ENHANCING PROBLEM-SOLVING SKILLS IN A GRADE 10- MATHEMATICS CLASSROOM BY USING INDIGENOUS GAMES

by

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DECLARATION

I, TSHELE JOHN MOLOI declare that enhancing problem-solving skills in a grade 10- mathematics classroom by using indigenous games submitted for the Doctorate degree, is my independent work and that I have not previously submitted it for a qualification at another institution of higher education.

TJ MOLOI DATE

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DEDICATION

This thesis is dedicated to my late parents Napo and Maditaba, my mother who passed on in the verge of completing my studies, for being there for me all the time, even on her last days she inspired me to work hard. To my wife Deliwe. J, for her unwavering support, encouragement and dedication to taking care of the children and family in my absence. To my children, Wetsi and Napo, your support has not gone unnoticed and even for reminding me that '*ha o so etse mosebetsi wa Prof*'. Lastly, special thanks my next of kin Mrs Moloi Nomalanga L and her late husband Moloi Enock, and your family for playing a significant role in my life.

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SCENARIOS

LIST OF ACRONYMS AND ABBREVIATIONS

SACMEQ	Southern African Consortium Management Education Quality
AMESA	Association for Mathematics Education of South Africa
PSS	Problem-solving Skills
CAPS	Curriculum Assessment and Policy Statement
ATM	Australian Teachers of Mathematics
DCSF	Department for Children, School and Families
AAMT	Australian Association of Mathematics Teachers
MERGA	Mathematics Education Research Group of Australasia
DBE	Department of Basic Education
HoD	Head of Department
SuRLEC	Sustainable Rural Learning Ecology
TIMSS	Third International Mathematics and Science Study
IEA	International for the Evaluation and Educational Achievement

ABSTRACT

The study aims at designing a framework for using indigenous games to teach problem-solving. In formulating the framework, one school was identified in the Thabo Mofutsanyana Education District. In pursuance of the aim of this study, the following objectives were identified as key:

- To demonstrate a need for a framework for the teaching of problem-solving at grade 10 using indigenous games
- To identify components of problem-solving for use of indigenous games in grade 10 mathematics classrooms to enhance learner performance;
- To indicate the conditions under which teaching problem-solving using indigenous games can be successfully implemented;
- To identify possible threats that may adversely disturb the teaching of problem-solving using indigenous games in grade 10 mathematics classrooms, so as to build the mechanisms that will resolve the anticipated threats;
- To trial and test the teaching of problem-solving using indigenous games so as to produce evidence of its effectiveness.

The study is conducted within community cultural wealth as a theoretical framework, which acknowledges that there is no deficiency in the marginalised knowledges of the excluded people. The theoretical framework validates and acknowledges the knowledge that the marginalised possess, as very rich in the teaching and learning of problem-solving. Thus, the study tapped into the marginalised knowledge to teach problem-solving, using the participatory action research (PAR) method in generating data. PAR is compatible to the principles of community cultural wealth, recognising community members as experts and empowering communities to find their own solutions to local issues (Moana, 2010:1). Hence, the involvement of participants such as community members (parents, traditional leaders), education experts (teachers, mathematics subject advisors, lecturing staff from institution of higher learning) and learners themselves were very important in the designing of the framework of teaching problem solving.

The research coordinator reviewed the literature from the local context (within South Africa), neighbouring states, SADC (Southern African Development Community), the continent and internationally. This helped achieve a sense of good practices in the teaching of problem-solving around the world. The reviewed literature was compared and juxtaposed to empirical data, with the common issues and disagreements that transpired discussed. Recommendations were made for the framework. Among the challenges identified were: mathematical content is very abstract for learners to comprehend. This is exacerbated by the method used for teaching problem-solving. The major part of the teaching of problem-solving is controlled by the teacher. This approach to teaching is influenced by the assumptions that learners are empty vessels, which must be fed with knowledge into their minds. Also, there is a lack of engagement of parents in the teaching of problem-solving, with the teaching of it divorced from home environment and learners' background, which was very rich in mathematical content. This agrees with Pramling-Samuelsson's (2008:630) argument that, when playing, children learn mathematical concepts easily. Thus, the study used indigenous games to teach mathematical concepts to learners.

In addition, the study also looked into possible solutions to the identified challenges. That is, all the activities in the teaching of problem-solving were learner-based, and teachers, parents and subject experts scaffolded the processes of learning problem-solving. Conducive conditions include teachers not having to dictate to learners how to learn problem-solving, rather to allow them to explore and discover various mathematical concepts on their own through playing the indigenous games or visualising these mathematical concepts by observing others playing. The primary data was generated by using tape-recorder and video camera, analysed using Van Dijk's critical discourse analysis (CDA) to identify instances of 'discursive injustices' in text and talk, and signifies a form of resistance to unethical and unjust social power relations (Hakimeh Saghaye-Biria, 2012:509; Van Dijk, 2009:63; Dijk, 2003:352). CDA enabled the study to acquire deeper meanings of the text.

Based on the above, through the study it was found that the framework for using indigenous games helped learners to be creative in approaching problem solving. It also, assisted them to discover mathematical concepts, definitions, and mathematical content which are embedded with the indigenous games.

OPSOMMING

Die studie het ten doel om 'n raamwerk, wat gebruik maak van inheemse speletjies om probleemoplossing te onderrig, te ontwikkel. Tydens die formulering van die raamwerk is 'n skool uit die Thabo Mofutsanyana Onderwysdistrik geïdentifiseer. Die volgende kerndoelwitte is bepaal in navolging van die doel van die studie:

- Om die behoefte aan 'n raamwerk vir die onderrig van probleemoplossing aan Graad 10-leerders deur die gebruik van inheemse speletjies, te demonstreer;
- Om die komponente van probleemoplossing wat gebruik kan word tesame met inheemse speletjies om leerderprestasie in Graad 10-Wiskunde klasse te bevorder;
- Om die toestande waaronder probleemoplossing m.b.v. inheemse speletjies suksesvol geïmplementeer kan word, aan te dui;
- Om moontlike bedreigings wat die onderrig van probleemoplossing negatief kan beïnvloed te identifiseer en meganismes om die verwagte bedreigings te neutraliseer in plek te stel; en
- Om die onderrig van probleemoplossing deur die gebruik van inheemse speletjies te toets en dus bewyse t.o.v. die effektiwiteit daarvan te verkry.

Die studie is onderneem binne die rykdom van die gemeenskapskultuur as die teoretiese raamwerk, wat erken dat daar geen tekortkoming is in die gemarginaliseerde kennis en begrip van die uitgeslote persone nie. Die teoretiese raamwerk heg waarde aan en erken dat die kennis wat die gemarginaliseerdes oor beskik, ryk is in die onderrig en leer van probleemoplossing. Hierdie studie delf dus in die gemarginaliseerde kennis oor hoe om probleemoplossing te onderrig deur gebruik te maak van deelnemende aksienavorsing om data in te samel. Deelnemende aksienavorsing is versoenbaar met die beginsels van gemeenskapskultuur-rykdom, erken gemeenskapslede as kenners en bemagtig gemeenskappe om self oplossings te vind vir plaaslike probleme (Moana, 2010: 1). Gevolglik was die betrokkenheid van deelnemers soos gemeenskapslede (ouers, tradisionele leiers), onderrigkenners (onderwysers, wiskunde-

vakadviseurs, dosente aan hoëronderwys-instellings) en die leerders self baie belangrik in die ontwerp van die raamwerk.

Die navorsingskoördineerder het literatuur bestudeer uit die plaaslike gemeenskap (Suid-Afrikaanse), die buurstate, die SAOG (Suid-Afrikaanse ontwikkelingsgemeenskap), asook op kontinentale en internasionale vlakke. Dit het gelei tot ontwikkeling van ‘n begrip vir goeie praktyk wêreldwyd in die onderrig van probleemoplossing. Die bestudeerde literatuur is vergelyk met en naas die empiriese data geplaas. Die gemeenskaplike kwessies en verskille wat opgeduik het is bespreek en aanbevelings t.o.v. die raamwerk gemaak. Van die uitdagings wat geïdentifiseer is, is onder andere dat wiskundige begrippe baie abstrak is vir leerders om te begryp. Dit word vererger deur die metode wat toegepas word om probleme op te los. Die grootste gedeelte van die metode van onderrig van probleemoplossings word deur die onderwyser beheer. Hierdie onderrigbenadering word beïnvloed deur aannames dat leerders leë doppe is en dat hulle met inligting volgeprop moet word. Daar is ook ‘n tekort aan ouerdeelname t.o.v. probleemoplossing aangesien die onderrig van probleemoplossing ver verwyder is van die bekende omgewing van die leerder se huislike omgewing en agtergrond, ‘n milieu wat ryk is aan wiskundige inhoud. Dit stem ooreen met Pramling-Samuelsson (2008: 630) se argument dat kinders wiskundige begrippe makliker leer deur speletjies. Daarom het die studie gebruik gemaak van inheemse speletjies vir die onderrig van wiskundige begrippe.

As byvoeging het die studie ook ondersoek ingestel na moontlike oplossings vir die geïdentifiseerde uitdagings. Met ander woorde, alle aktiwiteite in die onderrig van probleemoplossing moet leerdergesentreer wees en die onderwysers, ouers en vakkundiges moet die aanleer van probleemoplossing begelei. Gunstige omstandighede sluit in onderwysers wat nie nodig het om aan leerders te dikteer hoe om probleemoplossing aan te leer nie, maar eerder die leerders toe te laat om ondersoek in te stel en verskillende wiskundige begrippe op hul eie te ontdek deur die speel van inheemse speletjies, of deur die visualisering van wiskundige begrippe deur waar te neem hoe ander speel. Die primêre data is gegenereer deur ‘n bandopnemer en videokamera te gebruik, en geanaliseer m.b.v. Van Dijk se kritiese diskoersanalise om momente van “diskursiewe ongelykhede” te identifiseer, en dui op n vorm van weerstand t.o.v. onetiese en ongelyke maatskaplike magsverhoudings (Hakimeh Saghaye-Biria, 2012: 509; Van Dijk, 2009: 63; Dijk, 2003: 352). Kritiese diskoersanalise het die studie in staat gestel om ‘n dieper betekenis van die teks te bekom.

Gebaseer op bogenoemde, het die studie gevind dat die raamwerk van inheemse speletjies leerders gehelp het om kreatief te wees in probleemoplossing. Dit het die leerders ook in staat gestel om wiskundige begrippe, definisies en wiskundige inhoud wat in die inheemse speletjies opgesluit lê, te ontdek.

LIST OF KEY CONCEPTS

Teaching of problem-solving

Using Indigenous games

Participatory Action Research

Community cultural wealth

Ethnomathematics

SLEUTEL WOORDE

Probleemoplossing-onderrig

Gebruik inheemse speletjies

Deelnemende aksienavorsing

Gemeenskapskultuur-rykdom

Etno-wiskunde

CHAPTER 1

OVERVIEW OF STUDY

1.1 INTRODUCTION

Mathematics results in South African secondary schools are very poor, as illustrated in the Department of Basic Education (DBE) Reports of 2009 & 2010, which show that nationally the grade 12 mathematics results in 2008, 2009 and 2010 stood at 45.95%, 46.0% and 47.4% respectively. In Free State province the results for the same years were 77.63%, 53.3% and 48.4% respectively. The Annual National Assessments (ANA) Report (2011) showed that 74% of Free State learners in grade 6 failed mathematics. To a large extent, the results were influenced by poor performance of mathematics in lower grades, but the DBE Report (2009) posits that many of the mistakes made by learners in answering the Mathematics assessment tasks had their origins in poor understanding of the basics and foundational competencies taught in the earlier grades. The report further suggests that intervention to improve learners' performance should concentrate on knowledge, concepts and skills learnt earlier and not just in the final year of the Further Education and Training (FET) phase.

This study focused on enhancing problem-solving skills in a grade 10 mathematics classroom using indigenous games by paying attention to certain topics in which grade 10 learners do not perform well. These include patterns, functions, trigonometry in two and three dimensions, and analytical geometry, topics that the DBE Report (2009) shows also produced poor performance in grade 12.

Van De Walle, Karp,Bay-Williams (2010:32,33) argue that a problem-solving approach allows learners to build meaning for the concepts so that they shift to abstract concepts readily. The researcher and participants understood problem-solving as an approach to the teaching of mathematics in a creative way, whereby learners are taught skills and knowledge of counting, solving equations and how to interpret geometric figures. To date, learners, especially those in the rural areas, have experienced the greatest difficulty in understanding and problem-solving areas of mathematics. Gaigher, Rogan and Braun (2006:15,16) point out that most perform badly because of teacher-dominated approaches, and the learners expected to remain passive recipients of rote learning. Troutman and Lichtenberg (2003:11, 12) state that teachers need to provide students with stimulating problem-solving activities.

This study therefore uses indigenous games as a stimulating activity in the teaching and learning of mathematics. Mosimege (2000:427) clarifies misconceptions about indigenous games, namely that they are usually perceived from the narrow perspective of play, enjoyment and recreation, whilst there is actually more to them than just the three aesthetic aspects. Indigenous games can reveal mathematical concepts associated with them. According to Van De Walle, Karp Bay-Williams (2010:33) and Leonard (2008:6,7,8)in the traditional modes of teaching, learners do not learn content with deep understanding and often forget what they have learned. Thompson (2008:34) suggests that teachers should capitalise on the background of learners for performance to be enhanced. Children meet mathematical concepts every day and operate in rich mathematical contexts even before they set their eyes on a mathematics worksheet.

The researcher and the research participants argue that the teaching of problem-solving is abstracted and treated as if knowledge of it ends only with memorising mathematical formulae. Troutman et al. (2003:55) support the latter statements that teachers need to strive to build a foundation and master important teaching techniques related to problem-solving. Van De Walle et al (2010:34) points out that problem-solving knowledge content grounded in an experience familiar to learners supports the development of advanced mathematical concepts and provides them with access to meaningful mathematical reasoning, thus they are able to learn it successfully.

On the other hand, Hirsh (2010:154,155) argues that if mathematics teachers continue to teach what they know and ask learners to memorise and regurgitate content it is impossible to expect any advancement to be made in problem-solving. To a large extent the teaching of problem-solving seems to lack the element of relevancy to real life, hence learners do mathematics for the sake of passing tests or examinations, and with little understanding. They are not taught the skills of deriving formulae and functionalising knowledge derived from them. Koellner Jacobs, Borko, Schneider, Pittman, Eiteljorg, Bunning, Frykholm (2007:273) points out that problem-solving should not be merely taught as a set of procedural competencies but rather mathematics teachers had to help learners gain adequate conceptual knowledge along with a flexible understanding of procedures to become competent and efficient problem-solvers. Learners are thus limited in terms of creativity and self-discovery as a result of this way of teaching problem-solving. Thompson (2008:34,35) argues that children are likely to be creative when they use ideas and experiences, and make new connections through play.

Human (2005:303) indicates that for problem-solving to succeed it should put a high demand on the teacher's subject-matter understanding. Van De Walle et al. (2010:60) and Ridlon (2009:190) suggest that, before class, the teacher has to make proper preparations and planning. This will help the teacher to reach out to learners. Activities prepared should allow for learner-centredness, take learner background into account and clear connections between mathematics concepts and learner experiences. The connection is to align the indigenous games played at home with the mathematical content taught in class. Conditions to be taken into account are that both learners and teachers are familiar with indigenous games, however, Mhlolo & Schafer (2013:1,2) point out that there are learners who will be reluctant to work on activities and expect the teacher to provide them with answers.

1.2 PROBLEM STATEMENT AND RESEARCH QUESTION

Poor learner performance in Mathematics and invariably ineffective strategies used in teaching problem-solving tend to ignore learners' background and experiences. Based on the above discussions, the research question is therefore:

- How can using indigenous games enhance problem-solving skills in a grade 10 Mathematics classroom?

1.2.1 Research aim

In an attempt to answer the above-stated research question, the research aim of the study is to develop a framework for enhancing problem-solving skills in grade 10 mathematics classroom using indigenous games.

1.2.2 Research objectives

In order to accomplish the research aim the following objectives were addressed:

- To demonstrate a need for a framework for the teaching of problem-solving at grade 10 using indigenous games
- To identify components of problem-solving by which indigenous games are used in grade 10 Mathematics classes to enhance learner performance
- To indicate the conditions under which teaching problem-solving using indigenous games can be successfully implemented
- To identify possible threats that may adversely disturb teaching problem-solving using indigenous games in grade 10 Mathematics classes, so as to build the mechanisms that will resolve the anticipated threats;
- To try and test the teaching of problem-solving using indigenous games so as to produce evidence of its effectiveness.

1.3 THEORETICAL AND CONCEPTUAL FRAMEWORKS

The study is grounded by community cultural wealth theory, focussing on the wealth of knowledge which the marginalised groups possess. Such knowledge is key to the teaching and learning of problem-solving. The study is further guided by ethno-mathematics as the conceptual framework, concurring with a theoretical framework that posits that teaching and learning of problem-solving is created by the marginalised groups out of their everyday lives experiences, rather than being taught in a formal setting.

The proposed research is viewed through the lens of community cultural wealth theory. Lynn (2004:156) and Yosso (2002:98,100,102; 2005:69) argue that community cultural wealth concentrates on and learns from the range of cultural knowledge, skills, abilities and contacts held by subaltern groups that often go unrecognised and misunderstood. In this study, the use of indigenous games in teaching problem-solving skills in grade 10 Mathematics classes is a way of recognising and acknowledging the cultural practices of various communities.

Van Oers (2010:23,26,27) agrees with Yosso by introducing the cultural-historical theory of Vygotsky, which views learning as a process of qualitative change of actions that may take place when people take part in cultural activities and receive guidance for refining or appropriating actions. Van Oers argues that within the cultural-historical context, problem-solving can be defined as a cultural activity that arose somewhere in cultural history and went through a rich and significant cultural–historical development, to end up in the multidimensional and highly advanced discipline as we know it today. On the other hand, Leonard (2008:59, 60) contend that mathematical problem-solving, like all other forms of knowledge, is located within a cultural context. Subsequently, counting and numeracy can be conceptually understood as both a knowledge form and a cultural practice that enables learners to manage and organise their world. Employing cultural norms in the classroom is at the heart of teaching cultural relevance.

Yosso (2005:78,79) argues that community cultural wealth theory has various forms of capital, such as aspirational, navigational, social, linguistic, familial and resistant. These draw on knowledge of learners from homes and communities being taken into the classroom environment. The researcher supports Yosso's theory of community cultural wealth and Van Oers's cultural-historical theory, in that using indigenous games in to teach mathematics problem-solving skills is a way of bringing the immediate environment and experiences of the child to the classroom. Van Oers (2010:13,14) points out that children learn problem-solving optimally when their learning is deep-rooted in playful activities.

In this study, ethno-mathematics as the conceptual framework is used to operationalise the theoretical framework. The marginalised knowledge of the subaltern communities is contextualised through its use and is underpinned by Wittgenstein's philosophy of knowledge as multifaceted (Chilisa, 2012:40; Vilela, 2010:345), rather than perceiving human knowledge through scientific laws that objectify human beings (Ryan, 2006:21, 22). Wittgenstein' s philosophy came as a reaction to the relativity theory of Einstein, which saw human beings as objects to be studied and controlled (McGregor, 2010:424; Penco, 2010:2; Vilela, 2010:344).

Whilst it is evident that the teaching and learning of problem-solving should be viewed from a humanistic point of view (Barker, 2012:20; Bush,2005:3;Vilela, 2010:249), through the lived experiences of marginalised groups there are many mathematical concepts that are formulated. Problem-solving meanings are not fixed or predetermined, and meanings are not indifferent to linguistic practices (Lynn, 2004:154, Vilela, 2010:347; Yosso, 2005:80). The link between mathematical content and the cultural practices (such as play of indigenous games) helps learners to see and appreciate the relevance of problem-solving skills in their day-to-day activities (Chikodzi & Nyota, 2010:4).

1.4 CONCEPTUALISING OPERATIONAL CONCEPTS

It is necessary here to clarify the two key terms as they are understood and used in the study.

Problem-solving has been variously defined, but in the context of this study it is regarded as a topic of instruction on mathematical content (Posamentier & Kruik, 2008:4). Examples considered include, algebra and geometry (Grinstein & Lipsey, 2001:648), with the former a mathematical topic using variables to represent situations as a main focus, and patterns described to illustrated certain relationships, at times described as functions. The latter focuses on different shapes of geometric figures, and space occupied by them.

The **indigenous games** are examples of cultural practices played by various communities. The study focuses on the ones commonly played in South African schools, such as *morabaraba, dibeke, diketo* (Department of Sports and Recreation, 2006:1). As the learners play they also learn mathematical content embedded within them (Pramling-Samuelsson, 2008:630). Capital wealth includes socialising skill and ability to strategize, of use when learning problem-solving (Lynn, 2004:156; Yosso, 2005:80).

1.5 OVERVIEW OF LITERATURE REVIEW

In order to operationalise the objectives of study, the literature is reviewed from the good practices of learning problem-solving using indigenous games. The literature reviewed is local, regional (the Southern African Development Community, SADC), continental, and global. Key concepts arise, as constructs to be used in chapter four to make sense of the empirical data.

1.5.1 Demonstrating and justifying the need to develop the framework

According to various reports, such as the Annual National Assessment (ANA) Report of 2011, the Department of Basic Education (DBE) Report of 2009, 2010; the Southern and East Africa Consortium for Monitoring Educational Quality (SACMEQIII) Report of 2010, and the World Economic Forum Mathematics Report of 2011, learners' performance on problem-solving has been poor. Learners find it difficult to comprehend problem-solving in schools as they are taught in a very abstract way, with outdated modes of rote learning. Teachers tend to ignore the home background and context of learners, thus denying them full access to mathematical content (Anthony, 2009:153).

Based on the above scenarios, this research was conducted to address specific challenges and develop a framework to enhance learners' performance on problem-solving.

1.5.2 Determining the components of the framework

After a needs assessment was carried out certain challenges were identified. In addressing them the research team was constituted, from the school community, members of society, education officials, and individuals from institutions of higher learning. In coming up with comprehensive solutions to the identified problems, expertise from various sectors was needed. The framework of using indigenous games to teach problem-solving focused on the concept of a playing and learning child (Mhlolo, Venkat, Schafer, 2012:1,2; Mosimege, 2000:11; Mosimege, 2000:457; Pramling-samuelsson, 2008:629). As learners play there is much that they can learn about mathematical concepts. There were strong connections between indigenous games and problem-solving, and learners easily identify the mathematical concepts illustrated by indigenous games. For much of the time there were natural interactions between learners, and learners and teachers. The involvement of subaltern parents in the teaching of problem-solving assisted the researcher to operationalise the theoretical framework.

1.5.3 Exploring the conducive conditions for the framework

For the components of the framework of using indigenous games to teach problem-solving to yield results, there are contextual factors that must be considered. The teaching and learning environment must replicate the home environment. Learners were engaged in the playing of indigenous games, which simulated them to network and learn from others in their interactions (Yosso, 2005:79). Rather than the teacher transferring knowledge into their minds the teachers' and parents' role was that of providing scaffolding where necessary. For instance, as learners played they observed many patterns in the indigenous game, which were translated into mathematical content that resulted in linear patterns and quadratic patterns. Learners were given power and voices (Lynn, 2004:154, Mahlomaholo, 2010:17) to reiterate these mathematical concepts in their own understanding.

1.5.4 Identifying the risks factors that might derail the framework

It is important that in the process of implementing the framework the research team has to reflect on and identify risks factors that might derail the framework. The research participants have to think of ways of avoiding these threats in future. This is amplified by Keeley and Tobey (2011:89), who argue that as one reflects it is important to identify good practices and risks factors, such that the feed-forward process can be cautious of threats that exist and how to circumvent them. The study was mindful of excessive powers of teachers and education officials, and language used did not accommodate other participants (Akuno, 2013:66; Mukhopadhyay, 2013:94; Lynn, 2004:165). These threats impacted negatively on the framework, as some parents absented themselves from sessions, and others had wrong perceptions that there were other ulterior motives about the study or believed that they were being manipulated for certain individuals' benefit.

1.5.5 Demonstrating the indicators of successes of the framework

The framework of using indigenous games to teach problem-solving showed positively changes in learners' performances, and the method of teaching encouraged learners to have interest in problem-solving (Waege, 2009:90; Williams & Forgasz, 2009:96). Table 4.6.7 (d) shows that learners' performance has significantly increased (refer to 2.5.2.4 and 4.6.4 for details). The materials which were developed by the research team were also used to train Mathematics teachers in other clusters in the district.

1.6 METHODOLOGY AND DESIGN

The study utilised participatory action research (PAR), which recognises community members as experts and is empowering for communities who are enabled to find their own solutions to local issues (Moana, 2010:10). In the context of this study, the researcher and participants were empowered in using indigenous games to solve problems and identify mathematical concepts embedded in them.

The marginalised capital was explored to understand problem-solving by using cultural games, particularly indigenous ones. As Yosso (2005:69) argues, there is much cultural capital in the communities which is not being adequately utilised.

The researcher assembled a team of grade 10 learners in one school located in the rural area of QwaQwa, one deputy principal, one head of department (HOD), three grade 10 Mathematics teachers, two life orientation teachers, two district officials from the Department of Basic Education (DBE), one in the sports section and two Mathematics subject advisors, ten parents with knowledge of various indigenous games and two members of the royal family who were custodians of cultural games, and a lecturer in the school of Mathematics, Science and Technology Education from the university.

The framework was implemented in one school in the rural area of QwaQwa, in Thabo Mofutsanyana Education District. For confidentiality and anonymity the school and participants were given pseudonyms. In stimulating the debate the Free Attitude Interview (FAI) (Buskens, 2011:1) was followed, so as to ensure that participants were central to the study and their voices heard, rather than being perceived as objects to be manipulated and regulated in a setting detached from the real world of their lived experiences and practices (McGregor et al., 2008:199; Stinson, 2012:46).

1.7 ANALYSIS OF THE DATA

The study used Van Dijk’s critical discourse analysis (CDA) in analysing and interpreting data, to get deeper meaning of the spoken words of the research participants. The research team used the CDA because it is compatible with the theoretical framework of the study, and allows for various ways of arriving at the truth (Wodak & Meyer, 2009:3). CDA is a type of discourse analytical research that mainly examines the way social power exploitation, dominance, and inequality are sanctioned, reproduced, and opposed by text and talk in the social and political context. It seeks to pinpoint occurrences of ‘discursive injustices’ and denotes a form of resistance to unethical and biased social power relations (Hakimeh Saghaye-Biria, 2012:509; Van Dijk, 2003:352; 2009:63;).

The following is an example of a text, with translation, from chapter four:

> '…. *ha le letsoho la mme MmaTumelo le phahama ho bonahala hore kgati e phahamela hodimo, moo le letsoho la hae le theohelang fatshe, le kgati e theohela fatshe...'* As Mrs. MmaTumelo's hand goes up, it shows the rope is going up, and as the hand goes down, it illustrates that the rope goes down).

The above text was analysed and interpreted so as to get the deeper meaning form the playing of *kgati*, as follows:

> The mathematical knowledge/content extracted from this movement of the rope is that, as it (resembling a curve) rises it shows that the curve is increasing, and as it descends it shows that the curve is decreasing. Also, the phrases '*hand goes up and hand goes down'* demonstrates the movement of the rope or curve along the vertical axis or y-axis. The extract further demonstrated that learners or participants are able to visualise the movement of the skipping rope (*kgati*), and with the assistance of linguistic capital they possess they are able to describe the movement portrayed by the skipping rope.

1.8 IMPLEMENTATION OF THE FRAMEWORK

The framework used indigenous games to teach problem-solving in a grade 10 mathematics classroom in the following stages: preparation, planning and actual implementation. The preparation stage was followed in two stages, as when the research initiator met with the supervisory team and cohort of students to present the research proposal, after which the contributions from the supervisory team and cohort of students were incorporated into the research proposal for submission to the committees for Title Registrations and Ethics, and finally to the Faculty Board for approval. In the second stage the research initiator met with the possible participants to hold discussions and secure their participation (Chilisa, 2012:250; Dodson et al., 2005:953) in the research. Since the study was taking a post-positivist approach it was important that voices of participants were listened to, rather than objectifying their feelings and thoughts, whereby they are manipulated for the researcher's benefit (Barker, 2012:12; McGregor et al., 2010:422,423). The planning phase sees all the stakeholders who have interest in the research being invited and the team being formally constituted. Based on analysis of the strengths, weaknesses, opportunities and threats (SWOT) the team mapped the way forward by drawing up an action plan that guided our activities. Implementation followed, with the activities planned being put into practice. The stages are covered in greater detail in chapter five.

1.9 FINDINGS, SUMMARIES AND RECOMMENDATIONS

The main ideas and constructs which were formulated from the theoretical and empirical data were analysed and interpreted using critical discourse analysis (CDA) in chapters two and four. The analysis in chapter four assisted the study to craft findings, compile summaries and make recommendations. These showed how the study contributed to the body of knowledge. Chapter six presents the findings, summaries, conclusions and recommendations.

1.10 THE VALUE OF THE RESEARCH

The research helped identify serious challenges in the teaching and learning of problem-solving in grade ten mathematics classes. The method used by teachers in teaching problem-solving greatly inhibits learners' memorising of mathematical content and formulae with understanding, and hence their accessing it. Parents who possess rich capital in the teaching of problem-solving were alienated. It was found that the cultural capital possessed by learners and parents assisted in concretising the mathematical content, which initially looked abstract. As a result, learners' performances showed significant improvement (refer to Table 4.6.7(a,d)). The cultural capital of parents helped learners to extract mathematical content infused in indigenous games and encouraged parents to realise that they could play a significant role in the teaching of problem-solving. There were good practices that sustained the framework to teach problem-solving and achieve good results.

1.11 ETHICAL CONSIDERATIONS

In ensuring no harm befell the research participants it was necessary that they be protected from physical, mental or psychological injury (Chilisa, 2012:86), or any form of harm that might make them feel uneasy. In making certain that the study was not harmful to the participants and the institutional guidelines were adhered to, the Ethics Committee of the University scrutinised the proposal and ultimately granted approval for the research to continue to conduct the study. A similar request was made to the Free State Department of Education for permission to conduct the study at the local school of Thabo Mofutsanyana Education District (see Appendix B1).

All the research participants were given consent forms to sign (see Appendices D1-D6, and E1. The forms stated that participation in the study was voluntary, with no member forced to take part. If at any stage a participants wished to withdraw he or she was at liberty to do so. For learners who were underage, parents were given the consent forms to sign on their behalf of such learners. For parents and learners who did not understand English well these consent forms were translated into the home language. Participants were assured anonymity with regard to the information they would supply.

1.12 STRUCTURE OF THE THESIS

Chapter one has provided an overall picture of the study. The background to the study and problem statement were discussed, the research aim stated and research questions posed. For achieving the research aims five objectives were identified. Finally, the research methodology, findings, recommendations ethical considerations, value of the study, and structure of the thesis were briefly presented

Chapter two focuses on the details of the theoretical and conceptual frameworks, that is, community, cultural wealth and ethno-mathematics) that anchor the study. The operational terms used in the study are defined, and the related literature reviewed. This assists in formulating main themes (constructs) that operationalise the research objectives.

Chapter three describes the research methodologies used in generating the data for the study. Participatory action research (PAR) was used as it matched well with the theoretical and conceptual frameworks of the study. It allows participants to contribute meaningfully to the study. Data collection procedures and ethical considerations are addressed.

Chapter four discusses the analysis of data, presentation and discussion of the findings of theoretical and empirical data. The critical discourse analysis is used in analysis data so as to get the deeper meaning of the texts.

Chapter five explains how the framework of using indigenous games to teach problem-solving was successfully implemented, and on the stages that must be observed when implementing it.

Chapter six presents the findings, draws conclusions and makes recommendations of the framework to teach problem-solving effectively. This is an illustration of how the objectives were met, responding to the research aim and research question.

1.13 CONCLUSION

This chapter has provided a background to the study, as well as posing the research question, and outlining its research aim and objectives. The significance of the study, ethical considerations and structure of the thesis were provided. It provided the gist of the framework of using indigenous games to teach problem-solving, that is, the research participants collectively formulated the framework, and all had ownership of it. In ensuring sustainability the study included local residents, that is, subaltern parents, community members and traditional leaders.

The school community and society began to realise the knowledge they have is rich in concretising mathematical concepts. The community’s cultural wealth helped form a lens that assisted in inculcating the support of parents in formulating the framework. The lens is consistent with the post-positivist paradigm, as it placed the human beings at the centre. Again, it illuminates and validates people’s background knowledge, and relies on people’ words as its primary data (McGregor, 2010:421; Ryan, 2006:21), rather the viewing human beings as objects to be manipulated and controlled and to reach hidden motives of the researchers.

CHAPTER 2

THE FRAMEWORK OF USING INDIGENOUS GAMES TO TEACH PROBLEM-SOLVING

2.1 INTRODUCTION

The study formulates a framework towards the enhancement of problem-solving skills in a grade 10 mathematics classroom using indigenous games as a strategy. In pursuance of the aim, this chapter first presents both the theoretical and conceptual frameworks as a lens through which to analyse and operationalise the objectives. Yosso's (2005:70) community cultural wealth was found to be the most appropriate theoretical framework to launch this discussion because it talks to issues of aspirational, navigational, social, familial, linguistic and resistance capitals which are so central when analysing learning from the perspective of subaltern people (Ladson-Billings, 2005:117; Odora-Hoppers, 2000:286) whose cultural wealth has been marginalised in the learning of mathematics in particular (Chilisa, 2012:40,41; D'Ambrosio, 2009:89; Ernest,1989:558,559;Mosimege, 2000a;457; Mosimege, 2000b:11; Yosso,2005:77-88). The discussion of the above entails understanding the challenges faced by learners and teachers in grade 10 mathematics classrooms when learning problem-solving skills, as well as how they can be overcome using indigenous games as a strategy.

Once the theoretical framework has been clarified, as the chapter will proceed to analyse ethno-mathematics (Chilisa, 2012:131; D'Ambrasio,2009:91; Ernest,1989:556; Gerdes,2009:11; Mosimege, 2000a:456; Odora-Hoppers, 2000:285,290) as the logical conceptual framework within which to understand the abovementioned challenges of learning problem-solving skills in a mathematics classroom. It is within this conceptual framework that the use of indigenous games in the learning of mathematics is thoroughly researched (Chikodzi & Nyota, 2010:3,4). Such a conceptual framework enables the study to demonstrate how tacit community cultural wealth of the subaltern communities is rich with materials for learning the much valued problem-solving skills in mathematics.

The definition of the operational concepts, ‘problem-solving skills in mathematics classrooms’ and ‘the use of indigenous games to enhance them’, will be provided in the context of both the theoretical and conceptual frameworks referred to above. Focus will be on the aspects of problem-solving such as patterns, functions, as well as trigonometry in two and three dimensions. Lastly, through a review of the related literature from best practices of learning problem-solving skills using indigenous games from the Southern African Development Community (SADC), the African continent and internationally, the study will develop constructs to be used when making sense of the empirical data in chapter four.

2.2 THEORETICAL FRAMEWORK AS A LENS THROUGH WHICH TO ANALYSE AND OPERATIONALISE THE OBJECTIVES

The study is couched by community cultural wealth theory, interrelated (Lynn, 2004:156; Yosso, 2002:98,100,102) with other theoretical frameworks embraced within critical theory. Importantly, as demonstrated Barker (2012:12), Lynn (2006:20) and Stinson et al. (2010:44-46), it provides tools to examine inequalities and their root causes, and to work toward development of frameworks aimed at transforming the existed inequalities and crisis in education. Habermas, one of the critical theorists, argued that critical theory also critiques capitalism and liberates human beings from exploitation and expression. As the result, community cultural theory is relevant for the research as it makes the marginalised communities aware that the knowledge they possess is very rich in the teaching of problem-solving.

On the other hand, community cultural wealth challenged the Bourdieuean cultural capital theory that contends that the knowledge of the upper and middle classes has esteemed capital value. In other words, if one is not born into a family unit whose knowledge is already highly respected, then one accesses the knowledge of the powerful class through formal schooling (Yosso, 2005:70). The marginalised knowledge that learners and communities bring from home is relevant for understanding and master problem-solving.

2.2.1 Origin of community cultural wealth theory and the operationalisation of the framework to teaching problem-solving

The research team utilises the community cultural wealth theory to operationalise the objectives of the study, and to help understand the problems or crisis that exists in the teaching of problem-solving. For the study to use the positivist approach would be detrimental to reaching its objectives, as demonstrated by McGregor and Murnane (2010:422), that in the middle of the twentieth century Karl Popper and Thomas Kuhn challenged the scientific community regarding human beings as objects to be researched in a controlled environment. Rather, their hopes, values, expectations and feelings were divorced from the research processes (Royal Lybeck, 2010:96), and positivists believe that knowledge is true only if it was constructed by utilising scientific methods, by which data is derived from experiments in controlled settings and human beings are detached from real world of lived experiences and practices. The positivist paradigm dominates and alienates the knowledge of marginalised societies. In the sense that human beings are dehumanised they are seen as numbers to be manipulated rather than people who have opinions and objections to be listened to.

The study therefore follows the post-positivist paradigm, appropriate to demonstrate the objective of the study to identify the components of the solutions to the challenges. Barker (2012:20) and McGregor et al. (2010:423) cited Ponterotto (2005) and Zammito (2004) in illustrating that the term ‘post-positivism’, originated in the 1960s, presumes that there are various ways of knowing rather than commissioning scientific methods. It maintains that there is a place for the voices and the key roles of the researcher and research participants in providing possible solutions to the identified problems. In the post-positivist research paradigm, human beings benefit from the research, and research conducted happens in communities, that is, in natural settings rather than in experimental settings (Barker, 2012:20; McGregor et al., 2010:424).

The community cultural wealth theory acknowledges the marginalised background knowledge of subaltern communities in the teaching of problem-solving. The strengths enable the research team members to formulate the components of the strategy that effectively teach problem-solving. For the components of the strategy to succeed, Lynn (2006:23) argues that teachers and parents need to intertwine the indigenous games and mathematical content embedded within the games.

Jordan (2003:186) and Aparico (2012:40,46) cited Paulo Frere (1998) and Mato (2000) in contending that community cultural wealth is a powerful theory (Perez Huber, 2009:706) as it works with subordinate or oppressed groups to better their circumstances within society. Community cultural wealth embodies the principles of democracy, whereby the opinions and knowledge of the marginalised are valued. They further point out that the main problems the participants should deal with is to combat against the power of the principal neo-liberal ideology that offends against and attacks human nature while reproducing itself socially and historically, threatening dreams, hopes and aspirations of disfranchised communities. These include the rejections of other intellectual and cultural practices and marginalisation of excluded communities. However, they noted that the theory promotes the cultural capital of the poor and disenfranchised.

In addition, Perez Huber (2009:706 and Yosso (2005:69) posit that community cultural wealth refutes the claim that marginalised communities are replete with cultural poverty. Rather, community cultural wealth theory recognises and validates the range of cultural knowledge, skills, abilities, and interactions retained by socially marginalised communities. It affirms the value of the cultures of all peoples, irrespective of their social class and socio-economic standing.

On the other hand, it can be noted that community cultural wealth theory draws from critical theory. Backer (2012:202) and Sudersan (1998:254) argue that critical theory runs against the dominance of science over other disciplines, as scientific method ignores the human element, that is, personal reflection, insights, and bias. Critical theory disrupts and challenges the status quo, by which human beings are objects to be studied. This agrees with Stinson and Bullock (2012:47), quoting Ernest (2010), when they argue that the teaching and learning of problem-solving is designed to reshape the school mathematics so that it is empowering for all marginalised people, and also edifying for the human spirit of all. By the same measure, community cultural wealth theory validates and acknowledges the marginalised knowledge possessed by the subaltern communities as powerful in the teaching and learning of problem-solving (Lybeck, 2010:97, Yosso,2005:69), and validating teaching and learning of problem-solving as accommodating human experience.

Early critical theorists, Max Horkheimer, Theodor Adorno, Herbet Marcuse and Jugen Habermas, pointed out that positivism relegated freedom, beauty, justice, peace and love. It is evident that community cultural wealth inducts the conditions of comprehensibility, trustworthiness and credibility (McGregor, 2010:423; Sudersan, 1998:260,263). They maintain that research conducted within critical theory should not be neutral, value-free and unbiased, but be value-laden, subjective, inter-subjective and value-driven within the critical paradigm (McGregor et al., 2010:423). In addition, Chilisa (2012:46, cited in Guba & Lincoln, 2005) contends that critical theorists wanted to rupture the hegemonic scientific methods that see the world in one colour. The community cultural wealth theory was considered an appropriate lens for the study as it empowers and emancipates the marginalised communities. The background knowledge possessed by learners is relevant in contextualising the subject matter in learning problem-solving. The indigenous games played by learners in the homes are very rich in mathematical content. For the components of the strategy to succeed, teachers and parents need to intertwine the indigenous games and mathematical content embedded within them (Lynn, 2006:23).

In order to derail the risks factors, Lynn (2004:162, 2006:18) and Yosso (2002:100) evince that the experiences of learners must be central to the teaching and learning of problem-solving rather than at the periphery, and learners, teachers, and subaltern communities' prior knowledge should be considered. Teaching and learning of problem-solving be understood within the social activity context (Stinson, 2012:40), and community cultural wealth theory seeks to circumvent the repressive social structures and practices that scuttle the components of the strategy. Community cultural wealth allows teachers, learners and subaltern communities to see the wider social and political picture of problem-solving that embrace emancipation and deconstruction of social phenomenon, rather than perceiving problem-solving as a "White institutional space' or belonging to upper class of the society (Stinson et al., 2010;44; Yosso, 2005:70).

It can be concluded that community cultural wealth theory assists the study to realise its objectives, in the sense that the cultural practices of the marginalised communities (playing of the indigenous games) are used in the teaching of problem-solving. Also, the theory relates to interdisciplinary approach of or Kimberle Creshaw (1995) and concept of intersectionality of Lynn (2004:156), which helps the study to link the mathematical content to cultural practices. The use of indigenous games to teach problem-solving will help the marginalised communities to reaffirm and validate their indigenous games as an approach appropriate to teach and learn problem-solving effectively. The cultural context of the learners (Martinez, Saenz, Yamamura, 2010:131) is acknowledged and emphasised. In order to concretise the abstractness of mathematical content, the lived experiences of learners are considered in the learning of problem-solving.

In contextualising the problem-solving, Dika and Sigh (2002:48), Lynn (2004:162) and Yosso (2002a:100) assert that parents are the first teachers at home, as they better understand the familial environments and background knowledge of the child. Similarly, Kellaghan, Greany, Murray (2009:120) and Yosso (2002a:142) regard the valuable knowledge of parents as an extra resource in the teaching and learning of problem-solving skills. The teaching of the problem-solving using indigenous games is acknowledging the huge wealth of knowledge possessed by the families and communities. Thus, the research team included marginalised groups, such as parents and community leaders, so that their skills and abilities could be used in extracting the mathematical content infused in the indigenous games. It was anticipated that the community cultural wealth theory would help in the analysis of the mathematical content embedded within the indigenous games, and challenge myths about the abstractness of problem-solving and the subordination of the background knowledge and lived experiences (Lynn, 2004:154; Stinson et al., 2010:46). The use of indigenous games in the teaching of problem-solving should maintain and develop team members' cultural identities, formerly gagged and disempowered by suppressive theories or deficit-based theories which lessen the role of social structures in the teaching of problem-solving.

2.2.2 Tenets of community cultural wealth theory

Larrotta and Yamamura (2011:76,77), Martinez et al. (2010:131), Perez Huber (2009:710) and Yosso (2005:77) characterise six types of community cultural wealth(CCW) theory, that is, aspirational, linguistic, familial, social, navigational, and resistance, that learners may bring to school in various measure, irrespective of their socio-economic status, rural or urban origins, ethnicity, race, or creed (Mahlomaholo, 2010:16).

They are characterised as dynamic processes that build on one another, interconnect and overlap, and make learners aware that the type of education they receive from home can be used to understand the problem-solving skills in a mathematics classroom. In teaching mathematical concepts they must be treated in a manner that shows connectedness or integration of mathematical content knowledge. It has been emphasised (DBE, 2011:5; DoE, 2003:3) that mathematical content must be treated in a manner that shows progression within a grade and within phases, with consideration of how to infuse indigenous knowledge in the teaching of mathematical content. The theory posits that previously marginalised communities, with the knowledge and skills they possess, can be empowered through being connected to knowledge and skills gathered in formal classes to be used fruitfully by learners in mathematics classes.

Each principle is now discussed in greater detail, particularly as it relates to the study.

2.2.2.1 Aspirational capital

Aspirational wealth is the competency to maintain hopes and aspirations for the future, irrespective of noticed obstacles (Yosso, 2005:78). A positivist approach did give disadvantaged communities leeway to air their views in the research process, buts human beings were regarded as object to be studied (McGregor et al., 2010:423; Stinson et al., 2012:46). The critical theorists identified the challenges imposed by the positivists on exclusion of the human element, and social practices and ideologies that attempted to conceal asymmetrical power relations. They maintained that after the problem had been identified it should be changed rather than merely talked about (Barker, 2012:66; Royal Lybeck, 2010:95; Stinson et al., 2012:46).

In the same way, the teaching of problem-solving underplayed or ignored the background knowledge of the learners (Lynn, 2004:154; Yosso, 2002:96). As a result, schools in the teaching of problem-solving work from the premise that learners from subaltern communities possess inadequate knowledge of problem-solving (Yosso, 2005:70). During the teaching of problem-solving the teacher played a major role in transmitting the knowledge to the minds of learners, thus the community cultural wealth through aspirational capital brought hope and solutions that, regardless of barriers, allowed marginalised people to take charge of their destiny and exercise their human right to participate in the emancipation project (Moeketsi, 2011:155; Stinson et al., 2012:46).

As Mapadimeng (2011:253) and Moeketsi (2011:155) illustrate, regardless of their cultural practices being relegated and ignored by colonial masters, *ubuntu/botho* culture brought hope and light that their aspirations for community cultural wealth espouse the courage that together more can be achieved, despite obstacles faced. For Mahlomaholo (2010:16), learners from depressed and poverty-stricken environment, despite the odds, are determined and remain focused in the learning of problem-solving. During the process, learners are engaged in activities which require thought, and it takes time to arrive at the correct solutions. As they have the aspirational capital they can continue working on the activity until they arrive at a reasonable solution. To analyse the mathematical content imbued in the indigenous games, teachers and parents need patience with the learners so that they have time to explore various options.

2.2.2.2 Navigational capital

Navigational wealth describes skills of manoeuvring through social institutions and establishments or structures of inequality penetrated by racism (Yosso, 2005:80). These are inner resources which one acquires without attending formal institutions, including inborn qualities and skills that help one survive in hard times

(Huber,2002:706; Ruthven, 2001:358; Tondi,2011:295). In the context of the study, during the teaching of problem-solving learners are assigned activities that require them to work independently of the teachers. In most cases, the activities are learner-centred, with the teacher providing assistance only when learners are stuck. The assistance provided is not straight to the answer, but the facilitator has to ask questions that stimulate ideas to solve or work on the activity at hand. Learners have to manoeuvre through how the mathematical formulae are formulated, rather the teacher giving ready-made ones. This helps learners to use their background knowledge to discover mathematics concepts, definitions and axioms (refer to section 4.2.5).

2.2.2.3 Linguistic capital

Linguistic capital refers to the intellectual and social skills and experiences learned at homes. At times, children turn up in schools with multiple language abilities and communication skills, having been exposed to storytelling skills, such as listening to and narrating oral histories, memorisation and attention (Moeketsi, 2011:150-152; Tondi, 2011:294; Yosso, 2005:79). Thus, in the teaching of problem-solving it becomes easy for learners to use their home language to describe the mathematical processes and content illustrated by the playing of indigenous games. For instance, learners can describe the process of playing *kgati* in any language they feel comfortable with, and teachers can assist them through scaffolding to write the process symbolically, using the mathematical symbols. Learners need to be able to move from mathematical expressions and symbols to describe the process verbally.

Essien and Setati (2007:217,218) found that learners struggle to comprehend the teaching and learning of problem-solving in a language that is foreign to them or in language that is not their mother-tongue language. This can be viewed as one of the factors that contribute to underachievement in mathematical content by learners in most rural schools. Language is important as it develops conceptual analysis of mathematical processes and skills and ways of thinking about their lived realities and everyday lives (Chilisa, 2012:57; Tondi (2011:295).

2.2.2.4 Familial capital

Familial capital refers to cultural knowledge developed through interactions with families and friends in sports and social community settings. It carries a sense of community history, memory, and cultural intuition, and models lessons of caring and coping, with moral support from the families and friends (Perez Huber, 2009:710; Yosso, 2005:79). Within the context of the study this allows parents and communities to play key roles in the teaching of problem-solving. The team members in the study were able to demonstrate affective domains, and as argued by Mahlomaholo (2010:17), teachers and parents create a conducive learning environment in the teaching of problem-solving, such that learners acquire confidence through motivation, and sense of ownership of and control over their own learning and themselves. Also, parents and teachers give learners advice on how to cope with peer-pressure that might distract them from learning problem-solving. Ultimately, learners will gain confidence in themselves, which boosts their self-esteem (refer to 4.2.3 and 4.2.4).

2.2.2.5 Social capital

According to Perez Huber (2009:710) and Yosso (2005:79), social capital involves the interactions in networks of people within the society. As they collaborate they learn from others how to navigate through social institutions. The community takes the responsibility of looking after its members, developing a sense of trust and openness among in confiding on sensitive matters when they require assistance. In most cases, the teaching and learning of problem-solving is approached from the learners' perspectives, with activities designed in such a way that they work cooperatively in small groups. As they interact they learn from others and share ideas. Peer teaching becomes very effective. Learners who are not following certain mathematical concepts are still able to ask for clarification.

2.2.2.6 Resistance and resilience capitals

Mahlomaholo (2012:102) and Yosso (2002:93) argue that these cultural capitals enabled learners to challenge instances of inequity, unfairness, discrimination, oppression and marginalisation instituted against them. With the assistance of navigational capital, learners exert pressure in challenging the societal practices that promote social injustices. For Lynn (2004:154), they are demonstrated when members of the community resist and challenge the hegemonic leaders who unfairly oppress them. They are important forms of capital as they allow learners to enter into debates that promote critical thinking, equipping them to raise questions such as 'why?', and 'how?'. For those instances during the class discussion when the conclusion is reached that any number to power zero is one, there must be a logical justification made.

2.2.3 Formats of community cultural wealth theory

The various theories, whether of community cultural wealth, critical race, feminism, critical pedagogy, Critical Emancipatory Research (CER), bilingualism, multiculturalism and Afrocentric curriculum, amplify the principles of critical theory. They work collectively to fight for freedom for all, practising good democratic values and principles, and equal distribution of power. Also, they fight to cleanse society of the evils plaguing it, and challenge inequalities and any form of subordination imbedded in social structures, practices and discourses (Lynn, 2004:162; Stinson et al., 2012:46; Sudersan, 1998:254). They serves as a guide for teachers and practitioners to reveal and confront all forms of racial inequality, which are hidden as "impartial" and "objective" structures, processes, and dialogues of mathematics curriculum content. The community cultural wealth theory as a framework draws strengths from other convergent critical frameworks used to tackle any form of subordination in the teaching and learning of problem-solving (Perez Huber, 2009:705; Yosso, 2002:93,102; 2005:70).

