after</u> the launch of the Rocket)

Calculate the maximum height of the Rocket during each launch:

Launch number:	Rough Sketch	Height Calculations
1		
2		
3		
4		
5		
6		

Tabulated Data (to be done after the launch of the Rocket)

Complete the following Table:

Launch number	Angle of Inclination	Maximum Height
1		
2		
3		
4		
5		
6		

Displaying Data (to be done after the launch of the Rocket)

Display your data (Height vs. Angle of inclination) in the form of a Line Graph on the next page. Remember what the **independent variable** is displayed on the horizontal x-axis, and the **dependent variable** on the vertical y-axis.

Add all necessary labels and headings.

Day 5 (Friday, 1 July 2016)

Write a short essay to address the following:

- **Describing** the data that you have. (Discuss range, maximum height, minimum height, the relationship between maximum height and the angle of inclination etc.)
- Do you think it was necessary to use **Trigonometric ratios** to determine the maximum altitude? Explain.
- Do you think the data captured is 100% **accurate**? If not, give reasons/examples for your statement.

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End of mission