The collaborative styles of different theoretical frameworks gives strength in collectively addressing the challenges of teaching problem-solving, rather than the teacher alone crafting solutions to the identified challenges.

2.2.4 Epistemology, ontology and implementation of the framework to teach problem-solving

The theoretical framework adopted falls within the post-positivist paradigm that maintains that knowledge is authentic if it can be transformed into practice that empowers and transform the lives of the subaltern people (Chilisa, 2012:36), unlike the positivist view that knowledge is true if it discovered ‘out there’ by using scientific methods (McGregor et al., 2010:423). Thus, the study used community cultural wealth theory that perceives the truth as instituted in the context, that is, social, historical and cultural habits. The theory postulates that knowledge is emancipatory, generated around critical probing the way things have always been figured out (McGregor et al., 2010:431). For Barker (2012:20), knowledge is perspectival in character, and allows multiple viewpoints. Thus, in the teaching of problem-solving, the interactions between teachers and learners and between learners and learners are emphasised, so as to generate multiple ‘knowledges’, to help learners more easily understand the subject matter under discussion. Stinson et al. (2012:44) cited in Lerman (2000) contend that teaching and learning of problem-solving is conceptualised within social activity context, hence the marginalised knowledge of learners is considered in making mathematical content more accessible.

The theory allowed the study to include teachers, learners, and community members in the teaching of problem-solving. As Yosso (2005:69) argues, community cultural wealth shifts away from an insufficiency view that communities of colour are places disadvantaged by deprivation, and instead focuses on and learns from a wealth of knowledge, skills and capabilities captured by socially marginalised people. There are multifarious realities modelled by human rights values, democratic and social justice values, and political, cultural, race and gender values. Human beings have the ability to exercise jurisdiction over social arrangements and institutions to generate new reality and actuality (Chilisa, 2012:40; McGregor et al., 2010:432). As a result, the theoretical framework acknowledges the use of indigenous games to teach problem-solving as an appropriate way to discover multiple realities for learners to understand mathematical concepts and content. For Alaranta (2006:2), the community cultural wealth allows the collaboration of teachers, parents, district officials, and learners in the teaching of problem-solving, and empowers them with self-belief. Consequently, they have confidence and the will to conduct research on their own reality, using their ways of knowing, and to use research findings to embark on positive social change (Chilisa, 2012:235).

2.2.5 The role of the researcher, relationship with the research participants and the framework of teaching problem-solving

According to Solorzano and Yosso (2002:37), the theoretical framework of community cultural wealth uses multiple methods to access knowledge, often the unconventionally excluded as an official part of academia. For instance, the researcher and the participants are part of the research processes, that is, they are means to an end (Stinson et al., 2012:42; Sudersan, 1998:256). For Chilisa (2012:22) cited in Weber-Pillwax (2001) and Ercikank and Roth (200:15), the method used in the research should help to build respectful relationships between the research topic, participants and the researcher. In developing the framework the researcher was careful not to use personal powers to suppress or marginalise the lived experiences of the participants.

In addition, post-positivism seeks truth of knowledge through experiences and perspectives (Dodson & Schmalzbauer, 2005:955) of the research team, that is, research participants and researcher. Community cultural wealth theory accepts that the experiential knowledge of subaltern people is justifiable, legitimate, suitable and critical to understanding and analysis of problem-solving. Bungane (2014:33) and McGregor (2010:423) recommend that the researcher and the research participants operate as equals, to think, interpret and create meaning mutually towards the attainment of the goals set under the objectives of the framework. Human beings (researcher and participants) are key to research activities and processes, rather than isolated from it, so research partners such as parents, teachers and learners work collaboratively in the teaching and learning of problem-solving. Bungane (2014:33) cited in Mahlomaholo & Netshandama (2013) and Nkoane (2006) regard the role of the researcher as that of a convenor who creates spaces in which people can work on a solution to the problem.

2.3 CONCEPTUAL FRAMEWORK AS A LENS THROUGH WHICH TO ANALYSE AND OPERATIONALISE THE OBJECTIVES

The conceptual framework guiding the study is ethno-mathematics, a domain seen as the interplay between culture and mathematics. It is the study of the mathematical practices of cultural groups over the course of their daily lives, and mathematical ideas observed within a culture before being treated formally by western-style mathematicians (Chikodzi & Nyota, 2010:3); D’Ambrasio, 1999:132,143; Murray & Tillett, 2011:172). The link between mathematics and culture helped rural Shona learners see and appreciate the relevance of mathematics in their day-to-day lives (Chikodzi & Nyota, 2010:4). Rowlands and Carson (2002:84) state that the focus of ethno-mathematics is on cultural groups and people creating their own mathematics out of their everyday lives, rather than teaching and learning it as a formal academic discipline.

2.3.1 Origin of ethno-mathematics and operationalisation of the framework to teaching problem-solving

In operationalising the objective of the study to understand the challenges in the teaching of problem-solving, the philosophical basis for ethno-mathematics is elucidated by examining Wittgenstein's philosophy (Vilela, 2010:356). The radical change by Wittgenstein against relativity theory of Einstein is perceived as the solution to the identified challenge of objectifying human knowledge (McGregor et al., 2010:424). In the 1930s, Wittgenstein’s theory was reinforced by neo-positivism (Penco, 2010:5), hence his reluctance to endorse scientific points of view that mathematics was a product not a process (Vilela, 2010:345). Further, Relativity theory takes the role of scientific laws, with truth determined by fixed rules and procedures that do not accommodate social or human perspectives. As a result, Wittgenstein was dissatisfied with the problem of rule-following, which forms the foundations of mathematics. He took a radical stance, that learning of problem-solving meant being able to see the truth in problem-solving in different ways (Penco, 2010:11-12). This confirms Vilela's view (2010:356) cited by Gottschalk (2008) that Wittgenstein's philosophy perceived the truth in problem-solving as practical and concrete to life situations, that is, a broadening of meanings and humanistic focus (Overgaard, 2005:250; Vilela, 2010:349) are considered, rather than fixed interpretations. It can be noted that Wittgenstein's radical stance against the notion of viewing rules as incisive and metaphysical was replaced by accommodative solutions. Solutions embrace problem-solving as a social practice, that is, practice that involves contextual factors, and also use language as a means of expressing knowledge (Vilela, 2010:348).

Penco (2010:11) and Vilela (2010:346) show that Wittgenstein' s philosophy is associated with ethno-mathematics such that meanings are not fixed or predetermined, and quoted the claims of D'Ambrosio (1999) and Barton (1998) that ethno-mathematics is a humanistic domain. The study adopted ethno-mathematics because it gives voice to the learners when faced with a challenging situation during the teaching and learning of problem-solving. Learners are more able to invent solutions than to teach learners how to find solutions to activities for problem-solving (Alexander & James, 2005:17; D'Ambrosio, 2009:94-96). Consequently, ethno-mathematics is an appropriate lens through which to affirm the marginalised knowledge of subaltern communities (Lynn, 2004:155; Yosso, 2005:70) in the mainstream curriculum of mathematics. Also, the conceptual framework is used to uncover the deficit-informed research that suppresses and distorts experiences of excluded communities (Solórzano & Yosso, 2002:37).

Ethno-mathematics and Wittgenstein are associated with Lave's notion of situated learning (Overgaard, 2005:251; Vilela, 2010:356), whereby situated learning is regarded as the disagreement between mathematics as an end product or domain of knowledge and as process, displayed in the mathematical activities or events of an academic or layperson in everyday situations, that is, mathematics as used in diverse practices. In addition, D'Ambrosio (2009:94) points out that viewing mathematics as a product or Platonist approach is dominated by memorising formulae, such that learners master the instruments to find correct answers. However, the challenge of this approach is that the understanding of problem-solving is deficient, hence Mukhopadhyay's (2013:100) contention that ethno-mathematics incorporates culturally embedded mathematical practices into school mathematics.

2.3.2 Epistemology, ontology and implementation of the framework to teach problem-solving

As indicated above, ethno-mathematics is informed by the principles of Wittgenstein's philosophy in which meanings are broadened by humanistic elements (Vilela, 2010:349). This suggests that the knowledge is relational, and is guided by a set of numerous relations that one has with the universe (Chilisa, 2012:40). Also, McGregor et al. (2010:423) point out that there are many more ways of knowing than using scientific methods. Humans are central in the study, as their voices are listened to. The framework of using indigenous games to teach problem-solving validated the knowledge of teachers, learners, and subaltern communities in constructing mathematical concepts. Thus, problem-solving knowledge is generated through inductive reasoning (McGregor et al., 2010:425). The patterns and commonalities, placing experiences in words displayed by various indigenous games are critically analysed, so as to uncover hidden mathematical concepts.

On the other hand, the ontological stance of ethno-mathematics is the same as that of the theoretical framework, confirming Chilisa (2012:36) quoting Neuman (2010) that reality is created by social location. Various versions of reality are validated and recognised, and reality has multifaceted layers, the surface and deep structures being indiscernible. Ethno-mathematics helps to unmask the deep structures. McGregor et al. (2010:426) contend that ethno-mathematics as the conceptual framework can be used to search for meanings in distinct cultural and social contexts, as opposed to using general laws (Sudersan, 1998:263). The general laws are used to marginalise the wealth of knowledge possessed by the subaltern communities in accessing mathematical content, as argued by Solórzano and Yosso (2002:37) when they point out that experiential knowledge of excluded people is legitimate, justifiable, suitable and critical to understanding problem-solving.

2.4 DEFINITION AND DISCUSSION OF OPERATIONAL CONCEPTS

This section defines and discusses operational concepts that are used in the study.

2.4.1 Teaching of problem-solving in the grade 10 classroom

According to Grinstein and Lipsey (2001:69,70) in the *Encyclopaedia of Mathematics Education*, Posamentier and Kruik (2009:1) and Stacey (2005:341) indicate that 1989 National Curriculum of Teachers of Mathematics (NCTM) revised their mathematics curriculum with focus on mathematics as problem-solving, and is at the heart of mathematics. More importantly, problem-solving has been an integral part of mathematics throughout recorded history. Teaching of problem-solving focuses on instruction in mathematical concepts, which can be instructed in three stages, that is (i) concrete, hands-on or enactive; (ii) representational, pictorial, graphic of as mathematical concepts; and (iii) as mathematical concepts in words, symbols and abstract information.

Thus, the teaching of problem-solving in a grade 10 mathematics class is inspired by constructivist theory and situative theories of learning, the latter of which defines learning as active participation in socially organised activity, as the principal content of what is to be learned is not given but must be discovered by learners. In this way learning is made meaningful to learners. Both theories can be viewed as interconnected and interrelated, as learning involves both creation and enculturation (Koellner, Jacobs, Borko, Schneider, Pittman, Eiteljorg, Bunning & Frykholm, 2007:278).

2.4.1.1 Problem-solving

As shown by Bayazit (2013:1920), Mamona-Downs and Downs (2005:386), and Posamentier and Kruik (2008:4; 2009:2), problem-solving mean different things to different people. It may be considered as a topic of instruction, that is, as a subject in the Mathematics curriculum that must be taught to children in the same ways that multiplication, long division, and percentages are taught. It is a subject for study in and of itself. Stacey (2005:345) defined it as teaching content for later use in solving mathematical problems, *for* and *through* problem-solving. It could be teaching standard mathematical content by giving details to unfamiliar problems and teaching heuristic strategies to enhance overall skills and ability to solve problems, or teaching *about* problem-solving.

Others view problem-solving as a mode of instruction, using problem-solving as the underlying thread to unite all the mathematics taught. It provides a rationale for teaching the skills of arithmetic. It may be used to conceptualise problem-solving as a way of thinking, as learners cannot expect to learn to be problem-solvers without careful structure of the process. Although some intuitively may be good problem-solvers, most must be taught how to think, how to reason, and how to solve problems.

As stated in this section 2.4.1.1, above, problem-solving is considered a topic of instruction or mathematical curriculum to be taught to learners. This is the definition that will guide the study in conceptualising the problem-solving. In the study the problem-solving and mathematical content or topics are used interchangeably. The DBE (2011:9) and Grinstein and Lipsey (2001:648) outline the following problem-solving, which the research team in the study considers in using indigenous games to teach them:

Algebra: is part of a mathematics curriculum in which the use of variables to represent situations is a main focus.

Functions: learners learn to model real-world events and represent equations with graphs of functions.

Geometry from a synthetic perspective: the focus is on application of geometric properties of figures used to model situations.

Geometry from algebraic perspectives (analytical geometry): this involves more of a coordinated geometry approach and encompasses transformations and working vectors.

Trigonometry: among other content, the learners focus on application of trigonometry to solving problems involving triangles and periodic situations using sine, cosine, and tangents functions.

Statistics: all learners expand their ability in using statistics to include drawing inferences. Also, they have an understanding of randomness and an ability to use simulation to generate or estimate probabilities.

Moloi (2013:192) and Vankus (2008:112) show that in the course of playing of learners actively convey their mathematical knowledge and skills needed to play the game. The language used in the play of indigenous games epitomises the algebraic, functional and geometric meanings. For instance, the play of *diketo* shows the relationship between the throwing of a stone into the air and the number of stones scooped out and placed in the hole. The play of *kgati* demonstrates repeated patterns demonstrated by play. All these practices touch on problem-solving, as mentioned above.

The next three sections illustrate how the teaching of problem-solving has evolved.

2.4.1.2 Teacher-centred problem-solving

Teacher-centred problem-solving is informed by Skinner’s behavioural theory, which elucidates learning as a well-structured behavioural responses to physical stimuli. It includes stimulus or reinforcement approaches to learning, and emphasises behaviour as the ultimate result of learning (Meko, 2013:24). Skinner defines two categories, as positive reinforcers and negative reinforcers. The positive are the stimuli that tend to increase the probability that a particular behaviour will be repeated or strengthened when presented, while the negative stimuli are those that when removed tend to strengthen the behaviour.

This type of teaching regards learners as empty vessels that the teacher fills with information. The activities are teacher-centred, as he or she is playing an active role and learners are passive. In most cases, the teachers seek correct answers to confirm their progress in problem-solving. In a way, these assumptions of behaviourists violate the principles of ethno-mathematics (Vilela (2010:357), in that the learners' environmental background is rich in mathematical content embedded within indigenous games. The teacher assumes power in the teaching of problem-solving and suppresses and silences learners' lived experiences in the classroom discourses (Lynn, 2004:154; Yosso, 2002:100).

Fosnot and Perry (2005:38) and Meko (2013:25) write that educators using behaviourist framework maintain observations, listening to explanations from teachers that result in effective learning of problem-solving. Teachers using this theory explain mathematical content on the chalkboard, while the learners are attentively observing and listening. The major part of the lesson is carried out by the teacher, whilst to a great extent the learners' creativity is suppressed (Lynn, 2006:21). As indicated in section 4.3.1, this approach makes it difficult for learners to comprehend abstract mathematical concepts as they are not given chance to explore or create their innovative thinking around the matter.

This theory contradicts the principles of community cultural wealth, that is, it oppresses and marginalises the wealth that learners own. In addition, Nasir, Hand and Taylor (2008:208) contend that social justice factors such as equity and freedom should be respected in the teaching of problem-solving. Nasir et al. (2008:188) cite the NCTM standard that "Eminence and excellence in mathematics education requires equity—high expectations and immerse assistance to all learners", and as learners learn by connecting new ideas (new mathematical content) to prior knowledge or background knowledge it is imperative for the teacher to be familiar with such wealth resource of knowledge. The lessons need to respond to and build on this knowledge rather than forcing the new mathematical content knowledge into the minds of learners. Also, learners are heavily dependent on the teacher to construct correct answers to problem-solving,

2.4.1.3 Cognitive theory in the teaching of problem-solving

Burgos (2007:152) and Ojose (2008:29) indicate that cognitive theory by Piaget views children's development as occurring through a continuous transformation of thought processes. Children improve progressively and gradually right through the changing stages and experiences in one stage form the foundation for movement to the next. All children pass through each stage before starting the next one, and none skip any stage. Older children, and even adults, who have not progressed to the succeeding stages process and administer information in ways that are typical of young children at the same developmental stage.

Campbell (2006:8) and Meko (2013:25) state that cognitive theory signifies to a learner's characteristic preferences for thinking, perceiving, managing and recalling information. It makes an assumption that human beings are logical, with the capacity to think for themselves and make meaning out of what they see. In this theory, learners in the teaching problem-solving are viewed as active participants in the learning process.

In this study, the thinking and processing of information is not done in a linear way, as the theory suggests, but rather by adopting navigational capital (Yosso, 2005:80), which gives learners freedom to manoeuvre and explore multiple dimensions (Lynn, 2006:19) of mathematical contents infused in indigenous games. They are taught in the higher grades (refer to section 4.3.2), when learners discover that where the second and third difference are constant they use quadratic and 3^{rd} degree. Kauchak and Eggen (2012:xi) and Lynn (2006:19) contend that the instruction of problem-solving lies in learners' different perspectives. This suggest that they need not be restricted in their navigational skills, rather than allowed to exert a remarkable degree of control over their learning of problem-solving. Lynn (2006:19) cites Ferreira (2002) in claiming that learners by their nature have the aspirational capital that give them the possibility of hope as they are engaged in navigational exercise.

2.4.1.4 Constructivist theory as learner-centred teaching of problem-solving

Constructivism understands learning as an interpretive, recursive, multidimensional building process by active learners intermingling with their surroundings ecology, the physical and social world. It is a theory of learning that depicts how systems, language, activity, and meaning-making come about, instead of simply characterises the structures and stages of thought, or isolates behaviours learned through reinforcement. It is a theory grounded on the sophisticated models of evolution and development (De Corte, 2012:35; Fosnot et al., 2005:8).

There is a strong interaction between learners if they learn in collaboration. The activities are learner-centred and teachers regard learners as thinkers and knowledge constructors. The teaching of problem-solving is interactive, mediating the environment with learners. De Corte (2012:36) believes that this guarantees a good performance of learners in problem-solving, whilst Yosso (2002:96) contends that the knowledge of learners is viewed as powerful in the learning of problem-solving. At the same time, the teacher considers learners' knowledge and these conceptions are for use in subsequent lessons.

This theory amplifies the principles of community cultural wealth, as learners are not restricted in their thinking (DoE, 2003:61, 62) in analysing the mathematical content embedded in the indigenous games. The DoE (2003:3,61; DBE, 2011:8) and Kauchak et al. (2012:ix) add that in the teaching and learning of problem-solving the teachers must have high expectations of the learners. The use of indigenous games as a teaching strategy for problem-solving allows learning in the classroom to be maximised. This is in line with the 'centrality and intersectionality' concept in community cultural wealth theory, positing that the marginalised knowledge of learners and parents on problem-solving needs to be put at the centre and at the same time affirmed equal status to other knowledge of the ones in power (Lyn, 2004:162).

2.4.2 Using Indigenous games to teach problem-solving

The underlying principles of using indigenous games to teach problem-solving are anchored in views of problem-solving as a humanistic discipline (Grinstein et al., 2001:332), and community cultural wealth theory (Doug, 2004:250; Yosso, 2005:70) Presently, in most schools, the humanistic dimension is often limited to its private world.

Thomson and Chepyator-Thompson (2002:54, 55) and Osborne (2009:1) indicate that the study of games is a branch in mathematics known as 'game theory' that aims to assist researchers and research participants to comprehend situations in which decision-makers interrelate and network. A game is a practical activity that consists of sets of moves between players. The players in a game compete with each other according to a set of rules. In the context of this study the focus was on indigenous games, and analysis of mathematical concepts embedded in them was made by learners. In using the indigenous games to teach problem-solving it was anticipated that the challenges faced by learners could be minimised significantly.

According to the Department of Sports and Recreation (2006:1), an example of indigenous games commonly played in South African schools include *Morabaraba, Dibeke* (refer to section 4.3.1 examples listed by learners), and they develop the following skills: socialising, ability to strategize, team work, accuracy, coordination, speed, concentration, coordination, rhythm, high levels of fitness, ability to catch, and recreational activity.

Thomson and Chepyator-Thompson (2002:55) view strategy as organised steps which a player intends to adhere to while playing a game, with which one must demonstrate certain knowledge or skills. If knowledgeable about the rules the skills gained correspond to mathematical skills or knowledge, such as the resulting outcome, which cannot be predicted in advance. Mathematically it can be described in terms of a statistical notion of average or expected value, for example, a two-person, zero-sum game. For Murray (2000:113) people are reconfigured by global economics and the world system, so postmoderm cultural globalisation also reshapes the dimensions of knowledge, erasing and rearranging disciplinary boundaries.

On the other hand, the empiricist approach or new wave adopts a descriptive view of problem-solving, as consisting of a motley array of human activities driven by goals, intentions and purposes. Problem-solving is viewed as the humanistic activity practiced by all cultures, amplified and challenged over time through social interaction, and thus opens to change. It has its roots in culture and it is advisable that mathematics practitioners take examples from cultural contexts or through the use of indigenous games (Ernest, 2006:556,558; DBE, 2003:9; Gerdes, 1998:36; Barton, 1996:201-233; O'jose, 2008:29).

This new perspective of mathematics maintains that human context is key to problem-solving. D'Ambrasio (1999:139) and De Corte (2012:34) contend that there is a "mathematics community" which must reach out to the whole community, that is, parents, learners, teachers and other members, or the decline in mathematics performance will continue. The abovementioned scholars espouse a process-oriented view of mathematics with heuristics as central. The mathematics curriculum delivered by the practitioners of this view would consist of exposition and practices enriched with occasional problems for motivation or application (Ernest, 1989:558).

2.5 FRAMEWORKS ON INDIGENOUS GAMES IN PROBLEM-SOLVING

This section reviews the literature from the best practices of learning problem-solving skills using indigenous games from South Africa, Southern African Development Community (SADC), the African continent and internationally. The discussions will be given with respect to each of the five objectives and criteria formulated which succinctly explain each one. These constructs are to be used in chapter four when making sense of the empirical data.

2.5.1 Challenges in the learning and teaching of problem-solving

In this section, the challenges in the teaching and learning of problem-solving will be discussed, with reference to the best practices of teaching and learning problem-solving as articulated by the educational policies and theories of learning.

2.5.1.1 Content is too abstract in the teaching of problem-solving skills

According to Chilisa (2012:57) and Lynn (2006:21) it is clear that the cultural wealth that learners brought from home is relegated to the margins of the classrooms in the learning of problem-solving. The navigational capital affords learners an opportunity to understand their environmental background and contexts easily. In the same vein, the manner in which problem-solving is taught in school tends to lead teachers to ignore the cultural background and context of learners. As a result learners do not comprehend the presented mathematical content (Anthony, 2009:153).

The DoE’s (2003:2) National Curriculum Statement (NCS) states that in order to convey precise meanings to the teaching of problem-solving its content should not exist in isolation from the context of the world in which the learner lives. Campbell (2006:2) cited in Keller (2000) and Graven & Schafer (2013:4) added that it is important to affirm what learners know from their home environment, so that it becomes easy for them to acquire problem-solving skills added to what already exists in their knowledge schema. This will assist learners in the subaltern community to gain access to mathematics with regard to problem-solving skills

Similarly, the DoE (2003:2) through outcomes-based education (OBE) principles, and Su et al. (2013:2) assert that prior learning needs to be recognised in the teaching of problem-solving skills. This helps to integrate the new knowledge to the cultural background knowledge gained through interaction with the society. New knowledge taught in class to learners is easily concretised. In such instances the teaching and learning of problem-solving skills do not rely solely on prescribed textbooks, but diversified on various teaching and learning materials, which include hands-on activities. Such activities stimulate the mental thinking of the learners to contextualise knowledge and retain the gained information for longer duration (Tulving, 1985, quoted by Campbell, 2006:7). The gained knowledge on problem-solving skills is applied to unfamiliar problem-solving skills with which learners are confronted in an authentic environment. Grinstein el at (2001:332) contend that the teaching of problem-solving should reflect the humanistic dimension, which is often limited in the content of problem-solving. In most cases, teachers focus on content in an abstract way such that learners do not have access to the mathematical parts.

2.5.1.1.1 Lesson preparation

The lesson preparation is important in presenting a lesson in a coherent manner, and covering all the details to be carried in class. Details such as the objectives of the lesson, all the material content and the resources to be used must be in place. In lesson preparation, the teachers are able to conceptualise and reflect on theories of learning that guide it. In the teaching of problem-solving, teachers need to be aware that not only mental structures be considered, but also the sociocultural views of the learners. For instance, the constructivists elucidated that the learning process has allowed learners to construct new knowledge using the prior sociocultural knowledge. Learning is cumulative, which allows learners to link the new knowledge to their lived experiences (De Corte, 2012:35; Kellner, Attorps, Theron, Tarneberg, 2008:257; Perez Huber, 2009:710).

The content of problem-solving is taught at the abstract level, where it becomes difficult for learners to get the gist of the matter. The challenge is lack of time to create a plan on what materials and resources to use in class. The method used is not even their own, and does not consider the sociocultural views of the learners. It not enough to concentrate on mental structures of learners. As learning is cumulative it is important for the teacher, during the planning sessions, to consider the lived experiences and surroundings of the learners.

2.5.1.1.2 Classroom presentation

According to Anthony and Walshaw (2009:153), the DoE (2003: 14), Keeley and Tobey (2011:10) and Situation Analysis and interpretation of Outcomes Module (2007:49), classroom presentation is the interactive or teaching phase of a lesson, in which the actual teaching and learning, the execution of activities, and assessment take place. In the classroom presentation there are clear roles that must be performed by the teacher and learners. The roles are informed by the detailed tasks to be performed in class, guided by the lesson outcomes to be demonstrated by learners at the end of the lesson. For active participation of learners during the lesson presentation various teaching methods and clear activities, resources, as well as teaching media or teaching aids should be used throughout. Also, the various forms of continuous assessment are effectively applied in ensuring that learners can demonstrate the outcomes. It is expected that the teacher engage all learners in appropriate tasks that elicit deeper thinking and self-discovery. This will assist in sustaining the interest of learners to probe for logical answers and in turn learners will acquire high order knowledge on the problem-solving skill (PSS).

On the other hand, Jones, Jones and Vermette (2010:846) warn the teachers should refrain from 'getting the task out of the way'. Rather, the presentation should be designed in a way that it promotes self-discovery from the learners. They also complain that many teachers tend to teach continuously, without stopping, with learners given limited time to interact or forced to remain passive throughout the learning sessions. This type of lesson presentation encourages learners to learn by memorisations, that is, without deep understanding.

2.5.1.1.3 Assessment of activities

According to Ahmed Al (2006:125), assessment *for* learning and assessment *as* learning occur throughout the learning process, and learners are engaged in assessment in a more active way. It is expected that the teaching and learning process on the problem-solving skills forms an integral part in checking the understanding of learners continuously, and that they are engaged in the assessment process. Assessment is a activity of generating and collecting data and shed light on learners' results in order to determine their growth in learning problem-solving skills (PSS) and to make informed judgement about the learner's performance (DoE, 2003:63, 2007:5). It is expected by DoE policies and Ahmed Al (2006:125; McMillan, 2004:4) that assessment take many forms or that it be multidimensional in gathering information from several contexts, thus enabling learners to be critical connectors between learning and assessment. Teachers have the overall responsibility to assess the progress of learners in achieving lesson outcomes (Keeley & Tobey, 2011:3) and finally demonstrate the critical outcomes. Even for learners who experience barriers to learning of problem-solving skills (PSS) can be assisted adequately. This implies that assessment must be a continuous process in teaching and learning (DoE, 2003:14). Assessment is made before commencing the lesson presentation or pre-assessment teaching (Youdalle, 2010:30), during lesson presentation and during the reflection phase. Thus, the manner in which assessment is made or understood by teachers is designed to ensure that learners make connection between new ideas and prior knowledge. In this way, the quality assessment is key for making informed decision about the progress of learners (Keeley & Tobey, 2011:4).

2.5.1.2 Method of teaching is teacher-centred

Teaching methods have a real and pervasive effect on learning. It is expected that teachers use variuos teaching modes, making class becomes vibrant and encouraging the learners to ask questions and share inputs with other learners and the teacher. Learners should demonstrate the achievement of critical outcomes, in which they learners cooperate effectively with others as a team , and it is the responsibility of the teachers to reflect and explore a variety of strategies for learners to learn effectively. Several teaching approaches focus on different learning outcomes, whilst OBE inspires a learner-centred method and hands-on activity to the teaching of problem-solving skills in the mathematics curriculum in all grades (DoE, 2003:2; Samuelsson, 2010:36; Thomas & Brunsting, 2010:25,27).

The use of collaborative learning allows learners to interact among themselves to share their prior learning, so that it be can be linked to the new content with which they are engaged. The lesson will be easily followed by learners, as the activities are learner-oriented. On the other hand, it is still expected of the teacher to model, that is demonstrate the key issues, thus allowing for a review of the lesson presented with the learners. By so doing the teacher will be empowering learners, allowing them to discover and reflect on realistic experiences. As Mahlomaholo (2012:102) and Nkoane (2012:98) argue, this is a way of empowering the excluded ones and giving a voice to the voiceless. This type of teaching approves the learning styles of learners, and also shows that the teacher is using their perspectives.

Teachers have defended their use of the teacher-centred approach in the teaching of problem-solving, because learner-centred methods takes time to complete the planned class activities and learners may be reluctant to participate in class discussions. Van de Walle et al. (2010:2,3) suggest that the classroom culture restored in the teaching and learning of problem-solving has to afford learners opportunities to learn problem-solving with understanding, such that they are able to build new knowledge from experience and prior knowledge. The teacher has to understand that mathematics is a human endevour (DoE,2003:62), whereby social construction of ideas is filtered in problem-solving.

2.5.1.3 Lack of motivation among learners

According to the DoE (2003:2), Johnston (1996:23, 31), and Waege (2009:83, 84) the progress of learners during the learning process embraces a three-way theory of the mind, namely, emotions, thoughts and behaviour. OBE one of the principles of the NCS is emphasis on teaching and learning as holistic, in the sense that teachers need to reflect and assess all the facets of learners development in the learning process. On the other hand, it involves being aware that effective teaching is about not only knowledge and skills acquisition but also the influence of affective domain, which examines how learners perceive their capacity to learn. It is therefore imperative for the teachers and learners to understand that appreciation and acknowledging the worth and importance of oneself is key. This will boost the self-esteem and self-efficacy of learners (Campbell, 2006:12, 13; Williams & Forgasz, 2009:95).

The above argument suggests that all parties, especially teachers, learners and parents, need to support learners in the process of teaching and learning the problem-solving skills. When assessing, forms have to help assessors not only establish how well learners performed but also determine their needs. In turn, learners will develop interest and enthusiasm to engage in class deliberations and activities. If they are motivated, either extrinsically or intrinsically, then values of sharing, caring, self-discipline, acknowledging one's own competencies, and cooperation between individuals in classroom will arise. Mahlomaholo (2010:13) and Vankúš (2008:106) contend that it is expected of the teachers in the teaching of problem-solving skills to create, construct and maintain a sustainable empowering learning environment, in which the teacher shows that learning content is flexible in terms of time. When learners are struggling to understand concepts of PSS, the teaching time can be extended to accommodate the individual concerns and needs. This will encourage learners to work at their own pace and harder.

It is evident that once learners are not motivated in learning problem-solving it becomes difficult for them to cooperate with the teachers in effectively learning problem-solving skills. Hence, the teacher has to look at the child in totality, including his/her immediate background environment, as stimulating the child's interest in the topic as it relates to what learners like about it most (Hodkinson, 2005:111).

2.5.1.3.1 Class participation

The DoE (2003:3, 4; 2005:8), Jansen et al. (2014:61) and Kellaghan et al. (2009:1) envisage the type of learner who is instilled with the values and acts in the interest of society, based on respect for democracy, equality, human respect and dignity and social justice factors. During class participation, these values must be demonstrated by learners for sustainable learning (Jones et al., 2011:845;

Mahlomaholo, 2010:13). It is the responsibility of the teacher to create an enabling environment that helps learners to stay motivated and focused, so as to achieve the critical outcomes such as using problem-solving skills, science and technology, and demonstrate critical responsibility towards the learning ecology and the health of others.

In promoting motivation and self-esteem among learners in class, the teacher has to ensure the teaching of problem-solving skills illustrates the importance of cultural practices of individuals, and self-discipline with regard to respecting the views of other learners in class. Encouraging learners to acknowledge multiple realities enables them to participate optimally, hence their interest in problem-solving skills is generated and self-esteem is boosted.

Thus, it becomes imperative that the teacher has to teach the learners in totality, that is, not focusing only on the intellectual domain but also on the affective domain and sociological environment (Ambrose, Bridges, DiPietro, Lovett, Norman, 2010:187). In this manner, the level of classroom participation is enhanced, and affords different learners an opportunity to interact freely with other learners and the teacher.

2.5.1.3.2 Learner performance on assessments tasks

It is important for the teacher to assess learners' performance throughout the learning sessions. The DoE (2003:63); Kellaghan (2009:119), Sacmeq Report by Moloi and Chetty (2011:1, 7), the TIMSS Report (2011) by Provasnik et al. (2012:1,4) point out that many social partners have keen interest in how learners perform with respect to problem-solving skills. Social partners include the learners themselves, educators or practitioners, biological or legal parents, guardians, district, provincial and national departments of education, employers, and higher education and training institutions. Learner performance is assessed continuously for the purpose of scrutinising progress and giving constructive comments and valuable feedback, diagnosing obstacles to learning, selection. The teacher and learners are expected to be conversant with types and methods of assessment in the teaching of problem-solving skills.

In addition, the DoE (2003:64) and Kelley and Tobey (2011:4) mention the following types of assessment that can enhance the learners' understanding of problem-solving. (i) Formative assessment inform teaching and learning, and immediately provides feedback to teachers and learners about the progress on activities performed. (ii) Diagnostic assessment identifies preconceptions, errors, types of reasoning, and learning difficulties that learners have. (iii) Summative assessment gives the overall performance of children in class in achieving the learning targets.

It is therefore crucial that assessment be used before and throughout the learning process, so as to enhance learner performances. Rather than using the assessment at the end of the instruction, if it is used it has to feed forward the process of teaching and learning (Kelley & Tobey, 2011:90).

2.5.1.4 Drilling of mathematics formulae

The departmental policies expect that the teaching of problem-solving skills (mathematics formulae) need to be taught in a way that they make meaning to learners (Anthony et al., 2009:154; Averill et al., 2009:175,181; Battista (2004) cited by Su et al., 2013:1). Learning does not occur in vacuum, but it must be contextualised in real life situations. Problem-solving skills taught in this fashion are easily remembered by learners, and learners are able to give their own correct interpretations and meanings. It is expected that the teacher use pictorial, symbolic, and/or language skills in numerous approaches to concretised the problem-solving skills. (DBE, 2011:5, 2003:2).

Graven (2003:29) writes that drilling of mathematics formulae is common in South African schools, with learners repeating after teachers in rote learning. Reasons include teachers having become mathematics teacher not by choice but as the result of a serious shortage. Currently in South Africa, 50 percent of teachers of Mathematics have less than grade 12 mathematics qualifications. In some cases, they resort to drilling of formulae from the textbooks, which result in meaningless definitions and/or conjectures for learners. This is despite the prescripts of the DoE that teaching be conducted with awareness of the cultural context of learners.

2.5.1.4.1 Lesson planning

The interactions and discussions which take place in class can be taken as a springboard to craft the lesson plan that involves learners' lived experiences (Lynn, 2004:154). In making lesson planning learner-centred, Keeley and Tobey (2011:89, 90) suggest that the feedback process has to help the teacher to incorporate the learners' backgrounds. This acknowledges the pedagogy at home (Yosso, 2002:94) and enables learners to choose the mathematical content to be learnt on their own, with minimal assistance from the teacher.

Graven (2003:31) and Vorderman (1996:519) indicate that use of tessellations in the teaching and learning of problem-solving helped learners to construct different geometric shapes and further enabled them to define these interlocking shapes. Learners are seen as dominating the discussions in the classrooms rather than the teacher, whose role is interactive, and rooted in negotiations with learners about the mathematical concepts they discover. Rather than the teacher alone giving the definition directly, the lesson is a collaborative effort by teacher and learners, thus encouraging the latter to conceptualise a definition, in the case of tessellations by comparing and juxtaposing several patterns and shapes of real objects. As a result the knowledge of problem-solving does not come from the teacher only, but from the learners as a dynamic, ever-changing manifestation of their lived experiences. According to Graven (2003:30), collaborative lesson planning helps to make activities creative. Sometimes learners are not even aware when time has run out for the lesson until the next teacher knocks on the door. There is fun anticipated in the teaching and learning of problem-solving, and as Graven (2003:32) adds, teachers no longer decide to leave the profession because of the burden of lesson planning and dull lessons.

2.5.1.4.2 Class Activities

If activities in the class are learner-centred they will create a conducive atmosphere in which all learners will be highly engaged (Ambrose et al., 2010:83), combining real life experiences of the learner with the mathematical content to be learnt. As Anthony and Walshaw (2009:154) and Vorderman (1996:490) posit, learners' minds are not blank slates upon which problem-solving knowledge is etched. Rather, learners come to learning situations with activated ideas, understanding, and a wealth of problem-solving knowledge, which helps them to work out activities. For instance, Vorderman (1996:490) evinces how a teacher presented an activity in which learners had to measure a 'mayflower' in which the teacher, rather than ask them to use a ruler, stimulated their ideas and allowed them to reflect and construct their own methods of measurement (Averill et al., 2009:168,169,171). In this activity some used feet to measure, some their hands, others a ruler. This gave them a chance to discuss why they used particular methods. In such an activity, the teachers listened and provided minimal assistance when needed.

Tennant (2004:1,3) writes that in recent years mathematics teachers have investigated teaching methodologies for introducing problem-solving with the help of cultural connections. For example, Islamic tiling (Alhambra tilings) was used to teach symmetry and other transformational movements in geometry. Ambrose et al. (2010:83) argue that this endows authentic, real-world activity and assigns hands-on activity that allows learners to vividly and concretely see the relevance and value of otherwise abstract concepts and theories. Class activities of this nature allow learners to create new understanding for themselves. The teacher coaches, scaffolds (Averill et al., 2009:175) and suggests alternative solutions, whilst at the same time giving the learners room to experiment, discover, ask questions, and try things that seem not to work.

2.5.1.5 Non-involvement of parents in teaching problem-solving skills

Many educational legislative imperatives and policy directives of the democratic South African state dictate that schools be integral parts of the communities within which they occur. The structures within public schools must change to empower teachers, teacher mentors, and community members in the teaching and learning of problem-solving skills. Matthews, Watego, Watego,Cooper, & Baturo(2005:6) assert that this will make the schools form part of their communities rather than separate entities. This is further demonstrated by the stipulations of the South African Schools' Act (RSA, 1996:1,11) which emphasises this involvement and cooperation in the teaching and learning of problem-solving skills. It also deconstructs apartheid educational policies, which separated schools from communities under the pretext that communities were not sufficiently literate to inform academic practices in schools (Matthews, 2005:6; Jansen, 2002:203,204; Yow, Morton, Cook, 2013:101). There was a deliberate alienation of the subaltern communities in the teaching and learning of problem-solving skills. As Rocha-Schmidt (2010:346) posits, schools should not be observed as reproducers of unequal social power relations, but rather as institutions that encourage parents in the teaching and learning of problem-solving skills.

Potgieter et al. (1997:9) point out that learners and teachers cannot expect the provincial department or regional administration to give entirety to the school, but rather, parents and other members of the society are in the best situation to see what the schools need and how to tackle challenges in teaching. Lynn (2004:162) and Yosso (2002:99) caution teachers (school community) not to discredit the wealth of knowledge parents possess, as it can help learners to understand better. If parents are regarded as uninterested in their children's welfare they do not place high value on their learning. These perceptions persist when teachers disregard the importance of knowledge parents possess to enhance the teaching and learning of problem-solving. They argue that the mistakes committed by the school community are that subaltern parents are judged by the standard of the middle class, hence making derogatory remarks about marginalised parents.

2.5.1.5.1. Lesson planning

According to Mncube (2010:233), parents expressed much gratification with the substantial improvement of school activities with regard to the teaching of problem-solving and their relations with the educators in running school activities. To a large extent, as Dika and Singh (2002:5), Lemmer (2011:100) and Yosso (2002:94) agree, parents were more knowledgeable about the teaching of problem-solving because of their inherent capital, whether human, navigational and/or social. Teachers and parents make a strong team on mathematical content and knowledge of cultural knowledge, such as indigenous games, mastered by local communities. This is when the school works together with local communities in preparing lessons, and parents take responsibility for preparing the resources and materials for demonstrating the indigenous games in teaching mathematical content (Averill et al., 2009:176).

Lynn (2004:162) and Yosso (2002:95) see the involvement of parents in lesson planning as the breaking of social inequalities that existed between the school and the marginalised communities. In turn, this hampered the teaching and learning of problem-solving. Parents were misconstrued as possessing backward cultural practices (including indigenous games), and immoral and ignorant knowledge, which might impact negatively. The involvement of parents helped make the lesson practical, for learners to comprehend. Parents were able to relate the real life indigenous games with which learners were actively involved. At the same time, teachers provided the mathematical knowledge linking them (Tucker, 2010:146).

2.5.1.5.2. Lesson presentation

According to the Assembly of Alaska Native Educators (2002:15), the teaching and learning of problem-solving must be a collaborative effort between teachers, families and elders in the community. This will encourage the learners and parents to work together in stimulating the mathematical ideas or contents infused in cultural activities, among others, indigenous games. In addition, Lynn (2004:154) and Yosso (2002:94) assert that involvement of communities in a classroom situation is an inclusion of pedagogies at home in the teaching and learning of problem-solving. This will help learners make connections between environmental background knowledge and the new mathematical content presented in class. Averill et al. (2009:169) show that involvement of parents in the lesson presentation is of great assistance to learners. Parents brought *tukutuku* (woven panels), for learners have to illustrate the mathematical content and processes embedded within the weaving. The mathematical content extracted included sequential patterns, algebraic equations, relationships, names and properties of shapes, parallel lines, perpendicular lines and fractals.

It can further be argued that the social capital that parents possess can help them to interact freely with both teachers and learners in sharing their knowledge of indigenous games. The parents demonstrate how the games are played, then it becomes easy for the teachers to draw up activities for the learners. In their small groups, learners visualise mathematical processes and content illustrated by the games, enabling them to make conjectures that help establish mathematical formulae and definitions. In the presence of parents and other stakeholders, learners are given a chance to make presentations of mathematical content observed during the demonstrations by parents (Department for Children, Schools and Families (DCSF), 2008:5).

2.5.1.5.3. Assessment

According to Muijis and Reynolds (2011: 84) and Tucker (2010:144), assessment is seen as an activity integrated with teaching and learning of problem-solving. For assessment to continue smoothly, the learning environment should be as light and attractive as possible, such that the parents and teachers interact freely in the classroom setting. The interactions between teachers, parents and learners is masked by critical analysis, with all parties seeking clarification on why the indigenous games are structured in a particular fashion. Also, as learners give feedback on the mathematical content observed in a particular game, they are requested to justify their answers to the large groups.

Tucker (2010:146) argues that the assessment process is seen by parents and teachers as sharing knowledge about the child’s mathematical development, and the implication of this dialogue is far-reaching. It becomes easy and possible for parents at home to assist their children in assessing their problem-solving skills. The DCSF (2008:8) and Tucker (2010:140) found that for children to regard themselves as proficient mathematicians they need to feel assertive and positive about themselves as learners and problem-solvers. In engaging parents in their child's problem-solving plays a significant role in shaping how learners feel about and engage with the subject.

2.5.1.6 Limited content knowledge among teachers

It is expected that mathematics teachers will be lifelong learners in the teaching and learning of problem-solving skills, and competent in their area of expertise (DoE, 2003:3, Tamsin, 2013:484,485). A qualified mathematics teacher has acquired the content knowledge over years, in their schooling and at tertiary level, so it is important to attend mathematics developmental workshops conducted by departmental officials (mathematics subject advisors) and professional bodies. It is not enough to have content knowledge, but rather pedagogical content knowledge is key in teaching and learning of problem-solving skills. This will enable the individual teachers to use strategies that allow learners to interact freely, meaningfully and to extend knowledge of the learners to the limits, in the teaching and learning of problem-solving skills (DoE, 2003:5; Haverhals, 2010:334; Nutti, 2013:57; Pascuzzo, 2013:233).

On the other hand, Keeley and Tobey (2011:12) point out that, traditionally, mathematics teachers were considered the providers of content that learners had to consume without interrogating the teachers. This kind of approach to the teaching of problem-solving limits the learners' potentialities in understanding problem-solving. Rather, effective teaching of problem-solving requires understanding of what learners' prior knowledge is to be linked with the new content presented. For teachers to present high quality problem-solving they need to have deep understanding of mathematical content. This can let learners on their own discover it through interaction with the indigenous games (Van de Walle et al., 2010:2). Educators and mathematics practitioners are revived to present problem-solving in an interesting and creative way to learners, using design activities that allow learners to discover the definitions, formulae and mathematical content on their own, with minimal guidance from the teacher.

According to Ogbonnaya (2010:50), functions constitute the basis of many branches of mathematics, science and engineering, a concept clarified in the

South African Grade 10 – 12 Mathematics National curriculum statement (DBE, 2011:10, 11, 12; DoE, 2003:8). However, most secondary school learners have difficulties in understanding the topic. The teachers indicated that parabolic functions is one major area of learners' difficulty in mathematics. The possible implication is that the teachers find it difficult to present the topic in a way learners understand, since what they learn in a topic is mainly influenced by the way it is presented to them by the teacher. There is a need to find better ways of presenting this important topic.

2.5.1.6.1 Lesson plan does not show high order questions

The manner in which lesson plans are drawn up is monotonous, in that the teacher alone is active, while learners are taking notes. In this way, learners learn in an instrumental way, that is, problem-solving is seen as endless lists of isolated skills, concepts, rules, and symbols that are not related (Bansilal, 2011:93; Van De Walle et al., 2010:26). In addition, Van de Walle (2010: 23) and Yosso (2005:70) suggest that the lesson be planned in such a way that learners are able to use their marginalised wealth, that is, their navigational capital allows them to explore various possible solutions.

For instance, Bennett, Burton, Nelson (2010:135,136) and Van De Walle (2010:26) assert that in order for the lesson to show high cognitive levels of thinking the activities in the lesson plan must stimulate learners' thinking abilities. The 'why and how' questions dominate activity questions (Bansilal, 2011:99). Bansilal (2011:93,98) recommends that the class activities be within real life contexts, which adapt the innumerable learning styles and needs of the South African learner populations. In one case learners were given an activity of Robben Island and had to give justifications for the calculations of various dimensions and areas.

2.5.1.6.2 Class activities/tests, assignments

Class activities serve as formative assessment, which helps the teacher to know the progress of learners with regard to their understanding of problem-solving. Also, it allows the teacher to apply other teaching strategies that will help learners to perform well (Kelley & Tobey, 2011:12; Van De Walle, 2010:76). They further argue that timely feedback gives learners useful information to adjust their current learning activity. According to Bansilal (2011:94) citing Bansilal, 2006; Brodie, 2004, 2010; Graven, 2004; Mattson & Harley, 2003; Taylor & Vinjevold, 1999; Vandeyar & Killen, 2007) research proposes that South African teachers by and large embrace the philosophy and values behind the assessment of activities in the teaching and learning of problem-solving. On the other hand, whilst many South African research studies relate that most teachers speak positively in supporting learner-centeredness, critical thinking skills and becoming facilitators of learning, classroom observations reveal that the same teachers teach in a procedural and shallow manner, use drill work, encourage group chanting, do not use group work effectively and do not encourage insightful thinking amongst their learners. No activities are designed to stimulate the interest and creativity of learners.

2.5.1.7 Limited motivation among teachers

The working environments need to be conducive for sustainable teaching and learning to occur. Some of the elements that constitute a conducive environment are interrelationships between teachers and other human beings, not limited to learners, teacher, parental community, and the employer. This interrelationship concedes to a collaborative way in the teaching and learning of problem-solving skills, as opposed to top-down and an autocratic power culture. Also, interrelationships between the teacher and resources materials for the curriculum, which might include the resources, are necessary for effective teaching and learning. It is then expected that teachers demonstrate competencies of various roles and critical outcomes, such as a lifelong learner in designing their own materials for implementing new changes in the curriculum development of problem-solving (Norms and Standards, DoE, 2003:2). Demonstration of these roles will allow creativity of teachers, as they regard learning programmes as a guide that inspires innovation and motivation in designing programmes. As professionals, teachers should belong to professional bodies that assist them in developing mathematics knowledge for teaching (Pascuzzo, 2010:233) and researching effective methods for teaching and learning. That will help them become agents of change in transforming the bad conditions of teaching and learning in schools.

2.5.1.7.1 Lesson planning

Van de Walle (2010:58) quoted Burns (1992) in claiming that it is vital that teachers give the necessary time for learners to work all through the activities on their own, and that teachers not slide into teaching-by-telling for the sake of effectiveness. The lesson planning in the teaching and learning of problem-solving has to acknowledge the diverse needs of learners. In order to accomplish this, teachers need to include learners and parents in their planning sessions. The lesson planning has to take into consideration the prior knowledge of learners, and their background knowledge. For example, if the teacher intends to teach the formula for circumference of the circle, during the planning sessions he or she may request learners to bring circular objects of different sizes from their immediate environment, as well as measurement tools or instruments to be used. They may then design the activities around what they have brought to determine the perimeter of the circle. This activity will further concretise the meaning of pie (π).

Storeygard (2012:9) contends that proper lessons provide learners with an opportunity to be actively involved in class participation. Teachers provide support or scaffolding (Averill, 2009:175), which allows all learners to be engaged in problem-solving tasks. Learners feel that they are part of the class community and are free to make inputs and contributions to enhance the learning of problem-solving. However, Van de Walle (2010:72) warns that teachers need to be aware that learners do not understand in the same way, so the use of the same teaching approach may disadvantage some learners.

2.5.1.7.2 Class interaction is teacher-dominated

It is imperative that teachers find ways to involve all learners in the teaching and learning of problem-solving, not with emphasis on those who are eager to learn. Muijis and Reynolds (2011:83) and Van de Walle et al (2010:106) citing Mau and Leitze (2001) assert that in the learning of problem-solving teachers should ensure that learners are expected to talk and listen. As a result, more mathematical concepts and arguments are raised, while the teacher plays a scaffolding approach, that is, he or she talks less and encourages more input from learners. The established learning environment allows learners to discover knowledge on their own.

On the other hand, it has been observed that many mathematics teachers have a problem with learners not enjoying their lessons because they remain passive, with The lesson are dominated by the teacher presenting the problem-solving concepts. Activities are from textbooks with no creatively on the side of the teacher. Hence, learners dislike problem-solving, seeing no practical application of the topic being presented. Teachers have to be researchers and lifelong learners in the teaching of problem-solving (DoE, 2003:5; Anthony & Walshaw, 2009:154).

2.5.1.7.3 Assessment based on low order questions

The involvement of learners in the teaching and learning of problem-solving creates a space in which learners are able to raise questions such as *but tell me why is the area of the circle equal to* π r^2 ? *and why is* a^0 = *1* ?'. These questions help the teacher realise that learning of problem-solving is about helping learners to construct their own meaning, not about getting the right answers (Haylock, 2010:9; Muijis & Reynolds, 2011:79,83). These questions further help the teacher to understand the logical reasoning of the learner, and as such the activities designed can address such questions, rather than always providing the learners with right answers. These questions should be viewed in the positive manner as they trigger the critical thinking of learners.

Haylock (2010:9) points out that learners must be encouraged to ask ‘difficult questions, not routine types, such as *what is the area of the circle?*, to the whole class, and it does not matter if they give a wrong answer. The aim is to stimulate higher thinking and not let learners feel that they are asking stupid questions, as this causes them to shy away from asking any. Van de Walle et al (2010:77) adds that it is essential that teachers target high order questions by simply rephrasing those asked by the learners.

The assessment conducted before, during and after the teaching and learning should teach and enlighten each individual learner and the teacher about problem-solving ability and development toward conceptualising mathematical concepts, not just mastery of procedural skills. Thus, the assessment should be used as the feedback to ‘feed-forward’ (Keeley & Tobey,2011:89), that is, the high-order questioning used during teaching and learning is a way of saying to learners in future assessment do expect questions of this nature. The learner-centred activities allow learners to freely ask questions that are going round in their minds. In some cases, the frightening environment of teaching problem-solving prevented learners from asking “difficult questions or perceived as stupid questions” (Haylock, 2010:8). A frightening environment in the teaching and learning of problem-solving may be characterised by the mathematical language used by the teacher, as too technical, too specific to the subject, or not reinforced through language used in everyday life (Bush, Joubert, Kiggundu, Van Rooyen, 2010:164; Haylock, 2010:9). For instance, concepts such as ‘mapping’, ‘discrete data’ and ‘reflective symmetry’, are very difficult to comprehend when asked in assessment tasks, but when explained to learners, typical reactions are “*oh, is that what they mean?, why don’t they say so, then?, why do they have to dress it up in such complicated language ?*” (Haylock, 2010:10).

2.5.1.8 Limited expertise of classroom practices

Haylock (2010:3) points out that in England interactive whole-class teaching was emphasised in the 1990, and encouraged teachers to engage more with learners. There were explanations, questions and answer by learners and class discussions, aimed at promoting understanding and confidence in problem-solving. The technical language used in problem-solving was related to the background knowledge of learners, and allowed them to construct their own meanings where acceptable in the mathematics arena.

Setati and Adler (2002:130) and Tapson (1999:161) illustrate that for learners to have deep understanding of mathematical terms or concepts they need to be aware that many of the words used in English were originally taken from or based upon the ancient languages of Greek and Latin. The teacher has to make connections of such concepts to daily use language so as to have deeper understanding. For instance 'adjacent angles', where adjacent is '*adjacere*' in Latin meaning 'lying near', 'isosceles triangle', where isosceles is '*isoskeles*' in Greek, which means 'equal legs'. Thus, the technical languages used in problem-solving made these mathematical concepts more difficult for learners to comprehend. It is acceptable that to concretise mathematical concepts or definitions 'code switching' be used, that is the home language is interspersed with the language of learning and teaching (LOLT) to clarify or emphasise meaning. Teachers need to instil the basic meanings of mathematical concepts and terminology so as to facilitate understanding.

In many cases, teachers lack basic classroom practices in the teaching of problem-solving. Haylock (2010:3) maintains that for learners to enjoy problem-solving it is essential that they understand mathematical content, concepts, definitions and terms. They should make meaning of what they are doing in the mathematical content instead of learning to reproduce procedures and recipes that are low in meaning and purpose. For instance, if they understand that 'isosceles triangle' relates to triangle with equal legs it would be easy to come up with basic features of an isosceles triangle, and so not to confuse it with a scalene triangle, which is unequal, from '*skalenos'* in Greek, or tetrahedron, which means ‘four faces’, each an equilateral triangle (Bolt & Hobbs, 1998:124; de Klerk, 2004:121; Haylock, 2010:322; Tapson,1996:160).

2.5.1.8.1 Lesson planning lacks resources/activities

Teaching and learning of problem-solving using cultural practices and historical stories may help to concretise mathematical concepts which seem too abstract for learners. The resources used in the learning of problem-solving ensure that learners experience a sense of pleasure, stimulating curiosity to formulate their own discoveries (Haylock, 2010:17). Thus, in this research the use of indigenous games will be used as resources that enable learners to draw out the mathematical contents embedded in them. As Haylock (2010:18) and Van de Walle (2010:21) contend, learners are fully accommodated in lesson planning that draws resources and activities from their home environment. The real-life contexts that draw a wide range of mathematical ideas are used to introduce or teach new mathematical concepts, and stimulate learners to extend their problem-solving skills to abstract mathematical concepts. During the play of indigenous games, teachers give learners activities that describe the use of the terms: *impossible, almost impossible, not very likely, fairly likely, almost certain, certain* (Haylock, 2010:380; Van de Walle, 2010:64). The understanding of these terms develops the concept of probability used in mathematical fields and in turn, learners begin to take ownership of their learning as they are empowered in making sense of mathematical concepts within their surroundings.

2.5.2 Components of the strategy in the teaching of problem-solving skills

Section 2.5.1 above was an in-depth discussion of challenges in the teaching of problem-solving. In resolving the above-mentioned challenges, the literature is consulted so as to come up with the components of the framework in the teaching of problem-solving skills. The components of the framework in the teaching of problem-solving will address each challenge identified.

2.5.2.1 Meaningful subject-matter in the teaching of problem-solving skills

Jonassen, Strobel, Gottdenker (2005:15) citing Mayer (2002) note that meaningful learning in the problem-solving is regarded as a conceptual change, in the sense that it is a process of constructing and reorganising personal conceptual models, and treating learning as an premeditated, vibrant, and constructive process that embodies developmental differences among learners. Thus, for subject-matter to be meaningful the teacher has to integrate the teaching of problem-solving skills with prior knowledge of the learner (Anthony & Walshaw, 2009:109, 153). Also, Sriraman (2007:151), Sriraman and Lesh (2007:59) in Dienes (1973, 1983) on the theories of how mathematical content can be effectively taught. They contend that by using indigenous games, stories, and dance, learners become actively involved in the learning of problem-solving. Thus, in this study, prior knowledge is brought in by using indigenous games to teach problem-solving skills and assist learners in constructing new knowledge. This is in line with Yosso’s (2005:70) claim that learners possess a wealth of marginalised knowledge that can be tapped into when teaching problem-solving skills using indigenous games. They help learners to associate the new knowledge on problem-solving skills to be presented with the practical examples from their home environment.

For the content to be meaningful teachers have to take into cognisance the intuitive problem-solving skills learners bring to school, as it is rich in the teaching of problem-solving skills and incorporates their cultural practices (DBE, 2011:9; DoE, 2011:8; Ernest, 1997:22;Gerdes,2001:321,322; Yow,Morton,Cook,2013:101) that learners encounter on a daily basis. The legislative imperatives (DoE, 2003:9; DBE, 2011:10) in the NCS and Curriculum and Assessment Policy Statement (Gerdes, 2001:323) regard problem-solving skills as human activity practised by all cultures. This implies that if the learning of problem-solving skills is contextualised in the background environment of the child it is easy to learn and retain the content previously thought to be too abstract to comprehend.

2.5.2.1.1 Lesson preparation

It is good practice that lesson preparation be done well in advance of the class. For it to be effective it should be designed collectively, including the learners themselves, as well as brining expertise and experiences from other teachers and parents (Assembly of Alaska Native Educators, 1998:24; Averill, Anderson, Easton, Te Maro, Smith, Hynds, 2009:165; Theobald,Curtis, 2000;106,107).

Font, Godino, and D'amore (2007:2-3) illustrate, the effectiveness of an onto-semiotic methods in the teaching and learning of problem-solving skills encourages design of the lesson preparation through communication among members, rather than individuals taking the onus of preparing their own work. An onto-semiotic approach can take in multi-facetted modes, including the idiosyncratic and institutional. The teaching of problem-solving involves the participation of learners, teachers, and community members in sharing lesson preparation. The individuals in a school meet together with various partners to plan. Partners can be from groupings in a school and/or in the society, among others community leaders and representatives of the DoE to share ideas on how to use particular instruments and tools in the teaching of problem-solving skills content. The onto-semiotic approach to personal-institutional facets makes the teaching and learning of problem-solving accessible to all parties involved. This means that communities and mathematics teachers take ownership and responsibilities for continuous proper teaching and learning. This way learners have access to sophisticated content as they have a chance to provide input.

The Early American Life (2001:78) and Horsman (1900:5) used the checkerboard in the teaching and learning of problem-solving, a game comprising a square board and an odd number of rows and columns, each with the same number of playing squares. The playing pieces comprise two pieces of a first geometric configuration, two pieces of a second geometric configuration, and one piece of a third geometric configuration. The first geometric configuration is a cube, the second a tetrahedron and the third a sphere with a flattened base. These different geometric shapes have unique properties, which learners are able to identify in their small groups

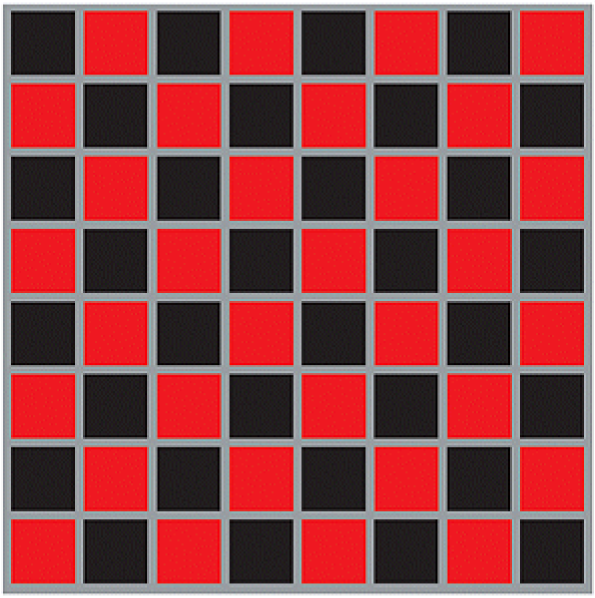

Figure 2.1: Checkerboard

2.5.2.1.2 Lesson presentation

Following the involvement of learners in the lesson preparation, their interactions with peers and teachers is remarkable (Haylock, 2010:3). In most cases, such vibrant atmospheres allow discussions on problem-solving to be pioneered and led by learners. This is also alluded to by Wainess, Koenig and Kerr (2011:24), who found that the Game Play Model and the Player Interaction Framework epitomise important elements in the design and development process for training games, and provide a convenient lens for the effectiveness of instruction and learning of problem-solving within games.

Fellows and Koblitz (1993:371,372) and the National Council of Teachers of Mathematics (NCTM, 2010:1) point out that the *Kido Krypto* mathematics card game stimulates learners to raise critical issues which are often taken for granted by learners, but when these issues are analysed it appears that they are very rich in mathematical content. Also, they assert that the fundamental ideas of *Kido Krypto* makes it possible to expose learners to some electrifying mathematical content and concepts. They further suggested that Krypto is a most excellent vehicle for presenting fundamental mathematical concepts to children. This game has the potential to stimulate research and can give fresh perspective to the teaching and learning of problem-solving. Such games provide a stimulating context for logical and mathematical modes of thinking, and the mathematical content embedded within *Kido Krypt*o included the order of operations, for example, the cards show: 1, 2, 4, 5, 6, so these numbers must be worked out to produce the target number 7, cubic graphs, number theory (Borovik, 2002:24) and hash functions (Schlaffer, 2011:1,2). It must be noted that the hash functions are abstract concepts, which are made accessible to learners.

2.5.2.1.3 Assessment of activities

Chinn (2012:9) contends that assessment is an integral part of the teaching and learning of problem-solving. All the activities prepared and presented by learners and teachers are assessed throughout the teaching-learning sessions (Sheng & Basaruddin, 2014:301; van de Walle, 2010:88). In addition, the DoE (2003:66) has stated that learner interaction allows the various methods of assessment to take place, *inter alia*, self-assessment, that is, learners pre-assess their contributions to activities before the teacher makes final comments or assessment. Another method of assessment used during learner interactions is peer assessment, which helps both learners being assessed and those who are making the assessment to gain a deeper understanding of the mathematical content under discussion. Also, it encourages learners to assess themselves and others' performances. Meanwhile, group assessment becomes clear when learners provide feedback to the larger group. It requires considering evidences that the group of learners cooperated, assisted one another, divided work, and combined individual inputs into a distinct composite assessable and measureable product.

For Sheng and Basaruddin (2014:302), the application of these methods of assessment motivates learners to actively become involved in classroom deliberations and thus create a cooperative environment. They are taught to be self-regulated and take the opportunity to challenge assessment tasks, practice learning, develop deep learning and understanding of subject content and achieve academic success.

2.5.2.2* Method *of teaching problem-solving skills is learner-centred

From the onset, that is, at the preparation of the lesson and the lesson presentation, leaners are part of the process. As a result, the activities designed are generally driven and performed by learners. Borovik (2002:24) advised that the learner-centred approach not be forced onto learners, but rather allowed to arise spontaneously (Chepytor-Thomson, 1990:18), as learners enjoy the full benefits of playing the games. There is much interaction among learners and their small groups in learning problem-solving. Chepytor-Thomson (1990:24) and Mosemege (2000:11) illustrate the dual purposes of indigenous games in which learners enjoy and learn to visualise the mathematical concepts embedded in them. According to Borovik (2002:23) and Habler, Hennessy, Lubasi (2011:27), the learning of problem-solving should involve two processes, that is, interiorisation and reproduction. The former is at an exciting level, where one visualises abstract mathematical concepts, underlined by a rich wealth of knowledge (Yosso, 2005:69) that is often marginalised in the teaching and learning of mathematical content in the mainstream curriculum. It helps learners to understand and handle mathematical concepts. Meanwhile, the latter process, reproduction (Chepytor-Thomson, 1990:18) involves articulating and communicating the mathematical ideas, and an ability to reproduce one’s own mental understanding about the mathematical content extracted. It can be noted that these processes are concurring with the ontological viewpoint of the community cultural wealth theory as socially constructed multiple realities (Chilisa, 2012:35,36).

Picture 2.1 (below) shows the playing of *Nakutambekela*, an indigenous game that involves the passing on of a stone object or any other similar object to a participant in the game (Mukela, 2013:36). The mathematical contents and skills embedded within this game are thinking and coordination skills, a friendly environment for creating interpersonal relationships, speed, and accuracy (Mukela, 2006:113). These skills are essential in the teaching and learning of problem-solving.

Picture 2.1: Children demonstrating the ***Nakutambekela*** play game song (Mukela,2013:36)

2.5.2.2.1 Lesson presentation

Cultural practices, including the indigenous games, illustrate that learners possess a wealth of skills and knowledge. Even the social settings, that is environmental backgrounds, influenced their acquisition of skills that are useful in the teaching and learning of problem-solving. Such indigenous games as riddles and story-telling generally promote listening, comprehension and speaking skills. On the other hand, stone catching or passing, such as *Nakutambela* and *diketo*, stimulate speed, accuracy, concentration and collaboration (Haylock, 2010:19; Zaslavsky, 1999:104,132; Roux, 2009:583,591). Such skills and knowledge allow learners to network with others and learn from them (Yosso, 2005:70). It is thus imperative for the teacher to use these indigenous games as resource materials when designing activities that make use of learners' capital, such as social, aspirational and navigational (Chinn, 2012:118; Yosso, 2005:75). Similarly, Chinn (2010:118) writes that teachers have to encourage discussions in the learning of problem-solving, rather than rely on imparting knowledge into the minds of learners. The playing of indigenous games affords them an opportunity to interact and network among themselves and the teacher. Also, interaction with materials helps develop conceptual understanding.

2.5.2.3 High motivation/interest among learners

Learners are motivated by being aware that indigenous games contain mathematical content. Roux (2009: 583-591) quoted Lennox, Pienaar and Coetzee (2007) that regular play of indigenous games lead to perfected echelons of mental wellness and the development of a positive self-esteem, as well as enhancement of social and cognitive development of learners. The teachers are advised to use indigenous games in the teaching and learning of problem-solving, so as to address affective outcomes imbedded within them. For instance, affective outcomes such as self-acceptance help learners to accept their wrongdoings, and through role-play vested in the games, learn from their peers or teachers. Similarly, the tolerance leaves them more open-minded and accepting of various possible solutions raised by others members of a group.

Thus, the teaching and learning of problem-solving using indigenous games satisfies a variety of physical, psychological, social and cultural needs, which are essential in boosting the self-esteem and confidence of learners in the learning of problem-solving (Roux, 2009:591). This is in line with the view of Chen, Looi, Lin, Shao (2012:355), that once learners are motivated it is easy to accomplish their goals in learning the problem-solving or achieving goals of a set activity. Again, the descriptive feedback, that is, giving scaffolding or advices to learners to aid him/her obtain the correct response, can amplify the motivation of learners to work out the new tasks and new or unfamiliar mathematical problems. Hence, the motivation they have developed gives them the zeal and zest to persevere in overcoming the barriers they are confronted with in problem-solving.

2.5.2.3.1 Class participation

The moment learners are motivated to learn problem-solving they demonstrate affective outcomes, such as own-ability, assertiveness, leadership, trust and security (Roux, 2009:591). Affective outcomes deal with emotions, feeling and interest, demonstrated by individuals being either happy or sad. Since learners showed interest in and enjoyment of learning problem-solving, during the feedback sessions they show their own understanding by explaining mathematical concepts and definitions, and an ability to defend their arguments. They show assertiveness and confidence in formulating their own mathematical conjectures. For example, learners were assigned the activity: "*Fill in the figure in the expression*", designed to encourage them to realise that various groups had different methods to find the answer. One group had five ways of calculations, that is, individual calculation, comparison, relay, assisted calculation and throughout calculation. Some presented four several methods of calculating: free, from the top, from the bottom, and from both the top and bottom (Roux, 2009:365).

Roux (2009:364) also found that the collaborative learning groups made a huge impact than individual learning groups, which implies that collaborative learning may have better the learning of problem-solving. Also, it can be noted that the underachieving learners gained a lot in this cross-number game. One mathematics teacher reported that most learners had been encouraged to have more discussions in their class, rather than be fed information by the teacher. On the other hand, it was noted that one of the best performing learners had conflicts during the discourse because all learners revealed high confidence and expectation. It is then the responsibility of the other groups and the teacher to show this group that tolerance can help them to alleviate their conflicts. Another medium achieving group showed vast interest in collaborative learning with one of learners playing the facilitation role. As the group facilitator, he/she showed others that they must have patience in listening to alternatives raised by other members, and also allow for discord to happen and agreement at the end of discussions

2.5.2.3.2 Learner performance on tests, assignments, homework

According to Roux, Burnett, Hollander (2008:91), indigenous games content and skills serve as the medium of socialising learners and developing them to reach their optimal potential as social beings. The potentialities in problem-solving appear during the class interactions as they present their solutions. It is evident that 87.5% of learners affirmed that it was less demanding to complete the calculations with collaboration than individually. Those learners with no confidence in mathematics found it easier to share their ideas with others and complete the calculations together. All learners concurred that they benefitted from deliberations with their peer groups. This finding was shared by Sheng and Basaruddin (2014:305), who posit that there are several strategies that can be implemented to enhance learners' performances in problem-solving. The school and home environment can be unified by using the indigenous games to teach problem-solving, as it brings the parents closer and give them a chance to demonstrate certain indigenous games in which learners can derive embedded mathematical representations. The Salem Press (2008:225) cites several indigenous games played in Turkey, France and Japan, such as '*The Coyote and the Chickens', 'Achi', Pryamid Ckeckers', diagonal checkers,* and *Seega*. The playing of these games requires a grasp of certain mathematical concepts, such as geometric configurations and patterns, from which learners can draw their own mathematical conclusions and make conjectures.

It was found by Cheng et al. (2012:358) that in pre- and post-test results of learners taking the assessment tasks on problem-solving higher average scores were recorded in the latter, from 50.29 to 63.29. Learners made greater progress through playing the game collaboratively and it was observed that the collaborative groups were playing indigenous games, suggesting that the low performers in problem-solving made the most significant progress. The incorporation of indigenous games in the teaching and learning of problem-solving inspires the interest and love of attending classes. During these interactions the learners showed competitive spirit and strong self-motivation, which helped them to perform optimally in all the assessment tasks. As they enjoy these indigenous games they learn and visualise the mathematical ideas and concepts infused within them, thus motivating them to show their full commitment in learning problem-solving. According to Roux, Burnett, Hollander (2008:91) quoting

Andersen and Taylor (2004), using indigenous games to teach problem-solving may also promote intercultural understanding and instil respect for the diversity of cultures.

2.5.2.4 Self-discovery of problem-solving skills formulae and processes

Averill et al. (2009:160,161) contend that successful teaching of problem-solving creates learning environments conducive to learners, and allows them to invent mathematical definitions and formulae on their own. Also, teachers acknowledge and validate ways of interacting, learning and knowing, and the authors highlight the importance of collaboration among teachers, parents, and other community members and leaders. A learning environment that uses indigenous games is central to learning the subject matter, to the extent that teachers teach to demonstrate how teaching problem-solving can incorporate them. They may enhance feelings of stimulation through activities that develop mathematical ideas, thinking and discovery of problem-solving conjectures, while drawing upon rich resources that learners bring with them to the classroom (Haylock, 2010:324; Sheng & Basaruddi, 2014:305; Yosso, 2005:69,70; Yosso, 2002:98).

The use of indigenous games in the teaching of problem-solving makes a learning environment conducive for learners to explore their abilities. The class interaction helped learners learn many things among themselves without relying on the teacher. In most cases, the teacher was there to clarify misconceptions and validate their findings (Chinn, 2012:4, 5, 6, 45; Muijis & Reynolds, 2011:79, 80; Van de Walle et al., 2011:B-2). Generally, they learn problem-solving through exploration and finally make their findings and draw mathematical conclusions. Ultimately, learners draw valid conclusions through their observation. The remarkable part of using problem-solving to teach and learn problem skills is that the learners are not given formulae by the teacher, but make the discovery on their own, through interaction is small groups.

The NCTM (2008:2) revealed that with the playing of *Krypto*, learners could formulate the scenario: playing cards have the numbers: 23, 5, 7, 9, 4 , and the target number: 13. They could not work out the solutions, but in the process they realised the sum of the playing cards: 23 + 5 + 7 + 9 + 4 = 48 (even number), and the target number = 13 (odd number). They then came to the conclusion that the sum of numbers on the card was even and the target number odd. Thus, no answer exists.

2.5.2.4.1 Lesson planning

According to Averill et al. (2009:162), in Calman and Sinclair (2001) and Glynn (1998), reflecting on a Maori worldview of the roles of learners and teachers, lesson planning is intertwined and interchangeable. Since the roles of learners and teachers are interlinked, the contributions made by learners in the planning will help the teacher to design activities that will stimulate the learners' interest in problem-solving. As shown by Van de Walle et al. (2010:B-2), such collaborative planning draws on learners' diverse background experience and dispositions (The Australian Association of Mathematics Teachers, 2006:2,4). The pedagogies of Maori learning of problem-solving is through participation, song, storytelling, metaphor, repetition, and observation, including routines and patterns of behaviour. The collaborative lesson planning (Averill et al., 2009:174; Van de Walle et al., 2010:63), including parents, learners and teachers, helps bring together the richness of the marginalised knowledge. The metaphor that describes collaboration in lesson planning is the interaction shown in the playing of indigenous games, where participants work together with passion and focus to create a vibrant lesson presentation and activities.

For example, some resources materials can be prepared for learners to teach problem-solving using the *muyato* game. This Zambian game of stone-throwing is played mostly by girls, and involves digging a hole into which small stones, seeds or round objects are thrown (Mukela,2006:32). The preparation of the resources can save time on tasks in the learning of problem-solving, as the resources are within the reach of everyone in the community.

2.5.2.4.2 Class activity

When learners form part of the planning team the resources and materials that are prepared by learners and teachers can help the teacher to design activities that stimulate learners' mathematical thinking. The teacher can include the learners' styles in the learning of problem-solving and help learners to make connections between the prior background knowledge and the new mathematical content to be presented (Haylock, 2010:16). According to Chinn (2012:65), the involvement of learners in planning and full engagement in the learning of problem-solving has ripple effects

Learners are playing but learning at the same time (Pramling-Samuelsson,20008:628,630). In such cases, interactions among learners come spontaneously. It alleviates mathematics anxiety among learners, as any mismatch between styles of learners and teaching are minimised. Further, Chinn in Skemp's (1971) recommended that over-reliance on rote learning as a dominant culture in teaching problem solving be avoided, because learners learn mathematical content and concepts with no understanding and merely memorise the content, which in the long run results in anxiety.

2.5.2.5 Good level of involvement of parents

The Australian Association of Mathematics Teachers (2006:3), Brigham (2012:45), Roux (2009:585), Roux et al. (2009:391) assert that excellent mathematics teachers do not leave out the parents or communities from teaching and learning of problem-solving. Their perspective is to deconstruct the dominant paradigms, and accommodate the new methodologies that open possibilities for alternative ways of knowing. The teaching and the learning of problem-solving using indigenous games has a significant transformative potential, particularly for people of colour (cited in Solorozo & Yosso, 2002). Also, Brigham (2012:45) argues that the use of indigenous games in the teaching and learning of problem-solving is a political act that brings the parents as role-players in the game from the margins to the centre, with the potential to empower the them as well as the listeners (learners, teachers and other community members).

Brigham (2012:46), citing Delgado (1988-1989, 2002), Villalpando (2002) and Tuhiwai Smith (2001:214) opine that the indigenous games of people of colour or subaltern communities (Leonard, 2008:157), which are often untold, can challenge the meritocracy. Thus, the inclusion of parents in the teaching of problem-solving using indigenous games serves to trouble traditional ways of research, which limit our understanding of the learning experiences of marginalised people.

The indigenous games known and played by parents can serve as a means for school communities to create collective transformational spaces, constructing new knowledge and deepening our understanding about problem-solving. The stance of community cultural wealth theory is that there is neither a single truth nor single method which can capture the complexity of human experiences in the learning of problem-solving. The interaction of teachers, parents, community members and leaders helps us understand problem-solving, and within its social contexts. Thus, the involvement of parents adds another dimension to perceiving the nature of reality and knowledge.

2.5.2.5.1 Lesson planning

In the planning sessions for the teaching and learning of problem-solving it is essential that parents form a core of the planning teams. Parents as socialisation agents (Roux, 2009:585) are of great assistance in exemplifying that problem-solving is a human endeavour (DoE,2003:4), practised by all cultures and communities. However, the mathematical processes, content and principles embedded in indigenous games (social contexts) are taken to for granted. The collective planning requires that parents be coherently organised, and sufficiently flexible to allow voluntary, self-driven learning. The learning occurrences and experiences incorporate significant mathematical content, which, as demonstrated by Averill et al. (2009:169), can be seen within *tukutuku* (panel weaving) of Maori people. Also, a variety of appropriate teaching strategies of problem-solving are incorporated in the planning sessions. Lived experiences of learners and parents again form part of the resources that will enhance the teaching strategies.

2.5.2.5.2 Lesson presentation

In arousing curiosity, challenging learners' thinking, and engaging them actively in learning, parents are given a platform on which to demonstrate the indigenous games. For instance, in the throwing of stones, learners may visualise patterns (relationship between variables), chances of winning or losing (probability), and dependent and independent variables, as mathematical content that can be seen or extracted. The involvement of parents in teaching problem-solving using indigenous games reinforces informal learning of learners. Informal learning arises as the consequences of learners establish connections with the experiences they come across in authentic situations(The Australian Association of Mathematics Teachers, 2006:4; Brigham, 2012:83; Coquin-Viennot, Moreau, 2007:70; Habler, Hennessy, Lubasi, 2011:23; Roux, 2009:585; Van De Walle et al., 2010:B-2). For Lynn (2004:161), Roux (2009:585) and Yosso (2005:78), parents are very resourceful during the reflection process on the indigenous games played. The linguistic capital they bring to the classroom environment is known to the majority of learners, which in turn helps in understanding mathematical contents and ideas.

2.5.2.5.3 Class interaction

Brigham (2012:83), Rieber and Noah (2008:80) and Van De Walle (2010: B-2) contend that the learning environment created by the presence of parents encourages the development of mathematical skills and content. Interaction among learners, parents and teachers is efficient (Anthony & Washaw, 2009:159), as they used dialogue and critical reflection to evaluate and deepen their learning of problem-solving. Learning is perceived as a consequence of a person choosing to partake and stick around in a task (game) they find pleasurable and interesting (Vankus, 2008:105). The mathematical concepts are simplified for learners by using or relating the lesson to indigenous games with which they are familiar. Also, learners are able to construct mathematical knowledge on their own (Vankus, 2008;105). For instance, the games played by learners showed silent learning of the relationship between acceleration and velocity, which are mathematical content and physics content extracted in the game (Rieber & Noah, 2008:84).

According to Rieber and Noah (2008:79), indigenous games and play are closely interconnected. They cite Huizinga’s (1950) belief that the construct of play was inextricable from humanity. As humans we might be *referred as Homo ludens*, or “man the player”. By nature, human beings play an active role, hence the interaction in the learning of problem-solving happens spontaneously. This suggests that the teaching strategy used for teaching problem-solving should be linked to games and play. Thus, Rieber and Noah (2008:79) quoting Piaget (1951, 1952) assert that games are a distillation of play. As a result, play has long been regarded as one of the imperative to human learning ways. Parents, as rearers of their children understand them better, where they can give them a chance to elucidate and substantiate their ideas orally and in writing and by acknowledging various modes of presentation.

2.5.2.5.4 Assessment

Theories maintain that assessment is an essential component of teaching and learning, so learners' engagement during the playing of indigenous games should be assessed (Chinn, 2012:9; Vankus, 2008:106). In essence, assessment happens continuously, either before instruction, during the interaction between learners and teachers and parents, or amongst learners. Rieber and Noah (2008:81) quoted Glynn (1995), Glynn, Duit and Thiele (1995), Mayer (1979) that teachers and parents have long before employ metaphors and analogies to assist learners narrow the gap between old and cutting-edge knowledge. As a result, using indigenous games to teach problem-solving is way of using the known to understand the unknown. Hence, during that process there are reflections made and questions posed for clarification, so as to reach the "aha moments" as conceptualised by Liljedahl (2005:220), as ideas spring to mind with characteristics of brevity and sudden clarification. Such experiences are the discoveries of new understandings and new theories, exemplified by the statements such as "oh! Is that what you mean?" or "Why don't they say so, then?" (Haylock, 2010:10). These "aha moments" signify that there are multiple interpretations of reality (Chilisa, 2012:40) and suggest that learners knew this information earlier. In simpler terms, this illustrates assessment which is continuous whilst teaching and learning is happening. It is clear that interaction in class in intertwined with assessment.

Rieber & Noah (2008:81), Van De Walle et al. (2010;B-2) and Vankus (2008:106) believe that the teaching strategies in the teaching of problem-solving must ensure that there are means or mechanisms in place to encourage discourses, dialogue, and deeper levels of learning that go beyond the playing of indigenous games context. The game may be of benefit to learners to organise mathematical content into well-expressed patterns and support them set up and monitor learning goals.

2.5.2.6 Adequate content knowledge among teachers

According to the DoE (2005;5) and Van de Walle et al. (2010:9), it is essential that teachers have profound, flexible, and adaptive knowledge of problem-solving, so that it becomes easy to teach problem-solving in a manner that develops interest among learners. This is in line with Pascuzzo's (2010:233) claim that the mastery of content knowledge alone is no t enough, but rather the teachers have to couple content knowledge with good pedagogical methods. Pascuzzo (2010:234) refers to this as 'content pedagogical knowledge', by which teachers demonstrate sound and significant knowledge on problem-solving. They understand the theories of learners' development across a spectrum of diversity and are able to utilise various modes of teaching, learning and assessing of problem-solving effectively.

In the previous sections it was argued that all activities in the teaching of problem-solving were jointly crafted (involving learners and parents), and assisted teachers to move away from teacher-centred approach to interactive learning (AATM, 2006:5). This allowed teachers to learn from their peers and learners ways to teach certain content areas differently, and in an effective way for learners to comprehend them. Among other modes of learning is the inquiry-based approach (Habler et al., 2011:170), an alternative to the 'chalk and talk' method, which forces learners to learn without understanding (Ellis, 2013:94). In order to make learners enjoy problem-solving, the use of culturally and locally contextualised resources materials such as indigenous games are to be considered for deep understanding of problem-solving).

2.5.2.6.1 Lesson plan shows high standard

In order to build on prior knowledge of learners it is crucial that the lesson plan incorporate contextualised resources, such as using indigenous games to teach problem-solving. AAMT (2006:4) and Samson (2013:47) emphasise that these contextualised resources afford an opportunity for learners to explore a diverse array of mathematical ideas in a meaningful and engaging manner. Not only do the use of indigenous games to teach problem-solving often lead to serendipitous moments of mathematical discovery, they also provide an opportunity to nurture curiosity and creativity, both of which are important components of a healthy mathematical disposition. It is for this reason that it is important that the lesson plan designed by teachers be an interactive one, and that it allows for spontaneous, self-directed learning. For instance, NCTM (2008:1) and Van de Walle(2010:C-7,C-8) cite the *Nextu* as a game that can be used to stimulate mathematical thinking of learners. The board has a symmetrical pattern of regular polygons, and the teacher can ask learners to describe and analyse shapes and so develop an understanding of congruence and symmetry. The *Nextu* board game has important factors required for a child's geometric education.

2.5.2.6.2 Class activities/tests

Haylock (2010:16), Moon-Heum and Jonassen (2009:117) and Van de Walle et al (2010:33) contend that class activities should ensure that learners enjoy problem-solving and also engage learners' intellect by developing mathematical understanding and skills. They suggested that after the learners have played the *Nextu* game board game, the following activities can be suitable for learners to work on: Which regular polygons are adjacent to each other on the board?; Which regular polygons are the strongest game pieces? Explain your answer; why do you think a regular triangle, square, and hexagon (with sides of the same length) were chosen as the shapes for the game board? and Why do you think the game designer chose a tessellated game board? (NCTM, 2014:1).

Samson (2013: 47) supports the above arguments that as learners work in groups they are going to generate many different solutions, to the extent that the number and variety of solution strategies arising in the discussions surprised many of the learners. This also supports the statement that the teacher should not be seen as a powerful human being who has all the answers. Rather, the teaching and learning of problem-solving should be guided by a perspective that all learners are capable of producing multiple realities and knowledge shaped by their experiences (Chilisa, 2012:40).

2.5.2.7 High motivation among teachers

According to Naidu (2013:1) and Keleş, Haser and ve Koç (2012:735), Olatunde Yara and Omondi Otieno (2010:127) the motivation of teachers in boosted by several factors in the teaching and learning of problem-solving skills, such as, sustainable learning environment and content activities that engage the learners throughout the learning session. These help learners in learning problem-solving skills in a more meaningful way and games and other activities play a vital role in linking the subject matter of problem-solving to real-life situations. The diversity of teaching methods in the teaching of problem-solving skills, which focus on learner engagement and arouse and maintain the interest, serves as a prime motivation for the teacher. The use of new materials in the teaching and learning of problem-solving skills allows the teacher to move from theoretical exercises that may not create interest to learners. It engages learners in new or exciting and practical activities that will ultimately enhance their performance. For Yildirim (2012:167), a positive motivation of teachers is more likely to be influenced by motivated learners, those who are actively engaged in problem-solving skills, and those who perform well in assessment tasks. In turn, learners expect teachers to be supportive in their learning, and innovative and creative in designing programmes.

2.5.2.7.1 Lesson planning

According to Elvis (2013:44), Keeley & Tobey (2011:35) lesson planning in the teaching and learning of problem-solving is important. The planning stage focuses on objectives of the lesson, justifications (why is it important to be learnt or taught), mathematical concepts, strategies for teaching and learning, assessment and resources. Lesson planning ensures that high standards of teaching and learning are maintained, through which learners will make progress. Learners' feedback is valuable as it enables the 'feed-forward' process to take into account their aspirations. The indigenous games encapsulate the rich wealth of resources (Yosso, 2005:69) which learners are exposed to or with which they are familiar. In this way the subject matter and the methods of teaching are matched to the learners' needs.

It is important that in the planning stage the teacher is able to reshape the ideas from learners and parents and represent knowledge in different ways, so that it makes meaning to learners. All these consolidated learning objectives are realised in the end, but this is only possible if the teacher has confidence in his or her own knowledge.

2.5.2.7.2 Class interaction is learner-centred

As lesson planning involves all parties, that is, learners, teachers and parents, it is imperative that interaction is made possible in the teaching and learning of problem-solving (Anthony & Walshaw, 2009:154). This concurs with the claim of Elvis (2013:44) that class interaction is the transaction stage, that is, when the execution of the plans are made earlier. It is the interaction or interplay between learners and teachers, and more importantly among learners themselves, necessary for effective learning of problem-solving to take place. This confirms Jonassen's (2003:267) belief that mathematical problem-solving involves situated and socially negotiated methods to authentic, intricate problems.

According to Jonassen (2003:267) cited Riley, Greeno, and Heller (1983), story problems penetrate mathematics curricula, from preschool or preprimary to higher education sector. From simple combined problems in beginning mathematics (e.g., *Kwambi has three token cows. Thabo gave Kwambi 3 more token cows. How many token cows does Kwambi have in the end?)* to complex problems in thermodynamics. Story problems are commonly used in formal setting education and there is no indication that suggests this dominance is declining. The underlying factor of story problems relates the learning of the child to lived experiences (Lynn, 2004:184). It can thus be argued that the use of indigenous games incorporates the marginalised knowledge of the child into the learning of problem-solving. They enable the interaction among learners to happen spontaneously, while in the process teachers monitor the progress by supporting learners where possible.

2.5.2.7.3 Assessment based on high order questioning

As demonstrated above, in section 2.5.7.2, within lesson planning the assessment is central. Chinn (2012:9) demonstrates that assessment; teaching and learning processes should be interlinked and continuous. They are also used to identify problems with the learning of problem-solving and diagnose problems that hinder their understanding. In addition, Chinn (2012:0) and Elvis (2013:44) point out that assessment is about measuring the learners' achievement, skills and deficits, as well as checking whether the intended outcomes or objectives in the teaching and learning problem-solving have been reached. If not, then in next cycle of planning remediation can be attempted. In reiterating section 2.5.2.7.2, once learners have an understanding of simple problems it becomes easier to handle complex problems, as demonstrated by Fellows and Koblitz (1993:372, 382), in that we use *kido Krypto*, but are unaware of doing so. It would be interesting to adapt these ideas in the teaching of problem-solving, so as to convey some of the richness and interconnectedness of problem-solving and at the same time give oneself flexibility when using the game in the classroom. The mathematical content embedded within the game include hash functions and cubic functions. These are high order concepts which are needed in understanding basic rules of *Kido Krypto*.

2.5.2.8 High expertise with regard to classroom practices

For the effective teaching and learning of problem-solving, teachers must be able to manage and control the didactical situations in the classrooms. A deep knowledge of problem-solving makes the teacher confident about himself or herself and able to understand the appropriate theories of learning. Teachers who are aware of different approaches will make sure that learners learn mathematical content (Tamsin, 2013:491; Van De Walle et al., 2010:B-2). In addition, the learning environment should be conducive to learning and the created learning context will encourage the development of mathematical skills and proficiency. For instance, Fergusson (1968;1-7) cited Whinihan (1963), indicates that learners were playing *Nim* (any type of "take-away game" in which players alternate in removing counters, and the player who takes the last counter wins). The analysis of this game resulted in the Fibonacci pattern, given as $F1 = 1$, $F2 = 2$, and $Fn+1 = Fn + Fn-1$ for $n \geq 2$, that is, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Thus, teachers need to reflect regularly on what and how they are teaching problem-solving, by using games that stimulate learners' thinking. Also, by providing regular feedback to learners they can help learners to discover mathematical content (Harringto & Brasche, 2011:23).

2.5.2.8.1 Lesson planning has adequate resources and activities

As the lesson planning is carried out in collaboration with the learners, this helps to place the content within their context (NCTM, 1999:5). There is a high level of interaction among learners, most of whom are familiar with the teaching resources (Harbin & Newton, 2013:542) used, such as storytelling and indigenous games. Leornard (2008:135) argues that these familiar resources, for example, the *Village of Round and square Houses*, equip learners with mathematical content such as measurement, area and perimeter. These resources are rich in mathematical concepts which improve learners' interest and motivation to learn problem-solving.

2.5.3 Conditions for components of emerging framework to work

In section 2.5.2 the eight components of the framework for using indigenous games to teach problem-solving were discussed. In this section the contextual factors corresponding to each of the components will be discussed, that is, those that ensure that the components of the framework of using indigenous games to teach problem-solving are implemented without any distractions.

2.5.3.1 Conducive conditions for meaningful subject matter to enhance the learning of problem-solving skills

According to Lynn (2004:154), Preus (2012:75); the Republic of Rwanda Ministry of Education (2006:91) and Yosso (2002:94), for meaningful subject-matter in the learning of problem-solving, the home environment must be replicated in the classroom. This can be done through the playing of indigenous games, in which learners are able to find embedded mathematical content. Through the playing of such games as *Kido Krypto, nextu* and *morabaraba* (Mosimege, 2000:457; NCTM, 2008:1), learners identified mathematical concepts of operations, cubic, hash functions and different geometric configurations. This intermingling of home and school cultures helped them to understand the problem-solving.

However, Haylock (2010:9) and Yosso (2005:78) point out that the linguistic capital and familial capital possessed by learners means they are able to define mathematical content for themselves and learn from others through interaction. On its own, mathematical language is too technical and too specific to the subject, and thus reinforcement or clarification is made through the language used in everyday life. It is argued by Bush et al. (2010:164) Setati and Adler (2002: 135) and Wong and Lipka (2011:823) that code switching is appropriate for learners to gain deeper meaning (Averill et al., 2009:181) of the mathematical concepts. This encourages learners to construct their own discoveries of problem-solving concepts (Dennis & O'Hair, 2010:5).

2.5.3.2 Contextual factors making learner-centred methods of teaching problem-solving skills conducive to the emerging framework

Learners are given a chance to play indigenous games or observe their parents play them, which allows them to explore and visualise mathematical content and concepts infused within the games (Republic of Rwanda Ministry of Education, 2006:89). Yosso (2002:95) citing Bell (1992) points out that by giving learners liberty to visualise and extract mathematical content they are actively involved in the learning of problem-solving. They enjoy playing indigenous games, while at the same time learning of problem-solving is taking place (Mosimege, 2000:11, 12). Also, this is viewed as the building of problem-solving from the marginalised knowledge of the people at the bottom of the society. The excluded knowledge of the learners is acknowledged and validated in the mainstream curriculum.

Cole and Griffin (1987:13) assert that the circumstantial factors are such that learners interact not only between teachers, but also among learners themselves. There is enough time allocated to class activities and presentations to authenticate time on task, that is, time learners spend engaged in activities, and to increase their understanding of problem-solving. For Preus (2012:62), the conditions reigning in the classroom environment are such that learners engage with their teachers and peers on the subject matter in a way that builds on shared understanding of mathematical content. The teacher as the group facilitator asks probing and persistent questions that ensure learners discover information and definitions on their own. The facilitation by teachers and parents in different small groups ensures that there is no ‘off task’, where learners deviating from the activity discuss matters not related to the given task, but time ‘on task’ prevails during their interactions.

2.5.3.3 High level of motivation/interest among learners

The contextual factors that influence learners to develop interest in the learning of problem-solving are to a large extent influenced by the teaching methodology used by the teacher. Learners are motivated to pursue the goal that has the highest value to him/her (Ambrose et al., 2010:74, 75). The adequate support given to learners in their interactive groups, that is monitoring their progress closely and valuing the prior knowledge they possess, are the factors that expedite high motivation (Preus, 2012:61). The comments given by the group facilitators (teachers) are specific and build learners' own understanding of problem-solving, and encourage them to make further exploration, rather than the teacher giving general comments, such as “good work', which is positive but does not pinpoint the learner's specific strengths.

Dennis et al. (2010:12) write that extrinsic motivation from the teacher can result into intrinsic motivation for the learners. The groups are instituted in class and more time is spent on activities that motivate learners in their learning. They may also form small groups when studying problem-solving, and benefit from working with teachers and parents in learning problem-solving. Hence, learners display affective outcomes (such as tolerance, self-acceptance, trust and security) when interacting with teachers and their peers (Roux, 2009:591). In class presentation they have tolerance, whereby they allow different views in their learning of problem-solving. They show self-acceptance by defending their convictions on certain definitions of mathematical content. For instance, Wong and Lipka (2011:823,827) show that rather than verifying the square using Euclidean proof, that is a square has four equal sides and all angles are right angles, from a Yup'ik perspective, learners verify the square using transformational geometry. To do so, it was imperative that teachers understood the cultural activities (such as indigenous games) and mathematical content embedded within them.

2.5.3.4 Circumstantial factors for learners to discover problem-solving skills formulae and processes

The atmospheric conditions in the classroom make it legitimate for learners to interact freely among themselves and not depend on the teachers to transmit information to their minds. As Yosso (2002:96) quoting Delgado (2000) illustrates, it is advisable for teachers to teach problem-solving in a manner that learners are seen as holders and creators of knowledge. Learners use the wealth of knowledge they possess to discover mathematical ideas embedded in indigenous games. As a result, teachers need to use various approaches to teach problem-solving. This supports Dennis et al.’s view (2010:5) on disciplined inquiry as a method that uses prior knowledge of learners in an attempt to understand problem-solving on a deeper, less superficial level.

Dennis (2010: 4) and Preus (2012:62) argue that activities designed for learners ensure that high order thinking is maintained, in such a way that teaching and learning of problem-solving involves learners in manipulating, synthesising, generalising, explaining and arriving at informed conclusions that produce new meanings and understanding for them. This is illustrated by Mosimege (2000:12, 17), in the play of a string game that enables learners to discover a pattern that shows relationship between quadrilaterals and triangles. The relationship between triangles and quadrilaterals is given as y = 2x+2 (Mosimege, 2000;16).

2.5.3.5 Appropriate conditions for parents to be highly involved in the teaching of problem-solving skills

The conducive conditions reigning in the schools are of the nature that the knowledge of the excluded people is more central than peripheral (Lynn, 2004; Yosso, 2002:99). Parents are permitted to collaborate with teachers in the teaching and learning of problem-solving using indigenous games. The connections and relationship between parents, teachers and learners exemplify the democratic atmosphere in the teaching and learning of problem-solving (Dennis, 2010:8). This concurs with Bush et al. (2010:164) and the Republic of Rwanda Ministry of Education (2006:91) that the involvement of parents in the teaching and learning of problem-solving creates a rich and positive learning environment at home and at school. The advantage of engaging parents is that they bring their wealth of knowledge of indigenous games, which encourages learners to experience mathematical ideas infused within them. The presence of parents promotes confidence and help learners believe that they can think mathematically. In this way the conditions are such that teachers and parents talk about problems with particular learners and their learning and/or disciplinary issues (Averill et al., 2009:181; Dennis, 2010:11).

2.5.3.6 Contextual factors that enhance content knowledge among teachers on problem-solving skills

The DoE (2003:5) and AAMT (2006:2) expect that all teachers teaching problem-solving must have high content knowledge and high skills, and demonstrate mastery of pedagogical content knowledge, that is, how the problem-solving content is effectively taught to learners. In imparting the content knowledge to learners it is important that teachers understand how problem-solving is represented and communicated to the majority of learners. Of importance here is understanding the connections within problem-solving, between mathematical content areas and other subject area, and how problem-solving is related to the society.

This concurs with the view of Posamentier and Krulik (2009: viii) that the teaching of problem-solving, if conducted more easily and neatly, and with greater understanding, will be more enjoyable to learners. Also, an encouraging and enabling parental involvement helps provide effective role models for learners. Wong and Lipka (2011:822) examined teaching measurement, patterns, numbers and algebraic thinking and cultural artefacts used in *Yup'ik*, in which parents were invited to be part of a Mathematics Cultural Context (MCC) project. This encouraged learners to view problem-solving with positive attitudes as they realised that the resources were from real-life situations.

Bush et al. (2010:165), Habler et al. (2011:18) and Dennis (2010:19) write that teachers who engage in lesson study in collaboration with other teachers, to plan, present lessons, and make after-class presentations in which they comment on strengths and weakness observed for future improvement, increase their chances of exploring variety of ways of teaching and assessment of problem-solving effectively. Teachers learn from their peers how to contextualise mathematical content for learners to comprehend it easily. This ensures that teachers remain lifelong scholars and subject specialists in their fields of specialisation.

2.5.3.7 Conducive conditions for high motivation among teachers

According to Dennis (2010:15), the conducive factors that contribute to high motivation among teachers are working together in the teaching and learning of problem-solving, seen as a driving force that accelerates momentum to work harder. Teachers' involvement in professional groups or lesson study sessions, in which they plan together, learn from one another and give them confidence and self-esteem to present problem-solving in various way for learners to comprehend in a simpler way. The feed-forward technique (Kelley & Tobey, 2011:89) provides an opportunity for teachers to reflect on how they use feedback received from their peers to improve on their next lesson presentations. By the time the peer feedback is given in such a motivating way it has built the confidence of the respondent teacher, particularly if the weaknesses identified are not put in a way that kills the spirit of others. Another circumstantial factor is the support teachers receive from the SMT and district officials (Bush, Jouberty, Kiggundu, Van Rooyen, 2010:162-165). The support is developmental, that is, wanting to identify the cause of the problem, such as why learners do not perform well in problem-solving and also think aloud or work together with teachers to arrive at the solutions.

This supports what Bush et al. (2010:166) and Kelley and Tobey (2011:99) refer to as ' friendly talk probes' that SMT, district officials, teachers, learners, parents, other community members engage in on the misconceptions of learners with regard to mathematical concepts or content. This encourages teachers to realise that they are not alone in assisting learners to attend problem-solving. In addition, Dennis (2010:9) points out that such conducive factors breed more conducive factors, such as better attendance by learners, and learners cooperate with teachers in their respective classes. This is motivating for the teachers to excel in their teaching practices of problem-solving. As learners show commitment and dedication towards their schoolwork, there is a higher pass rate in district standardised assessment tasks, and final examinations tasks (Bush et al., 2010:164; Moloi & Chetty, 2011:1).

On the other hand, Dennis (2010:15) asserts that prevalence of the conducive conditions in the teaching and learning of problem-solving reduces teacher burnout, that is, they lose interest and do not make an extra effort to help learners to perform better in problem-solving. In most cases there are more reasons than resolutions as to why learners do not perform well in problem-solving (Bush et al., 2010:167).

2.5.3.8 Circumstantial factors that enhance teachers' expertise with regard to classroom practices

According to Bush et al. (2010:165,167 and Dennis (2010:15,19) teachers' expertise in the classroom practices is expedited by their involvement in the professional development groups, that is where teachers plan and present lessons while their peers assess them. Their peer teachers focus on the strengths and weakness that they need to improve on. Bush et al.(2010:168) noted that in South African context, the middle leaders, such as senior teachers and HoDs, need to ensure that fundamental tools for managing teaching and learning of problem-solving are implemented and monitored effectively. Bush et al. (2010:165) in Southworth (2004) and Dennis (2010:10,17) illustrate the following tools for middle managers: modelling, in which the middle manager leads by example, that is presenting the lesson in the presence of his/her peers so that they copy good practices; and dialogue, that is observing teachers presenting the lesson in class, and thereafter providing feedback on strengths (comment of good practices of the teacher) and weakness of the teacher, so as to improve in the next presentation. Again, the middle manager together with teachers, parents, district officials and community members are tasked to set up structures and systems that ensure effective teaching and learning of problem-solving in classes. They also see a close link between good monitoring and good teaching (quoting Office of Standards in Education(Ofsted), 2003).

On the other hand, Habler et al. (2011:17, 18) and Preus (2013:70) argue that the conditions prevailing in class are such that there is close interaction and collaboration among learners themselves, and between their teachers in the learning of problem-solving. The types of questions or activities in which learners are engaged allow them to construct their own meaning for mathematical concepts. For instance, Habler et al. (2011:240 demonstrate the play of *Kechui* (a strategy game) as an example of indigenous games that teach learners to weigh alternatives in a series of additions and subtractions of small numbers. As Habler et al. (2011:24) and Wong and Lipka (2011:825) state, in the classroom environment the learners, teachers and parents identify and clarify the mathematical content embodied in the cultural activity, such as indigenous games and cultural artefacts, then re-contextualise the knowledge of elders to fit a modern mathematics curriculum. They further contend that learners come to school with well-developed informal systems of mathematical content by Romberg & Carpenter, 1986).

2.5.4 The factors that threaten the implementation of the emerging framework

The previous section explored conducive conditions in detail. These are the conditions that one must take into consideration for the emerging framework of teaching problem-solving using indigenous games. Thus, it is important for one to be mindful of possible threats and the risks factors that might prevent the framework of teaching problem-solving using indigenous games working effectively as anticipated. As a result, this section presents a detailed discussions of possible threats, and gives an account of how to circumvent these risks factors for the framework to produce desirable results.

2.5.4.1 Risks factors that impacted on subject-matter in the teaching of problem-solving skills

The use of indigenous games in the teaching of problem-solving, amongst other things, is intended to make the subject matter more easily accessible and meaningful to learners. The exciting part of using the framework is that learners learn in a fun way, and use their cultural wealth knowledge to understand mathematical content more easily. The framework tapped into the cultural background and the context of learners when explaining the mathematical concepts (Graven et al., 2013:4; Murray et al., 2011:72).

On the other hand, there are risks factors that threaten the meaningfulness of the subject-matter. Bush et al. (2010:164,166), Chick (2002:474) and Preus (2012:77) point out that the noisiness of learners might disturb other classes. As a result, this might lead the principal and SMT to withhold support for the framework of using indigenous games to teach problem-solving. In schools there are certain leaders who do not encourage innovation among teachers.

Insufficiency of materials is cited by Wedin (2010:145) and Yosso (2002:101) as helping teachers to extract mathematical concepts from the indigenous games. Meanwhile, another problem for Hornberger (2005:3) is the common use of English as a language of access in the teaching and learning of problem-solving, which leaves non-English-speakers, particularly parents, disadvantaged and effectively excluded from the classroom discussions. As a result, certain parents feel isolated in the teaching and learning of problem-solving. This supports Chick's (2002:463) argument that at times dominant groups use English to stigmatise the contributions of parents, who are not fully conversant in English.

Thus, in surmounting the risks factors for the framework of using indigenous games to work effectively, Bush et al. (2010:164) and Chick (2002:463,470) argue that code-switch is allowed so that all team members are accommodated in classroom discussions. It is wrong to perceive the use of indigenous language in the learning and teaching of problem-solving as rebellious. This supports Hornberger (2005:5,6) citing US Supreme Court 1954:Syllabus[(e)] Title VI of the Civil Rights Act of 1964 and the Equal Opportunity Act of 1974 and the Constitution of Republic South Africa Act 200 of 1996:14-15 that eleven listed languages are official and they can be used in any informal and formal settings. Use of any language is a basic human right and no state or educational agency or schools shall deny educational opportunities to individuals because of not being conversant in English. The use of indigenous languages is useful to explain mathematical concepts in a more understandable way. this is amplified by Setati et al,(2002:143) that teachers used code-switching frequently because learners do not understand English. Also, Setati et al(2002:129) found that teaching problem solving in the mother-tongue of learners is easier for learners to understand and ask questions freely.

Chick (2002:473,475) argues that some school principals or SMTs believe in ‘one-at-a time discourse’ in the teaching and learning of problem-solving, by which the teacher is active and learners are passive. The conditions are such that learners are negatively sanctioned for calling out that they know the answer asked by the teacher. Learners are urged to keep quiet until nominated to respond by the teacher. To make matters worse, side-talk with other learners is discouraged. Then, if one is not following this approach in the teaching and learning of problem-solving, the class is labelled as noisy. Bush et al. (2010:166) point out that in schools in certain part of Mpumalanga province the instructional leadership is poor and weak in teaching and learning matters. As a result, teachers are not well supported on new National Curriculum matters introduced by the DoE. In addition, the DoE (2003:2) encourages teachers to actively involve learners in the teaching and learning situations. Chick (2002:475) argues that once learners are put in small groups to work on class activities there is quick progress made as the faster and brighter explain to others what is required. Teachers are expected to have knowledge on mathematical content and encouraged to attend content workshop organised by the departmental officials. It is further to their advantage to be members of professional bodies, international bodies such as National council of Teachers of Mathematics (NCTM) and national bodies such as Association of Mathematics Educators of South Africa (AMESA).

2.5.4.2 Risk factors that threaten the methods of teaching problem-solving skills

According to Dada et al. (2009:24, 26) and Dennis (2010:11), teachers complain about the lack of time to prepare, plan and present problem-solving. At times they have to stay long hours after school, preparing for the next lessons. The ineffective teachers will derail the framework of using indigenous games to teach problem-solving. In most cases teachers who are about to retire are not willing to change for new approaches in the teaching of problem-solving. Teachers and parents need to be effective facilitators in the teaching and learners, such that they monitor each group (learners in their small groups) to ensure it is working on the given tasks. Group members (learners in a group) will not be in the positions to discuss irrelevant matters, or in a situation where only one member (learner) is active during feedback sessions to a larger group. In addition, Lynn (2006:18) and Yosso (2002:102) point out that teachers need to be mindful of the traditional ways of teaching problem-solving, which tend to be oppressive to learners. It also perpetuates the inequality if the few learners receive more attention than others in learning problem-solving. This concurs with Chick's (2002:476) use of one-at-a time discourse, in which the teachers discourage interaction among learners, and only concentrate on the few learners when asking questions in class.

In circumventing the observed risks, Preus(2012:62) argues that from the onset once the preparation , and planning is properly done, these can be duplicated for future use. Also, Hardman, Ackers, Abrishamian (2011:12) assert that for effective teaching and learning, teachers must ensure that learning outcomes in the classrooms are achieved. That in itself suggests that it is important for team members to do thorough preparations on the class activities to be performed by learners. The team members must ensure that time on tasks is not compromised during learner interactions. Time on tasks is time fruitfully spend by learners when engaged in activities. All other members need to assist by assisting in various groups, not be given them answers

2.5.4.3 The risks factors that derail high level of motivation among learners

According to Averill et al. (2009:179), there are teachers or educational practitioners who perceive the use of indigenous in teaching problem-solving as unimportant, irrelevant and unsophisticated. Hence, learners even forget to note mathematical concepts or mathematical content extracted in the indigenous games during the class interactions. However, Habler (2011:23), Lynn (2004:162) and Mbalula (2012:3) point out that it is the duty and responsibility of teachers to encourage the youth to embrace indigenous games, as they are important in contextualising the abstract mathematical content. There is a myth that marginalised communities are disadvantaged in terms of their cultural practices, that is, their playing of indigenous games is construed as ‘barbaric’ and ‘backwards’. Cultural practices can include language, ways of surviving and various indigenous games played in rural societies. The indigenous games were deliberately neglected by successive regimes of South Africa and they were also suppressed, to the extent of those playing them being labelled as lazy, immoral, unintelligent, and ignorant.

The DBE (2009:8) advocates teaching and learning of problem-solving as realistic and not contrived, and the contextual problems to include issues relating to social and cultural context whenever possible. Likewise, the Constitution of the Republic South Africa (Act 200 of 1996:14-15) acknowledges that everyone has the right to participate in any cultural practices and indigenous games of their choice. It is imperative for Mathematics teachers to weave these rich cultural stories into their teachings, to give learners the complete picture of mathematics as a living, breathing, exciting, and adventurous endeavour of the human mind and spirit (Tennant, 2003:6).

2.5.4.4 Factors that threaten the self-discovering of problem-solving skills formulae and processes

The new curriculum encourages learners to discover mathematical contents and skills on their own (Chick, 2002:474; DBE, 2011:4; DoE, 2003:2). At times teachers are tempted to be quick to give learners correct answers before they have explored the problem. As Yosso (2002:101) argues, this type of teaching limits learners’ access to deep understanding of problem-solving skills. On contrary, Bush et al. (2010:164, 166) citing Henefeld (2004) and Dennis (2010:10) assert that when teaching problem-solving, teachers need to pay attention to the learners’ involvement and stimulate deep learning, rather than using rote learning. In assisting with implementation of the framework to use indigenous games to teach problem-solving, the SMT should give teachers support in classes (DBE, 2011/12:52; Dada et al., 2009:8). This may be achieved by making lesson observations, so as to contribute positively to the implementation of the framework. In a study conducted in Tanzania, Uganda and Madagascar by Bush et al. (2010) the SMT's oversight was found to be mostly perfunctory, with slight observation of teaching taking place. Thus, SMTs are urged to promote and support new innovations taking place in classrooms.

Bush et al. (2010:164) contend that because the framework of using indigenous games to teach problem-solving is demanding for all parties, that is, teachers, learners and parents, learners use shortcuts in learning problem-solving. In small group discussions, some might not be involved on time on task, that is, they discuss matters outside the activity given. During the feedback sessions they want to copy down everything said by other groups, without deeper understanding, or drag deliberations by asking already answered questions. This is tantamount to rote learning. The cited threatening factors can also cause learners not to complete homework or assignments, and absent themselves from classes because nothing has been completed for homework (Bush et al., 2010:166).

In overcoming the threatening factors to the framework of using indigenous games to teach problem-solving, Bush et al. (2010:165) suggest that team work be instituted in the teaching of problem-solving. There will be enough facilitators to monitor the effectiveness of small groups, such that learners are not engaged on off-task (Preus, 2012:162) activities. The teamwork in classrooms helps teachers to be present all the time and use effectively teaching and learning that will make learners engage in tasks throughout the lesson sessions, without any deviations.

2.5.4.5 Factors threatening the involvement of parents

According to Lynn (2004:161) and Yosso (2002:100), a mistake made by the school communities is that parents are not listened to or judged according to the standards of the white middle class parents. Ultimately, these parents are labelled as uninterested in the education of the child, which results in their absenting themselves, citing numerous reasons. Chicks (2002:470,471) and Hornberger (2005:3) quoted Finlayson & Slabbert (2004) in that the use of English only marginalises subaltern parents who are not conversant in English. There is also a common perception of English as the language of access in the teaching and learning of problem-solving, and use of it stimulates critical thinking and restores good manners. These myths are threatening factors that isolate subaltern parents in the teaching and learning of problem-solving.

In circumventing these risks factors that threaten the involvement of parents in the framework of using indigenous games to teach problem-solving, Chick,(2002:470); Constitution of Republic South Africa Act 200 of 1996:14-15); Hornberger(2005:,6) citing US Supreme Court 1954:Syllabus[(e)] Title VI of the Civil Rights Act of 1964 and the Equal Opportunity Act of 1974, illustrate that the use of indigenous languages is useful in explaining the mathematical terms or concepts. No one must be prevented from using any of the official languages, either in formal or informal settings. Teachers and other educational practitioners must be aware that language is a basic human right for one to access mathematical content. Parents possess linguistic capital (Yosso, 2005:70) that can help learners to understand problem-solving more easily. Hornberger (2005:3) quoting Joseph and Ramani (2004) found that a home language project in Limpopo, which was a parent-based initiative, taught Ex Model C school's parents Nguni and Sotho languages as a way to value the mother tongue in the teaching of problem-solving. It also helped them to code-switch without difficulty.

2.5.4.6 Factors threatening the adequate content knowledge of teachers

The strong pedagogical content knowledge of teachers in very important in sustaining the framework of using indigenous games to teach problem-solving. However, there are risk factors such as ineffective teachers (Dennis, 2010:10) who do not prepare or plan well for their classes, so that they present coherent and substantial subject matter to learners. This normally occurs with the newly appointed staff members (Dada et al., 2009:9), and those who are close to retirement. It must be acknowledged that newly appointed teachers who do not have enough teaching experience normally experience certain challenges with the high demands of the new curriculum and heavy workload for teaching (Bush et al., 2010:164,166). In addition, Bush et al. (2010:165) and Hardman et al. (2011:12) strongly argue that teachers need to be supported in classrooms by ensuring that resource materials are available (Dada et al., 2009:8; Wedin, 2010:145). They should be encouraged to attend professional development workshops on subject content organised either by education officials or professional bodies. Also, incentives should exist for teachers as a way of motivating them to make an extra effort in conducting extra classes for learners on problem-solving.

Cole and Griffin (1987:13) and Dada et al. (2009:55) argue that the teaching of problem-solving using indigenous games is an integrative approach to that incorporates all mathematical topics, subject areas, and hands-on opportunities, by which learners gain greatly. Moloi and Chetty (2011:1) and Provasnik, Kastberg, Ferraro, Lemanski, Roey and Jenkins (2012: iii) write that learners' mathematical content is of such a level that they are able to compete at national level (final mathematics examinations), at regional level (that is, taking part at SACMEQ tests) and at international level in TIMSS competitions.

Booker (2004:16,20) and Republic of Rwanda Ministry of Education (2006:90) point out that the use of indigenous games in the teaching of problem-solving provides an opportunity for integrating the cognitive, affective and social aspects of learning. Learners develop abstract and critical thinking through the interaction in a play. In the process the manipulation of indigenous games helps with the discovery of patterns and verbalisation of actions, thoughts and interpretations while playing can help in the construction and formation of mathematical concepts. In turn, learners are encouraged to realise that they have discovered mathematical content and concepts arising out of the playing of indigenous games. Also engendered is an exchange of ideas as learners endeavour to make sense of their actions and thoughts. Thus, learners discover that mathematics can be a social process of sense-making and understanding rather than a set of rules handed down from an authority on high.

2.5.4.7 Factors threatening the motivation of teachers in the teaching of problem-solving

According to Bush et al. (2010:166), Dada, Dipholo, Hoadley, Khembo, Muller, Volmink (2009:8) and Dennis (2010:120), teachers complain of being overloaded with many periods and many subjects to teach. This leave them with too little time to prepare and plan for their classes and in some cases they fall behind the work schedule (Dada et al., 2009:26) for learners to be ready for common tests and June's trial examinations. This concurs with what Yosso's (2002:102) observation of criticisms and resistance to implementation of radical curricular discourse or new approaches to teaching that would not work out. As Bush et al. (2010:167) argue, the abovementioned risk factors made it impossible for teachers to attend classes on a regular basis, which results in absenteeism, or ineffective teaching and learning methods that weaken teacher subject matter. Exacerbating matter is that South African principals are mainly concerned with financial management, human resource management, and policy issues, though the core business of the school, managing of teaching and learning, is left to teachers only (Bush & Heystek, 2006; Dada, Dipholo, Hoadley, Khembo, Muller,Volmink, 2009:8).

In responding to the abovementioned risk factors, Bush et al. (2010:166) and Hardman (2011:2) assert that all stakeholders, including teachers, HoDs, parents, district officials and learners, need to work collaboratively in the teaching and learning of problem-solving, displaying sound and proactive leadership and management of learning (Bush et al., 2010:162 in Bush & Glover, 2009; Spillane, 2004; Taylor, 2007; and Dada, Dipholo, Hoadley, Khembo, Muller, Volmink, 2009:8).

On the other hand, the middle managers (HoDs) and subject advisors must stop criticising teachers for lack of commitment and unwillingness to provide extra classes. They must stop blaming learners for laziness and ill-discipline. Rather, the middle managers, SMTs and subject advisors must help teachers to set up structures and systems that allow the proper functioning of the framework of using indigenous games to teach problem-solving to function effectively. All research partners have a direct responsibility for the quality of learning and teaching and for learners' achievement in problem-solving. This suggests setting high expectations and monitoring and evaluating the effectiveness of learning outcomes (Bush et al., 2010:163 quoting National Assembly for Wales, 2005; DBE, 2011/12:52; Dada et al., 2009:8).

2.5.4.8 Factors threatening classroom practices

Booker (2004:16) and Bush et al (2010:164) argue that teachers dominate the teaching and learning sessions, to the extent that little time is allocated for learners to interact and share activities on problem-solving with their peers. As a result, the teachers tend to be very quick in their presentations and have too many activities within a single period, leaving learners behind. The authors advise teachers to create a conducive learning environment in the teaching of problem-solving, such that teachers ask questions rather than provide answers to learners. They allow learners to provide answers after they have collated and shared their ideas, and the answers provided are justified so as to show their understanding. Booker (2004:16) contends that the teaching and learning of problem-solving should be perceived as the construction of ideas rather than transmission of mathematical content into the minds of learners. It would be easy for them to forget the knowledge imparted knowledge to their minds. The main responsibility of the teacher (DoE, 2003:2,3) is to help learners to reconstruct different ways of arriving at an answer. In order to understanding learners' thinking and understanding they should be given resources and materials that stimulate their mathematical thinking, of which games are an example. In such cases, learners are given space to discuss, negotiate and reflect on the class activities.

2.5.5 Evidence that the strategies to use the indigenous games to teach problem-solving have yielded good results

In this section, indicators of success, namely milestones that signify the achievements of the framework, will be fully discussed. Evidence that justified the framework of using indigenous games in teaching problem-solving will be provided. Averill (2009:179) points out that for the framework to work effectively and fit logically for the purposes, indicators of success must be given. The results show that the framework of using indigenous games to teach problem-solving is successful, because it has been tried and tested.

2.5.5.1 The mathematical content is accessible to learners

Considering learners' background in the teaching of problem-solving helps learners to establish connections of mathematical concepts in their minds. It is argued by Haylock and Cockburn (2013:253) that learners develop a sense of equivalence, that is, they are able to relate mathematical objects to real life contexts, and transformations, that is, recognise the differences, which enables them to organise and make sense of their observations and their practical engagement with mathematical objects and symbols. As a result, the mathematical skills and competencies show significant growth, and learners demonstrate higher understanding of mathematical content than mere repetition of learnt procedures and rules (Haylock, 2010:24; Haylock et al., 2013:253; Nyingi et al., 2003:491; Van De Walle et al., 2010:B2).

2.5.5.2 The method of teaching and learning problem-solving skills is learner-centred

According to the DoE (2003:6), learners are encouraged to take control of their own learning. Teachers need to create an enabling learning environment that allows learners to interact freely among themselves and their teachers. This concurs with the claim of Thomas and Brunsting (2010:10) that a mathematics classroom that is humming with thought and activity, and where the learning environment encourages problem-solving through dialogue, collaborations and cooperative learning, will be a success. At certain intervals, teachers provide scaffolding to assist learners in promoting divergent thinking to master mathematical concepts (Haylock, 2013:7; Shatzer, 2008:652).

Chick (2002:473) illustrates that divergent thinking inculcated in learners stimulates them to ask teachers unexpected questions on problem-solving. In such cases, it is important that teachers do not just give them correct answers but also use scaffolding so that learners discover mathematical concepts on their own (Haylock,2010:47; Sorensen & Nebraska, 2006:4).

2.5.5.3 High motivation among learners

For learners to learn problem-solving effectively, a high level of motivation is needed among all learners. According to Nyingi et al. (2003:489), motivation refers to cognitive and affective psychological processes that influence learning. Also, Sorensen et al. (2006:3) adds that motivation impels and directs learners to engage in academic activities. Once motivation is instilled in learners they do your utmost to understand the subject matter, improve performances and persist in tasks even in the face of failures. Thus, the teaching of problem-solving using indigenous games combines *intrinsic*, that is, personal interest that encourages learners to engage in learning activities, and *extrinsic,* that is, external rewards, such as teachers' praise or approval of participation in the lessons, as motivations. These categories are crucial in making the learning of problem-solving effective and sustainable.

Nyingi (2003:487,490), Shatzer (2008:650) and Sorensen et al. (2006:3) contend that a good grade in problem-solving, coupled with constant feedback from teachers, will motivate learners to perform better. On the other hand, they also point out that the use of various instructional approaches in the teaching of problem-solving inspires interest among learners. The school social set up allows learners to learn from the social background, so as to associate the mathematical content with their prior knowledge from home. A school environment should be friendly for learners to learn problem-solving easily and also sensitive to their needs. The use of indigenous games in the teaching of problem-solving elicits the following dimensions: interest, relevance, confidence and curiosity to learn, which all play important roles in making learners fully engaged in the learning and teaching process.

2.5.5.4 Self-discovery of problem-solving skills formulae and processes

The use of indigenous games in the teaching and learning of problem-solving revealed a self-discovery processes, in the sense that all activities learners engaged in involved trying all possible solutions, while some did not reveal the expected outcomes. Learners learn the benefits of continually exploring and trying various possibilities on problem-solving (Sorensen, 2006:5). This supported the findings of Haylock (2008:24) and Sorensen (2006:5), that in most cases the learning activities involve the manipulation of concrete materials (e.g. use of indigenous games), symbols, language, and pictures (structural nature of the indigenous). Connections between these four types of experience constitute important components of mathematical understanding. Thus, the network of connections between concrete materials such as indigenous games, language used the interactions, structural nature of the indigenous games and symbols used constitute a mathematical concept. Also, Shatzer (2008:650) highlights that this symbolises the interdisciplinary connections between problem-solving and real life contexts. In addition, Bintz, Moore, Wright, Dempsey (2011:59, 69) quoting Carter (2001) illustrate that the connections of contextual factors and problem-solving help learners to develop deeper understanding of abstract concepts.

2.5.5.5 Good level of parent involvement

Mncube, Harber du Plessis (2011:216) and Sorensen (2006:5) point out that working with parents in the teaching of problem-solving using indigenous games is very important. Teachers who collaborate with parents understand their learners better; are able to generate distinctive, rather than routine, solutions to problem-solving, and can reach a common understanding with the parents and learners. As such, parents support learning of problem-solving inside and outside the classrooms. Parents demonstrate the play of indigenous games at home, the learners are assisted by their parents in extracting the mathematical content visualised through the play of indigenous games.

Mncube et al. (2011:216) cited Gutman & Midgley (2000) contend that parents who are involved in school activities tend to develop a greater appreciation of their role in the schooling of their children. This also results in a close relationship between the family and school. Parental engagement in the teaching of problem-solving using indigenous games has influenced the academic performance of learners, and is connected with a variety of positive academic outcomes, including higher grade point averages. The intense engagement of parents in the teaching of problem-solving motivates learners to perform well (Meaney & Evans, 2013:483; Sorensen, 2006:11, cited Marchant, Paulsen & Rothlisberg, 2001).

Mncube et al (2011:217) suggest that the involvement of parent of learners from lower social-economic status does not benefit learners as the gap in performance is not reduced. In other subjects in which parents feel confident there can be positive changes observed, but for mathematics tests the gap of achievement does not narrow. In its research, the Assembly of Alaska Native Educators (2002:9) cautions teachers not to use parents selectively, only when circumstances favour the school. The community cultural wealth theory accommodated the expertise of parents in the teaching of problem-solving by using indigenous games, where learners have interest in extracting the mathematical concepts visualised by playing them.

2.5.5.6 Adequate content knowledge among teachers

According to Bintz et al. (2011:63) and Yosso (2002:102), the interaction between teachers and parents on the teaching of problem-solving using indigenous games empowers teachers in understanding mathematical concepts embedded within them. This is supported by De Cruz and De Smedt (2013:4), who found that the demonstration of indigenous games by parents and other community members showed many mathematical realities and concepts can be found within human cultures. They also argued that human beings are equipped with unlearned problems-solving skills. As illustrated by Bintz (2011:63) and Haylock (2008:7, 16), the mathematical content which comes out during the play of indigenous games helped the teacher to extract mathematical concepts from the tacit knowledge that learners bring from their home environment.

Bintz (2011:64) and De Cruz et al. (2013:6) assert that the mathematical content and objects which are abstract for learners are handled in a simplified way. Also, Hugh Maxwell and Iman Chahini (2013:63) contend that the framework of using indigenous games to teach problem-solving is perceived as culturally relevant activities that engage learners in hands-on explorations of meaningful learning, and also provide opportunities for teachers to teach mathematical content in an interesting way. Quoting Gay (2000), they further argue that using indigenous games to teach problem-solving is understood as culturally responsive teaching. It utilises cultural knowledge, learners' previous experiences and performance styles of diverse learners to make learning of problem-solving more appropriate and effective.

2.5.5.7 Motivation among teachers

Bush et al. (2010:167) cited Leithwood et al. (2006), Robinson (2007) and Hardman et al. (2011:2) to argue that efficient leaders (SMTs) are able to raise the standards of teaching and learning problem-solving by motivating and inspiring teachers. Teachers are supported in their classroom by the SMT through classroom observations and monitoring and ensuring that after such support visitations teachers are providing with feedback. It is advisable that the feedback be on strengths and weakness identified not on weaknesses of the teacher. Also, the direct engagement of parents and local community members in the teaching of problem-solving encourages teachers to open up whenever they experience difficulties and challenges, so that they can be jointly confronted. This is also supported by Yosso (2002:102) citing Delgado (1994) and Diaz (2000), who argue that parents of colour hold high esteem for and actively engage in their children's education.

On the other hand, Preus (2012:6) and Sorensen et al. (2006:4) point out that good learners' performance in problem-solving is the inspiring factor for teachers to work harder in executing their duties. For Nyingi et al. (2003:496), mathematics teachers could enhance their learners' motivation to learn problem-solving by creating their interest in problem-solving, making it relevant to them and increasing their probability of success and satisfaction by using indigenous games to teach problem-solving. This agrees with Wong and Lipka (2011:821), who reiterate that addressing underperformance of problem-solving in the use of Mathematics in Cultural Context (MCC) is crucial.

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2.5.5.8 High level of expertise with regard to classroom practices

The learning environment created, in which learners are free to talk and interact with their peers in the teaching and learning of problem-solving, allows them to control their own learning. Learners are active participants, and as a result they construct their own knowledge on problem-solving using their lived experiences of playing indigenous games. The physical space is where learners are taken to the playing field to play indigenous games and appropriate resources materials that facilitate the learning of problem-solving skills. The created atmospheric conditions are those of respect and value for the learners to learn problem-solving effectively. Also, learners collaborate with their peers, teachers and parents to develop plans to improve the instruction of problem-solving (Bintz et al., 2011:59; Sorensen, 2006:11; Van De Walle et al., 2010:B-2).

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY: THE FRAMEWORK OF USING INDIGENOUS GAMES TO TEACH PROBLEM-SOLVING

3.1 INTRODUCTION

This chapter deals with the operationalisation of the objectives of the study, that is, developing a framework for enhancing problem-solving in a grade 10 mathematics classroom by using indigenous games. It begins by outlining the research design and methodology of Participatory Action Research (PAR), which subscribes to the principles and objectives of community cultural wealth theory. The marginalised knowledge of the excluded communities was utilised in developing the framework with a research population composed of parents, community leaders, learners, and education officials. The multiple forms of knowledge brought by these participants is highly acknowledged and valued in the study.

PAR is the relevant methodology for this study as it views participants as human beings (Barker, 2012:201; Lybeck, 2010:91) and intends to improve their lives. Unlike methodologies which perceive people participating in the research as objects of the research, PAR subscribes to what Denzin (2010:306) refers to as crucial to research, that is, marginalised people must have ownership on the research conducted. Denzin posed eight questions:

1. What research do we want done?
2. Who is it for?
3. What difference will it make?
4. Who will carry it out?
5. How do we want the research done?
6. How will we know it is worthwhile
7. Who will own the research
8. Who will benefit?

These questions are equally posed to indigenous and non-indigenous scholars, and Denzin suggested that they be answered in the affirmative, some arising frequently when conducting preliminary visits. People consulted showed willingness to participate in the research, and one sensed in the discussion what Denzin (2010:300) posited as an acceptable methodology that has tolerance for public debate, in which alternative views are embraced.

The chapter describes how participants of the research gathered data as a collective unit. As de Vos (2005:29) and McGregor (2010:420) argue, the gathering of data is the responsibility of all the participants, and to this end collective work and collaboration were encouraged for all. The data collected was categorised into constructs emerging from each objective, then analysed and interpreted, using critical discourse analysis (CDA). Cohen et al. (2001:298) and Wodak and Meyer (2009:29) view CDA as the construction of meaning in social context, not simply sentences that are disjointed from context. CDA will be more thoroughly unpacked in chapter four.

3.2 PRELIMINARY VISITS

The community cultural wealth theory, as the theoretical framework that underpins the study, indicates that the subaltern communities had abundant wealth of knowledge which traditional research had marginalised but which the framework to be developed needed to tap into. One required support from the community at large, so the researcher made visits to various role players in the community. de Vos (2005:28) argues that a researcher may be mindful of a specific challenge in a community, or the community may speak to the researcher to assist in identifying and formulating the problem. In this situation I was the one who had to go out to the community to generate interest so that they would take part in the development of the framework to enhance learner performance in mathematics by using indigenous games. According to Chilisa (2012:250) and Dodson et al. (2005:953), such initial meetings are important for the researcher, because they build up trust and rapport with the community. Interacting on repeated occasions, with community representatives and discussing research constructs and aims, goals, conducting theme gathering or preliminary focus groups, and including research-trained community representatives on research teams, helped in enhancing the relationship between the researcher and members of focus group.

The recruitment of role players into the research project went well, as there was an overwhelming response. The Traditional Council introduced me to other community members who were actively involved in indigenous music and who were willing to be part of the research project. Chilisa (2012:250) confirms that the process of recruitment of participants is more effective if community leaders assist in persuading others of the benefits of the research idea to the community. At the chosen school, in which the majority of learners were from farms or inhabitants of this rural village, both the teachers and learners showed an interest in being part of the project.

There were still concerns from other sectors that such research initiatives were useful, but after the researcher had received the information he/she would ‘disappear’ and leave the community with no tangible benefit. It was explained to the members that this research was for the benefit of all participants, and that all role players, that is, me, as the initiator of the research project, the community leaders, parents, the school at large and departmental officials would be empowered. All participants would realise that with their knowledge they could contribute significantly to the teaching and learning of mathematics in public schools. The established relationship with all role players would stay for long and be sustainable through continuous interactions.

There is a distinction between traditional methodologies and PAR, as argued by de Vos et al. (2005:428), in that the latter is the continuity of a relationship of continuous involvement. In some instance, the established relationship between participants may fade over time, but it should not be broken altogether. Furthermore, exponents of PAR call for marginalised communities to be treated with special care, unlike traditional researchers who leave the field after their research has been completed and view participants as objects whose emotions and interests are not to be respected.

Dodson and Schmalzbauer (2005:949) showed that acquiring precise life experiences and learning from people's knowledge in marginalised communities can be demanding, because these excluded people tend to have doubts in sharing their world experiences. They suspect that they might be sharing valuable information with 'the enemy'. This was observed at the initial meetings with traditional leaders, as they were hesitant about being involved in the research. In our discussions they raised concerns that academics used to steal their information without acknowledging their inputs: "*le ya re nanarela, le ntano nka tsebo ena ya rona le tsamaye ka yona, ntle le hore le ananele seabo sa rona le diphehiso tsa rona."* (You come swiftly to us, get our information and disappeared from the scene"). This is a clear indication that it is important to gain trust and loyalty of the co-researcher, as some have bad experiences of being used as numbers rather than as people (Lybeck, 2010:91, 96).

These initial meetings were helpful in levelling any power dynamics before the research project was formally started. The community leaders understood that there was much work ahead in making sure that the teaching and learning of problem-solving could be done cooperatively with other stakeholders in the DoE. de Vos et al. (2005: 432) and Dodson et al. (2005:953) confirm that members of the community are empowered to solve their own problems. All Individuals participating in the project learn to acquire self-confidence and obtain the resources needed to function more effectively, so that the project is sustained. They further indicate that integral to PAR is a strong consciousness of societal and local conditions and how they transform the lives of excluded communities.

3.3 RESEARCH PARTICIPANTS

The research study looks into developing a framework of teaching problem-solving using indigenous games and its nature is in line with what Denzin et al. (2011:683) refer to as indigenous, collaborative and community-based. This study was conducted in a setting in which indigenous games are most popular. It worked with participants from indigenous communities whose skills, experiences and knowledge were marginalised. During the reflection sessions some were unaware that there were mathematical concepts in these games, and most parents did not see themselves playing a significant role in enhancing learner performance in problem-solving. In participating, community cultural wealth theory indicates that research participants' minds will be liberated, and they begin to acknowledge the richness of their experiences. Denzin et al. (2011:683) assert that their participation will enable them to use their own experiences and knowledge as vehicles for pushing against structures of racial, sexual and class oppression and exploitation (Barker, 2012:12), and be invited to become agents in their own biographies. Their participation will bring new understandings of critical inquiry into classrooms.

Participants can be in a position to help learners to see how everyday life can link to the teaching of problem-solving. Garegae (2007:47) asserts that classroom involvement of participants is demonstrated in the field of ethno-mathematics, which merges culture, education and problem-solving. Ethno-mathematics focuses on understanding marginalised communities' mathematical ideas in the formal school curriculum subjects. Activities and artefacts in which some of these mathematical concepts are found include house or hut construction, wooden utensils, carpet laying, weaving string bags, baskets, mats, and traditional or indigenous games.

The aim of conducting preliminary visits was to have wider community members represented in the focus group structure. de Vos et al. (2002:421) write that the PAR model concentrates on the engagement and mobilisation of research participants as active agents in the process of constructing knowledge, reaching a shared objective and solving problems. The participants were community members, school community, the DBE at district level, and higher education institutions (see Figure 3.1 below). The composition was informed by the principles of community cultural wealth, which according to Leonard (2008:8) validates the voices of the marginalised communities, as a single voice, abstract idea, or thought cannot explain the experiences of an entire society. Similarly, Ferreira et al. (2012:47) argue that the wealth of social indigenous knowledge that community members possess should be recognised, which implies that the collective voice of the community can help find a sustainable solution to problems faced by the society. Leonard (2008:9) and Ladson-Billing (2012, presentation on Sustainable Rural Learning Ecology) argued that classrooms are microcosms of larger society and schools and society should collaborate in the teaching and learning ecology.

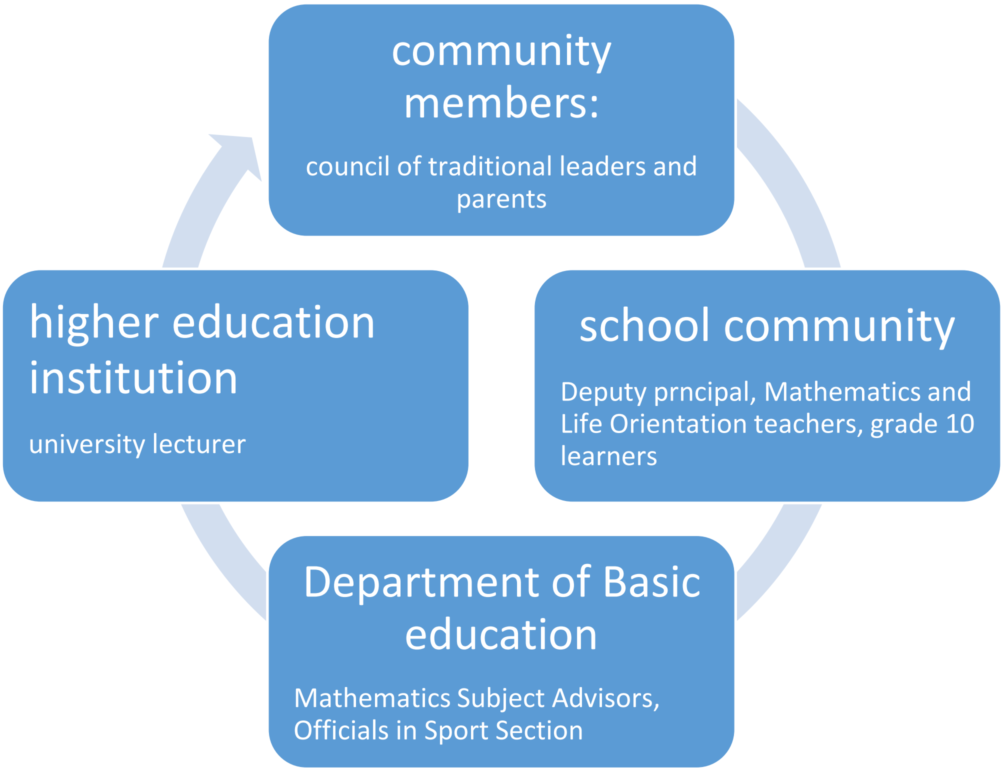


Figure 3.1: The structural nature of the research participants

Each division in Figure 3.1 (above) will be unpacked and the interactions between them demonstrated in the next paragraphs.

3.3.1 School community

Within this study, the school community division comprises the participants, deputy principal from the school management team (SMT), one Mathematics teacher, one Life Orientation (LO) teacher and two learners from grade 10 classes. The two learners attended scheduled meetings on behalf of all grade 10 learners, but during the class interaction all grade 10 learners were involved.

The school was situated in the rural area of QwaQwa, the majority of learners coming from neighbouring farms, in the area of Harrismith, van Reenen. Other learners were staying in the vicinity of Tsheseng village in which the school was located. There was a diversity of culture in this school, but the most spoken languages were isiZulu and Sesotho. Through interaction with learners it was discovered that most were familiar with various indigenous games, particularly in the home environment. Initially, it was seen as a potential threat to the framework that most of the learners might not know how to play the games, as this would have required time being devoted to teaching the games instead of actual mathematical content. Another challenge was that they used different languages to refer to an indigenous game, for example *Morabaraba* in Sesotho and *umlabalaba* in isiZulu.

The invitation was extended to the school to take part in the research towards developing a framework to enhance problem-solving skills in the teaching of grade 10 mathematics class using indigenous games. The SMT accepted the call but invited me to make a presentation to other staff members. There followed support from the principal, deputy principal, HoD of Natural Sciences, and the Mathematics teacher. They felt that this would boost learner performance in Mathematics (see 4.6 for details) and they suggested that such a framework could also be implemented for grade 11 and 12 Mathematics learners. After a lengthy debate it was agreed that as the grade 10 learners had started with the new curriculum (Curriculum and Assessment Policy Statement, CAPS) it would be ideal if the focus were initially on grade 10 learners then gradually trialled and tested as learners progressed to the next grades. Grade 10 learners were at the entry point of FET schools, and had three years before they progressed to higher education. This would help to redesign and realign the framework before learners complete the grade 12.

Within the school level, the focus group consisted of the deputy principal, grade 10 Mathematics educator, one LO educator, and two grade 10 Mathematics learners. The principal, HoDs for Mathematics and LO and two classes of grade 10 learners were part of the larger group.

3.3.1.1 Criteria for inclusion of school community in the focus group

The meeting suggested that criteria of the focus group members include alignment with PAR values, portrayed by Jordan (2003:189) as democratic engagement, transparency and openness, and well-defined conviction to issues concerned with growth and sustainability. Participants should have knowledge of Mathematics and Life Orientation (LO), and also cultural matters, such as playing of indigenous games. LO is important in this research as its content areas, such as physical education, includes indigenous games. LO teachers were important, in that, according to the CAPS document, indigenous games should be addressed from grade 10 up to grade 12. It is also stated in the *Government Gazette* (2011:58) that LO is an inter-disciplinary subject that integrates knowledge, values, skills and processes entrenched in several disciplines. Those acquired in LO can be used to enhance learner performance in Mathematics.

Since the introduction of the new curriculum in schools, learners have performed better in LO than in Mathematics. The HoD of LO commented that: "*eeh, e se ka moralo ona o tla etswa wa ho sebedisa di-indigenous games ho rutweng mmetse, o ka etsa hore results tsa mmetse di be ntle, jwaloka tsa Life Orientation*." (Translated into English: with the framework to be developed using indigenous games to teach problem-solving, the developed framework will contribute in making the Mathematics results as good as the Life Orientation results).

The school community agreed that the deputy principal, one Mathematics teacher, one LO teacher and two grade 10 learners would be members of the focus group. In terms of learner component it would be voluntary, but with both genders and both isiZulu and Sesotho languages represented. After the presentation to grade 10 classes two learners agreed to be part of the focus group. PAR is democratic and extols openness, as pointed out by Chilisa et al. (2010:624). In traditional research, children were not consulted but spoken for by others. New critical research is exemplifying the international contexts in which children's voices and opinions are foregrounded in research on gendered violence. The school community division therefore understood that it should not leave out the learner component from the focus group. Importantly, the study was designed in such a way that learners would receive adequate attention in the teaching and learning of

Mathematics. As Leonard (2008:5) states, societies had created social structures that exclude learners from the decision-making process, and improvements have not always been of benefit to them. This research was aimed at changing this.

3.3.1.2 Profile of focus group members

The deputy principal had 10 years' experience in teaching Mathematics and English. He had taught Mathematics from grade 10 to 12 in previous schools before joining the present one. In 2001 he was promoted to HoD in charge of Mathematics, Physical Sciences and Life Sciences. At the former schools, results in Natural Sciences had improved significantly. He was highly committed to community services, and in the former school had encouraged parents and the community at large to support teaching and learning in school. Because of his dedication and commitment he was selected for a Teacher National Award, as 'teacher of the year'. At the time of the study he was deputy principal at the school in which the research was being conducted. It was anticipated that his expertise and knowledge in the field of mathematics and his involvement in indigenous games would be of great benefit to the research study. He personally believed his involvement in the research study would make him a better person in the teaching of Mathematics. In our school meetings we continually encourage parents or guardians to take part in the education of their children, and with this research study more emphasis would be placed in parents becoming involved in the education of their children: "*Ke thaba hobane,batswadi batla kgona ho nka karolo e kgolwanyana thutong ya bana ba bona*" (happy that parents will take a major role in the education of their children).

The Mathematics teacher had started teaching the subject in school six years previously. She had appropriate qualifications and taught Mathematics in grades 10 to 12. On a weekly basis she was offering extra classes, so as to revisit aspects covered during the course of the week. Currently, she was teaching Mathematics to all the grade 10 learners and indicated that she was delighted to be part of this group, as she felt the framework would help in the teaching and learning of Mathematics. Most of the learners did not see the connection of mathematics and daily life activities such as indigenous games. There were many mathematical concepts embedded in these indigenous games, such as patterns and proportions, that would promote interest among learners and they would be encouraged to engage in higher order activities. Problem-solving skills and critical thinking, motor coordination skills and eyesight would be enhanced. She pointed out to the meeting that on this morning learners could not work out the properties of a kite or even say what a kite was. However, having been told that this was similar to the toy they flew when playing at home they quickly remembered its characteristics and were able to define it.

The Life Orientation teacher was keen to be part of the research and did not hesitate to join the focus group. He attended all research gatherings/meetings and shared much wealth of knowledge in them. The HoD of LO attended some research gatherings, acknowledging that LO would no longer be at the periphery in the main curriculum of the schools. This supported the view of Wane (2009:171) that decolonising the curriculum involves discarding the compartmentalisation of knowledge with regard to disciplines such as biology, psychology, and philosophy, as all knowledge forms are interrelated and overlapping. The involvement of the LO teacher was very important in this research, as the DBE (2011:8) has conceptualised the learning area or subject as one of the four fundamental subjects essential for the National Curriculum Statement (NCS), making it obligatory for all learners in grades 10 to 12. The DBE (2011:8) further indicates that LO focus on skills, knowledge, and values about the self, the environment, responsible citizenship, a healthy and positive life, social engagement, recreation and physical activity, careers and career choices. These skills and forms of knowledge addressed in LO are also utilised in the teaching and learning of Mathematics, and in solving mathematical problems. This idea of integrating other subjects with mathematics helped to bridge a longstanding gap. LO gives learners an opportunity to participate in the development and practice of a multiplicity of life skills to solve problems, to make well-informed decisions and choices and to take right actions to live importantly and successfully in a speedily changing society. It not only stresses on knowledge but again it emphasises the importance of the application of skills and values to real-life situations, participation in physical activity, community organisations and initiatives (DBE, 2011:8).

The research links the LO content area of physical education under the sub-topics: skills in playground and/ or community and/or indigenous games, and skills in traditional and/or non-traditional sport, with the teaching of mathematical concepts and skills. Hence, the research is on developing the framework in teaching problem-solving skills in grade 10 Mathematics classroom using indigenous games. The notion of integrating Mathematics with subjects such as LO is supported by Garegae (2007:48), as the aim of research in the ethno-mathematics programme (Vilela,2010:350) is to unearth, interpret and apply mathematical ideas from other cultures into the school curriculum, giving a human face to classroom mathematics. In addition, Jansen (in a presentation addressing community members at UFS, Qwaqwa campus, 2012) argues that the type of education delivered in classroom must have a human factor and the community members be part of it. This is in line with Moana (2010:1), who posits as acceptable practice of research a recognition of community members as experts on the phenomenon of interest, and allowing the meaning and usefulness of research to be informed by everyday experience and understanding.

The arguments raised above demonstrate that the use of PAR is appropriate for the study, as it locates it within the 'third-space', conceptualised by Chilisa (2012:25) as the broader community in research discourses, and a space that recognises and essentialises the views of subaltern people, whose voices were previously excluded on the basis of gender, race, ethnicity, dis/ability, health, socio-economic status, sexual orientation and age. The study aims to bridge the gap that existed for many years, whereby the teaching of mathematics was divorced from day-to-day activities of the subaltern children. This approach of bringing mathematics closer to the environment of the child is emphasised by Vankúš (2008:104), as a demand for development not only of pupils' knowledge but also of all key competencies. Teaching methods should fulfil these needs, in particular using the method of teaching through games. Here Vankúš is in line with Wane (2009:174), who articulated as the responsibility of indigenous scholars the reconstruction of teaching methods to ensure that they address the needs of the indigenous communities they are serving as social workers.

As the study explores the usage of indigenous games in teaching mathematics, the observations of the participants during the contact sessions concurred with Vankúš (2008:104), namely that games embrace educational goals and bring to pupils happiness and pleasure. Moreover, Vankúš (2008:105) asserts that games are an asset in the area of learner' attitudes towards problem solving and the process of teaching and games are useful tools for better construction of mathematical knowledge. Therefore, there is need to find new games for realisation of numerous educational goals and to inquire into their efficiency and influences on the teaching process.

The two grade 10 learners had attended the previous grades at other schools, so this was their first year at this school. They contributed well to the discussion, and believed the use of indigenous games would bring the home environment to the classroom setting, which was too formal and intimidating at times. This continuation of home education into the school curriculum concurs with what Rowlands and Carson (2002:80) termed 'ethno-mathematics', in that it promotes a pedagogy that relies on the knowledge that the learner brings to the classroom from the learner's everyday experiences. Garegae (2007:55) and Pramling-Samuelsson (2008:629, 630) posit that the researcher(s) should know that children learn by listening, observing, and doing. They should be patient and be involved, observing as well as taking part in the process. The researcher should not ask questions as if he or she is cross-examining someone at court. Knowledge exchange takes place in an atmosphere which by Western standards would be termed 'informal'.

One learner said that "*uko sebenzisa la ma –games izo susa uzwalo loku funda imetse*", which means using these indigenous games would help alleviate the fear of doing mathematics. They both mentioned that the use of indigenous games could help one enjoy it. Wilbert (2008:22) reiterated that mathematics anxiety can be devastating in the classroom, at home and in society. Most learners believed that mathematics anxiety is caused by their teachers. Also, Wilbert (2008)'s study exposed that at all grade levels learners do experience mathematics anxiety. Wilbert (2008:73) indicated that mathematics anxiety is a cause of poor and lagging mathematics achievement. Thus, Bush (2005;3,4,8) argued that teachers need to be creative in their teaching, that is make problem-solving an enjoyable activity and use instructional methodologies that appeal to learners' interest, and meaningful methods that make mathematical content sensible to them.

Learners believe that there are many resources to consult when they encounter difficulties in mathematics problem-solving, for example study buddies from class, older siblings and parents whose knowledge has hitherto been marginalised in the mainstream. The integration of mathematics with indigenous games encourages learners to keep mathematics reflective journals when on the playing fields. This approach to teaching of mathematics is not too formal, but rather touches on an informal way of life, that is playing whilst learning.

Wane (2009:160) sees the connection between home environment and the teaching and learning of problem-solving as important in drawing on the interest of learners. As a result, learners will learn problem-solving with understanding, not needing to memorise mathematical content or formulae. Learners' participation in the study would allow learning to focus on their terrain and lived experiences.

Community members included the parents and traditional leaders as part of the research team, as they possessed the knowledge and skills that learners were brought up with. The traditional leaders are seen as the custodians of cultural beliefs, and values such as mutual sharing. As Wane (2009:160;170) found, in most thriving community schools there is an emphasis on local relevance and education relevant to local communities has been successful. The community becomes part of schooling because the learners identify with what is being taught and become involved in learning. Indigenous education is specific to learners as well as the need for the participation of community members who will bridge the gaps that textbooks leave.

Ferreira et al. (2012:48) view the community as a group of people or families residing in the same geographical area under common laws. This group is typically described by friendly groups, joint sharing, and common interest, cultural beliefs, values, heritage and religion. The community members possess certain skills and values that are greatly needed in the teaching and learning of mathematics. From this perspective of how community members cooperate, one can conclude that community members work in a conducive environment, which can also be translated into the teaching and learning of mathematics. These are delicate skills which parents possess, as their children were brought up and nurtured in a healthy environment. Leonard (2008:153-154) argues that a school can have few resources, be under-funded and have outdated facilities but still not regard these as barriers to learning. Of importance, rather, is for schools to create caring, and safe learning environments, similar to what they are exposed to at home. The environment should be culture-rich, academically rigorous, and family-strong. Leonard (2008:165) suggests that creating and maintaining a legacy of achievement in Mathematics requires a major overhaul of the current educational system, and substantial investment of capital, that is, human, cultural and social. Yosso (2005:78), and Leonard (2008:165) agrees that these forms of capital are possessed by subaltern communities, and the research study seeks to work with these marginalised communities such that they are utilised in the teaching and learning of problem-solving.

It is evident that the knowledge and skills possessed by excluded communities linked well with certain teaching strategies used by teachers in the classroom, which Leonard (2008:155), quoting Haberman (1991), termed the ‘pedagogy of poverty’. This is characterised by the strategies of collaborative learning, inquiry-based teaching, and hands-on activities or project-based learning, embraced by the different forms of capital of community cultural wealth theory. The social capital demonstrates the use of cooperative learning skills, whilst navigational capital has a direct link with inquiry-based instruction. With no guidance provided one can navigate through and arrive at a convincing conclusion, with project-based learning linked to cultural capital.

The traditional leaders were targeted for this research because they are the main custodians of culture in communities. Community leaders possess extensive knowledge of indigenous games, which they are expected to share with other members in a group. Furthermore, Garegae (2007:53) argues that the seniority reputational research approach acknowledges that the elderly have words of wisdom. Africans believe that elderly people are more knowledgeable than any other person in the community. For instance, Akinpelu (1988:2) writes that the words of the elderly are both witty and pregnant with meaning. Such words of wisdom find expression in proverbs, incantations, or oracular and prophetic sayings. They are treated with deep respect because old age is generally held as synonymous with wisdom and experience, and the elderly are regarded as clever, and familiar with their historical background. They put their teaching in context and when they pass on wisdom to the younger generation the message will be metaphorical, with the information embedded in the story and the listeners having to glean the message therein.

In addition to respecting the elderly as sole bearers of traditional knowledge, a seniority reputational research approach recognises the African way of knowledge retrieval and knowledge transmitting throughout generations, The oral tradition of narratives is an advantage of seniority or age-specific reputational approach, considered better than the other traditional methods of researching ethno-mathematics because it produces data that is contextualised. This information will tell the ‘whole’ story about the artefact, such that when it is used in the classroom, learners will come to know its origins and hence appreciate the part played by problem-solving in the developments in different cultures. Also, contextualised information about the patterns on the artefacts would consolidate the idea that problem-solving is a socially constructed type of knowledge and thus when teachers employ the theory of constructivism in classrooms the learners would be responsive.

This is indication that traditional leaders will play a significant role in concretising the mathematical content which learners perceive as abstract. Learners will observe patterns that can be generalised into mathematical conjectures, unlike memorising the general conclusions given in various textbooks.

Inclusion of parents validates Vilela’s (2010:347) view that mathematics is a social practice. Through interaction with the community leaders I was introduced to parents who were passionate about the indigenous games and music. This concurs with Garegae’s (2007:54) view that they also know people who value culture or have preserved cultural practices such as religious rituals. They were happy to be part of the focus group and indicated that they had formed an indigenous music group, whereby they participated in various competitions under the auspices of the provincial Department of Sports and Culture. Parents had another aim to play indigenous music to teach their children culture and to keep their bodies fit.

Such playing of indigenous music can be useful in the teaching of mathematics, as noted by the Rwandan Ministry of Education in their national curriculum guide (2006:81). Mathematics is a human activity in which all are involved from birth. In daily lives there are patterns in which people are engaged, such as musical patterns, words and phrases, and ways people behave. Parents' knowledge is incorporated into the mainstream of education, as they are prepared to contribute to making the framework of teaching problem solving using indigenous games a success in improving Mathematics results in schools. They noted with amusement that the cultural activities they performed made a contribution to Mathematics performance.

Parents admired as useful and relevant things that were done in the past or at home, in the teaching of mathematics in an informal setting: "*rona ba tsamaisi ba dipapadi tsena ha re tsebe mmetse o teng kahare"* (as organisers of these games, we are not aware of the mathematics addressed by these games). Vilela (2010:353) found that parents thought that the street mathematics they possessed could be used in everyday life and could find room in the school curriculum. Now they understood that the formerly excluded knowledge they possessed was highly valued in formal education. Similarly, Garegae (2007:48) quoting Gerdes (1985) sees that main purpose of research in the ethno-mathematical programme as being to improve the school mathematics teaching. Through ethno-mathematics, mathematics education can value the scientific roots embedded in students' culture (Garegae, 2007:48-49).

The linguistic element of Yosso (2005:78?) and the Rwandan Ministry of Education (2006:83) validates the contribution of parents by citing that talking with children and listening to what they have to say, by telling stories, reading rhymes and singing songs with numbers in them is an excellent way of communicating through the use of mathematics. Murray and Tillet (2011:172) regard parents as operating from a rich environment of ethno-mathematical perspectives, within which mathematical ideas can be observed. These mathematical ideas or concepts are the ones which are taught formally by Western-style mathematicians.

The Department of Basic Education district level officials were based within Thabo Mofutsanyana education District. In terms of the number of schools and learners it had the largest numbers in the Free State Province. The DBE included Curriculum Directorates and (in full) SYRAC directorates. Officials were responsible for curriculum implementation for all subjects and in various grades. For the purpose of the study, mathematics subject advisors were consulted in the curriculum section to take part in the development of the framework of teaching mathematics using indigenous games. In SYRAC directorates, only officials responsible for the Sport section were consulted. In the Sport section, only officials responsible for indigenous games and indigenous music accepted invitations to be part of the focus group. The four departmental officials from Curriculum and Sport sections welcomed being part of the research study. One official in the Sport section said that despite working in different sections of the DoE "re *mona molemong ngwana*", (we are all here for the benefit of the child).

Two **Mathematics subject advisors** were part of the team, one in charge of Mathematics in General education and training (GET) and one for the Further Education and Training band (FET). Both made school visits in supporting Mathematics educators in content, assessment and classroom management. Their experience and expertise would help strengthen the framework be developed, and after its being operationalised they would be in position to popularise the model to be implemented in schools that are not performing well in Mathematics. Various mathematical concepts are addressed by indigenous games, such as addition, counting, knowing types of numbers, knowing number names, remainders, and mental calculations, as well as skills such as eye coordination.

Teaching through use of indigenous games does not mean that teachers will be taking the whole semester without tackling the 'real' mathematics, claiming that the second semester he or she will handle work as per work schedule. After each activity on indigenous games there must be reflection, giving learners a chance to looks at what transpired in the playing of the games or what they have learnt in the play. This will be an ideal chance for learning to share with their peers the underlying mathematical principles applied in the games. Therefore, teachers need to make thorough lesson preparations, or the lessons might fail and learners not achieve the expected outcomes. They need to plan in advance and find out how the physically challenged learners will be engaged in the games.

Two **officials from the Sport section** participated, one in charge of indigenous games and the other indigenous music. They conducted workshops based on their expertise in the respective fields. The research study will encourage the cooperation between different departmental officials within the district. In that their programmes will be coordinated, the activities in the district will be conducted in such a way that there are minimal clashes for sporting activities and curriculum matters.

3.3.2 The higher education sector

The core responsibilities of higher institutions are teaching and learning, community engagement and research. As the Faculty of Education is based in a rural learning ecology it is the responsibility of all academic staff members to provide support to public schools in the teaching and learning of mathematics. The principles of PAR illustrate that as we enter into partnerships with school communities we work together as colleagues and equal partners on problems that schools face, not as authorities who are going to give instructions and orders. This is confirmed by Ferreira and Ebersohn (2012:50), that:

> … based on current South African policy and legislation, community engagement forms part of the core priorities of higher education institutions, together with research, and teaching and learning. Within this context, community engagement involves the actions of tertiary institutions in establishing and maintaining supportive relationships and links with the community. The main purpose of the relationship is to benefit partners, that is, the community and tertiary institution.

The emphasis is increasingly placed on integrating community engagement with research and teaching and learning activities, instead of approaching community engagement as a free-standing entity.

As the researcher in this study I viewed myself as contributing in public schools by teaching problem-solving skills in Mathematics using indigenous games. As Ferreira, quoting Bender et al. (2006:11) argue, community involvement is seen as the blend and integration of teaching and learning. As the researcher is also researching towards a higher degree, this study combines the three core responsibilities of the academic staff at higher institution. The practices researched together with participants were to be tried and tested in the grade 10 mathematics classroom.

As stated in South African policy and legislation on Higher education (Green Paper on Higher Education Transformation of 1996, Higher Education Act of 1997), it is essential that higher education and training sectors give support to communities, such as public schools in the basic education sector, which is the feeder for higher education sector. It is imperative that there be a collaborative partnership between the two sectors, as recommended by Ferreira et al. (2012:52), as a traditional view of higher education institution as providing a service to communities that were merely on the receiving end in a subordinate role no longer stands. The higher education sectors should focus on forming partnerships with the community, including schools, where needs are jointly identified, plans of action jointly crafted and progress of involvement planned, taking action to address identified difficulties.

The study embarked upon a way of fulfilling the core responsibilities of academic staff members in higher institutions, in particular that of developing communities to sustain themselves. Community engagement is effected through integrating research and teaching and learning. The researcher was part of a focus group engaged in teaching problem-solving skills in a grade 10 Mathematics classroom using indigenous games.

3.4 PLAN OF ACTION

The focus group mapped the way forward, that is, it agreed on activities to be implemented in teaching problem-solving using indigenous games. This showed that the focus group owned the plan of action. Denzin (2010:307) writes that if the oppressed, marginalised and silenced groups take charge of their own lives and humanity then the project of emancipation can become a reality. Chilisa (2012) asserts that the community action plan should show the goal of the research project, the objectives to be achieved, strategies to achieve the objectives, timeframe, persons responsible, resources required, and monitoring and evaluation framework. In the same vein, the target group agreed to follow the format as suggested by Chilisa. The participants' activities are outlined in the table below.

Table 3.1: Plan of action

Activities	Person responsible	Monitoring	Evaluation	Timeframe
Phase 1: Playing of *Morabaraba* game	Two pairs of learners and parents volunteered to play the game	Mathematics Teachers and other research participants	Observation sheet used to record observations	45 minutes
Phase 2: Reflection on the lesson learnt from playing the game	Grade 10 class and the focus group	Mathematics Teachers and other research participants	Discussion on mathematics concepts/skills/knowledge identified from the game	30 minutes
Phase 3: Presentation of the lesson	Subject teachers: The Researcher or the grade 10 teachers or Mathematics subject advisor	Mathematics Teachers and other research participants	Lesson plan is crafted based on the identified mathematical concept	60 minutes
Phase 4: reflection on the lesson presented	Grade 10 learners and focus group participants	Mathematics Teachers and other research participants	The conclusions is done by the presenter, consolidating all the feedbacks given	30 minutes
Phase 5 Assessment of the lesson	Teacher assessment, peer-assessment, and parent assessment	Teachers , Subject experts, and parents monitor the completion of activities	Give feedback on activities done during class, after class or at home	45 minutes

As illustrated in the plan, each activity was to be executed in five phases, coherently structured in a manner that diverse dimensions or facets would be implied. These facets are viewed by Godino, Batanero and Font (2007:1) as ontological, that is looking at types and nature of objects in the teaching and learning of mathematics; epistemological and cognitive facets, looking at how knowledge is accessed when doing mathematical activities and how mental processes operate during the teaching and learning of mathematical concepts;

and socio-cultural and instructional facets that deal with the teaching and learning of mathematics in school institutions and society. This socio-cultural facet concurs with the theory of community cultural wealth, which is guiding the study, as it conducts the teaching and learning of mathematics in collaboration with stakeholders. This is in line with Denzin's (2010:304) view of indigenous theory on the academy and its classrooms as blessed spaces in which indigenous and non-indigenous scholars network, share experiences, take risks, investigate alternative modes of interpretation, partake in a mutual agenda, and come together in a spirit of courage, love, and shared community. The community of cultural wealth theory, meanwhile, demonstrates that social capital is manifested in a network of people and community resources, learning from others and collaborating throughout the interaction. The aspirational capital is noticeable in what Denzin (2010:304) reiterated and realised when hopes for the future are maintained.

Godino et al. (2007:2, 7) and Font, Godino and Contreras (2008:157) write that these facets are depicted in an onto-semiotic approach, viewed as a home-grown model that assumes a systemic and interdisciplinary perspective, in the sense that various fields are engaged, such as psychology, pedagogy, sociology and epistemology, each with a focus on the teaching and learning of mathematics. The key roles shown by onto-semiotic approach are to construct and communicate mathematical knowledge to various parties, such as school community members and the society. Specifically, Godino et al. (2007:7) refer to ecological suitability, whereby teaching and learning of mathematics processes fit the educational project.

The study demonstrates interdisciplinary notions by including LO subjects, Mathematics, Sport section personnel, traditional leaders, parents and learners. This further demonstrates what Wane (2009:169) regards as the inclusion of indigenous voices in the main curriculum, signifying that the positivist forms of knowing, educating, and doing science and research are challenged (Denzin, 2010:300).

3.4.1 Phase One: Playing of the indigenous game

In Phase One the chosen group members play indigenous games scheduled for that day. As Sancar-Tokmak (2013:3-4) stated, this phase can follow a juxtaposition model, with teachers and parents watching the children during their free play and noting key points that can be used in designing a mathematics lesson. The group members engaged in the indigenous game can be learners from the school community or local community. As some are involved in the playing of the indigenous game others observe how it is played and record their observations on a specially designed sheet. Vankúš (2008:105) states that learners' active involvement during the games is important for better comprehension of mathematical knowledge embedded within them. This phase is conducted in such a way that the actual play is demonstrated, important observations are noted, and children are encouraged to explore new mathematical ideas and skills (Sancar-Tokmak, 2013:3-4; Vankúš, 2008:107). At the end of the phase the teachers arrange activities to teach mathematical concepts or skills noted during play, or concepts identified by the learners, teachers and parents.

3.4.2 Phase Two: Reflection on the lesson learnt from playing the game

At the end of game, all members share their observations and attitudes towards the indigenous game played. This is when teachers note teachable moments or mathematical concepts which became apparent during the children's free play or parents' play. Sancar-Tokmak (2013:3-4); Vankus(2008:107) called this the 'integration model', when contextual issues demonstrated during play of indigenous game are linked to the mathematical knowledge However, in the study the discussion was free for all participants to share observations. As Godino, Batanero and Font (2007:127) indicated, the discussions touch on various facets, such as socio-cultural skills demonstrated by players, and epistemological matters, on how knowledge is accessed through play, not to a particular group. All members in a group are on the same level during the discussion, although at the onset of this project some members felt that the reflection processes was targeting learners only. Such statements as: "*ha re utlweng bana pele hore bona ba reng"*, (let's hear first from learners what are their comments) helped all agree that anyone was free to lead the discussions. Denzin (2010:298) argues that if the notion of learners first is emphasised then the indigenous persons are excluded from discussions. Denzin (2010:298) propagates the decolonisation project, which supported the idea that all participants are on an equal level in terms of knowledge, and everyone is free to lead the discussions. This emphasised the importance of social wealth, sharing of which means participants can learn from one another.

During the reflection stage, the discussions are not limited to state of the play or likelihood of winning, but include children's conceptual understanding of intertwining the game-structure and game-play with the mathematical concepts (Booker, 2004:18).

3.4.3 Phase Three: Presentation of the lesson

The teacher prepares the presentation of the lesson in such a way that contextualisation of mathematics is taken into account. The teacher chooses the mathematical content from the ones which were suggested in Phase Two, as the main topic for the day, as this will allow spontaneous integration of topics which were mentioned in before. The class activities are designed in such a way that learners can work together in finding solutions to the activities assigned to them. Vankus (2008:106) and Alexander and James (2005:16) stated that placing the indigenous games as the context in presenting the mathematics lesson enhances learners' interest in mathematics. Also, they are motivated and encouraged to take part in class discussion, and come up with innovative, creative ideas. Hence, there is greater likelihood of success in Mathematics.

3.4.4 Phase Four: reflection on the lesson presented

Learners in small groups are given a chance to share with the whole class what they have learnt in the activities. Reflections and feedback on the onto-semiotic approach provides dual dimensions of extensive-intensive facet, wherein several observations are made to reach substantiated conclusions (extensive case) and vice versa (that is, intensive cases). These are key in the construction and application of mathematical knowledge (Godino et al., 2007:131). These are important mathematical practices which provide learners with the means to develop mathematical arguments and justifications that lead to robust concepts and thinking (Booker, 2004:18; Alexander & James, 2005:16). The teachers and parents play a key role in guiding the process of reflection, and also provide input on issues that are confusing to learners, the exciting moments during this session and when learners find ways to take the centre stage in the learning and teaching, making excellent contributions. In essence, all the parties need to be prepared for the lesson presentations, not only the teacher. This concurs with the claim of Alexander and James (2005:15) and Booker (2004:20), that this session of feedback encourages learners to work independently and enhances instructional group teaching. In this way instructional games induce learners to make sense of their ideas and the interpretations of others, and, significantly for this study, they are able to conceptualise the definition of mathematics, that "… it is human activity practised by all cultures…" (DoE, 2003:9; DBE, 2011:9; Vilela, 2010:349). It is a social process of sense-making, understanding, rather than a set of rules handed down from an authority on high (Booker, 2004:20). By the same token, learners discovered the 'frustum structure' by using the geometric figures in *morabaraba* (board game, which has a high level of mathematics covered at tertiary level).

3.4.5 Phase Five: Assessment

It is understood that assessment is conducted throughout the phases, as a way to of integrating it with teaching and assessment (DoE, 2003:63, 64). Booker (2004:17) regards it as essential that the effectiveness of the game in achieving the educational and mathematical outcomes be assessed. Assessment tasks are performed as a way of reinforcing the understanding of certain mathematical concepts.

3.5 RESEARCH METHODOLOGY

Participatory Action Research (PAR) was the appropriate method for the study, because it agrees with the principles of cultural wealth theory. It indicates that the subaltern communities possess wealth of knowledge and experiences which are not fully utilised. Dodson et al. (2005:953) assert that PAR can be used to promote research interaction with marginalised people, who willingly communicate precise information about their lives and viewpoints. Participatory methods are not just an expedient way of gathering useful data but also introduce opinions of community members into the fabric of the inquiry. This was evident with the traditional leaders, when they shared the knowledge on types of indigenous games and demonstrated how mathematical concepts could be applied to such games. Their demonstration was in line with the view held by Rowlands and Carson (2002:84) of ethno-mathematics as problem-solving in the sense of cultural groups and people creating their own mathematical content out of their everyday lives, rather than as a formal academic discipline. They even referred me to other people who could be part of the focus group.

As indicated above, the traditional leaders thought that the research was going to wipe out their valuable knowledge or exploit them. With the use of PAR they understood that the research was there to empower them to act effectively in their own interest. During the initial meetings the researcher explained to them that it was not a mission to steal their valuable information but rather that the information would benefit children who were struggling to perform well in problem-solving. They should view the research as a learning process characterised by flexibility, sustainability and total capacity-building approach. Once the community members could see themselves as researchers they would regain power over knowledge (Barher, 2012:66; de Vos et al., 2002:421). They need to perceive the research as an activity that will last for some years to come and will benefit them and their children.

PAR is chosen as an appropriate method for this study because it discourages domination, alienation and narrowness of knowledge. The research participants were seen as human beings, not as objects without feelings or ideas, to be manipulated in a controlled environment. Participants' ideas and voices were acknowledged and they feel free to make useful contributions to the deliberations (McGregor, 2010:423; Lybeck, 2010:91).

PAR features well in the third-space, seen by Chilisa (2012:25) as the space in which positivist methodologies are contested and declared invalid because they are based on a culture that has been made static. This complements Denzin's (2010:300-301) view of grounded and indigenous theories, that the spaces encompassed are constantly created, never bedrock solid but always nuanced, and possibly threatening. Creeping around the corners are countervailing conservative forces that seek to unsettle any agenda of social justice. Denzin (2010:302) regards grounded theory as useful in detecting unrest and unruliness, where other research paradigms perceive patterns, processes, and interconnections in research they undertake. This focus on disorder and illusion is planned to illustrate the arbitrary and unjust, the unfair practices that function in daily life.

For Pretty (1995:1250), one problem with other paradigms is that their absolutist position appears to exclude other methodologies, such as PAR, which allows pluralistic ways of thinking about the world. Different views of participants are acknowledged, as each possesses a different epistemology about the truth (McGregor, 2010:419,423; Sudersan, 1998:256). Pretty (1995:1250-1251) elucidates key principles that differentiate other methodologies to PAR approach, one of which is that any attempt to delineate concepts accurately is fundamentally defective. In the section of definition of concepts, various views on the concepts were considered, and thereafter common understanding was shown, even different arguments on the concept shown. The study conforms to the principles of PAR, that each individual has different values that must be acknowledged.

According to Chilisa (2012:40), Naser et al. (2008:199) and Pretty (1995:1250), PAR is open to various interpretations, but knowledge and comprehension are socially fabricated. What each of us knows and have faith in is a function of our own unique contexts and pasts, thus, there is no definite or correct understanding. In identifying the challenges of problem-solving the study considered diverse problems as crucial and was cautious not to force solutions upon the research participants. Rather, it allowed for disagreement that would finally lead to common understanding.

Again, the viewpoints of participants were tabled and discussed so as to reach a common understanding on how to teach problem-solving using indigenous games. At some stage parents did not see what significant roles they would bring to the group. Eventually, the whole group would realise that each had a meaningful role to play in the teaching and learning of problem-solving.

For Pretty (1995:1251), PAR posits that the resolution of one challenge certainly directs to another "problem-situation," as problems are prevalent. This matches well with the two objectives of the study, which anticipated the possible threats of the study and also suggests the possible solutions to circumvent the problems anticipated. The participants found that some problems emanating from the study could be pursued in future research. Pretty maintains that in a multifaceted and changing world there will always be ambiguities and modern interpretations.

These arguments justified the choice of PAR as appropriate to the study. All participants are empowered as they learn from others, and learn to acknowledge and validate the marginalised knowledge of the others.

3.6 DATA COLLECTION PROCEDURES AND ETHICAL STANDARDS

After the preliminary meetings, in which different stakeholders were extensively consulted on the use of indigenous games in teaching problem-solving in grade 10 mathematics classroom, the focus group members and the two Mathematics grade 10 classes were invited to the meeting, so as to outline the procedure to be followed when conducting the research. All participants were informed about the ethical procedures of the study. The study was structured to avoid any deleterious factors that might affect participants. Permission was granted by the Provincial Department of Basic Education that the study could be conducted in the school concerned. As the traditional council members were present they also gave permission for the study to be conducted in their area. The letter of permission from the Free State Department of Education was read to all the participants.

Community members were encouraged to support for the endeavour for the benefit of their children.

All members were given consent forms to sign, and learners or minors whose parents were not in the meeting were asked to take the forms home for the parents or guardians to sign on their behalf. Consent forms were written in two languages, English and Sesotho, and learners speaking IsiZulu were also catered for. These forms were explained to the participants, who were made aware that if they did not wish to continue with the study at any time they were free to withdraw.

In the discussion it was agreed that all events and activities would be recorded and video-taped using tape-recorder and video-camera. The extract of these texts was to be analysed (chapter four) using critical discourse analysis (CDA). These spoken or written words are seen as primary data (Ryan, 2006:21), which and are used to obtain deeper meaning and repertoires of each participant (Francis, 2012:18; Mahlomaholo, 2012:51).

3.7 DATA ANALYSIS

In analysing the generated primary data, Van Dijk's (2003) CDA was used. According to Hakimeh Saghaye-Biria (2012:509) and Van Dijk (2003:352, 2009:63), this is a style of discourse analysis that reviews the way social power exploitation, dominance, and inequality are endorsed, duplicated and contested by text and talk in the social and political context. As such, this approach seeks to identify instances of discursive injustice in text and talk and denotes a form of resistance to immoral and unjust social power relations. In addition, Terry Locke (2004:2) and Wodak and Meyer (2009:3) contend that CDA is seen as a political intervention with its own socially transformative agenda, throwing light on discursive aspects of societal disparities and inequalities, which deprive learners of access in understanding problem-solving. Even the resources used by teachers and methods of teaching made it difficult for learners to comprehend mathematical content. Voluminous of the prejudicial attitudes and beliefs of the public are originated from their interpretation of privileged discourse in the mode of media messages, textbooks, corporate discourse, and, particularly, political discourse (Hakimeh Saghaye-Biria, 2012:511; Jager & Maier, 2009:34).

Wodak and Meyer (2009:3) maintain that CDA entails systematic analyses, self-reflection at every point of one's research and distance from the data being explored. Also, it encourages new questions, new responses and new thoughts. Hence, the theoretical and conceptual frameworks and research methodology of this study matches with the critical discourse analysis. CDA was an suitable tool for deconstructing the ideological foundations of racial discourse in society (Hakimeh Saghaye-Biria, 2012:511, citing Henry & Tator, 2002).

During the deliberations by the research participants many arguments surfaced. Some reflect discourse injustices in the teaching and learning of problem-solving. Such convoluted, rambling and meandering discussions needed to be analysed to get the hidden meaning (Francis(2012:50). For instance, there follows an example of a conversation that transpired on the playing of the indigenous games:

> '….. *ha le letsoho la mme Motaung le phahama ho bonahala hore kgati e phahamela hodimo, moo le letsoho la hae le theohelang fatshe, le kgati e theohela fatshe* (As Mrs. Motaung's hand goes up, it shows the rope is going up, and as the hand goes down, it illustrates that the rope goes down).

Thus, the use of CDA will enable the research participants to obtain the deeper meaning of the text. In essence, the meaning and interpretations denoted mathematical content embraced within this text, which is illustrated by the play of *kgati*. This agrees with what Francis (2012:18) and Mahlomaholo (2012a:51, 2012b:124) argue for CDA as an enabler for the spoken/written words of the research participants being used as evidence for the interpretations made at both the levels of discursive practice and social structures, to extract deeper meaning and making repertoires of each participants.

According to Hakimeh Saghaye-Biria (2012:511) in Van Dijk (1997a) and Wodak and Meyer (2009:29), the levels of analysis, that is, meaning generalised references to inherent personalities or typical actions of minorities, may manifest social attitudes and ideologies as configurations. This analysis will look at the conversations that subaltern communities are incapable of illustrating as mathematical concepts in the playing of indigenous games. The second level of analysis is reference to in-groups' goals. These may dictate group interests and the general orientation of ethnic ideologies. With references to the study the conversations will include discursive practices.

The third level of analysis makes reference to pertinent (in-group or universal) norms and values, for instance, in argumentation. These may articulate the building chunks of ideological structures that organise attitudes about others. These are shown in the study during the preparation phase, when parents believe that if one has not gone for formal education one can be resourceful in the teaching of problem-solving. It is maintained by the societal norms that problem-solving is only for those who have good socio-economic status.

The fourth of level of analysis is semantic, which moves such as disclaimers. These play a role in impression management and persuasion while demonstrating the fundamental structures of ethnic attitude. The fifth level is storytelling about ethnic events incorporating intimate experiences with others. In the study, the hegemonic ruling class (Barker, 2012:66) used this discursive to discredit the mathematical concepts imbued in indigenous games, to the extent of labelling people playing these games as 'unintelligent' and 'lazy' (Lynn,2004:156). These convey mental structures of such events and the opinions storytellers have about them.

Lastly, is the argumentation structure level of analysis applied to support or counter racial policies. Wodak and Meyer (2009:29) referred to this as the strategy of intensification and mitigation: intensify or mitigate the force of declarations.

3.8 CONCLUSION

In this chapter profiling of research participants has been presented, and also the criteria that the members agreed upon in deciding upon members of the focus group. The PAR as research methodology was discussed, elucidating why it was preferred to other research methodologies. Using PAR was appropriate for generating large amounts of useful data, which will be analysed using Van Dijk's CDA.

CHAPTER 4

ANALYSING DATA, PRESENTING AND INTERPRETING RESULTS ON THE FRAMEWORK TO USE INDIGENOUS GAMES IN THE TEACHING AND LEARNING OF PROBLEM-SOLVING

4.1 INTRODUCTION

The aim of the study is to formulate a framework towards the enhancement of problem-solving skills among grade 10 Mathematics learners using indigenous games as a strategy. The problem-solving skills to be considered were functions; number patterns and sequences, algebra and equations, probability, Euclidean geometry and measurement, analytical geometry, trigonometry and statistics (DBE, 2011:8). In order to operationalise the aim this chapter focuses on the theoretical and empirical data, collected through literature review and participatory action research (PAR) respectively. The chapter uses critical discourse analysis (CDA) as appropriate in analysing and interpreting data, to get deeper meaning of spoken words of the research participants. This was conducted through analysing the dialectical relationship between the theoretical and empirical data, and by explaining reasons for the tension or agreement between both sets.

Francis (2012:50) regards it as crucial for the participants' stories to be studied against the backdrop of the literature reviewed, thus discovering and highlighting the areas that are consistent or contradictory. The issues and themes that emerged from the data are then organised into a framework to illustrate the relationship between the different variables and the participants' experiences. Meanwhile, CDA enables the participants to use the spoken or written word as evidence for the interpretation made at both the levels of discursive practice and social structures to extract deeper meaning and make repertoires of each participant (Francis, 2012:18; Mahlomaholo, 2012:51, 104).

In addition, the analysis demonstrates how the five objectives have been operationalised. The first objective looks into the challenges that exist in the teaching and learning of problem-solving skills. In addressing the first objective the research team identified the following challenges:

1. content too abstract
2. method of teaching too teacher-centred
3. lack of motivation/interest among learners
4. ready-made formulae drilled
5. non-involvement of parents
6. limited content knowledge among teachers
7. limited motivation among teachers
8. limited expertise with regard to classroom practices and meaningless subject-content.

4.2 ANALYSIS OF THE CHALLENGES

Each challenge will be discussed by referring to expectations in terms of good practices as indicated by the educational policy, research and theory of learning. The researcher will make reference to legislative imperatives which support the resolution of the specified challenge in the teaching of problem-solving skills and demonstrate how each of the said aspects (construct) constitutes a challenge to effective teaching of problem-solving skills. In substantiating the argument that a construct is viewed as a challenge, evidences in the form of spoken words, written words, pictures and scenarios painted by research participants will be provided. In obtaining the deeper meaning of the texts the CDA will be used to analyse and interpret the data. The evidence will be interpreted using Yosso's community cultural wealth theory in order to make sense of empirical data. A brief conclusion will be made looking at the findings of the empirical data, whether they correlate or refute the literature reviewed. Lastly, observations will be made as to the contribution of the findings in the body of knowledge.

4.2.1 Content is too abstract for the teaching of problem-solving skills

It becomes very difficult for learners to comprehend the mathematical content, as it is presented in an abstract manner. The teaching and learning of problem-solving ignores the cultural background and the contextual environment of learners. For the learners to attach meanings to the problem, Bush (2005:4,6) and the DoE (2003:2,62) point out that the marginalised background knowledge of learners needs to be brought in. In order to concretise the mathematical content, teachers should utilise the hands-on activities with which learners are familiar from their home environment. The contextual factors facilitate development of the child into understanding high level of mathematical content.

The observations were made in the research sites that the teaching and learning of problem-solving skills are taught in such a way that it is not easy for learners to comprehend and relate them to their daily life activities. In the following subtopics, lesson preparation, classroom presentation and assessment of activities will demonstrate how the teaching and learning of functions, algebraic equations and geometrical skills are abstracted in the actual classroom teaching. What follows is an example of the pattern of teaching problem-solving skills observed as problematic in the schools under investigation.

4.2.1.1 Lesson preparation

Lesson preparation entails an overview of what is going to happen in the lesson, for instance the teaching of numeric patterns and finding the general formula for one. Moeller, Dubitsky, Cohen, Marschke-Tobier, Melnick and Metnetsky (2011:3) suggest that collaborative lesson planning, whereby colleagues, learners and parents are invited to give inputs has ripple effects, in which preparation reaches out to cover wider scope of work, and help learners to understand the content with relative ease. Also, for the lesson preparation to be complete, it is expected of the teacher that specific content and contexts be addressed in the planning, and the lesson outcomes be shown and communicated to learners. In the sample lesson (see Table 4.1, below) compiled by the teacher (Mr Talana)[1] at one of the schools under investigation, it shows proper lesson preparation was not carried out. Mr Talana's preparation showed the content was presented in a very abrupt way, which can pose a serious problem if one Mathematics teacher has to present that content.

[1]Pseudonyms are used.

The lesson preparation left out the context, which is crucial in assisting learners to connect new knowledge to the prior knowledge they possess from their social background. It can further help the teacher to incorporate the community cultural wealth theory, which takes into consideration the socio-cultural background of the child. In that way it becomes easy for the learner to see the relevance of the numeric pattern with daily life activities.

The lesson plan (Table.4.1) does not respond to the expectations discussed above, as the lesson commenced by asking learners "what is ….number pattern…?". rather than establishing the prior knowledge of learners. In the lesson's introduction, which was not shown by Mr Talana, he could have capitalised on the navigational skills, familial capital and social knowledge that learners possess. Their marginalised knowledge could be easily integrated into the teaching of numeric patterns. The manner in which the lesson preparation is drawn up, number patterns as they are presented and prior knowledge of learners had not been integrated or tested. Thus, the context of lesson content did not capitalise on community cultural wealth theory. The outcomes, activities and resources should be considered in the lesson preparations.

The DoE (2008:5,18,19) and Watson and Mason (2007:206, 209-210) stipulate that for the lesson preparation to be a meaningful and useful tool to both learners and the teacher the onus is on the teacher to have enough time to plan and accommodate all learners in class. Contrary to policy stipulations, at Tlhabollo secondary, the school under investigation, Mr Talana crafted a rushed lesson preparation (Table 4.1, below), which was not adequately written to guide him on how to accommodate all learners.

> Mr Talana: *meniza Tshabane, mpolelle ke hona ho felang pirote thri kapa ke hona e qalang, ke tlo ya klaseng ka pirote fo. Ke elellwa hore ke misiplesitsi teksbuko yaka ya mmmetse, ako nkadime ya hao ke bone hore ke ilo ba ruta eng kajeno. Ke hopola eka ke ilo etsa disekwensi le bona.* (Mr Tshabane, tell me this is the end of period three or are we about to start period three. I am supposed to be in class by period four. I realised that I misplaced by mathematics textbook, can you borrow me yours, so that I

can quickly check what I am going to teach today. If my memory serves me well I had to start the sequences today).

Mr Tshabane: *ke hona perote thri e felang, ho qala perote fo. Ho bolela hore o wa kena hona jwale. Tshwara teksbuko ke ena*(this is end of period three, period four has just commenced, meaning that you are getting to class now.here is the Everything Mathematics textbook).

After Mr Talana has received a textbook, he hurriedly took a clean A4 sheet and scribbled the points on the number patterns, as shown below:

Topic: Number pattern

- What is number pattern

Given the sequence:

1;4;7;10;……

- What is the first term, second term, and the common difference
- Find the last term or general term of this sequence

2. Class activity

- Textbook: Everything mathematics; pg.105 – 106. Do No: 1(a) – (c), 2(a) – (d).

3. Homework:

- Textbook: Everything Mathematics pg. 106

Do number 3 and 4

Table 4.1: Sample of the lesson preparation on Number pattern, compiled teacher Talana

The phrase'… *ke bone hore ke ilo ba ruta eng kajeno. Ke hopola eka ke ilo etsa disekwensi le bona…'* ('…. so that I can quickly check what I am going to teach today. If my memory serves me well I had to start the sequences today…') shows that Mr Talane did not have time to plan properly before he went to class to teach problem-solving skills, such as number patterns. As evident in the lessons prepared, patterns are not going to be interesting to learners if taught as shown in the lesson preparation (Table 4.1). No creativity was stimulated in this lesson plan, and neither preparation nor presentation on number patterns allowed learners to use their prior knowledge or navigational skills, such that they could reflect on their social life to integrate the marginalised knowledge into acquiring problem-solving skills.

The teacher made the lesson preparation alone in the staffroom, with no consultation with other teachers, learners' prior knowledge, parents' input, and other community members. He omitted learning and teaching support materials (LTSM), teaching aids that might have concretised the teaching and learning of number patterns. Generally, such elements of lesson preparation can help learners to comprehend the content delivered, and their omission disempower learners. The teacher showed excessive power in classroom practices, as Mr Talana on his own decided that he would teach sequences. Learners were not given the freedom to voice their opinions or provide input in deciding on the aspects to be revisited before moving on to the next lesson on sequences.

The teacher's lesson preparation underplayed the rich wealth possessed by the collective of mathematics teachers at his school, marginalised parents, learners, and any other important partners who might have been helpful is sharing valuable knowledge. This is congruent with findings of Cunningham (2005:3); the University of the Free State Report (2012:11) and Yosso (2005:77), who termed it 'social capital', of value when sharing practical experiences on patterns in their life situations. It can be extrapolated from the extract '….I have misplaced textbook….' that the teacher, Mr Talana, was following the planning cycle.

According to the DoE (2008:5,18-20) and Van Wyk (2007:47) on lesson planning, the first thing to consult in lesson planning is the policy document of mathematics content, followed by looking at the work-schedule, which packaged the content to be covered each week. Thereafter, the teacher can draw up the lesson plan for the day, consisting of activities to be done in class. In designing such activities various sources, including textbooks, must be consulted, but this procedure was not followed by the teacher. The planning failed to meet the expectations of the planning cycle.

In conclusion, as illustrated in the literature review, teachers of various subjects, including the teaching of mathematics problem-solving skills, made inadequate or insufficient lesson preparation. Some teachers argued that crafting of the lesson plan was a waste of time as they were conversant with the subject-matter. Evidence from this study confirmed that lesson preparation was not consistently made, and if it was it did not follow departmental requirements. There should be a well-thought out class map, of help to the teacher and learners, and to produce a lesson that blends the relevancy of the knowledge learners possess from home with what is taught in the classroom. The preparations failed to capture the interest of learners in the planned content and the problem-solving skills were not clearly articulated to learners.

4.2.1.2 Classroom presentation

According to Anthony and Walshaw (2009:153), the DoE (2003:14), Keeley and Tobey (2011:10) and Situation Analysis and interpretation of Outcomes Module (2007:49), classroom presentation is the interactive or teaching phase of a lesson, in which the actual teaching and learning, execution of activities, and assessment takes place. In the classroom presentation there are clear roles that must be performed by the teacher and learners. The roles are informed by the detailed tasks to be performed in class and are guided by the lesson outcomes to be demonstrated by learners at the end of the lesson. For active participation of learners during the lesson presentation various teaching methods and clear activities, resources, as well as teaching media or teaching aids need to be used throughout. Also, the various forms of continuous assessment are effectively applied in ensuring that learners can demonstrate the lesson outcomes. It is further expected that the teacher engage all learners in appropriate tasks that elicit deeper thinking and self-discovery. This will assist in sustaining the interest of learners to probe for logical answers. In turn, learners will acquire high order knowledge on the problem-solving skill (PSS).

As Table 4.1 shows, the teacher did not have time to make a thorough preparation on subject matter or creative activities of learners, and presented no clear lesson outcomes which learners could work towards achieving. This reduced the chances of learners comprehending the lesson.

Mr Talana started the presentation in this manner:

> Mr Talana: Good morning, my learners.
>
> Learners: Good morning teacher.
>
> Mr Talana: Ehh, today, we are going to handle the number patterns. Who of you can tell me, what are number patterns?

The teacher waited a few minutes for learners to respond. None of the learners did so. Mr Talana gave learners the correct answer, as '….number patterns are numeric list of number given in this form: 1, 3, 5, 7…'

> Mr Talana: Are we together class?
>
> Learners (a few learners responded): Yes sir, we are with you.

The other group of learners were quiet, staring at the teacher or scribbling down the answer given by the teacher. The dialogue between the teacher and learners was not interactive. The majority of the learners were passive, whilst other were taking notes. It could be read from their faces that they were struggling to understand why the numbers 1,3,5,7 had been chosen as a pattern. The teacher failed to use the facial expression of learners to change the way he was conducting his presentation. He did not use concrete examples from the learners' experiences to explain the pattern.

Regardless of facial expressions by some learners, Mr Talana proceeded with the presentation as follows:

> Mr Talana: Right now learners, let have a look at this sequence: 1, 4, 7, 10… The first term is given as $t_1, which\ is\ a,$ …so in this example our t_1 is 1. The second which is t_2 , which can be described as $a + d$, where d is the common difference between any two consecutive terms.

Mr Talana: Ok, I think you are with me, who can tell me what is the third and fourth term of the sequence?

Mokete: Sir, let me try, the third term is three.

Mr Talana: Ok, my son can you tell us how did you get it?

Mokete: (Raised his hand and counted up to three fingers). That is how sir I got three as the third term.

Mr Talana: Ok, I see your problem, look you must consider the sequence that is given. Yes, Pulane, let's hear from you.

Pulane: The third term is 3 + 4.

Mr Talane: Quite correct, but why did you expanded 7 as 3 + 4. Anyway, let's carry on, the third term is 7, which can be described as t_3, which is known as $a + 2d.$ In this particular sequence the last term will be defined as $t_n,$ which simplify equal to $a + (n - 1)d.$

Dineo (Chatting to her friend,) *eish, o a bora, o tla qeta neng, wena o a utlwisisa*…. (he is boring us, when is he going to finish his presentation, or are you following what he is doing?)

The lesson presentation was teacher-centred, with no activities meant for learners to work cooperatively on. The way the teacher handled the class interaction denied learners an opportunity to share their answers first, before they responded to the teacher or to the large group. Such environment of teaching and learning is not conducive to effective learning, as some appeared not to follow the presentation, and others seemed to be disinterested on the presentation. Few learners were responding to his questions correctly, and were mostly passive, taking notes as the teacher wrote on the chalkboard.

Mr Talana was not focussing on the whole class or moving around, and thus did not hear comments such as 'is boring…'. The presentation did not encourage any learner participation there might be to flow freely. There were limited activities in which learners could be engaged and most of presentation was by Mr Talana himself. Learners were only asked questions, no activities which learners were engaged on their own or as group.

One learner posed a question:

> Temoho: …eh, sir, let me ask this: why are you talking about numeric pattern, can I also include the geometric pattern?
>
> Talana: Wait a bit; we are still coming to geometric patterns, which will be handled immediately after the numeric pattern, as for the geometric you had to divide terms to get the pattern.

The extract demonstrates that the interaction between the teacher and the class was one-to-one, with the learners asking questions and the teacher immediately responding without giving more thought to the question. The manner in which the teacher answered restricted learners' thinking of practical example of patterns. Number patterns were viewed from a numeric point of view only. In practical life situations, patterns can be found in nature, as shapes (geometric designs), events, and sets of numbers, hence the definition Mr Talana gave to learners was limited to numbers.

The extract shows the danger of planning alone, without adequate LTSM, and not listening to the learners. The classroom presentation disempowered learners and did not encourage them to think independently about dealing with patterns. The classroom activities were mainly teacher-centred.as the lesson presentation did not focus on the creativity of the learners. The lesson preparation made learners think in a limited rather than divergent way, the majority being left to understand the patterns in numeric form only, if they even followed it.

The familial capital learners gain in their home environment is oppressed, for example, the series of activities before they leave for school, such as waking up, making the bed, washing, putting on school uniform, breakfast, and picking up books for school. The manner in which the teacher presented the pattern, led learners to look at the pattern in one way. The response by teacher ('…wait a bit, we are still coming to geometric patterns…') shows that he ignored the multiple forms of knowledge displayed by learners on number patterns and the navigational capital that plays a crucial role in demonstrating them.

The findings correlate with the finding from the literature review that when the learner performance is poor it is because the teachers have failed to engage leaners optimally. It follows that teachers are dominating class presentation, as learners are passive throughout the lesson presentation. Such teachers believe that it is enough simply to cover the work prescribed in the departmental policy documents.

The extract above showed that the questions asked were too objective and did not allow for multiple answers, with time given for learners to share views on their considered responses.

After his lesson, Mr Talana informally met his colleague, Mr Tshabane, and reflected on what had transpired during class presentation:

> Mr Talana: *ke se ke kgutlile klaseng sititjho (monghadi), athe wena o kene ne neng* (I'm back from class, when is your period, Mister?)
>
> Mr Tshabane: *ke tlo kena ka lase piroto kajeno, feela o sebeditse jwang le bana ba hao.* (I will be in grade 10 (A) class by the last period, but how was your lesson with your class?)
>
> Mr Talana: *O wa tseba, jwale ka mehla feela, ha o ba botsa dipotso ba tla thola, ka ha ba senya nako, ke qetele ke ba holanya fatshe feela, hore ke tsebe ho qeta silabase*. (As always, the progress is very slow, especially when you ask them questions they keep quite. You end up going on continuously with your presentation, so that one can finish the syllabus in time.)

The informal conversation above showed that it normal for teachers to concentrate on completion of work rather than learners comprehending the subject matter. The lesson was not learner-centred and focussed only on getting through the syllabus. If this attitude persists the teaching of problem-solving skills will remain isolated from the life of the learners.

4.2.1.3 Assessment of activities

According to Ahmed-AL, 2013: 125) and DoE 92008:18), assessment for learning and assessment as learning occurs throughout the learning process, and learners are engaged in assessment in a more active way. It is expected that the teaching and learning process forms an integral part of the problem-solving skills, so as to check the understanding of learners continuously, and engage learners in the assessment process. Assessment is a process of collecting data and interpreting results in order to determine the learner's progress in learning PSS and to make informed judgement about learners' performance (DoE, 2003:63, 2007:5). It is expected by DoE policies, Ahmed Al (2013:125) and McMillan (2004:4) that assessment may take many multidimensional forms in gathering information from several contexts, enabling learners to be critical connectors between learning and assessment.

Teachers have an overall responsibility to assess the progress of learners in achieving lesson outcomes (Keeley & Tobey, 2011:3) and finally demonstrate the critical outcomes. Even learners who experience barriers to the learning of PSS can be assisted adequately. Assessment must be a continuous process in the teaching and learning situation (DoE, 2003:63, 64), made before commencing the lesson presentation, during pre-assessment teaching (Youdalle, 2010:30), during lesson presentation and during the reflection phase. Thus, the manner in which assessment is made or understood by teachers ensures that learners make a connection between new ideas and prior knowledge. In this way, the quality assessment is key for making informed decision about the progress of learners (Keeley & Tobey, 2011:4).

The lesson plan in Table 4.1 shows shallow understanding on number patterns, narrowly defined, thus making it difficult for learners to conceptualise their own understanding of them. Mr Talana did not contextualise the pattern to relate to environmental background (DoE, 2008:13) of the learners but responded by showing there is no link between the numeric and geometric patterns. As Moeller et al. (2011:3) suggest, the collaborative lesson planning approach would have helped Mr Talana to consider multiple realities about the patterns. In turn, it would be easy for learners to cite examples relevant to their daily life, that is make connections between their learning and assessment processes, hence being critical about their learning.

The definition of ‘polygon’ was given as independent of that for ‘polyhedra’, but the teacher did not assess the understanding of learners correctly as he failed to incorporate assessment *for* learning and assessment *as* learning. That would have helped him to use assessment for learning throughout the teaching and learning processes, at the same time focusing on assessment as a process not an event. The extract (‘….wait a bit, we are still coming to geometric patterns…..’) illustrated that the teacher had not prepared well for the class and hence was not in a position to answer the question raised by learners appropriately or in a convincing manner. As the teacher argued that patterns are in numeric form only the learner had a point when mentioning that a pattern could also take a geometric shape. The assessment of the teacher on that question misled the class into viewing patterns in numeric form only. The point raised by the learner on geometric and number patterns was a relevant point for linking geometry and algebra. The extract revealed that the teacher had failed to demonstrate the integration of problem-solving skills or help learners simplify abstract concepts.

It is clear from the previous paragraph that by ignoring the social wealth and navigational skills learners possess, learners could not view problems solving skills other than in an abstract way. Social skills would allow them to interact and share their understanding about the patterns and cite examples of relevance to their prior knowledge on patterns. This idea of sharing would enhance their aspirational capital, thus keeping them motivated and having a keen interest in the lesson. The comment “…when is he finishing the lesson, he is boring us…” signifies that aspirational capital of learners was being suppressed. They were denied access to their navigational skills, which would have helped them to contextualise the problem-solving skills with the pattern. They were not given freedom to air their understanding as to how patterns affect or surface in their life, or what significant role they play in their environment and coping with realities efficiently.

The lesson illustrated that the teacher dominated the discussion, as he taught as though he was pouring liquid into empty vessels. The teaching and learning did not incorporate assessment as learning and it was not evident which, if any, lesson outcomes the learners could demonstrate at the end of the lesson.

The informal conversation after class with Mr Tshabane could be looked at as a reflection on the lesson, but he did not use it as a way of assessing how it could be improved for the future, or to stimulate the interest of learners by taking centre stage in the teaching and learning of problem-solving skills. Aware of the weaknesses of the lesson his remarks were, "…as always, learners do not respond to questions…." but he did not devise means as to how to better learner involvement during lesson presentation or assessing them properly. The assessment tasks (homework) given to learners left out the interrelatedness of problem-solving skills on numeric patterns and geometric shapes that would have constituted numeric patterns when unpacked. This omission was to have a negative impact, as learners regard algebraic expressions and equations as not related to geometry or trigonometric concepts.

The empirical data showed that learners were merely copying the work on the chalkboard, not following the discussion in class. This confirms findings reported in the literature that in most cases teachers use traditional methods of teaching and assessing. The teacher merely modelled a procedure, and the learners copied the steps of the problem, raw as it was. In turn, they did not even reflect on the chalkboard workings or ask for clarity from the teacher. Also confirming the literature, such approaches to traditional approaches used in lesson presentation and assessment cause learners to adopt stereotypes and absorb raw information without making sense of it.

4.2.2 Method of teaching is too teacher - centred

Teaching methods have a real and pervasive effect on learning. It is expected of the teachers that they use variuos teaching modes, as the class becomes vibrant, and encourage learners to ask questions and share inputs with other learners and the teacher. For them to demonstrate the achievement of critical outcomes, where learnersand work effectively with others as members of a team or group, it is the responsibility of the teachers to reflect and explore a variety of strategies. Different teaching methods draw attention to different learning outcomes. As one of the National Curriculum Statement (NCS) principles, OBE encourages a learner-centred and activity-based approach in the teaching of problem-solving skills in mathematics curriculum in all grades (DoE, 2003:2; Samuelsson, 2010:36; Thomas & Brunsting, 2010:25,27).

The use of collaborative learning allows learners to interact among themselves and share their prior learning so that it be can be linked to the new content they are engaged in. The lesson will be easily followed as the activities are learner-oriented. On the other hand, it is still expected of the teacher to model, that is teach the problem-solving skills as a way of demonstratiing the key issues.This kind of mode allows the teacher to review the lesson presented with learners and by so doing empower the learners, allowing them to discover and reflect on realistic experiences. This concurs with what Mahlomaholo (2012:5) and Nkoane (2012:3,4) argue is a way of empowering the excluded and giving a voice to the voiceless. This type of teaching reinforces the learning styles of learners, and also shows that the teacher is teaching the problem-solving skills from their perspectives.

In one research site it was observed that the teacher, Ms Swane presented the problem-solving skils without synergy between shape, space or measurement, and marginalised knowledge possessed by learners.

Topic: Shape, space and Measurements

Introduction: definitions of polygons, polyhedra

Teacher activities: the following will be explained to learners

- How to calculate the area of any polygons such as square
- Surface area of different polyhedra

2. Class activity

- Textbook: Everything mathematics; pg.396 No: 1 (and 3.

3. Homework:

- Textbook: Everything Mathematics pg. 397

Do number 7 and 8

Table 4.2: sample of the lesson plan on measurement, compiled teacher Swane

As teacher Swane entered the class, she started the presentation in this fashion:

> Ms. Swane: *Dumelang bana*. (Morning learners).
>
> Learners: (As a group): *Kgotso titjhere.* (Morning teacher).
>
> Ms. Swane: In the previous grades you encoutered these defintions: area, surface area, polgons, polyhedra. So today we will work on them in a detailed fashion.

Ms Swane moved towards the chalkboard, on which she wrote definitions as the learners hurriedly took out their scribbling books in which to copy them down:

> Ms Swane: Area is the amount of space covered by the polygons. By the way, can you tell me, what are polygons?

No one responded. She pointed at one learner:

> Ms. Swane: Yes, Thokozane, what do you think?
>
> Thokozane: I think area deals with polgons are in triangular shape, rectangular shape.
>
> Ms Swane (writing on the board): yes, Thokozani, that is correct. (She writes on the booard) polgon: is a plane shape whose boundary is formed from 3 or more straight lines, such as rectangle, square, etc. like you mentioned Thokozani. Are we all together class?
>
> Learners (in unison): yes, mam.

The presentation of shape, space and measurement was teacher-based. Ms Swane dominates the discussion, giving all the answers. The chalkborad was covered with answers suggested by her as learners merely copied the defintions of concepts and formulae for area and surface area as raw, from the chalkboard. She was transfering the knowledge into blank minds of the learners, who simply memorised them without attaching any meaning.

Thus, it was important to engage them in various modes that would allow them to interact among themselves. They could also engage in reflection in their groups, and compare solutions obtained in their teams. Ms Swane appeared very powerful in terms of knowledge on shape, space and mearurement, but in turn learners were disempowered and unable to access or show their wealth of marginalised knowledge. They were deprived of the opportunity to use Yosso' s capital wealth, such as social wealth, to work collaboratively and share their understanding or multiple intelligence on shape, space and measurements. Familial wealth would facilitate use of their home environment to cite practical examples of polygons, whilst navigational wealth would have empowered them to discover concepts not raised by the teacher, such as noting that there are types of polygons, that is, regular and irregular. Also, navigational capital would have helped the teacher to teach from learners' realistic perspectives and learning styles would have been acknowledged and amplified. Operationalsing community cultural wealth theory, learners would be freed from the misconceptions around complex areas and surfaces of polygons and polyhedra respectively.

It is evident that the formulae presented to learners in a rote laerning mode denied them the chance to explore how the area of irregular polygons could be determined. If the teacher used problem-based learning it would encourage learners to complete these complex problems (formulae for polyhedra and irregular polygons) independently, either in class or at home. At times, in explaining the so-called 'abstract formulae on surface area of polyhedra', rather Ms Swane could use scaffolding to help learners up to a certain level, where they comprehend what to do, then proceed on their own to derive the formulae for irregular polygons. In so doing she should have created a home among learners in which they could work on mathematical concepts such as 'integration', which is taught at university level. The learner-centerd appraoch, perhaps problem-based, could enhance their motivation towards mathematics, and application of problem-solving skills creatively and used innovatively in problem-solving skills.

The empirical evidence confirmed what the PAR had indicated, that the use of various teaching modes helps teachers to stimulate learners' creativity in problem-solving skills. Marsh (2013:32) and Pascuzzo (2010:233) point out that pupils enjoy delving into university mathematics, and find the work on Boolen logic, different base number systems, and infinity exciting, as opposed to traditional ways of teaching which made it difficult for them to progress well in problem-solving skills with any interest (Samuelsson, 2010:38) or enjoyment. Ms Swane's class depended on her entirely with regard to the teaching of shape, space and measurement.

The lesson plans and lesson presentation of Ms Swane corresponded with what the finding in literature raised that it is not enough for the teacher to show competency on knowledge of content or knowledge of pedagogy, rather he or she must demonstrate the mathematics knowledge for teaching. This would assist in reaching a high level of understanding and the learners learning effectively.

4.2.3 Lack of motivation among learners

According to the DoE (2003:2), Johnston (1996:23, 31) and Waege (2009:83, 84), the development of the learners during the learning process includes the tripartite theory of the mind (feelings, thoughts and behaviour). OBE, as one of the principles of the NCS, regards teaching and learning as holistic, in the sense that teachers need to consider and assess all the aspects of learner development in the learning process. On the other hand, they should be aware that effective teaching not only involves knowledge and skills acquisition but also the influence of affective domains. The latter examine how learners perceive their capacity to learn, therefore it is imperative for the teachers and learners to understand that appreciation and acknowledging the worth and importance of oneself is key. This will boost the self-esteem and self-efficacy of learners (Campbell, 2006:12,13; Williams & Forgasz, 2009:95).

The above argument suggests that all parties, especially teachers, learners, and parents, need to support learners in the process of teaching and learning. When assessing, forms have both to help establish how well learners performed and determine their needs. In turn, learners will develop interest and enthusiasm for engaging in class deliberations and class activities. If they are motivated either extrinsically or intrinsically, then values of sharing, caring, self-discipline, acknowledging one's own competencies, and cooperation between individuals in the classroom will arise and be sustained. This is agreement with Mahlomaholo

(2010: 13) and Vankúš (2008:106), who argued that it is expected of teachers that they create, construct and maintain sustainable and empowering learning environments. Here the teacher needs to show that learning content on problem-solving skills is flexible in terms of time. When learners are struggling to understand concepts of PSS, the teaching time may be extended to accommodate individual concerns and needs and so encourage learners to work at their own pace.

4.2.3.1 Class participation

The DoE (2003:3, 4; 2005:8), Jansen et al. (2014:61) and Kellaghan et al. (2009:1) envisage the kind of learner who is imbued with the values and acts in the interest of society based on respect for democracy, equality, human dignity and social justice. During class participation, these values need to be demonstrated by learners for sustainable learning (Jones et al., 2011:845) Mahlomaholo, 2010:13). It is the responsibility of the teacher to create an enabling environment that helps learners stay motivated and focused, so as to achieve the critical outcomes, in the case of this study using problem-solving skills, science and technology effectively, and critically showing responsibility towards the environment and the health of others.

In promoting motivation and self-esteem among learners in class, the teacher has to ensure the teaching of problem-solving skills illustrates the importance of cultural practices of individuals, and self-discipline with regard to respecting different views of other learners in class. He or she should encourage learners to acknowledge multiple realities, and to participate optimally, thus their interest in problem-solving skills is generated and self-esteem boosted.

Mr Talana was making a presentation on linear functions, on the standard form: $f(x) = mx + c$. He posed a question in this way:

> Mr Talana: the standard form of linear functions shows the y-variables, which are known as dependent variables, and x-variables which are

independent variables, and c is the constant. Then, who can tell me, what do we mean by independent variables and dependent variables?

Two learners were arguing loudly over their answers, such that the teacher could not hear the response of the learner he pointed at for an answer.

> Mr Talana (aggressively addressing the two learners): *Hey, lona ba babedi, wena Tshepo le Nyakallo, ka hlooho tse kgolo le a hlodiya, ha re kgone ho utlwa karabo tsa bana ba bang*. (You two, Tshepo and Nyakallo, with swollen-headed you are making noise, we can't hear the response of learners, you are disturbing).

> Tshepo (politely responded back): But, sir we are arguing about our responses…

> Mr Talana (interrupting Tshepo): Keep quiet, you are lying….., (' *Le etswa ke hoba le shapuwe, ke hona le etsang tjena*) (It's because corporal punishment is abolished, hence you are behaving like this…'

The two learners were shattered by the comments from the teacher, and took no further part in the lesson. The comment '…you two with swollen-headed…..' was derogatory, and broke any rapport with their teacher. The self-esteem of these two learners was negatively affected, and led to their not completing class activities or assessment tasks. Such comments might develop in learners a hatred of the Mathematics teacher and the subject itself.

It is acceptable practice for learners to be reprimanded when they are misbehaving during class interactions but this should be in such a way that they show remorse for their wrongdoings. Unbecoming behaviour, when properly handled, can be a lesson to them and others, whereby they cannot repeat it. The remarks made by teacher should build self-confidence for learners to be self-disciplined and respect their peers when they are on the platform. Putting aside that the learners on this occasion were not misbehaving but discussing their answers, the manner in which Mr Talana handled them was inappropriate, and undermined their human dignity in front of the class. Such response displayed by the teacher might discourage learners from taking part in class activities. As their lesson continued these two boys were not more paying attention to the conversation taking place in class. Apparently, the teacher forgot that teaching the child is an holistic process, that is learners' feelings must be respected, and they must be given space and freedom to explain themselves. In this way learners may be empowered to be responsible citizens (DoE, 2003:8), a critical outcome to be demonstrated by learners in later life.

The comment '… keep quiet, you are lying…', indicates that the learners were not trusted and their self-esteem was further undermined. The manner in which the teacher reacted violated the values of treating learners with respect or caring for them, and failed to display the pastoral role (Norms and Standards for Educators in National Policy Act no 27 of 1996). Indirectly, the teacher was discouraging the sharing of ideas in class among learners. As Yosso (2005:76) illustrates, social capital is crucial for making learners learn freely with their team members, whilst linguistic capital the learners possess should be used properly in encouraging them to communicate effectively with their classmates.

The response '… you are making noise…' made learners feel their thinking was confined, rather than the teacher using class interactions as a way of amplifying the navigational capital and resistance. These capitals give learners hope to explore various alternatives to the solutions without giving up when they come across challenges.

The comments made by the teacher affected other class members, as they were very quiet, not taking part and being afraid that the teacher would be rude to them too. Immediately after the Mathematics class ended, one learner, Sebolelo, commented:

> Sebolelo: Cameraman, this is off record, ke mogozi wa rona. J*o*! (clapping hands, showing to be shocked), *bathong jwale class ya mmetse e se e le jwang na, ke hore motho o itholela ho tloha kajeno. Ha ke batle hoba lehlatsipa nek sitaeme*. (Cameraman do not capture this, it's off record. Colleagues! What is happening with Mathematics class these days, I think it's better for one to keep quite. I do not want to be the victim next time).

Tau: *motho, e sa le a le tjena, da is wae, nna ke tholang , ha ke na taba le yena,le mmetsenyana oo wa hae le homework nyane tseo tsa hae ha ke sa di ngola* (He is used to this behaviour, that is why always keep quite during his period. I don't even care about him and his subject anymore, to an extent that I do not even worry to complete the homework assigned to us.)

That the learner was demoralised by the rudeness and had decided to remain quiet during the class, no longer wishing to attend Mathematics lessons or complete the assessment tasks. The harshness of the teacher, failing to show the pastoral and parental care when handling learners, only served to instil fear among learners and a wish not to take part in class interactions. Moreover, learners viewed the Mathematics classrooms as a space of conflict. The unprofessional behaviour contributed to learners feeling of disempowerment in resolving conflicts amicably. Unresolved conflicts make learners pessimistic in problem-solving skills. They are not giving voice to air their views in a peaceful classroom environment.

4.2.3.2 Learner performance on assessments tasks

It is important for the teacher to assess learners' performance throughout the learning sessions. The DoE (2003:63), Kellaghan (2009:119), Sacmeq Report by Moloi and Chetty (2011:1,7), the TIMSS Report (2011) by Provasnik et al. (2012:1,4) point out that many stakeholders have an interest in how learners perform with regard to problem-solving skills. Stakeholders include the learners themselves, teachers, parents, guardians, district, provincial and national Departments of Education, employers, and higher education and training institutions. Learner performance is assessed continuously for the purpose of monitoring progress and providing feedback, diagnosing or remediating barriers to learning, selection, guidance, supporting learning, certification and promotion. The teacher and learners are expected to be conversant with types and methods of assessment in the teaching of problem-solving skills.

In the following transcript, the teacher was giving feedback to learners on the tests they had written three weeks previously:

> Mr Talana: It is important to show all your workings for you to get all the marks allocated. What surprised me is that all the work you wrote in the tests, all those questions are from the homeworks I gave you. Did you revise the work we did in class before you sit for the tests?
>
> Thato: I do not know sir what happened, when we do these things on the chalkboard we understand, but the problem is when one is alone one becomes stuck. As you are alone you end up being discouraged to try several ways of getting the answer.

The extract showed that the teacher relied on the traditional way of assessing. The assessment tasks that informed him about learner performances were tests only. These tests were conducted after a month, after much had been done. Formative assessment, which monitors the learners' progress daily was used only minimally. The previous extracts on lesson plans and presentations showed that the teacher was dominating the class. Teaching was not learned-centred and he even went to the extent of doing the homework exercises on the chalkboard, so learners could correct themselves from there. After the marking of the homework he continued with the lesson for the day. Thus, the remark '...all the questions I gave you are taken from previously homeworks assigned to you…', demonstrates that learning did not take place as expected. The teacher did not employ assessment for learning or assessment as learning models. If these models applied it would help the teacher to identify mistakes immediately during class interaction:

The above-mentioned lesson plans and presentations showed that lessons were teacher-oriented, which left learners heavily dependent on the teacher. That their voices in terms of the answers on problem-solving skills were marginalised, demoralised them and prevented them from being actively involved in completing the assessment tasks assigned to them. This was evident from the response of one learner ('….I do not even worry to complete the homework assigned to us…..'). The tendency was not to complete the homework and the teacher apparently was not monitoring through checking the work done or carrying out formative assessment effectively, *for* learning and *as* learning. In this way, the teacher was not using the assessment as an integral part of teaching and learning problem-solving skills. As the learner indicates in the excerpts ('…. end up being discouraged to try several ways of getting the answer…..'), the teacher was not helping them to comprehend the problem-solving skills. Learners were not given a chance in class to display their responses. They only consumed what was given by the teacher, and were not connecting it with their prior knowledge. This shows that social justices principles were not adhered to by the teacher, as the resource knowledge possessed by both learners and the teachers was equally distributed. Again, it is evident that the teacher as failing to capitalise on the wealth of knowledge learners possessed from their background environment. Navigational capital can help learners to voice many alternatives on the solutions of problem-solving skills, which will help them to make meaning with the subject matter presented or assessed. Since such wealth of capitals as not used, they ended up being discouraged from thinking about various alternatives to effectively work on the problem-solving skills.

These findings agree with what the literature. Traditional methods of teaching and learning have negative impact on the learner's performance, interest and enjoyment in problem-solving skills. It is imperative that teachers use various teaching modes in the teaching and learning of problem-solving skills, which not only address on knowledge skills but also affective domains. The manner in which the teachers conduct his/her class has a great bearing on learner performance, which is grounded on affective skills.

4.2.4 Drilling of mathematics formulas

Departmental policies expect that problem-solving skills (mathematics formulae) be taught in a way that they make meaning to learners (Anthony et al., 2009:154; Averill et al., 2009:175,181; Battista (2004, cited by Su et al., 2013:1). Learning does not occur in vacuum but must be contextualised in real-life situations. Problem-solving skills taught in this fashion are easily remembered by learners, who should be able to give their own interpretations and meanings. It is expected that the teacher use visual, symbols and/or language skills in various modes to concretise the problem-solving skills (DBE, 2011: 5; DoE, 2003:2).

4.2.4.1 Lesson planning

The lesson planning should show creativity and innovation from the teacher. One is not expected to take the raw information as it is from the Mathematics textbooks and transfer it to learners, rather the lesson planning must demonstrate how learners will be given freedom to discover formulae on their own. This is done through various teaching modes, such as scaffolding, by which assistance is provided to learners until he or she can manage to proceed unaided.

At the research site, it was observed that the lesson planning was evident but it focused on the teacher. The teacher did not prepare any activities for the learners so as to make them understand the distance formula:

> Ms Mokoena: Ok, learners, let's have a look at this concept in analytical geometry. Given the following points on the Cartesian plane, let us determine the distance between the two points, given $A(-2,0)$ and $B(3,9)$. Find the length of AB. We will use this formula:

$$d^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$

$$d^2 = (9-0)^2 + (3+2)^2$$

$d^2 = 81 + 25$, therefore $d = \sqrt{106}$, then is $d = 10{,}3$

> Tshepiso: Can I find out, teacher, why do you use x and y, is it fine if I use any other variables is to determine the length of AB?

> Ms. Mokoena: You have to use x and y as it the formula to use, and the formula is defined in terms of x and y.

The class conversation showed that the teacher only gave learners a formula to use to determine the distance, with no explanations as to how the formula was derived, or an activity that would make them discover it on their own. There was no justification for why the x and y variables were used.

The presentation did not allow creativity on the part of learners, who were expected to memorise the distance formula without understanding it. Ms Mokoena did not capitalise on the social capital that enables learners to share ideas among themselves, in the form of peer teaching, which allows learners to discover meanings on their own, and by using linguistic capital, that will make them to express themselves freely in understanding how the distance and the length of AB relate. The teacher suppressed the learners' self-discoveries, which would have helped them to conceptualise the distance formula. Relating this to their daily life events as a convenient way to utilise navigational capital would recognise the prior knowledge they acquired from the home environment.

Again, from the extract, it was evident that the teacher was using her power to enforce rote learning of problem-solving skills formulae. Comments such as '…..it's a formula, that you have to know it…'. Ignored Tshepiso's search for an alternative way to get the distance, demonstrating that she wanted to instil hope that as learners they could succeed in problem-solving skills.

4.2.4.2 Class activities

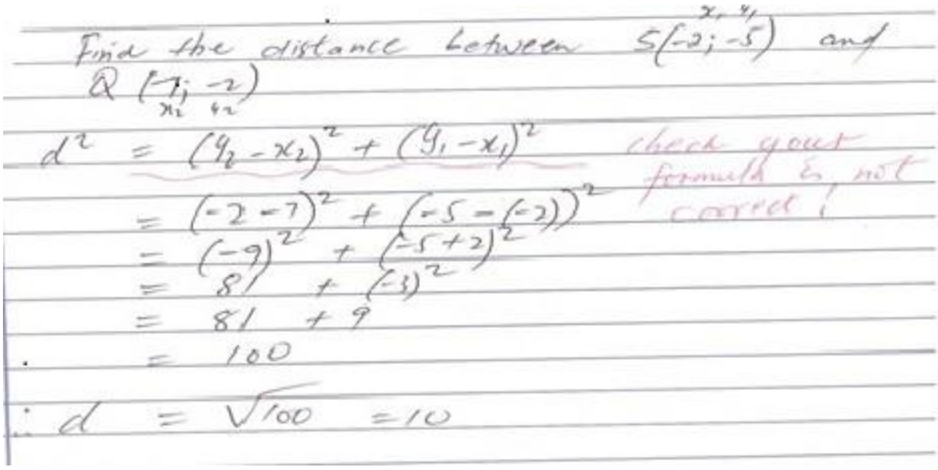
It is expected that class activities need to be learner-centred, with the teacher giving learners freedom to reflect among themselves (DoE, 2003:2; Gonzalez et al., 2013:28). Yosso's theory on community cultural wealth acknowledged that there is huge potential for this.

Below is the work of two learners, Zodwa and Moeketsi, who attempted the homework problems:

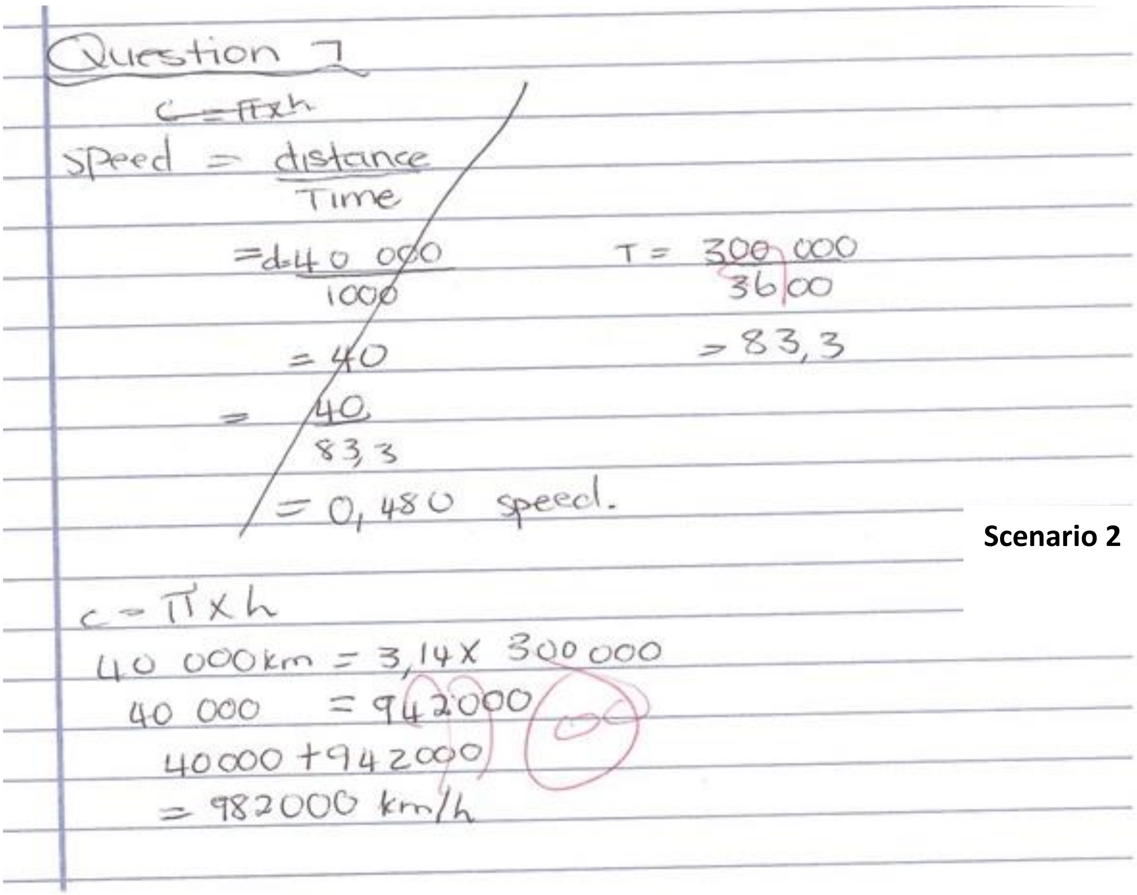
(a) Use the distance formula to find the distance between $S(-2,-5)$ and $Q(7,-2)$.

(b) Light travels at 300 000 km/s. How many times would a beam of light go around the erath in one second if it is given that the circumference of the erath is 40 000 km ?

Scenario 1 showed the workings of Zodwa and scenario 2 the answers of Moeketsi.



Scenario 1



Scenario 2

The two scenarios above demonstrate that not enough class activities were assigned to learners for them to acquire or use problem-solving skills. The navigational skills that could assist them in finding other alternatives to reach the correct answers were suppressed by drilling ready-made problem-solving skills and with the formulae. The learners lacked understanding of the formulae and the way they were presented followed no logic, hence the learners failed to use them appropriately. In the distance formula, learners did not write x and y values in an orderly way, where one has to start with y values and ended up with x values. The incorrectly memorised formulae led the learners to arrive at incorrect solutions. Their workings showed that they did not have hope that they would succeed in mastering the problem-solving skills formulae. The teacher did not use the classroom environment as a space in which class activities (on distance, speed formula) could be discussed in a small group to construct meaning and understanding. From the extract it was evident that the teacher did not use various modes of teaching, which showed that the prior knowledge of the learners was not fully used.

In conclusion, the empirical data agreed with the literature findings that a teacher-centred approach makes it very difficult for learners to comprehend formulae on problem-solving skills. These formulae were given outside the context of the child, which makes it difficult to relate to their background environment. The same problem on homework is complex, which can be simplified if it links to the immediate environment of the child (Asante, Mereku, 2012:23; Olatunde Yara, Omondi Otieno, 2010:126; Sriraman et al., 2007:152).

4.2.5 Non-involvement of parents

Many educational legislative imperatives and policy directives of the democratic South African State dictate that schools should be integral parts of the communities within which they occur(Averil et al., 2009:169; Chin, 2012:65,66). The structures within public schools must change to empower teachers, teacher mentors, and community members in the teaching and learning of problem-solving skills. According to Matthews, Watego et al. (2005:6), this will make the schools part of their communities rather than separate entities. This is further demonstrated by the stipulations of the South African Schools' Act (RSA, 1996:11,12) which emphasise involvement and cooperation in the teaching and learning of problem-solving skills. This deconstructs apartheid educational policies with regard to problem-solving skills, which separated schools from communities under the pretext that communities were not literate enough to inform academic practices in schools (Kim, 2009:80.82). There was a deliberate alienation of the subaltern communities in the teaching and learning of problem-solving skills. As Rocha-Schmid (2010:346; Lynn, 2004:154) posits, schools should not be observed as reproducers of unequal social power relations, but rather as institutions that encourage parents in the teaching and learning of problem-solving skills.

4.2.5.1 Lesson planning

Parents can make a contribution to designing the lesson plan that will help to engage learners throughout their learning. The familial capital that parents have can be used to contextualise the learning of problem-solving. The homework environment with which the learners are familiar can easily be brought into the teaching and learning of problem-solving which they find difficult to comprehend. For instances, if the teacher is planning alone it might be that he or she does not address the homework environment extensively, but rather with assistance.

For example, during the first session, when the team were recruiting parents to become participants in the study, the following conversation ensued:

> MmaTumelo: *a ko mphetise mona ngwanaka, jwale rona boholo ba batswadi ha re sa fihlela boemo bo hodimo ba dithuto tseo tsa lona, tsa boemo bo phahameng, re tla kgona jwang ho nka karolo projekteng na*. (Explain this to me my child, the majority of us, as parents, did not study up to your level of education, how is it going to be possible for us to take part in this project?)

The words of concern by MmaTumelo show that parents have accepted an unjustifiable excuse to be alienated from the teaching and learning of problem-solving skills, because they did not acquire formal education. In addition, they show the repercussions of past injustices, such as separating public schools from their communities. People lost confidence in themselves and the previous education system did not empower subaltern communities or give them a voice (Matthews et al., 2005:2) in the teaching and learning of problem-solving skills to their children. This text signifies the acceptance of unethical and unjust social power (Cameron, de Leeuw & Greenwood, 2012:181; Hakimeh Saghaye-Biria 2012:509; Van Dijk, 2009:63).

Using CDA, it was evident that Mma Tumelo's rhetorical question, "*how is it going to be possible for us to take part in this* project?" reveals a belief that those who have formal education have more power than those with non-formal education. The social injustices in the education system give more authority in the teaching of mathematics to people who have acquired formal education in the subject. The theory of PAR is used to indicate that the excluded communities (the likes of Mma Tumelo) need to be empowered and given authority and access in the teaching of problem-solving skills. Marginalised societies will be unchained if problem-solving skills are viewed as socially embedded and socially accountable, to differently engage those who are so often distant in the teaching and learning process. This confirms Cameron et al. (2012:182) and Ruthven (2001:355), that in order to acculturate the teaching and learning of problem-solving skills we need to bring in the 'everyday', historical and community cultures into the classroom.

The extract shows that parents do not see hope that they can be of great assistance in the teaching of problem-solving skills. The utterances of Mma Tumelo ' .. *re tla kgona jwang ho nka karolo* …') ('… how is it going to be possible for us to take part…'), supresses the resistance wealth they have, that against all odds they can be of great assistance in the designing a rich lesson plan, that is, one that can make learners enjoy the teaching and learning of problem-solving skills.

4.2.5.2 Lesson presentation

The involvement of parents in the teaching and learning of problem-solving skills makes it possible for learners to see the relevancy of problem-solving skills in daily life. The examples which can be made by parents will however help them to concretise the over-sophisticated concepts. The presentation and class interaction will be able to fit into broader learning, such as critical outcomes, which parents can demonstrate and cite examples from within the practical home environment of the child.

According to the school year plan, parents would be invited on quarterly basis to the school. They were briefed about learner performance in various subjects, the focus being on ‘killer subjects’, namely those in which learners were performing very badly, Mathematics included.

The mathematics teachers were given a platform on which to interact with parents about learner performance. One began the discussion:

> Mr Talana: Batswadi dumelang kaofela.Ka nnete re thabela boteng ba lona ho tlo shebisana ka tshebetso ya bana ba rona le lona thutong ena ya mmetse. (Good afternoon my dear parents. We are pleased that you manage to make it to this meeting to discuss the performance of learners in Mathematics.)

All parents showed appreciation and acknowledged comments:

> Mr Talana: *re kopa le thuse bana hae ka ho sheba hore ba etsa mosebetsi wa bona wa hae ka nepo, le ka botlala. Hoba re elellwa hore boholo ba bona ha ba etse di-homeweke tseo re ba fang tsona , ebe ba ba le mathata ha ba fihla ka phaposing tsa bona. Mosebetsi o wa di-homeweke ke wona oo re ba fang wona hape ha ba ngola diteko le dihlahlobo tsa bona*. (We humbly ask your assistance with regard to the homework given to learners, whether they are correctly done or completed. We are aware that most of the learners are not paying attention to such assessment tasks. In most cases for tests and examinations, we repeat the very same work but they do not perform well).

One parent commented as follows:

> Ntate Kgalapa: *Ke leboha sebaka seo le mphang sona mesuwe yaka. A ke le mpolelle mona. E be ha re itshunyake mosebetsing wa lona bomme titjhere le bo ntate titjhere, moo re tlo ferekanya bana teng.Jwalekaha rona re sa tsebe mmetse ona,na re tla kgona ho bona hore ba ntse ba etsa tsona ho ya kamoo le batlang mosebetsi ka teng na.* (Let me thank you for the opportunity afforded to me by my teachers. Can you clarify this for me: are we not interfering with your work, whereby we will end up confusing

> learners with their work? As parents, we do not have the understanding of mathematics like you.)
>
> Ms Mokena: Nka homeweke buka ya hae o tekene ho bontsha hore le wena o bone hore o ngotse, kapa mmotse hore o entse mosebetsi wohle oo a o filweng na. (Take the homework book and attach your signature as evidence that you did check the work or simply ask the child whether he or she has completed the work as assigned by the teacher)

The extracts show that parents see themselves as irrelevant sources with regard to assisting learners with problem-solving skills. As the question shows ('…*E be ha re itshunyake mosebetsing wa lona...'*)('… are we not interfering …?'), parents feel that they do not want to interfere with the work of teachers. They feel that they are of inferior social class compared to teachers. In addition, this is indication that parents marginalised the knowledge that have, as it is not equal to the knowledge teachers possess with respect to the presentation of problem-solving skills. This showed that they disrespect the knowledge they possessed, displaying no confidence in themselves.

On the other hand, the teachers failed to make them aware that they have potential in problem-solving skills. Instead, they limited their contributions and assistance to peripheral issues, merely appending their signature and asking whether they had completed their work as assigned. Teachers failed to convince parents that they were free from ignorance of problem-solving skills. They were supposed to give parents hope that the kind of knowledge they possessed could assist their children in the teaching of problem-solving skills to contextualise certain concepts, such that learners would understand these abstract concepts.

This confirms findings from the literature that there is still little that parents can do in the teaching of problem-solving skills. In most cases, they are only given governance and management issues. The curriculum matters, involving the teaching and learning of problem-solving are left to teachers, thus, as Eder (2007:279) points out, parents' knowledge of problem-solving skills is conveyed through social knowledge that is not relevant to the problem-solving skills offered in a Mathematics classroom.

4.2.6 Limited content knowledge among teachers

It is expected that teachers be lifelong learners, competent in their area of expertise (DoE, 2003:3, Tamsin, 2013:484,485). The qualified Mathematics teacher will have acquired content knowledge over the years, in their schooling and at tertiary level, so it is then important to attend mathematics developmental workshops conducted by departmental officials (mathematics subject advisors) and professional bodies. It is not enough to have content knowledge and pedagogical knowledge, but the latter is key to teaching and learning of problem-solving skills as it enables the individual teachers to use strategies that allow learners to interact freely, meaningfully and to extend knowledge of the learners to the limits, in the teaching and learning of problem-solving skills (DoE, 2003:5; Haverhals, 2010:334; Nutti, 2013:57; Pascuzzo, 2013:233).

4.2.6.1 Lesson plan does not show high order questions

The outline of the lesson plan needs to show the intentions of the teacher, as to what will transpire in the classroom on the teaching and learning of problem-solving skills. A lesson plan that portrays the subject content in a logical, coherent and meaningful way will be more likely to lead to a teacher achieving the lesson outcomes. The lesson plan and class presentation of the teacher should not be limited to giving learners formulae, but respond to questions such as why is the distance formula given in this format? and how can the formula be derived in a logical and meaningful way?

The teacher, Mr Debako was presenting the simplification of exponential laws in this fashion:

> Mr Debako: Let’s get these definitions as they will help us to understand the rules governing exponential laws:

- $a^n = a \times a \times a \times \dots\dots\dots. \times a \quad (n\ terms)$, ok, learners this definition will assist you to expand the terms such as $a^3 = a \times a \times a$

and $2^4 \quad = 2 \times 2 \times 2 \times 2$

- Ok, let us look at second definition as $a^0 \quad = 1 , if\ a \neq 0$, which can be used to work out problems of this nature $4^0 \quad = 1$,

- The third is $a^{-n} \quad = \frac{1}{a^n} , if\ a \neq 0$

Dineo: can I make sense of, $4^0 \quad = 1$, but will I be wrong if a say $4^0 \quad = 0$, because any number times zero is zero. How can it be equal to one?

Mr Debako: Look $4^0 \quad = 1$, we used the definition, and is not the same as $4^0 \quad \neq 0$

Teboho: But sir, why are they are they different? It does not make sense to me.

Mr Debako: You see, it's what the definition directs us to do.

Dimakotso: I agree with my fellow learners, why is the knowledge we were given in previous grades changes now, whereby any number times zero is zero?

Mr Debako: It's because here we are using a definition, I think you need to note that difference.

Teko: Are we correct to say any number times zero is one?

Mr Debako: that is not correct, sir.

The extract illustrates that Mr Debako did not have adequate knowledge of the exponential definitions that could help to clarify the misconceptions learners have with regard to $a^0 \quad = 1$, as compared to the incorrect argument of learners that $a^0 \quad = 0$. He gave learners shallow answers, which did not explain why $4^0 \quad = 1$, or why $4^0 \quad = 0$, are different. The inappropriate response to the learners detracted them from what they knew, and the teacher failed to illustrate how the definitions differed.

The teacher suppresses the quest of the learners to know by providing them with inadequate answers. These revealed that he lacked the pedagogical content knowledge on problem-solving skills. The answer given led them to memorise the definition without hearing a meaningful explanation.

Furthermore, the resistance capital and navigational capital demonstrated by learners, whereby exploring various possibilities to the answer and firmly believing that they are correct, was not used. The teacher used his powers to conclude that learners' analysis and interpretation was wrong. He ignored learners' multiple intelligences, as shown by the phrase '…it is not correct…').

4.2.6.2 Class activities, tests, assignments of low quality

All assessment tasks given to learners need to be the building blocks with which learners can acquire high knowledge and skills, enabling them to be independent thinkers and responsible citizens. For this to be possible, the kind of mathematics teacher envisaged is one who is a lifelong learner and researcher, and a designer materials that can be integrated in the teaching and learning of problem-solving skills.

The extract in section 4.2.6.1 '….it's like that… , it is the definition you need to know….' , did not challenge the critical thinking of the learners, rather their creative thinking was suppressed, forcing them to resort to rote learning. During his presentation, Mr Debako failed to clarify the misconception learners had with respect to why $4^0 \; = 1$, but not equalling zero. Learners are not given freedom to exhaust all their misunderstanding peacefully. Otherwise, the teacher tended to be very autocratic in forcing the answer on the learners.

The type of assessment tasks must accommodate various learning styles. The table 4.2 showed that learners are encouraged to memorise problem-solving concepts or formulas .The examination questions asked learners to recall answers without being engaged in analysis or high-order questioning. The manner in which the lesson was presented gave no evidence of thorough preparation having been made. Various alternatives on assessment and lesson presentation of concepts were not exploited. As a result of this kind of lesson presentation, assessment and tasks, the teacher as preventing learners from acquiring high knowledge. The assessment tasks designed by the teachers were one-sided; with focus on knowledge level one questions, which are more on memory and recalling type of questions. The learners were disempowered and not helped to be competent with high order questions, which require application, analysis, synthesis and evaluation.

Since learners' prior knowledge was not considered in the lesson presentations, assessment tasks (Table 4.3, below), this was seen as the marginalisation of the navigational, social capital they owned. These capitals help learners to interact among themselves and explore different avenues to find solutions. This caused them to lose hope in ever succeeding in problem-solving skills.

In summary, the evidence and literature reviewed leads to the conclusion that quality presentation dictates standardised assessment tasks. The lesson presentation encapsulates the mathematics knowledge for teaching. In such a teaching and learning environment, the needs of learners should be fully catered for.

4.2.7 Limited motivation among teachers

The working environments need to be conducive for sustainable teaching and learning to occur in the teaching and learning of problem-solving skills. Some of the elements that constitute conducive environment are interrelationships between teachers and other human beings, which are not limited to learners, teacher, parental community, and the employer. This interrelationship allows for collaborative approach in the teaching and learning of problem-solving skills, as opposed top-down and an autocratic power culture. Also, interrelationships between the teacher and resources for the curriculum might assist effective teaching and learning. It is then expected of teachers to demonstrate competencies of various roles and critical outcomes, such as a lifelong learner, in designing own materials for implementing new changes in the curriculum development (Norms and Standards, DoE, 2003:2). Demonstration of these roles will foster creativity in teachers, when they regard learning programmes as guides that inspire innovation and motivation in designing programmes.

Teachers are expected to belong to professional bodies that assist them in developing mathematics knowledge for teaching (Pascuzzo, 2010:233) and researching effective methods for teaching and learning. That would assist them to be the agent of change in transforming the bad conditions of teaching and learning in schools.

4.2.7.1 Lesson planning

Departmental formats of designing lesson plans are only guides to help teachers to see the broader picture of arriving at outcomes and Millennium Goals(2010:56).Outcomes can be the lesson outcomes focusing on the content delivered in class, and demonstration of critical outcomes, which looks into the kind of person the society is expecting after schooling years (De Jager & Nieuwenhuis, 2005:255,256; DoE, 2003:2; Van Niekerk & Killen,2000:90,93,98) is expected that a qualified individual can assist community members and globally to curtail the challenges they are faced with. For instances, how can the teaching of problem-solving skills assist in issues affecting global warming, access to education and reduction of poverty? The DoE (2003:2) added that problem-solving skills need not be seen in isolation from authentic situations but in understanding that the world is a set of related systems. In summary, the lesson plan should be understood as building blocks towards attaining critical outcomes, environmental and human rights issues, and Millennium goals.

According to the school departmental policy, every Friday mathematics teachers are expected to submit the weekly lesson plans to the Natural Sciences' HoD for control and monitoring purposes, and the following week he or she will conduct class visitations for lesson observations and randomly select Mathematics teachers. After controlling the lesson plans and class visits are completed, a meeting will be convened to provide feedback to all teachers under his or her supervision.

On the scheduled day, the HoD, Ms. Leribe, presented the report in this fashion, showing that classroom practices were not of the expected level. Lesson plans had not been made, or in some cases they were incomplete. The way the work was done by teachers showed they were demotivated in executing their duties and responsibilities as expected.

The extract below showed what transpired during meeting held by the HoD to give feedback to mathematics educators. Ms Leribe, the chairperson of the meeting commented as follows:

> Ms Leribe: Good afternoon, colleagues.
>
> Mathematics teachers (together acknowledging the greetings): Afternoon, mam.
>
> Ms Leribe: I really do not know where to start, I feel depressed about your work.
>
> Mathematics teachers: (all staring at her attentively):
>
> Ms Leribe: Colleagues, we need to be aware that the departmental prescripts, set only the minimums to be done, but as teachers you can go beyond that, but not below the prescripts and imperatives as stipulated in departmental policies. It is unacceptable for us not to do lessons plans. There are colleagues who presented the term one's lesson planning only in their portfolios, and we are about to end term two, before learners sit for mid-year exams. Then, one starts to wonder how do you teach our learners properly? Other colleagues do incomplete lessons plans, as you visit them in classes the lesson planned and the lesson presented do not correlate. The lesson outcomes do not speak to the lesson content presented. I expect you to draw a lesson plan that can be used by any mathematics teachers if you happen not to be at work. The other point is, execution of the lesson plan needs to talk to items you mentioned in the plan. There is this item CO [critical Outcomes] to be addressed, people just ticked the CO to be addressed in class but as you listen or observed the lesson presented there is no indication that such CO are attended to.

> Mr Debako: Thank you, chair for giving the chance to air my views, maybe I am one of the victims, who is depressing you.

All members in the meetings laughed.

> Mr Debako: eh, chair, as much as you are depressed, we are more frustrated about the unbearable workload on us, that is, there is lot of paperwork we have to do, as a result little on the teaching and learning of problem-solving skills is taking place in classes. The second point is, you are even frustrated by learners who do not care what you are doing class. Learners who do no complete the assessment tasks they are given, who do not want to cooperate in class. Also being expected to plan properly as if all is well. This is unrealistic situation, where we are being tormented.

> Mr Talana: Eh, thank you chair, I want to concur with the later speaker that being faced with the type of learners who are stubborn who are not willing to do the work as expected. According to my view lesson, plans will not make learners pass or understand problem-solving skills. For them to pass you have to repeat one concept for several weeks, before it is mastered. We are in the unfortunate situation in the teaching fraternity, where poor teachers are so discouraged of being overloaded with unnecessary paperwork. Very soon, you will be expecting us to submit the School Academic Performance Improvement Plan [SAPIP].

The extract above showed that there was no alignment between the lesson plans and the subject matter being presented by the teacher in classes. It is clear that the HoD was using top-down approach ('…..the Departmental (DoE) prescripts are …..') in managing her subordinates (Mathematics teachers). She was not talking of a collective plan from the school by Mathematics teachers which aimed at fulfilling the DoE's policies. The phrase '…..It is unacceptable for us not to……', shows that the HoD was aggressive when addressing her subordinates, which causes work anxiety among teachers and being depressed at the same time. Clearly, the extract showed lack of leadership skills from the HoD in picking up morale and work ethos of the teachers. She was not considering departmental policies to help her supervise her team of Mathematics teachers effectively, in the teaching and learning of problem-solving in classes. This confirms findings in in the Integrated Quality management Systems (IQMS), that the HoD or school leaders must acknowledge and recognise the hard work of teachers, and develop them after their shortcomings were collectively identified. Further, the excerpts ' …… there is lot of paperwork we have to do, as the results little on the teaching and learning of problem-solving skills is taking place in classes……', illustrates that there was no clearly articulated plan that showed Mathematics teachers how paperwork supports the teaching and learning of problem-solving skills in classes.

From the extract above, it can be argued that the HoD was showing an autocratic style and being very powerful in executing the duties of teaching and learning. In the extract, she frequently used 'you', which showed how she exercised her oppressive authority. Teachers were disempowered and denied freedom to air their views on how the teaching and learning could be improved to meet or go beyond the DoE's prescripts. The power and undemocratic management style displayed by the HoD might reduce the morale and work ethos of teachers. She failed to capitalise on navigational skills and aspirational capitals, which consider the vast experienced knowledges, which teachers possessed in the teaching and learning of problem-solving skills, classroom practices and management skills. The aspirational capital that teachers inherited from familial background could have been used to arouse the loyalty of teachers and work culture towards the teaching and learning of problem-solving.

On the other hand, the extract shows that the teachers viewed learners as problematic in the teaching and learning of problem-solving skills. Phrases such as '…. Stubborn learners, who do not want to work…', show that teachers do not understand how best learners can learn problem-solving skills effectively, so that class disturbances are minimised. Mathematics teachers do not use their creativity to emulate what other mathematics teachers are doing with regard to learner discipline, which happens to distract the teaching and learning of problem-solving. From the extract above it is evident that Mathematics teachers were not involved in various professional workshops, pedagogical content knowledge sessions done by the DoE or professional bodies, such as Association for Mathematics Educators of South Africa (AMESA). Such workshops could help equip them with classroom practices skills, and effective ways to present the content on teaching problem-solving skills. These workshops are platforms that are used to demonstrate how issues affecting society can be used, that is integrating critical outcomes, issues of human rights and global warming in the teaching and learning of problem-solving skills.

4.2.7.2 Class interaction is teacher dominated

It is expected that teachers make classrooms lively in the teaching of problem-solving skills. The teacher has to integrate issues with which learners are familiar. These issues should show progression (DoE, 2003:2) according to their level of complexity for learners to comprehend problem-solving skills. For instance, if one has taken human rights or social injustices, education and poverty (Chisholm, 2003:3,4), it is important to show how human rights impact on local society, national and international communities. This will bring the understanding that whatever one does in classroom in the teaching of problem-solving skills has an influence, even internationally.

South African learners are assessed internationally on Third International Mathematics and Science Study (TIMSS) and International for the Evaluation and Educational Achievement (IEA). The teacher has to understand that the kind of learner envisaged in the teaching and learning of problem-solving needs to be recognised as competent in international as well as local arenas (Kellaghan et al., 2009:118,119 ; SACMEQ Report 2010 and 2011 by Moloi & Chetty,2011:6,7). When Mathematics classroom situations are learner-centred they will help prepare learners to participate as responsible citizens in the life of local, national and global communities.

This is an example of the presentation on linear equations, which was presented to learners by teacher Debako:

> Mr Debako: A linear equation in one variable x is an equation that can be written in the standard form:$ax + b = 0$, where a and b are real numbers,

$a \neq 0$. A linear equation has exactly one solution. To see this, consider the following steps. (Remember $a \neq 0$.)

$ax + b = 0$ (Mr Debako: This is original equation)

$ax = -b$ (the teacher explains that you transpose b to the right hand side)

$x = -\frac{b}{a}$ (the teacher, explains you divide both sides by a, and get the answer as $-\frac{b}{a}$)

Mr Debako: Hope we are together class, Can you try this example

$3x - 6 = 0$

Dineo: (Raised her hand): But, sir how does the first example relate with this one? I had you saying a and b are real numbers. I am lost here…

Puleng: And also, sir what do you mean all of us must get the same answer?

Mr Debako: Hey, class you are frustrating me, can't you see that a and b are represented by 3 and 6. This is very simple. (He continued to solve the problem on the chalkboard.) By one solution, I mean only value of b will be obtained as a solution. Do not confuse issues here.

The presentation showed that the teacher made the explanation alone. No learner interaction was seen. The phrase '….I am really lost…', showed that learners were not engaged, hence they did not follow the workings on the linear equation. Learners' voices were marginalised and suppressed in the teaching and learning of solving linear equations, as they did not see any connection between $ax + b = 0$ and $3x - 6 = 0$.

The comments displayed the anger and frustration the teacher had towards the teaching professional, in particular the teaching and learning of problem-solving. The arrogance and lack of interest in assisting the learners was re-surfacing in this lesson presentation on linear equations. This is illustrated by the utterances directed to the class, '….hey, class you are frustrating me….', these utterances further intimidate learners and dissuade them from interacting freely with their teacher. The kind of teaching mode used by Mr Debako was even frustrating himself, as it became difficult for learners to comprehend the presented content. At the same time, learners did not understand the subject matter as intended.

The presentation shows that he had all the power and answers to the questions posed by a few learners. He did not create a conducive space in which learners could share their inputs among themselves. He was not giving learners activities to work on in small groups, which might assist him to utilise the social capital and aspirational capital. Through engagement in their small groups there would be some of the learners who might provide better explanations on linear equations. In turn, team members would be encouraged to have hope that as they persisted to collaborate they would finally master these problem-solving skills.

As transpired in the HoD, some educators did not draw up lesson plans to inform them on what or how to handle the subject matter in class. Even the presentation above gave no indication of lesson outcomes to be achieved by learners. These lesson outcomes were not spelt out in the lesson plan or communicated to the learners. The manner in which the teacher was explaining the terms in linear equations did not consider the linguistic capital that learners possessed. This would have made it possible for them to connect their prior knowledge with the new knowledge.

4.2.7.3 Assessment based on low order questions

In aligning with assessment *for* learning and assessment *as* learning, the teacher has to consider knowledge, routine procedures, complex procedures, and problem-solving when assessing. Also, Bloom's taxonomy is relevant in making assessment tasks meet the variety of learners' needs and skills. This will constitute fair and balanced assessments that allow the teacher to gather evidence about learners' understanding and inform future learning. Further, it is noted that assessment is used to advance learners' learning, and to improve learners' achievement. It is crucial that assessment tasks conducted inform the teacher as to which teaching strategies to use to improve learner performance (Breed, 2012:29; DBE, 2011:53; Hudson, 2012:48).

In the research sites, these are sample questions which were assessed in tests, homework.

It is Wednesday afternoon; the department of Mathematics is holding the subject meeting. The main item on the agenda is the submission of the examinations paper for mid-year examinations.

The chairperson of the meeting is the Mathematics senior teacher, Mr Tshwene. He raises a serious concern of the late submission of examinations papers:

> Mr Tshwene: Colleagues, hope you are all aware that the mid-year examinations are commencing on the Friday of this week. Last week you were made aware to make changes on the moderated papers and resubmit them for finalisation last Monday. It is unfortunate that some did not honour the re-submission date. May I make a humble plea that before school out today I get those papers ready?
>
> Mr Talana: Mr Chairperson, this is not fair to be undemocratic about the submission date, but you are fully aware that some of us were busy compiling the School Academic Performance Improvement Plan and finalisation of semester marks. After all mathematics papers are written next week on Thursday. If we submit this week on Friday, we are not late. According to the examinations timetable, mathematics examination papers are written next week Thursday. Are you trying to frustrate us with this awkward submission dates?
>
> Ms. Mokoena: and really chairperson, be reasonable with your decisions. We deserve to be treated like professionals,
>
> Mr Tshwene: although you showed your dissatisfactions, you were required to set standardised and quality examinations papers in terms of the cognitive levels. Apparently, most of your questions are addressing low

order questions. Most of your questions are on level one, that is, knowledge. You cannot include the true or false questions. You also need learners to show the calculations, where they can express their argument. Objectives questions like True-False can only be five marks, if ever you included them. Let us be realistic with our standard of question papers. Remember parents, and outside world will doubt our quality of our work. We cannot just focus on the issue that you will mark quickly and hand-in marksheets on time. The quality of assessment is very important for the image of our school and quality the teaching of problem-solving skills. The other point you need to bear in mind, photocopying of *these papers need to be done on time, and be sorted out at least one or two days* in advanced for distribution by invigilators.

Mr Tshwene projected the slide of questions that were included in the mathematics examination papers.

Mr Tshwene (pointing at the slide on the screen): This is a sad situation; even the mark allocation is not appropriately done

Table 4.3: Sample of examination question paper

1.1. Which is the is the standard form for linear equation (4 marks)

A. $ax + b = 0$

B. $ax + b = y$

C. $ax^2 + b = 0$

D. $ax^3 + b = 0$

1.2 The distance formula is given as: (4 marks)

A. $\sqrt{(x - y)}$

B. $\sqrt{a^2 + b^2}$

C. $(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} +$

D. $None\, of\ the\, above\ is\ correct$

.

From the deliberations above, it is clear that Mathematics teachers wanted to rely on low order questions, which would help them to mark quickly. Their major complaint was that they were overloaded with work, so they wanted to compromise the standard of problem-solving skills by assessing low order questions only. These questions were on recall types, not application type questions, and analysis questions are asked. That is why the senior teacher, Mr Tshwene returned all their papers to effect necessary changes and include high order questions on problem-solving skills.

These types of questions shown above revealed how badly assessment was conducted during class interaction. Learners were disempowered from having or demonstrating high knowledge on problem-solving skills. These questions encouraged learners to regurgitate the raw information without deeper understanding. As the senior teacher indicated it would be a shame for the department of Mathematics and the school to produce such question papers, which could contribute to parents not having trust in the standard of teaching and learning of problem-solving skills.

The phrase ('….are you trying to frustrate us with this awkward submission date……'), shows that teachers had developed a negative attitude and low morale towards the school work with regard to effective assessing of problem-solving skills. This tendency of dissatisfactions seems to disadvantage learners on acquiring high knowledge and high skill on problem-solving skills. The teachers focussed on low order questions (that is true-or false type questions) which are easy to mark, and inflating marks allocation, where each question was allocated 4 marks. It is possible that a learner can correctly guess the correct answers; such learner will be given a biased feedback that he or she is competent in problem-solving skills.

In conclusion, Noyes, Sealey (2011:180) and Olatunde Yara, Omondi Otieno (2010:126) found that the low morale of teachers has a negative impact on the assessment and teaching and learning problem-solving skills. This is in line with the literature that individual teachers tend to use shortcuts in assessing and teaching problem-solving skills. This compromise the quality of assessment tasks which do not meet the expected levels.

4.2.8 Limited expertise with regard to classroom practices among teachers

The classroom practices are the best ways of imparting the subject content to learners in creative and interesting manner throughout the learning sessions. The teacher is expected to create expanded opportunities where there is excellent involvement of learners with different learning styles are fully supported. A learner-centred environments takes the teacher away from the front of the classroom and enables him or her to be among learners, helping individual learners or groups that are struggling (ELRC, 2003: H-48; Garcia & Sylvan, 2011:396).

Garcia and Sylvan (2011:398) argue that the mathematics teacher has to adopt a dynamic pluri-lingual approach, which use the individual learner's languages to act on learning, and make sense of learning moment by moment. Rather than languages being strictly "assigned" a space, time, place, or person in the curriculum. It is important for the teacher to display competency in using variety of tasks with varying levels of complexity to sustain the learner involvement in inclusive teaching and learning atmosphere.

4.2.8.1 Lesson planning lacks resources/activities

The crafting of the lesson plan should be a demonstration that the classroom culture is inclusive to all learners' needs. The prior knowledge is of paramount importance as it is an excellent means to assist indigent learners to be fully involved. As Yosso's theory suggests, areas or capitals can be considered to cater for learners' needs and prior background knowledge. Social capital is demonstrated by allowing learner involvement to dominate the class discussion. Learners share their learning experiences with others, in turn the teacher assists to bring connection with the new knowledge to be presented. Linguistic capital is brought to the surface where at some point in their discussions they use the home language practices to make sense of the academic content (Garcia & Sylvan, 2011:398) to engage one another.

Beane (2010:64) and Garcia and Sylvan (2011:399) assert that linguistic capital is a moving force in learning an additional language and all learning of problem-solving skills. The teacher would learn the value of having learners use their home language practices to support learning. Rather than being told what language to use when and where, teachers must practice noticing the learner as he or she is engaged in meaningful instructional activities. In this way, teachers can learn to adjust their language and instructional practices to support learners' linguistic and cognitive growth. Linguistic capital has the power to break the barriers to learning problem-solving skills and culture of silence, where learners are free to articulate their thoughts.

What becomes clear here is that resources and activities need not be far-fetched, but the ones inherited by the learners and teachers suffice for excellent planning to be in place. The cultural wealth demonstrates that learners from home environments already possess these capitals.

As evident in previous extract, the lesson planning lacked resources and activities for active involvement of learners. The conversation held above showed the teacher failed to design activities that would captivate the interest of learners to interrogate misconceptions such as a $a^n \neq a \times n$ and also to trouble the facts such as why is $a^0 = 1$. Instead the teacher stifled the class interaction by not giving the convincing argument why the two ($a^0 = 1$ and $a^0 = 0$) statements are not the same. Mr Debako continued saying '… use the definition…', the cause of disagreement is that learners do not make sense of the meaning of the definition they are repeatedly referred to. Mr Debako used the linguistic skill unfairly to supress learners' thinking. Learners were not properly engaged or answered as to why their reasoning that '….any number times zero is one…' is not correct. The teacher gave them short answers, which are not making meaning to learners.

The class of grade ten learners were given homework to complete. The next morning the teacher pointed to one to demonstrate it on the board. As Yilmaz (2011:206) argues, the teacher did not use instructional materials meaningfully by identifying learner's mental model. The learning processes used by the teacher did not provide relevant context for learning in order to activate an existing schema. This is evident by the responses, which were provided by learners in their homework books. The questions they need to respond to were as follows:

Subtract the following fraction:

(a) $5\frac{1}{10} - 3\frac{9}{10}$

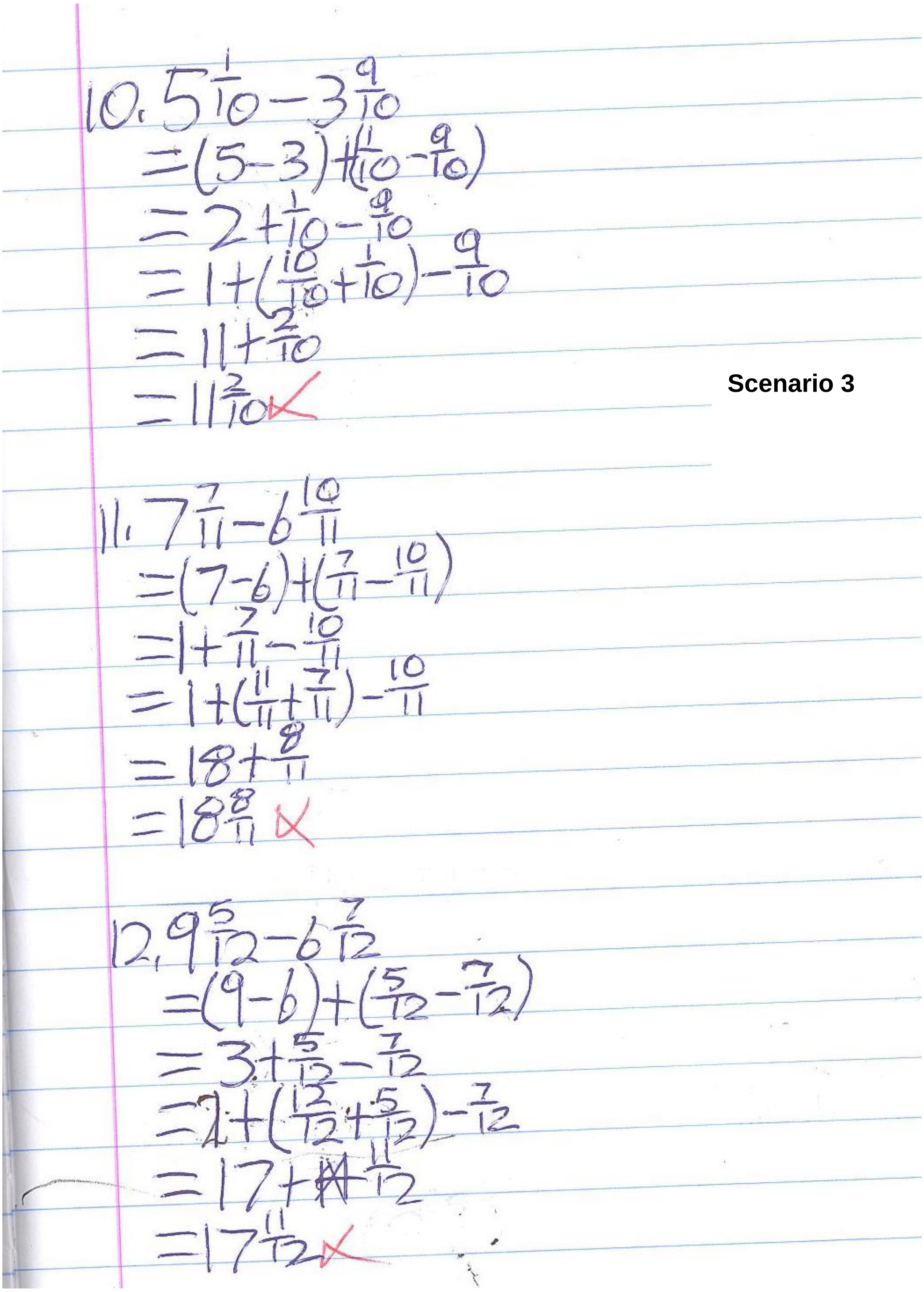
(b) $7\frac{7}{11} - 6\frac{10}{11}$

(c) $9\frac{5}{12} - 6\frac{7}{12}$

Most of the learners' responses were as follows, as shown in scenario 3.

The scenario 3 shows that the learning and teaching of problem-solving skills on simplification of mixed fractions was not well-received by learners. The teacher did not manage to create learning environments that encouraged learners to make connections with previously learnt material, using relevant examples. There were no teaching aids or materials used to foster understanding of subtracting mixed fractions. The learners were not engaged in how to develop rules that might assist them in reaching the answer, but rather they were made to memorise rules, which they happened to forget to apply correctly.

The learning environment created by the teacher did not make enough provision for distribution of resources to enable active involvement of the learners in the learning of simplification and subtraction of mixed fractions. All the responses were incorrect, which undermined hope in mastering the content presented.



10. $5\frac{1}{10} - 3\frac{9}{10}$
$= (5-3) + \left(\frac{1}{10} - \frac{9}{10}\right)$
$= 2 + \frac{1}{10} - \frac{9}{10}$
$= 1 + \left(\frac{10}{10} + \frac{1}{10}\right) - \frac{9}{10}$
$= 11 + \frac{2}{10}$
$= 11\frac{2}{10}$ ✗

11. $7\frac{7}{11} - 6\frac{10}{11}$
$= (7-6) + \left(\frac{7}{11} - \frac{10}{11}\right)$
$= 1 + \frac{7}{11} - \frac{10}{11}$
$= 1 + \left(\frac{11}{11} + \frac{7}{11}\right) - \frac{10}{11}$
$= 18 + \frac{8}{11}$
$= 18\frac{8}{11}$ ✗

12. $9\frac{5}{12} - 6\frac{7}{12}$
$= (9-6) + \left(\frac{5}{12} - \frac{7}{12}\right)$
$= 3 + \frac{5}{12} - \frac{7}{12}$
$= 2 + \left(\frac{12}{12} + \frac{5}{12}\right) - \frac{7}{12}$
$= 17 + \frac{11}{12}$
$= 17\frac{11}{12}$ ✗

Scenario 3

4.3 COMPONENTS OF THE STRATEGY IN THE TEACHING OF PROBLEM-SOLVING SKILLS

In section 4.2 numerous challenges were identified in the teaching of problem-solving skills. For each challenge I discussed the components of the solutions which our team formulated based on the actual classroom practices in an attempt to resolve and circumvent it. Consequently, the components of the strategy which respond to the challenges identified in section 4.2 are: learners are taught subject matter of problem-solving skills in such a way that they can understand it easily. Indigenous games will be used to concretise the problem-solving skills. On street corners one will see learners playing one type of indigenous games (such as hide-and-seek and many more), even before they attend school, and they are familiar with mathematical concepts. The second component of the solution will focus on how learners can be fully engaged during the teaching-learning sessions of problem-solving skills. As the indigenous games are used to contextualise the problem-solving, most of the learners are fond of playing and they know various ways for navigating through play for one or the team to be a winner. The third and fourth solutions focused on how we motivated teachers and learners so as to derive pleasure and interest in the teaching and learning of problem-solving skills, whereby both teachers and learners are committed to their duties and responsibilities in teaching and learning problem-solving skills. The fifth one concentrated on how learners can discover or formulate problem-solving skills formulae on their own or as team members. That would help them to solve the problem-solving skills sums in many ways with understanding. The sixth one looked into getting the parents involved in the teaching and learning of problem-solving skills. Parents can play a vital role in the teaching and learning of problem-solving skills. They possess a wealth of cultural knowledge, which the teaching of problem-solving skills will tap into. It is a way of validating the marginalised knowledge they possess in the mainstream curriculum. The seventh and the eighth solutions looked into how teachers can be helped to demonstrate high pedagogical content knowledge (Pascuzzo, 2010:233). It is not enough for teachers to have high content knowledge but the expertise with regard to classroom practices is also key for effective and quality of the teaching and learning of problem-solving skills

Each of these components to the solutions will be thoroughly discussed. References will be made from various theories of learning, policies, legislative imperatives and previous research findings based on literature reviewed that encourage good practices in the teaching and learning of problem-solving. These suggested good practices will be looked at against the practical evidence provided by research participants to overcome the identified problems. The practical evidence provided is in the form of texts by research participants, spoken words, pictures and scenarios. In getting the hidden and deeper meaning of the textual forms, CDA will be used to interpret the data. As the study is underpinned by community cultural wealth theory the evidence will be further analysed and interpreted within the context of the theory. Finally, the conclusion will be drawn against the literature reviewed.

4.3.1 Meaningful subject-matter in the teaching of problem-solving skills

According to Lynn (2004:154) and Yosso (2002:162) the teaching of problem-solving should draw on the strengths of learners nurtured from homes. The subject matter to be presented in mathematics class will then make sense to them. They argue that there is no deficit in the learners' language, culture and lived experiences, so it is important for the teachers to consider this capital wealth if the teaching and learning of problem-solving is to be simplified and be meaningful. Among others, the lived experiences of learners can include storytelling, indigenous games and family histories, but not limited to these few mentioned.

However, the DoE (2003:2), Haylock (2010:3) and Van de Walle (2010:13) contend that teaching of problem-solving must be learner-centred for learners to have access to mathematical content. This supports the ontological and epistemological stances of the community cultural wealth theory that knowledge and nature of reality do not reside within one powerful individual. Rather, there are multiple realities shaped by the set of multiple connections that human beings have with the environment, and nature of knowledge is subjective (Chilisa, 2010:40). The teacher should not take centre stage, trying to explain everything for learners. This relates to the view of Van de Walle (2010:13) citing Schifter and Fosnot (1993), that no matter how lucidly and patiently teachers explain the subject matter to their learners the truth is that they cannot understand for their learners. If the teacher explains everything to learners their potentialities are oppressed and marginalised. Hence, in the research the team members used indigenous games to teach problem-solving, to allow learners, teachers, and parents to unearth the mathematical content contained within these games.

4.3.1.1 Lesson preparation

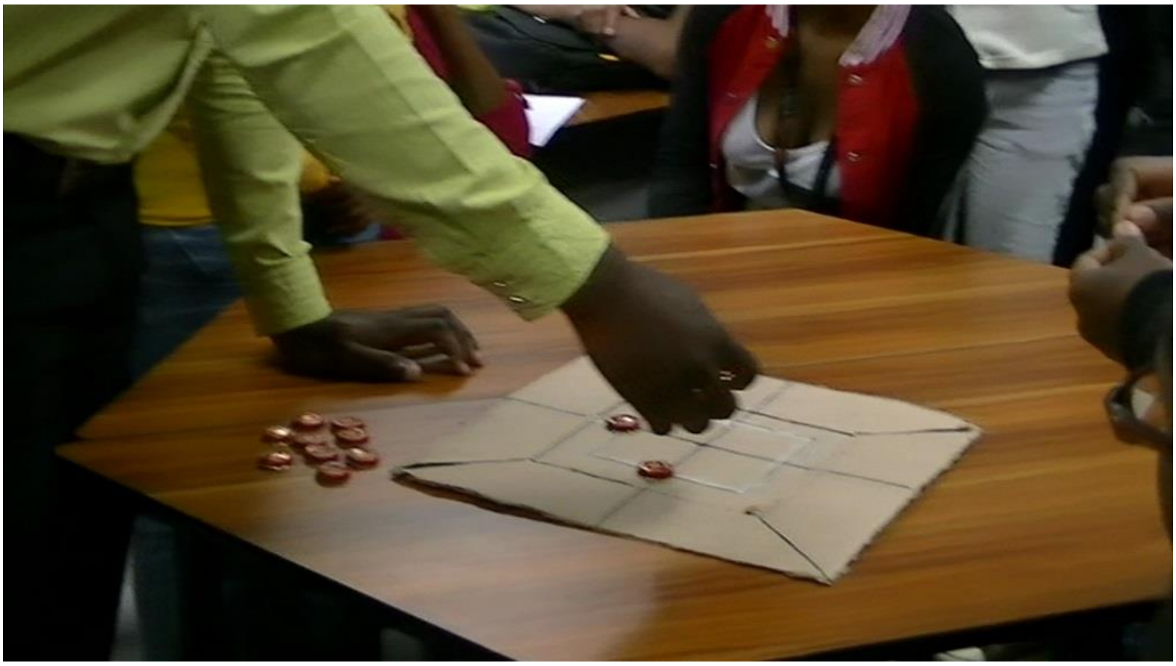
For learners to gain meaningful understanding in the mathematical content it is essential that in the lesson preparations they are included too. Such collaborative lesson preparation create learning environment that engage learners, while providing deep learning opportunities (Hamalainen, Manninen & Jarvel, 2006:47; Jorgensen & Lowrie, 2013:130). This is in agreement with what Muijis and Reynolds (2011:79) argue becomes easy for the teacher to teach from a learners' perspective, if learners and parents (Van de Walle, 2004) are involved at the preparation stage. This helps to avoid bringing in an unfamiliar context for learners in the learning of problem-solving. Also, it contributes to making problem-solving appear abstract for the learners. In addition, Lynn (2004:154) cited Delgado Bernal (1998, 2002) and Solórzano and Yosso (2002) contend that the lesson preparation involving learners assist in harmonising the relationship between domestic environment and mathematical knowledge taught in the classrooms.

In line with above trends, the study has adopted the approach in which various stakeholders took part in crafting the lesson preparation. The team in the study comprised Mathematics teachers, Life Orientation teachers, an HoD in Mathematical Sciences, deputy principal, traditional leaders, parents, district departmental officials (mathematics subject advisor and officials from Sports section) and Mathematics grade 10 learners. In making sure that the background of the learners would be taken into consideration when teaching problem-solving skills (Graven, 2009:31) the team came up with a plan of action *(see Appendix F1:Table 4.4)* that guided them to roll out the teaching of problem-solving skills using indigenous games.

The activities showed various indigenous games that were played in teaching problem-solving skills, such as number patterns, functions and geometry. This approach of planning helped the group to bring the learners' home background or real life situations into the classroom and so to make the problem-solving skills more easily understood by learners. The activity column had five phases or stages, which at times overlapped. For each phase there was proper collaborative planning beforehand to ensure that lesson outcomes would be reached as anticipated. For instance, if the team has decided to play *morabaraba* the people or small team to lead in playing the game would have to be organised in time. Tools, instruments and all resources needed were put together by the team. The worksheets (*refer to Table 4.5)* and observation sheets (*refer to Appendix F1 Table 4.14)* to be used had to be ready. All team members should be clear of the roles they had to play. As the lesson presentation is interactive, no fixed roles were assigned to members. Although there was one person leading the discussions or deliberations, every member was free to interject at any given point. The dominant roles played by teachers, experts, and parents were facilitating the small group formed by learners, in each of which was a teacher and/or parent, and/or mathematics subject advisor. They were there to monitor learners' progress and to do coaching, scaffolding, modelling, and reflection, only when necessary.

During the playing of the indigenous games it had to be very clear who would lead in the play and other team members were expected to use an observation sheet as they observed. This helped members to capture important issues that would be discussed during the reflection session.

After lesson preparation was smoothly handled by the team, phase 1 and phase 2 as reflected in the plan of Action were executed. For instance, Picture. 4.1 (below), shows that learners' and parents' teams were playing *morabaraba*, while other members of the team (spectators) were using observation sheets (Table 4.14: Appendix F1 and Figure 4.3) to write their observations. The observations ranged from the structural nature of the game and actual playing of it, whereby team members (learners in particular) had to mention any problem-solving skills embedded within the game played at that time. This helped the teacher to teach from learners' perspectives, and learners to discover and reflect on realistic experiences.



Pic.4.1: Parents and learners playing *morabaraba* (board game)

In phase 2, (referred to in *Table 4.4*: Appendix F1) reflections were made on the lesson learnt from playing the indigenous game, that is morabaraba. One group in class reported, with the report back captured on the observation sheet (refer to Figure 4.3), which had to address the main headings, such as the structural nature of the game and the actual playing of it. Under each heading they needed to specify any mathematical concepts, skills or knowledge observed, and mention any information they deemed fit to share with the whole class.

> Group 1*: rona ha re shebile straturale neitjha sa morabaraba, re bona rektengele e nyane, ho latele e kgolwanyane, e kgolo,(lebella Figure 4.2 ,Pic. 4.2 and Figure 4.3) jwalo-jwalo. rektengele tseo di entswe ka dilaene. Ha papadi e bapalwa re bona tsena, re beha dikgomo tsa rona ka ho fapanyetsana, o lokela ho nahana ka kelo hloko pele o beha kgomo ya hao, hore o tsebe o hlola enwa wa direng. O menahana mokgwa wa ho hlola papadi ena*
>
> (as we view the structural nature of *morabaraba* we see rectangle of various sizes, the big one, the bigger one, and the biggest one (see *Figure Figure 4.2, Pic. 4.2 and Figure 4.3).* These rectangles are made out of lines. On the actual playing of the game, these are apparent; we play by giving a chance to

each opponent to place his/her token cow on the board, you have to think strategically before you place the token cow on the board, so as to maximise the chances of winning the game, and also anticipate the movement that the opponent might take)

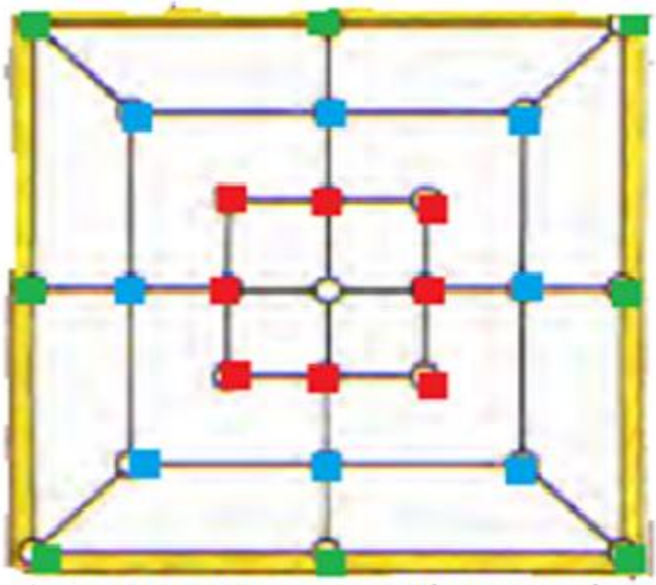
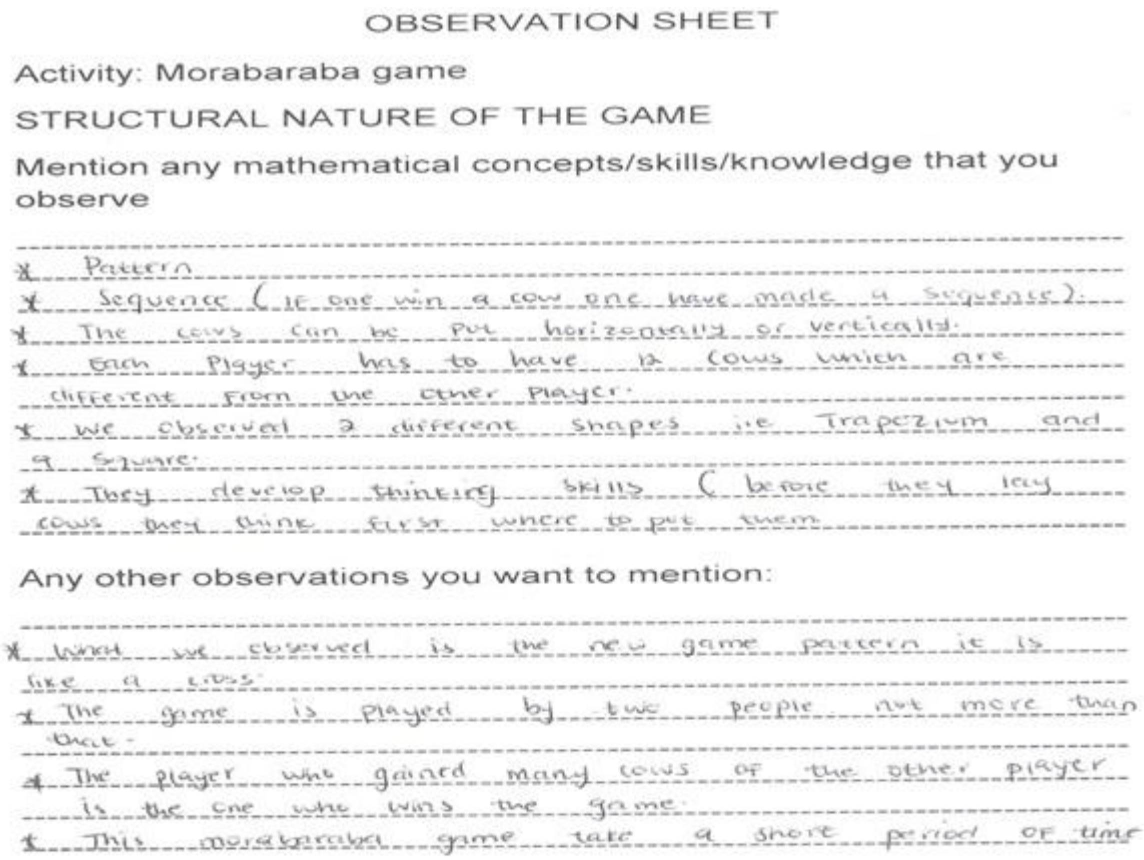


Figure 4.2 STRUCTURE OF THE BOARD GAME(MORABARABA)

Pic.4.2: Reporting back after their discussions



OBSERVATION SHEET

Activity: Morabaraba game

STRUCTURAL NATURE OF THE GAME

Mention any mathematical concepts/skills/knowledge that you observe

* Pattern
* Sequence (if one win a cow one have made a sequense).
* The cows can be put horizontally or vertically.
* Each player has to have 12 cows which are different from the other player.
* We observed a different shapes i.e Trapezium and a square.
* They develop thinking skills (before they lay cows they think first where to put them

Any other observations you want to mention:

* What we observed is the new game pattern it is like a cross.
* The game is played by two people not more than that.
* The player who gained many cows of the other player is the one who wins the game.
* This morabaraba game take a short period of time

Figure 4.3: responses from some learners

In the extract, it shows that learners were able to interact freely among themselves. The teacher had given them freedom to think freely of problem-solving skills embedded within the board game then they were able to design a lesson plan which included the problem-solving skills mentioned by learners. In this way it was flexible as one plan and accommodated the prior knowledge of learners.

Thus, the learners were empowered to decide on the content to be taught, and as such it was infused with the context with which they were familiar. Figure 4.3 above, showed the mathematics concepts or content mentioned by learners. As a result the teacher packaged the content raised by learners into the class activities to be presented. For example, in Figure. 4.3, learners mentioned patterns, then the teachers elaborated the concepts by showing that with *morabaraba* there were patterns, for example ascending or descending. In this way learners were given hope that the problem-solving skills they mentioned, such as sequences, rectangles, lines, and chances of winning the game, fell within the parameters of their syllabus. The playing of *morabaraba* resuscitated these concepts which they knew from their home experiences. As the teacher went into detail it was easy for learners to relate new knowledge to what they previously knew.

As learners or teams members interacted with their various groups to share their reflections they demonstrated that they possessed social capital. The navigational skills were illustrated as they made analysis and interpretations of the mathematical concepts or skills they observed from the playing of the game.

As the extract indicates ('…. *we see rectangle of various sizes, the big one, the bigger one, and the biggest one*…..'), what they conceptualised was true. They saw patterns or sequences of rectangular shapes, which differed according to size. The power of linguistic capital enabled learners to use words such as 'big', 'bigger' and 'biggest' to describe the pattern of rectangles they observed. The ability of linguistic skill helped them to understand mathematical concepts such as ascending order and descending order, and concentric geometric patterns. These are the problem-solving skills (algebraic expressions, equations, number patterns and geometry and analytical geometry) which are featured in the grade 10 Mathematics Curriculum and CAPS. Furthermore, they used linguistic capital to describe the shapes properly, revealing how the geometric figures were arranged in a particular sequence and explaining how these geometric figures were related. Mathematically, it shows that they realised that these geometric figures [see Figure 4.2] were ordered in concentric way.

In addition, the extract shows that learners were able to perform high cognitive skills of analysing and synthesis skills. The teaching and learning of problem-solving skills had to include such skills and assess them. From cultural wealth theory, learners already possessed these rich skills, but it is the responsibility of the teachers and community members at large to nurture and develop them further.

In conclusion, this argument agrees with the DoE (2003:29), Provasnik, Kasteberg, Ferraro, Lemanski, Roey, Jenkins (2012:1,4); TIMSS & PIRLS International Study Center (2009:24), accentuating that in the teaching of problem-solving; the reasoning skills such as analysing, selecting, synthesising, generalising and conjecturing are important skills for learners to have, but also important as life-skill for survival. The empirical evidence (SACMEQ Report by Moloi & Chetty, 2011:7; Su, Choi, Lee, Choi

& McAninch, 2013:2,3, quoted Baroody et al., 2004) evinced that the wealth of marginalised knowledge learners possessed enabled them to unearth and relate the problem-solving skills addressed by the CAPS. Problem-solving skills included shape, space and measurement, algebra, probability concept, and ways of placing the token cow on the board and the chances of winning the board game show the understanding of probability.

4.3.1.2 Lesson presentation

In the lesson preparation all members of the team were deeply involved. Learners were able to show that their prior knowledge (Su et al. (2013:2) citing Ausubel (1968), Linn, Eylon and Davis (2004) was intact. It links well with the content to be presented in grade 10 mathematical class as per departmental guidelines (DBE,2011:11-15), In the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) Report by Moloi and Chetty (2011:7), this allows learners to be hands-on rather than too dependent on textbook activities. This compels the teacher to plan the lesson in such a way that it is learner-centred and more activities are to be performed by groups rather than handled by the teacher alone.

The interesting point to be noted for planning as the team is that there was a clear integration (as illustrated in *Figure 4.4 below*) of problem-solving skills. These were taught in a logical way, which enhanced learner understanding. The use of *morabaraba* helped to contextualise the problem-solving skills and so make meaning from the immediate environment of the learner.

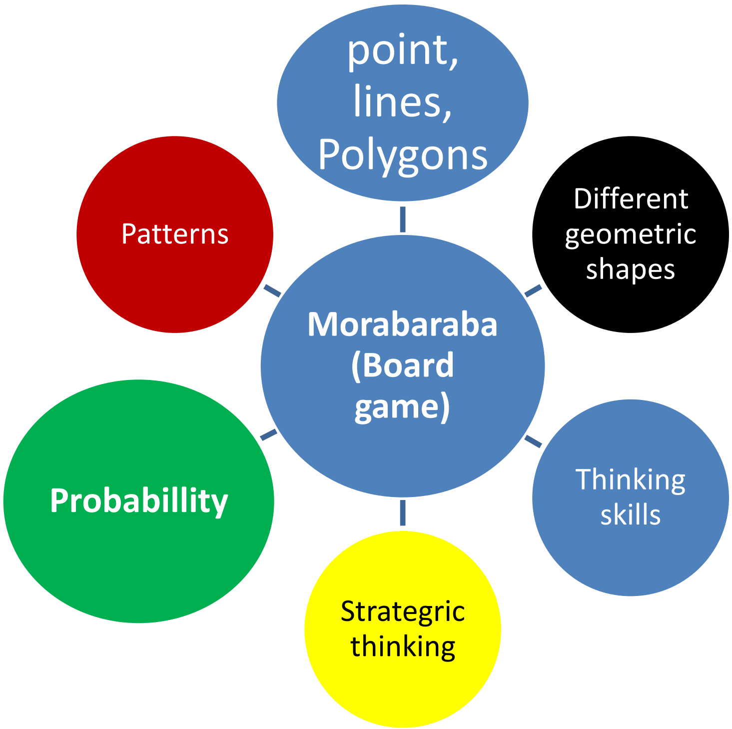


Figure 4.4: Some mathematical concepts which emanated from *morabaraba*

As the group members reflected, during phase 2 on the structural nature of the board game and actual playing of it, problem-solving skills and mathematics process skills emerged, as shown in *Figure 4.4 above*. These implied that the teaching of these problem-solving skills had helped learners to see the relevance of the subject matter in authentic situations. All group members were actively involved in the presentation.

Mr Debako issued the worksheets (Table 4.5: Appendix) to various groups, structured in the following manner:

WORKSHEET No. 1

1. Name all the shapes or figures (where corners are indicated in red, blue and green colours) in the board game.
2. To justify the answer in question 1 measure (in cm) the dimensions of the above shapes and compare your response with question 1.
3. How do the shapes or figures in question 2 relate?
4. Calculate the area covered by the shapes or figures and the perimeter of the shapes.
5. What deductions can you make from question 4?

Table: 4.5

Learners were given questions to which they had to respond to by referring Figure 4.2 above. As Pic 4.2 above. They were in small groups working on the activity. The activities were structured in such a way that learners had to explore various methods for them to reach the answer. Answers were not only from the teacher or the person presenting, but also arrived at through the class interaction.

Linda, the group leader from group B, presented solutions in this fashion:

> Linda: *we really enjoyed to work on the activity. Lots of answers came out, but finally we agreed on one solution. We will present our solutions there after we can take questions and comments from the floor.*

> Linda: *Question 1, the figures are squares, but after measurement were performed in question 2, we realised that the geometric figures showed are rectangles. Remember the properties of square and rectangles. That is what we provided as our motivation, and the table below:*

Figures	$\boldsymbol{Length\ x\ Breadth}$	$\boldsymbol{Perimeter}$	$\boldsymbol{Area}$
$figure1\ , where\ corners\ are\ in\ red$	$1{,}4\ cm\ x\ 1\ cm$	$4{,}8\ cm$	$1{,}4\ cm^2$
$figure1\ , where\ corners\ are\ in\ blue$	$2{,}6\ cm\ x\ 2\ cm$	$9{,}2\ cm$	$5{,}2\ cm^2$
$figure1\ , where\ corners\ are\ in\ green$	$4\ cm\ x\ 3\ cm$	$14\ cm$	$12\ cm^2$

Group B’s table of answers

After the questions and comments were entertained, group D was given a chance to give feedback on questions 3 and 4.

Tshepiso, the group leader took the platform to give their responses:

> Tshepiso: *Thank you group B, we really enjoyed your presentation, the excellent way you did the presentation. We hope our presentation will match yours.*

> Tshepiso: *Generally the breadths increase by* $1\ cm$ *every time. The length of the rectangle with blue corners is* $1{,}2\ cm$ *more than the length of the rectangle with red corners. The rectangle with green corners is* $1{,}4\ cm$ *more than the length of the rectangle with blue corners. Generally the pattern followed by the lengths can be described as follows:* $l_n = (0{,}1)n^2 + (0{,}9)n + 0{,}4$ *(where* l *indicates the lengths of rectangles and n indicates the number of rectangles).*

The general pattern of perimeter and area of the rectangles can be illustrated as follows: $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$. *(where P indicates the perimeter of the rectangles and n indicates the number of rectangles) and Area pattern is as follows:* $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ *(where A is the area of the rectangles and n is the number of rectangles).*

In Figure 4.4 above these are the mathematical concepts which are collated by the teachers, but these were mentioned by different groups of learners (illustrated by Figure 4.3: above. This shows that learners are empowered to determine concepts to be taught in class. The collated mathematical concepts (Figure 4.4) showed that the voices of learners are respected when deciding the content to be presented in class. This is an illustration that one is teaching from learners' perspectives. Also learners are empowered to be aware that integration of problem-solving skills concepts happens spontaneously, that is, they were aware that problem-solving skills do not occur as separate entities, rather Algebra, Euclidean geometry (shape, space and measurements), data handling and probability, and mathematical processes (e.g., critical thinking, communication) are linked.

Worksheet No 1 (Table 4.5) indicates that the class activities are learner-centred, with learners having to argue before reaching the decision on the answer. This worksheet No 1(class activity) encompasses the social capital which learners demonstrated excellently by freely sharing and networking on ideas to reach solutions. The phrase (' … we really enjoyed to work on the activity….') shows that there is joy and hope that they have a brighter future in the learning and comprehending problem-solving skills The aspirational capital they possess will definitely sustain them to maintain hope and dream for a brighter future. Even when they face real and perceived barriers in problem-solving skills they determined that they will overcome such obstacles. The navigational skills they have helped to manoeuvre through the patterns they observed in the class activity, they managed to come up with the general formulae like $l_n = (0{,}1)n^2 + (0{,}9)n + 0{,}4$, $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$ and $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ for lengths concentric rectangles, perimeters of concentric rectangles and areas of concentric rectangles in the board game, respectively.

The findings above agrees with Ewing (2013:135) in that learners recognise problem-solving skills such as pattern (repeating patterns), when they sing songs, dancing, learning how to weave and playing. Moreover, Su et al. (2013:2) citing Gagne, Briggs (1974) added that the preliminary or background knowledge that the learners possess are foundational to understanding more sophisticated problem-solving. The process of learning problem-solving skills is sustained if owned and framed by learners and school communities. Vankúš (2008:106) points out that if learners are actively involved in the learning of problem-solving skills they tend to develop self-reliance and become more creative .This is also evident when learners managed to discover the formula for the perimeter as $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$, given the rectangle with dimensions $1{,}4\ cm$ and $1\ cm$. Learners did not use the usual formula, that is $P = 2(l + b)$, which most of the textbooks used. This showed creativity and critical thinking that they had developed in learning problem-solving skills using indigenous games.

4.3.1.3 Assessment of activities

According to Kellaghan, Greaney and Murray (2009:52,53), the National Education Evaluation and Development Unit (2013:4) have shown that assessment needs to be made throughout the teaching and learning sessions of problem-solving skills. It should form an integral part of teaching and learning and help learners to be fully involved in the class discussion. As was shown in section 4.3.1.2, team members were free to interject at any point as the presentation progressed. The interjection can be in the form of follow-up clarification or posing a question that seeks clarity from others members on the floor. In most cases, when the question is asked it can be by any member of the team, whether learner, teacher, parent or any expert in the subject. The assessment types used can vary from formative, summative, baseline to diagnostic. These types are administered using methods and techniques such as self-assessment, peer assessment, group assessment, and teacher assessment. The individuals assessing activities or assessment tasks can use observation sheet, question and answer methods, class activities (classwork), homework, investigations, projects, assignment, tests and examinations.

In this solution, where the teaching of the content on problem-solving skills is made meaningful to learners, assessment by the team members was conducted throughout the lesson sessions. For instance, deciding on the indigenous games to be used in the teaching of problem-solving skills, learners were asked to propose the indigenous games to be used. They were asked to list indigenous games in the sheet (labelled Table. 4.15: Appendix F2). These games were not just imposed on the team but agreement was reached after the baseline assessment was made. According to the DoE (2003:6), baseline assessment is used to establish which indigenous games learners and team members already know and can do.

After the assessment, most of the team members agreed on the following ten indigenous games for the start: *morabaraba* (board game), *diketo* (coordination game), *kgati* (rope-jumping game), *ncuva* (board game), *melamu* (stick-fighting game), *malepa* (string game), *ditshomo* (story-telling), *dibeke* (*umabhorisha*), *Kho-kho*, and *jukskei* (target game).

In phase 2, the collective reflection is on the lesson learnt from playing indigenous games. The reflection is done within the context of infusing the problem-solving skills emanated from the play. As the Table. 4.14: Appendix F1 and Figure 4.3 above, showed, the reflections were written down by small groups on observation sheets. These reflections cut across many disciplines, but the main ones which were relevant to the team have to strongly speak to the mathematical concepts and skills embedded within the indigenous games. The completed observation sheets (Figure 4.3) showed that there are many mathematical concepts, skills and knowledges which are contained in the indigenous game, either on its structural nature of the game or the actual playing of it.

The discussions and interrogations on mathematical skill, concepts, and knowledges were then cited as to how they feature in the indigenous game performed. These are conversations which transpired during the collective reflections. In some instances team members would engage in the deliberations by asking questions for further clarifications or give comments to add to the pool of ideas.

One group wanted clarification on the cited mathematical concept which was observed as demonstrated by playing *morabaraba*. The question was posed by Ms. Ntswaki:

Ntswaki*: Mokgopi wa gropu D, ha le re lona le bona dipethene le ho nahana, a le ke le re hlalosetseng hore di bonahala jwang*

(Members of group D, you mentioned patterns and thinking, can you explain to us, how are they demonstrated in the play?)

The appeal about small group discussions before reporting or giving feedback to the larger group is that these answers were thoroughly discussed and all members took the ownership of answering if questions were asked seeking clarity. Voluntarily, any member can respond to the question, without being ordered to do so. This helped the team to bond well and members gained confidences to respond correctly to posed questions or give comments.

Thabiso responded to the posed question in this manner,

Thabiso: *re leboha sebaka seo re fuwang o arabela potsong e ntle e botsitsweng. Re tla hlokomela hore pele o beha kgomo ya hao fatshe o lokela ke ho nahana hantle hore lewa la hao le tla sebetsa hantle hore o tseba ho wina. Le ha o tsamaisa dikgomo tsa hao o lokela ke ho tseba hore wa direng o tlo mo tshwara kapa hona ho mo hlola ha a etsa eng. Ke hore o nahana ka pelenyana ho sheba hore ke tsela difeng tseo wa direng a tla dinka , jwale wena o tsebe ho khaonta -ekta jwang movu oo wa hae. Lebella ha dikgomo di behwa fatshe kapa di tsamaiswa re fana sebaka hantle ka tatelano. Motho ha o behe kgomo feela kapa wa tsamaisa dikgomo feela, o lokela ho nehana sebaka. Ke ka hona re buang ka pethene.*

(We want to thank the opportunity given to us to respond to the well -thought question. You noted that as you place a token cow on the vertex of board or moving the token cows, whatever movement or placement of these token cows, a well-calculated move or placement should be performed. You do not just move or place the token cow without thinking because that will jeopardise your chances of winning the game. At the same time, you think very fast looking at the movement or the placement your opponent that might be taken and see how can you counter act that, so as to minimise his/her chances of winning. The pattern is shown when you orderly give turns on placing or moving the token cows, you don't just place or move the token cow haphazardly but it is systemically done. Hence we spoke of patterns.)

Even during phase 3, lesson presentation, there were informal or formative assessments taking place within small groups and bigger group. When Tshepiso's group was presenting their solutions there were questions posed. For instance, one parent, Mr Lekau asked the question in this manner:

> Mr Lekau; *Thanks for the wonderful presentation. But I'm little bit lost here, how do the two formulas for the area of the rectangle differ, that is* $A_n = (0,1)n^3 + (0,9)n^2 + (0,4)n$ and $Area = l\, xb$

One member from Tshepiso's group responded in this way:

> Dineo: *Thanks for the accolades given to us, we really appreciate that as the group. To come to your questions, there is no much difference between the formulas. But what you must notice with the formulas is. Look at structural nature of morabaraba is, in essence there are three rectangles we identified. Moving on in a concentric way, let say you ended up having 1o rectangles. And you are asked to determine its area, then you can use this* $A_n = (0,1)n^3 + (0,9)n^2 + (0,4)n$ *, without dwelling into the process of measuring the sides. Because you calculating the area of the tenth rectangle, you just substitute n =10 in the formula and get your answer. On the other hand if you prefer to use the formula* $Area = l\, xb$*, then you must first get the dimensions of the tenth rectangle. Then you can see there are many ways than can used to get to the answer.*

In phase 4 and 5, that is reflection and evaluation on the lesson presented, instead of the teacher doing reflection in alone in our office the team did it together. Small groups met to reflect and evaluate the lesson presentation; thereafter the feedback was shared with a larger team. Every member of the team was given a chance to indicate the weaknesses or strong points of the lesson presented. Issues which teams members touched on varied from how the lesson was introduced, learner involvement in the activities, how effective teaching modes used for learners were in achieving the anticipated lesson outcomes, how diversity was handled, how learners with barriers to learning presented problem-solving skills in that lesson, and many interesting issues. These helped to inform the next lesson and how it was to be planned and structured for effectively teaching and learning of problem-solving skills.

One member, Pulane from group E reflected on and evaluated the presentation:

> Pulane: *We are so excited about today's presentation. We even learnt that there are various formulas to be used in calculating the perimeter of the rectangles and area of the rectangles. We really like the approach where we learn as collective. I did know that our peers, Life Orientation teachers, parents can be so helpful to broaden our understanding on the problem-solving skills –subject matter. We want to commit ourselves to teacher Debako, Talana to expect a hundred percent in the examinations*.

The Table 4.15: Appendix F2, assessment sheet, learners were given freedom to decide on the indigenous games they wanted to play during the teaching and learning problem-solving skills. Their choices of the games to be played in making content accessible to them were not oppressed but acknowledged. Learners used the familial capital to reach informed decisions on choice of the indigenous games, as some of these had been learnt from their parents, siblings and friends. Also, learners showed the navigational capital that helped them to discover alternative formulae for area, perimeter such as $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ and$P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$. They managed to navigate through the patterns they investigated to discover the alternative formulae. This indicated that learners are empowered to discover problem-solving skills on their own, rather than being given by their teacher. The social capital displayed by various group played a significant role in making it possible for learners to interact and network among the larger group in acquiring problem-solving skills with ease. As questions posed to different group members, for instance, ('…*you mentioned patterns and thinking, can you explain to us, how are they demonstrated in the morabaraba*…*?*'),the manner in which the questions was answered demonstrated a high level of linguistic capital and helped them to communicate their thoughts and ideas well.

The extracts by Thabiso and Dineo revealed that the navigational capital learners possessed allowed them to do critical thinking and deep analysis on problem-solving skills which are infused in the daily life of the learners. This helped them understand the subject-matter presented.

The observation sheet (Figure 4.3 and Table 4.14: Appendix F1) used demonstrated that teachers trusted that their assessment was of high standard. After Dineo had responded to the question: '……how do the two formulas for the area of the rectangle differ, that is $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ and $Area = l\,xb$…..', the explanation she gave to the class was well accepted. That means that the class (team) was capable of assessing that the responses given were correct or not correct. The remarks, such as ('….*thanks for the wonderful presentation*…') confirm the quality work done by learners, and are testimony that they were able to assess informally to indicate a good progress towards attaining lesson outcomes. Such remarks encouraged learners to work harder and gave them hope that there was a brighter future in understanding problem-solving skills.

The findings above concur with Ashton (2007:19), Nutty (2010:24) and Yosso (2005:79), who argue that navigational skills, social capital, and linguistic capital learners possess are critical in understanding abstract problem-solving skills offered in mathematics classes. For example, the navigational skills are evident even at very young age of children, when they are able to navigate the maps of their environments, such as map to the bathroom, path to the playground. Also, it is evident that the social home environments can be used effectively to teach problem-solving skills. The theory of community cultural wealth is enriched by the home environments. Consequently, home environments demonstrate the wealth of these capitals. Within the home environments, art is impossible with geometric shapes and lines, indigenous games and music without rhythmic patterns, and spatial movement without division. These cases amplify the richness of these capitals. This shows that learners learn various forms of content on problem-solving skills from everything that happens to them and in their immediate environments. Hence, in the teaching and learning of the so-called 'abstract content', it is apparent that these capitals need to be brought to the surface by locating the teaching and learning problem-solving skills within immediate environments of learners. That helped learners to be active and hands-on in the teaching of problem-solving skills.

In Addition, Ashton (2007:7) and Ewing (2013:131) argue that from home, as children acquire verbal language, parents must encourage them to relate play to mathematical language and concepts, as these capitals will be fully utilised when extracting mathematics concepts and skills in indigenous games.

4.3.2 Method of teaching problem-solving skills is learner-centred

The teaching modes that mathematics teachers use should incorporate the knowledge learners possess from the social environment with the knowledge they are about to learn in problem-solving skills. Thus, the teacher has to use various teaching methods to ensure that learners are actively involved in the learning of problem-solving skills (Assembly of Alaska Native Educators, 2003:7,10; Assembly of Alaska Native Educators, 1998:13; Kemp, 2006:34).

The study used indigenous games to teach problem-solving skills to ensure that learners were fully engaged. This concurs with Booker (2004:16) and Mosimege (2000:457) who maintain that the playing of these indigenous games should not be viewed from the narrow perspective of play, enjoyment and recreation, rather making certain that the learning of problem-solving skills is effective. Learners also show full participation during the lesson presentations. The use of indigenous game in teaching problem-solving skills contextualised the subject matter for learners to access it easily.

4.3.2.1 Lesson presentation

According to Kemp (2006:140), Mason (2010:6), Warren and Miller (2013:2) and Yosso (2005:80), the majority of learners enter school with a wealth of knowledge on mathematical processes such as critical thinking, analysis, navigational, reasoning, conjecturing and extrapolation skills. The teacher has to use the teaching methods that will activate these skills to be relevant in understanding problem-solving skills, whilst also encouraging learners to be actively involved throughout the learning sessions.

In phase 1, the team chose to play *diketo* (a coordination game) (refer to Pic.4.3 below). During phase 2, when the collective reflection was done .One of the group reported their findings of mathematical concepts, skills embedded within *diketo*. Teboho, spokepreson of the group, reported:

Teboho: re bona mokoti o rono, le majwe a chitja a tene. Ha o bapala o akgela keto hodimo o nto sutuletsa majwe ka mokoting. Ho seng one, kgetlo la pele o akgela keto hodimo, o ntano ntsha majwe a naene ka mokoting, jwalo-jwalo, ho fihlela o qeta seng one. Ha o qetile fetela seng two, jwalo jwalo ho latela pattern, ho fihlela o fihla seng 10.

(We see the round hole, ten small round stones and one big round stone (*ghoen*). In round one, you throw the *ghoen* into the air, at the same time you scoop or lift all the stones in the whole., and throw it in the air and place the nine round small stones, one follow this pattern until you complete round one and move on to round two, till you reach round ten).

Pic. 4.3: One Team member plays *diketo*

Subsequently, the team summarised the concepts and skills in this fashion (refer to Figure 4.5: below), The following concepts and skills came out strongly in most of the groups.

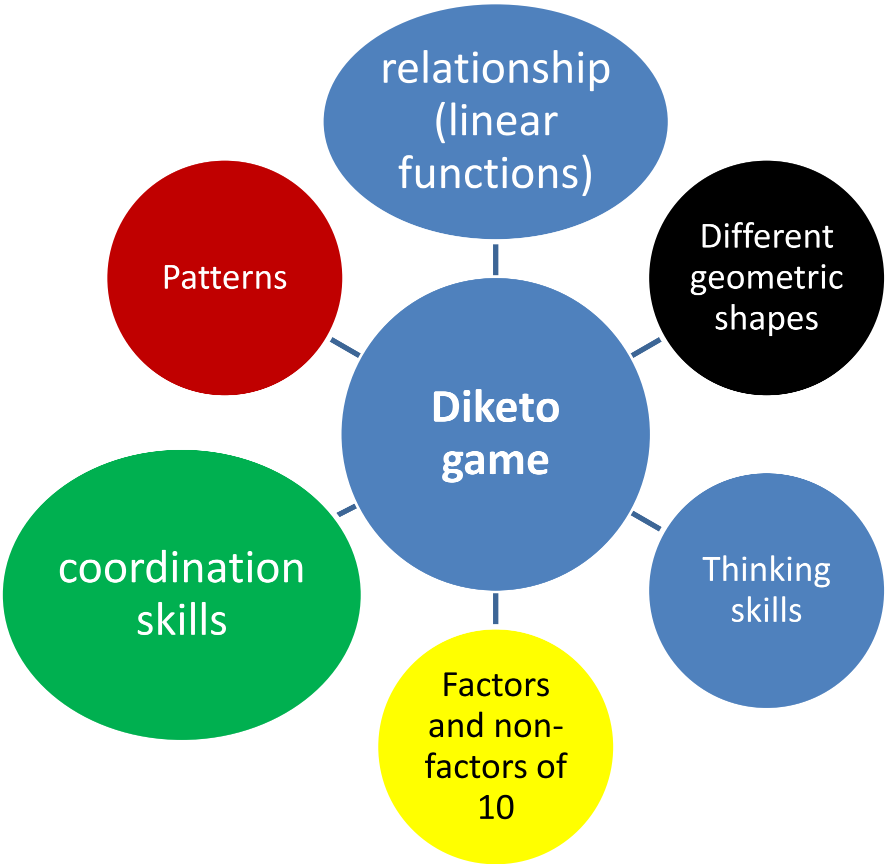


Figure 4.5: Some mathematical concepts which emanated from *diketo*

The teacher, Ms Mokoena who led the lesson presentation, took the patterns and linear functions (relationship) and the scenario painted by Teboho's group to design the class activity (Table 4.6: worksheet, below). This allowed various groups to be fully involved. The teachers, other experts and parents facilitated discussions in various groups. They gave assistances through empowering learners to discover and reflect on their own. The modes they used in assisting the groups varied from scaffolding, modelling, facilitating, coaching and reflection and any suitable ones. The emphasis of their teaching modes was on ensuring that learners discovered answers on their own.

WORKSHEET No. 2

Instructions:

With the help of the scenario, given by Teboho develop a strategy that will assist you to respond to the questions below:

(a) In the 2nd, 12th , 16th Throw-ins, how many stones will be pushed into the hole?
(b) In round one (seng one), how many throws of the ghoen(keto) need to be made to complete round one
(c) At which throws, will one scoops 5 stones, 3 stones out of the hole.
(d) If there is two stones in holes, how many throws were done
(e) (1st throw; 10), (2nd throw; 9)… (20th throw; 0). On the set of axes plot these points.
(f)(i)Suppose, that there are 20 stones to be used, on the 6th throw-in, how many stones will be put in the whole?
(f)(ii) In round one (seng one) given that there are 20 stones (diketwana), how many throws need to be done to complete round one successfully.

Table: 4.6

After 30 minutes, the teacher, Ms Mokoena allowed various groups to report back on their findings. The parent, Mma Tumelo, from group A, gave the feedback of their group:

> Mma Tumelo: Kgotsong banabeso. Re leboha sebaka seo re se fuwang ho fana ka dikarabo tsa akthivithi eo re e fuweng. Haeba o hloka tlhakisetso kapa tlatsetso o ka etsa jwalo neng kapa neng. Re ile ra bapala diketo hape hore re arabe ditsopo. Ra fumana hore ka kgetlo la bodedi majwe a kenang ka mokoting ke 9, 12^{th} ke majwe a 4, and 16^{th} ke ketwana tse 2.Potso ya bobedi, hore o qete seng one o lokela ho ba le makgetlo a 19.
>
> (Good morning colleagues. We want to thank the opportunity given to us to give feedback on our answers. If you need clarifications or you want to comment, feel free to do so at any point, my group members will assist to respond. To answer the questions, we demonstrated by playing *diketo* again. Through the play of *diketo* we deduce the following: for second throw, there are nine stones to be pushed into the whole, twelfth throw there are four stones and sixteenth throw there are two stones. The patterns can be followed in that way. To complete round one (*seng one*) there are nineteen throws to be made.

All present was interested in the manner in which the presentation was made. The other group gave alternative ways of answering the same question. Dineo, from group C, showed their workings in this fashion:

> Dineo: thank you chair, let me thank the previous presenters, that is group A for the wonderful manner they did their presentation. What we did in our group, we also demonstrated the play of *diketo* for few minutes, as we do that we recorded the throws done is this manner. The table below helped us to put our data in an orderly manner. It enabled us to respond to all the questions.

Throwing the *ghoen* up	No. of stones scooped out of the whole	Throwing the *ghoen* up	Number of stones placed into the hole
1st throw	$= 10 - \frac{(1-1)}{2} = 10 - \frac{0}{2} = 10$	2nd throw	$10 - \frac{(2)}{2} = 10 - 1 = 9$
3rd throw	$= 10 - \frac{(3-1)}{2} = 10 - \frac{2}{2} = 10 - 1 = 9$	4th throw	$10 - \frac{(4)}{2} = 10 - 2 = 8$
5th throw	…………………	6th throw	$10 - \frac{(6)}{2} = 10 - 3 = 7$
7th throw	…………………	8th throw	$10 - \frac{(8)}{2} = 10 - 4 = 6$
9th throw	…………………	10th throw	$10 - \frac{(10)}{2} = 10 - 5 = 5$
11th throw	…………………	12th throw	$10 - \frac{(12)}{2} = 10 - 6 = 4$
13th throw	…………………	14th throw	$10 - \frac{(14)}{2} = 10 - 7 = 3$
15th throw	…………………	16th throw	$10 - \frac{(16)}{2} = 10 - 8 = 2$
17th throw	…………………	18th throw	$10 - \frac{(18)}{2} = 10 - 9 = 6$
19th throw	$= 10 - \frac{(19-1)}{2} = 10 - \frac{18}{2} = 10 - 9 = 1$		$10 - \frac{(20)}{2} = 10 - 10 = 0$
$nth\ throw$	$= 10 - \frac{(n-1)}{2}$	$mth\ throw$	$= 10 - \frac{(m)}{2}$
	$f(n) = -\frac{n}{2} + \frac{21}{2}$		$f(m) = -\frac{m}{2} + 10$

> Dineo:Like the previous group, we used the table to respond to the questions, for second throw there are nine stones placed into the hole, the twelfth throw there are four stones to be placed in a hole and sixteenth throw there are two stones to be placed in a hole. The strategy of drawing the table really helped us a lot.

The extract '…*we see the round hole, ten small round stones and one big round stone (ghoen)…*.', illustrates that the learners were empowered to capacitate others on analysing *diketo* to demonstrate the existence of problem-solving skills in the game. Since learners were empowered to explain mathematical concepts and skills infused in problem-solving skills, that action boosted their self-esteem and confidence. This amplified by Dineo's comment: … $10 - \frac{(2)}{2} = 10 - 1 = 9$ …', as their alternative method, used to get the nine stones which were put in the hole for the second throw of *ghoen*. This is a demonstration that the power of teaching problem-solving skills is no more in the hands of teachers. Learners have gained power in understanding problem-solving skills in a simpler ways. For learners to come up with various ways of answering the class activity is helped by their possession of navigational capitals.

They further showed good interpersonal relations among team members when they honestly share their inputs with the rest of the class. The phrase '…*let me thank the previous presenters, that is, group A for the wonderful manner they did their presentation*…' shows that their interpersonal skills towards other members was good and they were at peace with one another. This is in line with Yosso's claim (2005:80) on aspirational capital, when they are capable of maintaining positive hope for the future. Everyone was given hope that they could manage to do well in problem-solving skills, irrespective of social class, age or gender.

Effective interactions and networking in understanding problem-solving skills is based on the social capitals of learners. Also, the richness of linguistic capital made them clear in explaining the structural and actual playing of the game. The manner in which they articulated the responses to the class activity was illuminating.

The arguments above relate to what the Assembly of Alaska Native Educators (2003:10) and D'Ambrosio (2009:93) (see 2.5.2.2.1) established. As the study confirms, they establish that the use of curricular and various instructional methods enhances the learners understanding of problem-solving skills. This encourages the learners to work together, not heavily depending on the teacher. Likewise, the use of cultural practices (such as indigenous games) in which the learners are situated helps to enrich and give meaning to what is being taught on problem-solving skills.

4.3.3 High motivation/interest among learners

In sustaining the high level of motivation among learners the teaching and learning of problem-solving skills has to build upon or strengthen what learners display in terms of motivation (Waege, 2009:84). For Olatunde Yara et al. (2010:127), learners, as they come to Mathematics class, already possess both intrinsic and extrinsic motivation, which they have within them and/or got through social interactions. The authors further assert that they should be motivated through various ways, which may include providing the required physical resources in problem-solving skills, and verbal encouragement from peers, teachers and parents and other stakeholders. Consequently, it becomes crucial that a mathematics classroom environment has to nurture and develop such motivations. This will assist in effective teaching and learning of problem-solving skills. Learners need to derive pleasure and interest from the teaching and learning of problem-solving skills. According to Morris, Matthews (2011:31); Mosimege (2000a:457,460, 2000b:11,13), as learners play indigenous games they show enjoyment and pleasure. As a result, the research team used the indigenous games to teach problem-solving skills, on the assumption that the interest, enjoyment and excitement displayed in the playing of the indigenous games would be restored as useful ingredients of effective teaching of problem-solving skills.

The teaching of problem-solving skills has to appeal to the interest of learners (Godino et al., 2007:8) and the teaching strategies of the teacher have to arouse learner's curiosity to discover new processes in problem-solving skills. They should also build learners' confidence in mastering problem-solving skills. This supports what Alexander, James (2005:16) and Van De Walle, Karp, Bay-Williams (2010:113) contend is the engagement of a variety of stakeholders (such as parents, community members and mathematics teachers) in enhancing and transforming learners' attitude towards problem-solving skills.

4.3.3.1 Class participation

During class participation, all stakeholders in the teaching and learning of problem-solving need be active. In particular, learners need to take a centre stage. Muijs and

Reynolds (2011:79) contend that learning of problem-solving must always be viewed as an active process (Campbell, 2006:9). Also, learning of problem-solving skills is about helping learners to construct their own meaning on mathematical content, rather than getting the correct answer without understanding the concept (Anthony & Walshaw, 2009:151,153). In addition, Campbell (2006:8,9) argues that the construction of meaningful or discovery of mathematical content is not an individual thing, but the learning needs to be understood as a socially constructed process, in which peers, teachers, parents and others interact. As they do so, emotions, interest and feeling towards the learning of problem-solving needs to be inculcated. Learning facilitation and coaching helps to boost the morale of struggling ones.

Consequently, the extrinsic motivation (Noyes and Sealey, 2011:180) that the teacher is exerting on learners to discover meanings on their own will help in instilling the interest and appreciation of mastering problem-solving. Taking advantage of affective domain components, such as values of sharing, developing self-esteem, confidence and caring for oneself and others (DoE, 2003:2) has a direct bearing on learners performing well in problem-solving skills. At the same time, this helps learners to avoid developing a phobia and anxiety toward problem-solving. Once the feeling of apprehension is there, learners feel demotivated to learn problem-solving skills.

In one lesson session all the groups presented shared feedback. In wrapping up the session as the way forward and highlighting important points for consideration, Ms Mokoena commented:.

> Ms. Mokoena: *It was really exciting and encouraging to listen to powerful presentation you did. Your presentation covered almost all what were supposed to do today. So many interesting points you raised and patterns derived from morabaraba. Points such as using the structural nature of morabaraba. Let us suppose that we have concentric rectangles of this nature* (see Table 4.7 below).

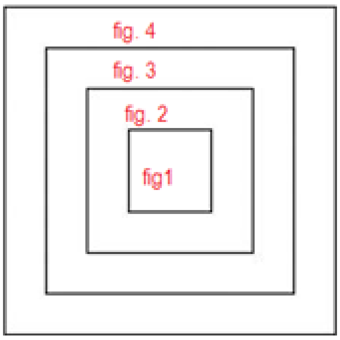


Table. 4.7: Concentric rectangles

> *For argument sake you are given figure one up to twenty of concentric rectangles, with the dimensions of the first three figures. Is it possible to calculate the area and perimeter of figure nineteen and figure twenty*.

Members of all various groups showed by nodding their heads, confirming that it is possible to calculate area and perimeter of such figures with the given data.

> Ms Mokoena: *Yes, basically these are types of questions that you are also expected to know and show understanding of*.

Mpho, a learner, commented:

> Mpho: *Yes, mam it is possible to calculate the area and perimeter. On behalf of the class, may we request you to provide us with scenarios problems of this nature. Then we can do them during our afternoon studies*.

Group leader form group E showed the problem scenarios they crafted from their group:

> Zanele: *Can we share what problem scenarios we visualised in the play of diketo. We discovered that functions like* $\boldsymbol{f(m) = -\frac{m}{2} + 10}$ *are classified also under pattern. This describe a relationship which is linear pattern. And general formula of this nature*, $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$, *describes the relationship which is a quadratic pattern*.

One parent, Mr Nhlapo sought clarity on certain issues:

> Mr Nhlapo: *Zanele and your group E, thanks for the demonstration you made. This is thought provoking and quite interesting. Tell me then, eh do you mean that functions are type of patterns?*

Eric from Group A, contributed to the discussion:

> Eric: *let me contribute by saying linear function* (like $\mathbf{f(m) = -\frac{m}{2} + 10}$), *is pattern which has been generalised from the Table 4.15: Appendix F2, then it is correct to say linear functions are special types of patterns. Then we can go on and deduce further, from morabaraba, we discovered that perimeter*: $P_n = (0,2)n^2 + (3,8)n + 0,8$, *and area*: $A_n = (0,1)n^3 + (0,9)n^2 + (0,4)n$. *Then, it is clear that quadratic functions and cubic functions are other types of patterns.*

Table 4.8: Patterns deduced from *diketo*

Throwing the *ghoen* up	**Number of stones placed in the hole**
2nd throw	$\mathbf{10 - \frac{(2)}{2} = 10 - 1 = 9}$
4th throw	$\mathbf{10 - \frac{(4)}{2} = 10 - 2 = 8}$
6th throw	$\mathbf{10 - \frac{(6)}{2} = 10 - 3 = 7}$
8th throw	$\mathbf{10 - \frac{(8)}{2} = 10 - 4 = 6}$
10th throw	$\mathbf{10 - \frac{(10)}{2} = 10 - 5 = 5}$
12th throw	$\mathbf{10 - \frac{(12)}{2} = 10 - 6 = 4}$
14th throw	$\mathbf{10 - \frac{(14)}{2} = 10 - 7 = 3}$
16th throw	$\mathbf{10 - \frac{(16)}{2} = 10 - 8 = 2}$
18th throw	$\mathbf{10 - \frac{(18)}{2} = 10 - 9 = 6}$
	$\mathbf{10 - \frac{(20)}{2} = 10 - 10 = 0}$
$\boldsymbol{mth\ throw}$	$\mathbf{= 10 - \frac{(m)}{2}}$
	$\boldsymbol{f(m) = -\frac{m}{2} + 10}$

Ms Mokoena remarked:

> Ms Mokoena: *Wow!, this is exciting indeed. We can develop these scenarios together. Let give ourselves a task where craft many of these scenario problems and we work in them collectively.*

The excerpts above revealed a high level of motivation displayed by learners in showing deeper meaning on problem-solving skills. This is supported by this phrase ‘

…. *linear function* (like $\mathbf{f(m)} = -\frac{\mathbf{m}}{\mathbf{2}}\mathbf{+10}$), *is pattern which has been generalised from the Table 4.8 above,….***'**. It illustrates that they are committed to work on problem-solving skills to craft their own problem scenarios. Ms Mokoena gave them problem scenarios to take note of. The problem scenarios given by Zanele's group '('….*and general formula of this nature* , $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$, *describes the relationship which is a quadratic pattern*….'), showed a high level of commitment towards understanding problem-solving skills. The tenacity, enquiring mind and believing in themselves displayed by Zanele and Eric's comments ('...*we discovered that functions like* $\boldsymbol{f(m)} = -\frac{\boldsymbol{m}}{\mathbf{2}}$ ***+ 10*** *are classified also under pattern….this describes a relationship which is linear pattern…')*, gave a clear explanation as to how patterns, relationships, and linear and quadratic functions are viewed as patterns and how they relate. The linguistic capital they portray helped them to establish the link between patterns, relationships, linear and quadratic functions. Also, it showed appropriately that indigenous games are imbued with limitless problem-solving skills.

This showed that learners were interested and motivated to learn problem-solving skills. The navigational capital helped them to come up with these deductions that patterns can be extended to relationships, functions. The phrase such as (Zanele: "…$P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$, *describes the relationship which is a quadratic pattern*…") (Nhlapo:"'…..*this is thought provoking and quite interesting…")* illustrated that they had enquiring minds and the aspirational skills such as courage and positive approach to learning problem-solving skills in different ways The team was so empowered that they had moved from an individualistic approach to collective empowerment or panoptic ideology, that is, whatever knowledge on problem-solving skills is not individual but shared by all team members. The contributions of Zanele and Mpho brought hope and fulfilment that problem-solving skills are easy to understand and derive pleasure from working on problem-solving skills.

In conclusion, the findings above support Ewing (2013:131-132), Tennant (2004:1), Troutman, Lichtenberg (2003:512), and Van De Walle et al (2010:107), that the use of indigenous games, brainteasers and mysteries boosts the learners motivation to perform and understand problem-solving skills. The practical context encourages them to stay focused and engaged throughout the learning sessions. The Learners who are highly motivated are hardworking and committed to assist others in understanding problem-solving skills. As they are more motivated and confident they do not show any anxiety towards problem-solving, but in most cases enjoy it. This is line with Ewing's (2013:135) argument that effective teaching of problem-solving skills, when learners are able to craft the problem scenarios, helps to build learners' confidence and self-esteem. They are free to articulate their views on what they have observed or discovered.

4.3.3.2 Learner performance on various assessment tasks

Campbell (2006:10) citing Mayer, Salovey, Caruso (2000) opines the motivated learners achieve good marks in their assessment activities and tasks. The affective domain components, such as courage, perseverance, confidence and self-esteem play a crucial role in the teaching of problem-solving skills. According to Keeley and Tobey (2011:5,11) argue that formative assessment should focus on the learners' progress, as it encourages continuous reflection on the teaching and learning problem-solving skills. Then, learners will be motivated to ask better questions, as their interest has been triggered.

Furthermore, it is important that the assessment techniques used should promote learning rather than measuring and reporting it. *Inter alia*, the assessment techniques are observations, problem scenarios, constructions, games, story-telling, journal reflections, panel discussions, demonstrations and research projects (McMillan, Venable, Varier, 2013:5). These help learners to consider alternative viewpoints and accommodate the learning styles of learners on learning problem-solving skills. Learners showed tolerance in understanding the multiple of realities in the learning of problem-solving (Campbell, 2006:7; Chilisa, 2012:36, 37; Kellaghan et al., 2009:141,142).

At the school under investigation, Mathematics teachers played a facilitating role. The lesson presentations were handled collective as the team, in particular the learners, did sterling work. The problem scenarios, reflection, panel discussions, and indigenous games played a great role in monitoring the progress made by learners with regard to problem-solving skills. Ms Mokoena requested them to craft problems scenarios which were collectively discussed in class.

Observations by group E, on the patterns portrayed by *diketo*, when on round two (seng 2) were led in the larger group by Napo, though others members commented by making additions and answering the questions from the floor:

> Napo: *It is pleasure to me to share our response to you in this manner. Considering the table 4.9 below. If one is playing diketo, on round two (seng 2), using one big round stone (ghoen) and ten small stones. We discovered that the general pattern of scooping the stones out of the hole is described by the pattern:* $f(x) = -x + 11$ *, following the observations on first, second etcetera throws of ghoen, thus what we have. In the same way, the general pattern of placing the stones back in the hole is described by* $f(x) = -x + 10$

Throwing the *ghoen* up	**Number of stones scooped out of the hole**	**Throwing the *ghoen* up**	**Number of placed in the hole**
1st throw	$= 10 - (1 - 1) = 10 - 0 = 10$	2nd Throw	$10 - 2$ =8
3rd throw	$= 10 - (3 - 1) = 10 - 2 = 8$	4th Throw	$10 - 4$ =6
5th throw	$= 10 - (5 - 1) = 10 - 4 = 6$	6th Throw	$10 - 6$ =4
7th Throw	$=10 - (7 - 1) = 10 - 6 = 4$	8th Throw	10 −8=2
9th Throw	$=10 - (9 - 1) = 10 - 8 = 2$	10th Throw	$10 - 10$ =0
	$f(x) = m - x + 1 = -x + 11$ (Note: m is the number of stones used, and x is the number of throws made.)		$f(x) = m - x = -x + 10$ (Note: m is the number of stones used, and x is the number of throws made.)

Table 4.9: Patterns extracted from *diketo* (round two)

After the presentation, all the team members applauded the discovery made and eloquent presentation. One member of the same group, Zanziba made the following contribution:

> Zanziba: *Thanks you colleagues let me just add few points on what my colleague has just said. It was interesting for us to realise that the pattern can*

further be explained in this fashion. Let consider the first two columns, that is: throwing of the ghoen up and number of stones scooped out of the hole. Check here, as the number of throws increases, what happens when the number of stones scooped out of the hole….?

The whole team answered by saying:

The team (in chorus): *the number of stones scooped out of the hole decreased.*

The faces of the members of the team portrayed amusement and appreciation, as there were smiles, indicating they were excited about the invention.

Zanziba: *Colleagues, thanks so much for listening to our presentation, we are also excited about this discoveries. If there are questions and comments about the presentation you are free to ask. My colleagues from the group will assist me to respond to your questions*.

Mr Tshabane raised his hand for comments

Zanziba: *Ok, let give mister Tshabane a chance to raise a comment or ask a question*

Mr Tshabane; *I really humbled, very excited to notice the huge work you did. In fact I do not have questions, but it is the contribution. Based on what you said on the relationship between throwing of the ghoen up and the number of stones scooped out of the hole, that illustrate mathematics concepts such as ratios and proportion. And interesting what you told us now, that as the number of throws increases, then the number of stones scooped out of the whole decreases, you have defined the mathematical concept" indirect proportion". Other problem-solving skills concepts are direct proportion.*

One member of group D, Ditlhare, let me also contribute in this manner:

Ditlhare: *It can be mentioned that if you play diketo, let me say starting from send 1 (round 1) up to seng 10 (round 10), that shows us the factors of 10, which are rounds 1, 2, 5, 10, because no remainders will be left out as you* *placed the stones back into the hole. Non factors of 10 are rounds 3, 4, 6, 7, 8, 9, there will be remainder left as you pushed stones into the hole*.

Mma Sebolelo, the parent, who is from group B, gave this contribution, although it showed another discipline:

> Mma Sebolelo: *I'm baffled by the deeper analysis that you are doing. Anyway let me ask will it be proper to say the 'indirect proportion' mentioned also signify 'demand and supply' scenario in Economics as the discipline?*

The district office, from Sport section, responded to the questioned posed by Mma Sebolelo:

> Ms Ntwa: *Thank you colleagues, it is just pleasing to see how our parents and the team at large can make a significant contribution in understanding problem-solving skills. In short, Me MmaSebolelo you are quite correct. Economist around here, you could further assist, like you mentioned earlier with stones, the analogy can even hold here in economics: 'as the price goes up, the quantity demanded goes down'*

The extracts above showed the high level of respect and acknowledgement by team members. For instance, the phrase '…*colleagues thanks so much for listening to our presentation, we are also excited*….' shows a spirit of cooperation, as team members respect and admire the inputs raised by others. Their social capital helped them to show social cohesion, team bonding and networking, whilst their interactions continued throughout the learning sessions. This is made possible by the linguistic capital which helped presenters to express mathematics concepts eloquently, and in such a way that content is simplified for everybody, irrespective of gender and age.

Consequently, the level of self-esteem and confidences was boosted. Members tended to be enthusiastic, that is, all felt free to contribute on their new discoveries or findings. The new discoveries demonstrated by the excerpts '…*we discovered that the general pattern of scooping the stones out of the hole is described by the pattern:* $f(x) = -x + 11$ *, following the observations on* 1^{st} *,* 2^{nd} *, etc throws of ghoen* …', this illustrated that members were empowered on problem-solving skills such that they could share it with confidence with others members.

The new findings and discoveries are made possible by the navigational capital and resilience capitals. From the identified patterns in *diketo* they were able to mention mathematics concepts such "…. *ratios, proportion*s, *indirect proportions, direct proportion, coordinates and factors*....'. These concepts constitute a large part of the problem-solving skills grade 10 learners are supposed to know. Learners and parents are given hope that there prior knowledge is validated, recognised and used to its full potential. The extracts convey the idea that the social transformation on the teaching of problem-solving is in place. This is shown by the contributions of Mma Sebolelo, as she understood problem-solving. She was empowering learners to realise that there is integration between problem-solving skills and other disciplines such as Economics and cultural practices, including indigenous games and people being formally disadvantaged to gain access to problem-solving

In conclusion, it is evident that the literature was validated. The DoE (2003:2) and Su et al. (2013:2) evince that critical outcomes learners have to demonstrate that the world can be seen as a set of related systems, such that problem-solving skills can be linked to other disciplines. In turn, learners will see the relevance of problem-solving skills to authentic life situations. This will help in developing enquiring minds and a wish to remain lifelong learners or researchers. Also, this will motivate them to conduct more research on problem-solving skills. All the problem scenarios learners mentioned were correctly solved, supporting the argument that when learners are motivated they happen to be top achiever (TIMSS report 2011, by Provasnik et al., 2012:3).

4.3.4 Self-discovering of problem-solving skills formulas and processes

In the teaching and learning of problem-solving skills, it is important to value, explore, and incorporate the home culture or environment. Contextualising mathematics classroom assist learners to be effective and creative in learning problem-solving (D'Ambrosio, 2009:94). Moreover, Booker (2005:49), D'Ambrosio, 2009:94), and Kemp (2006:133) demonstrate that problem-solving skills are not only about memorisation of set procedures and the solutions of routine problems, but the deeper understand of how and why the method is used to find the solutions. This indicates that the teaching and learning of problem-solving does not occur in vacuum. Learners are able to relate the content or subject matter with real life situations. As a result, this will encourage learners to ask challenging questions (Keeley & Tobey, 2011:5), and provide thoughtful responses. It suggests that these learners will be able to construct their own conjectures, which develop into valid conclusions or formulae.

It is significant that the learning activities were undertaken by the learners themselves. If they were always told or taught what to do their reflection and creativity would be diminished. In such instances, learners would come to depend on the teacher, henceforth such learners becomes passive (Campbell, 2006:2) in performing activities. The expectation is that learners must be able to devise alternative approaches to acquiring problem-solving skills.

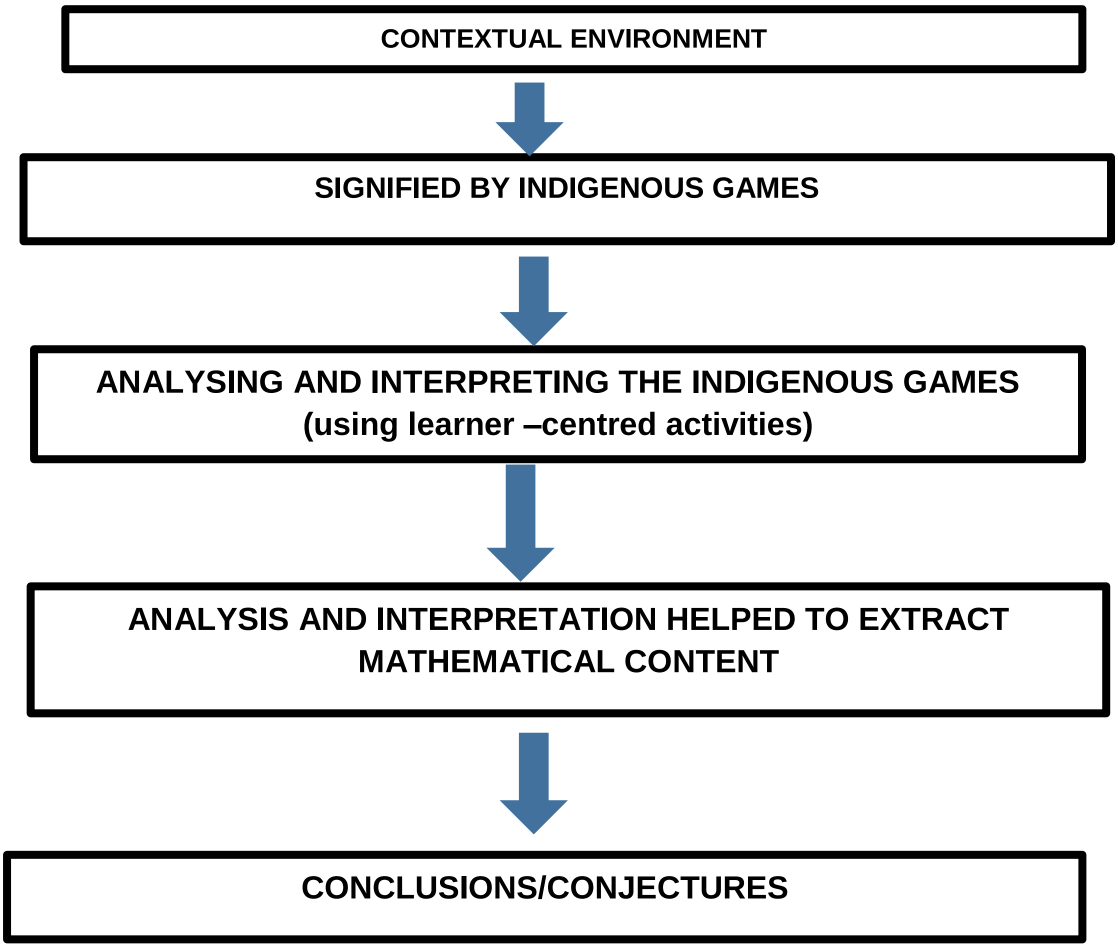
4.3.4.1 Lesson planning

The background environment makes it possible for learners to associate the prior knowledge with new information to be taught (Grootenberg, Sullivan, 2003:169; Meaney, Trinick and Fairhall, 2013:235). For the lesson to be learner-centred, from the beginning of planning phase up to end of the presentation, learners and parents need to be actively involved (Jansen & Blank, 2014:152,153). This will help make the lesson flexible and seen from learners' perspectives (Muijs & Reynolds, 2011:79). According to Kellaghan et al. (2009:1), continuous collective planning, should ensure that all learners take ownership of the lesson. It does not mean that the teachers are exonerated from their duties and responsibilities but rather that one empowers learners. Also, this will boost learners' motivation (Averill et al., 2009:173,181), thus improving their performance towards problem-solving.

At the site under investigation, the indigenous games were used to teach problem-solving skills. In the action Plan (Table 4.4: Appendix F1), there are indigenous games which are listed to use to teach problem-solving. Beforehand, the team met to decide on indigenous games to play on the scheduled date. This prior planning helped the group to identify the resources, tools, instruments, observation sheets and worksheets to be used, so that on the scheduled date everything needs is arranged. Even the small team that led in the playing of the games was in order, while others members observed the actual play.

For the lesson planning to be systematic and encompass all the necessary data, this is what transpired in the planning sessions. In the planning session, teams were given the template to complete (refer to Table 4.15: Appendix F2). The Fig 4.5 above (in section 4.3.1.1 above)**,** showed that through interpretation of the indigenous games listed on the first column of the Plan of Action, they managed to visualise the problem-solving skills such as patterns, functions, ratio and proportions. The social capital they possessed enabled them to interact well and communicate their findings convincingly using the linguistic capital. Also, the navigational capital ensured that they were able to understand concepts such as ratio and direct proportion. The relationship demonstrated by the *diketo*, (Table 4.8), was that ' …*as the number of throws increases, then the number of stones scooped out of the hole decreased'.* This is observation demonstrated that learners had been empowered to think critically.

The logic and sense displayed by the Table 4.8 above and Table 4.9 above helped them to contextualise problem-solving subject matter, and formulate meaningful conclusions or generalisations. It is summarised by the diagram below:



The social capital, which is deepened by contextual factors and/or environment such as indigenous games, empowered the learners to analyse and interpret the nature of games and actual playing. As the diagram above showed, the navigational capital allowed them to extract mathematical content, and thus make general conclusions (pattern/formulae) and further formulate definitions.

In summary, there is a correlation between the literature reviewed and the evidence in this section. According to Alexander and James (2005:17) and Godino et al. (2007:132), teaching problem-solving using indigenous games can be viewed as 'curriculum as an environment' or place-based education (PBE). These investigate the interconnectedness of sociality of individuals and cultural practices and problem-solving skills, in line with what Booker (2004:20), D'Ambrosio (2009:93), Seah (2009:484), Warren and Miller (2013:2,6) argue, namely that problem-solving skills and mathematical processes such as critical thinking, analysis, reasoning, conjecturing, and extrapolation are imbued within indigenous games.

Thus, the interaction between problem-solving skills and learner background (Assembly of Alaska Native Educators, 1998:3) is crucial as it gives learners deeper understanding of content (DoE, 2003:10). As a result, learners are able to make general conclusions from specifics. These processes of abstractions make it easier for participants to discover sophisticated formulae on their own. It is clear that learners were not just given the general formulae but managed to discover them. This further demonstrates the application of the onto-semiotic approach, the intensive-extensive dual dimension (Godino et al., 2007:132). Learners use several observations to justify the abstract processes.

4.3.4.2 Class activity

According to Muijs and Reynolds (2011:26), class activities on teaching and learning of problem-solving skills are to embrace different learning styles. To some extent, learners' needs are accommodated. Kolb's learning styles theory is applicable to the types of class activities conducted. In short, these learning styles touch on the activeness of learners. Concrete experiences assist learners in conceptualising abstract concepts. In general, these styles are centred round learner engagement. Learners throughout the learning sessions are active with hands-on activities, which allow for logical discovery of problem-solving skills formulae.

Hence, it is important that teachers and parents prepare a learning environment in which learners can discover (Vankúš, 2008:106) problem-solving skills formulae and procedures on their own. During class activity the teachers, parents and other experts in Mathematics need to guide and support learners towards new inventions. The teaching strategies to be used can include modelling, in which the teacher demonstrates a few steps and lets learners carry out the remaining activities on their own. Scaffolding and coaching provide support and encouragement to learners up to a certain level, after which the teacher withdraws (Lemmer, 2011; 101; Sacmeq Report by Moloi & Chetty, 2011:7).

On the research site, teaching of problem-solving skills is by using indigenous games. An exciting part of this process was that learners themselves extracted the problem-solving skills in the playing of the game, and were visualising of the structural nature of the game. In most cases they were able to formulate meaning of the concepts, then the teachers, parents and other experts in the team augmented where possible.

For instance, in section 4.3.3.2 above, the excerpt in which Zanele is quoted as saying '…*as the number of throws increases, then the number of stones scooped out of the whole decreases*…' illustrated that there was a pattern being observed, and she was able to draw a general conclusion. Other experts from the team augmented this by saying what was defined is the mathematical concept was 'indirect proportion'.

Learners were given class activities (worksheet 3 - Investigation 1: Table 4.10) below, which was structured in this fashion:

> Mr Tshabane: *On the activity task below [Investigation 1] projected on the screen, you are supposed to explore it and find out what other patterns can be observed and went on to make general conclusion, or come up with conventions.*

INVESTIGATION 1: On *Diketo*

Instructions: Explore various patterns within *diketo,* focusing on the table below. The patterns you discovered put them in general form or general conclusions or conventions.

Throwing the *ghoen* up	**Number of stones placed into the hole**
2nd throw	$10-\frac{(2)}{2}=10-1=9$
4th throw	$10-\frac{(4)}{2}=10-2=8$
6th throw	$10-\frac{(6)}{2}=10-3=7$
8th throw	$10-\frac{(8)}{2}=10-4=6$
10th throw	$10-\frac{(10)}{2}=10-5=5$
12th throw	$10-\frac{(12)}{2}=10-6=4$
14th throw	$10-\frac{(14)}{2}=10-7=3$
16th throw	$10-\frac{(16)}{2}=10-8=2$
18th throw	$10-\frac{(18)}{2}=10-9=6$
	$10-\frac{(20)}{2}=10-10=0$
$mth\ throw$	$=10-\frac{(m)}{2}$
	$f(m)=-\frac{m}{2}+10$

Table 4.10: Worksheet 3

Five different groups worked on the activities in each group, composed of learners, parents, teachers and other experts on the subject, mathematics. After 30 minutes they shared their observations with a larger team. Before the group presentations began, Mr Tshabane indicated the following points:

> Mr Tshabane: *Ok, we are now at the time where we want to reflect on the discussions you had. Let us share our discoveries with the whole team. Let us get any group volunteering to start*.

Members of group A, led by Ditlhare, volunteered to lead the discussions, and Picture 4.5: Appendix F3 with Ditlhare moving towards the front of the class to share their inputs:

> Ditlhare: *Good afternoon colleagues, we had a long of arguments, but finally we agreed on the following observations. Let us refer to Table 4.11 below .We discovered that as the number of throws increases, the number of stones placed into the hole decreases. This relationship is illustrated in column three. Further the relationship is described as number of throws times number of stones placed into the hole is equal to a pattern describing a general quadratic pattern.*
>
> *Then we concluded by saying let : number of throws be equal to* x *and number of stones placed into the hole is equal* y *, and pattern describing a general linear pattern be* k *, where* $k = -2n^2 + 20n$ *Thus, the relationship is described as* $xy = k$*.*
>
> Ditlhare: *For any questions that need clarifications, my group members will assist me to respond to your question.*

Throwing the ghoen up	Number of stones placed into the hole	Number of throws multiplied by number of stones
2nd throw	$10 - \frac{(2)}{2} = 10 - 1 = 9$	$2 \times 9 = 18$
4th throw	$10 - \frac{(4)}{2} = 10 - 2 = 8$	$4 \times 8 = 32$
6th throw	$10 - \frac{(6)}{2} = 10 - 3 = 7$	$6 \times 7 = 42$
8th throw	$10 - \frac{(8)}{2} = 10 - 4 = 6$	$8 \times 6 = 48$
10th throw	$10 - \frac{(10)}{2} = 10 - 5 = 5$	$10 \times 5 = 50$
12th throw	$10 - \frac{(12)}{2} = 10 - 6 = 4$	$12 \times 4 = 48$
14th throw	$10 - \frac{(14)}{2} = 10 - 7 = 3$	$14 \times 3 = 18$
16th throw	$10 - \frac{(16)}{2} = 10 - 8 = 2$	$16 \times 2 = 32$
18th throw	$10 - \frac{(18)}{2} = 10 - 9 = 1$	$18 \times 1 = 18$
	$10 - \frac{(20)}{2} = 10 - 10 = 0$	
$mth\ throw$	$= 10 - \frac{(m)}{2}$	$No. of\ throws\ be : x$ and $No. of\ stones\ placed\ into\ the\ hole : y$ $k\ is\ the\ product, where\ k = -2n^2 + 20n$ $where\ n = 1, 2, 3, 4$ $xy = k$
	$f(m) = -\frac{m}{2} + 10$	

Table 4.11: Patterns derived

Group D, led by Wetsi made their additions:

> Wetsi: *Thanks you ladies and gentlemen. I must commend group A for the wonderful work they shared with us. Well done. I want to thank our Mathematics teachers for us learning opportunities they created for us. I do not have questions, but to add on what you raised. This is what we noted, looking at the function we deduced, that is* $f(m) = -\frac{m}{2}$ ***+ 10,*** *where* m *is number throw the ghoen up, thus* m *is referred to as the 'Independent variable'. On the other hand,* $f(m)$ *the number of stones placed into the hole, as follows* $f(m)$ *is described as 'dependent variable.' I will request my group members to make additions.*

Joyce from the same group D, added the following points:

> Joyce: *Thanks, Wetsi, I think you said mouthful, let me point out to the definitions of the following terms: dependent variables: the occurrences of*

number of stones placed into the hole is influenced by the throwing of the ghoen into the air. Then the existences of the number of throwing the ghoen up is not influenced by any events, hence we labelled them as 'Independent variable'. Thank you so much. We will wait for the questions and comments.

Mr Tshabane, and other team members showed smiles on the faces, and Mr Tshabane commented as follows:

Mr Tshabane: *I really impressed with the feedback, you really touched on the most crucial problem-solving skills formulas. You mentioned $xy = k$, the of the function describing the "hyperbola'. In this linear function, $f(m) = -\frac{m}{2} + 10$, it is true that you identified independent and dependent variables. It can be noted that 10 is the constant, as the variable change, the constant is always fixed.*

Mr Talana: *Great work indeed, I am so motivated about the level of knowledge you are displaying of problem-solving skills formula. What you came up with, that is, $f(m) = -\frac{m}{2} + 10$ and $xy = k$ are the models that help you to predict the outcomes. 'Let us suppose, you are playing diketo, on round one. If someone has pushed 6 stones into the hole, can you tell which throw of the ghoen was complete?'*

Tsietsi, group leader of group C:

We can try it in this fashion sir, the answer can be read from the table, pushing six stones into the hole, one was performing the eighth throw of the ghoen.

Dimakatso came up with alternative method of getting the answer:

Dimakatsi: *We think of using the model (formula) we derived, like $f(m) = -\frac{m}{2} + 10$, since we know that $f(m)$ indicates number of stones pushed into the hole. Then in this case we are given $f(m) = 6$. We can substitute the 6 into the model and work out the throw which was completed.*

Dimakatso' s calculations as shown of the board

$6 = -\frac{m}{2} + 10$ (multiply the whole equation by 2)

$12 = -m + 20$

$m = 20 - 12 , then\, m = 8$

Thus, the logic is (showed the calculations on the chalkboard).

> Dimakatsi: *Therefore, it follows that the eighth throw of the ghoen into the air, there will be 6 stones pushed into the whole*.

One member of the team, Ms Putsi, the subject advisor for Mathematics, commented:

> Ms Putsi: *Wow, this is excellent. I am so delighted to notice the high standard of work performed in this class. My dear teachers and learners are you aware that you did integration of various problem-solving skills with this activity. To mention few, this is what we came up with* (see Fig. 4.6 below).

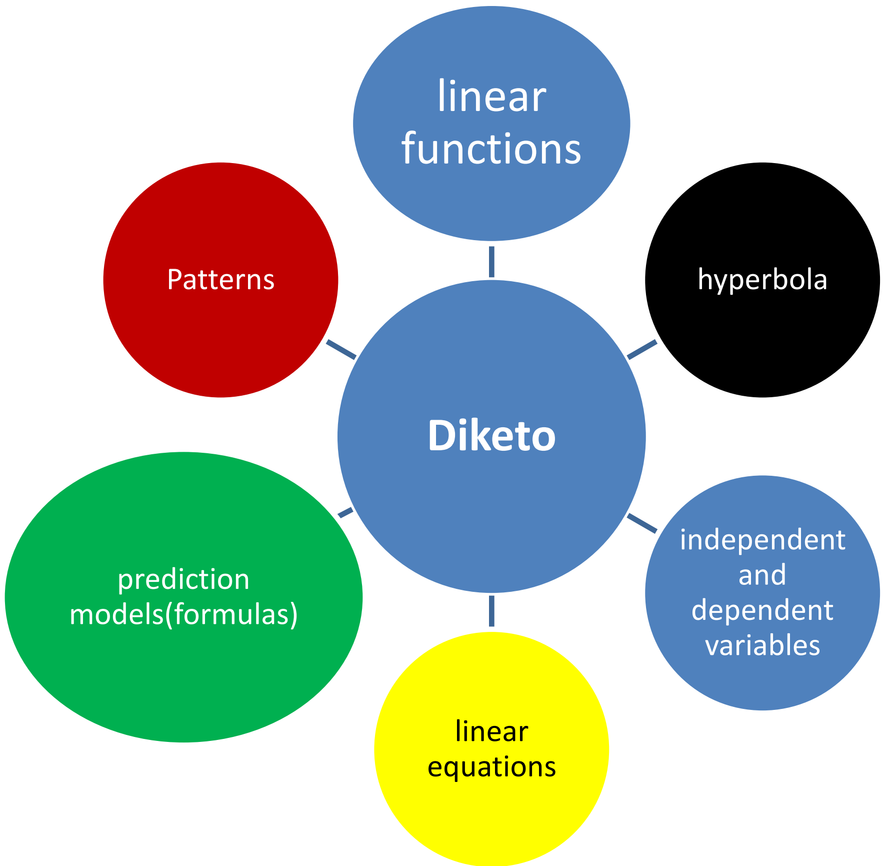


Fig. 4.6: Integration of problem-solving skills

The worksheet on Investigation 1 showed that learners were given activities which were empowering them in high level of thinking, and good communication of the discovered problem-solving formulas ($xy = k$). These types of activities stimulate learners to use navigational and linguistic capital. They have to manoeuvre through various options of deriving the formula. They do not just explore one option, formula or convention, but various ones need to be considered before deducing the formula or definition. Also, the navigational capital and resilience capital they possess made it possible for them to reflect critically on the findings of these options on problem-solving formulae and definitions. The linguistic capital helped them have open dialogue to express their thinking and workings in a logical way. This was shown by Table 4.11 (above) and phrases '….*looking at the function we deduced, that is* $f(m) = -\frac{m}{2} + 10$*, where* m *is number throw the ghoen up, thus* m *is referred to as the 'Independent variable'. Other other hand,* $f(m)$ *the number of stones placed into the hole……'* They were able to state conventions, definitions and general formulae ('…..$xy = k$, $f(m) = -\frac{m}{2} + 10$*,)* on problem-solving skills with understanding.

On the other hand, the excerpts '….*We discovered that as the number of throws increases, the number of stones placed into the hole decreases. This relationship is illustrated in column 3. Further the relationship is described as number of throws times number of stones placed into the hole is equal to a pattern describing a general quadratic pattern…'*, demonstrate the level of engagement, networking and interactions the learners entered into. These are amplified by their wealth of social capital. The interpersonal relations skills shown by the phrase '……*I really impressed with the feedback, you really touched on the most crucial problem-solving skills formulas*…'. , show that they respect and validate the inputs shared by other members of the group. In addition, these positive remarks such as *'…wow, this is excellent. I am so delighted to notice the high standard of work performed in this class…*' by Ms Putsi, the Mathematics subject advisor, showed that she acknowledged the aspirational capital of learners, and that they must maintain hope that their future was optimistic in problem-solving skills. The efforts they demonstrated can even be made irrespective of perceived difficulties.

In conclusion, it is noted that the findings above coincide with what D'Ambrosio (2009:93), the DoE (2003:9) and Warren and Miller (2013:5) assert is present from an early age as the child acts like a research worker. The navigational and social capital helped them to demonstrate an ability to be researchers in formulating problem-solving skills formulae and conventions, such as '… *dependent variables: the occurrences of number of stones placed into the hole is influenced by the throwing of the ghoen into the air. Then the existences of the number of throwing the ghoen up is not influenced by any events, hence we labelled them as 'Independent variable….'*

Again, learners engage in the experimentation and exploration processes in interacting with *diketo* to explore problem-solving skills. In carrying out experimentation and exploration processes, the learners tends to discover strange things or options (which are referred to as 'wrong things') and others things or options from the onset. As Yosso (2005:79) indicates, learners possess aspirational and navigational capital, an argument supported by Warren and Miller (2013:5). The alternatives formulae, which might look strange, provoked learners to go on with exploration or discovery of problem-solving formulae until reaching the right conclusions. This exploration, experimentation and early patterning helped the learners to understand the problem-solving skills.

4.3.5 Good level of involvement of parents

Cao, Bishop and Forgasz (2006:85), and Hawera and Taylor (2011:340) contend that the close relationship between school and the community members is considered important for supporting children's learning of problem-solving. Participation of parents is expected and valued. As Yosso (2005:68) and Malvin (2006:118), because of the familial capital that parents have, their cultural backgrounds (including the play of indigenous games) have a major influence on their children's learning on problem-solving. Also, parents' participation can be considered in order to develop a useful framework that can be employed to transform the teaching and learning of problem-solving skills.

Olatunde Yara, Omodi and Otieno (2010:127) cited Obama (2004) to assert that parents have the primary responsibility for instilling ethics of hard work and educational achievement in their children. By practicing the familial capital learnt from their parents, learners will have morale values of hard work which is needed in the learning of problem-solving. Such valuable ethics will make parents rediscover that every child can learn problem-solving skills and have faith and confidence that none of the learners are stupid, and that it is not impossible to understand problem-solving. In realising the good performance on problem-solving, parents, community members, and government officials can be used to teach problem-solving using indigenous games. Thus, problem-solving through the use of indigenous games can be practical and enjoyable to learners.

4.3.5.1 Lesson planning

The marginalised knowledge which parents possess on problem-solving can be used to demonstrate the mathematical content embedded in the indigenous games. Averill, Anderson, Easton, Te Maro, Smith, Hynds (2009:165); Mncube (2010:238,239) and Lemmer (2011:96) contend that it is important that during the lesson planning sessions parents' inputs be recognised, as this will increase their participation. As much as teachers are good on mathematical content knowledge, even parents and communities do excel on resources and artefacts for indigenous games which visualises the mathematical content imbued in them. Thus, the language used in the lesson planning sessions should accommodate parents, as the language of communication might worsen the non-involvement of parents during the planning sessions. This is against the notion that parents are ostensibly included in a team of teaching problem-solving, but being excluded by language that is not their home language.

The Table 4.4 in Appendix F1, showed a detailed Plan of Action and Picture 4.6: Appendix F3, depicted parents demonstrating how to play *kgati*. As the parents are making the demonstrations of *kgati*, other members of the team conceptualised or figured out the mathematical concepts illustrated by the play. This showed that the contributions of parents during the lesson planning is crucial in elucidating the mathematical content in various games.

The featuring of parents in the Plan of Action gave them voice in explaining the mathematical concepts easily.

One parent, MmaTumelo argued:

> *MmaTumelo: ke bona hore disebeswa tsa ho rutwa di bohlokwa hore bana utlwisise se bolelwang. Ke maikemisetso a rona batswadi ho nka karolo e kgolo mona thutong ya mmetse ka hore re bontshe dipapadi tsa setso*
>
> (I realised the importance of using resources and teaching aids in the teaching of problem-solving, as learners and other team members concretise the mathematical concepts. As parents we are more than prepared to be involved in the teaching of problem-solving using indigenous games).

Dineo, from group A, said:

> Dineo: *As we play these indigenous games or take part in various indigenous games festivals, we will take note of their contributions in understanding abstract mathematical concepts. We are fortunate to work with our parents. In case we need assistance at home, we can make use of your knowledge and expertise.*

Consequently, (Table 4.4 in Appendix F1, showing a detailed Plan of Action and Picture 4.6 in Appendix F3 depicting parents demonstrating how to play kgati) the activities freely encouraged parents to take part in the teaching of problem-solving skills. This demonstrated social justice as parents and teachers have equal responsibilities for increasing the performances of learners in problem-solving. On the other hand, it is an empowering endeavour to all team members to be actively involved in the lesson planning sessions. The inclusion of parents in the Plan of Action gave them voice in explaining alternative ways of understanding problem-solving.

The words of Dineo '… *we will take note of their contributions in understanding abstract mathematical concepts', indicated* that learners had gained confidence in working on mathematical problems using indigenous games. The social capital they possessed made it possible to learn from peers and other team members, without marginalising any team member in terms of gender, race, age or social class.

The above evidence concurs with that of Lemmer (2011:96) and Mncube (2010:238), who argued that parents have a feeling of ownership in the teaching and learning of problem-solving. They are highly motivated in helping with the process of teaching problem-solving using indigenous games. They expressed interest in learners' work and contend that learners learn problem-solving in three spheres (that is, family, school, community), drawn together to strengthen children knowledge on problem-solving.

The evidence above refutes that claim that the parents with a low level of education have little effect (Mncube, 2010:238), or that parents who are less educated do not assist their children in problem-solving (Lemmer, 2011:242). It is the responsibility of the teachers, parents and education policymakers to curtail barriers that might limit the involvement of parents in the teaching of problem-solving. In addition, Lemmer (2011:95) mentioned barriers to be considered, such as limited knowledge and skills about parents, restricted opportunities for cooperation with parents, and psychological and cultural barriers, where people think that indigenous games do not feature in the teaching of problem-solving. As the team using the framework to teach problem-solving, we need to view parents as equal partners.

4.3.5.2 Lesson presentation

According to Anthony and Walshaw (2009:159), Averill et al. (2009:165,172,174), Hawera and Taylor (2011:340), lesson presentations, teaching approaches, and strategies for teaching should be shared with parents and other members of the community, as they possess valuable skills, such as working in groups, storytelling, including the use of legends, use of cultural artefacts, and use of indigenous games and proverbs. These can be used to concretise mathematical concepts and content. They further point out that significant improvements in learning of problem-solving require the efforts of many participants. Team members, including but not limited to teachers, parents, community members, education policymakers and subject advisors, as well as learners themselves, have to a role play in enhancing learners' problem-solving proficiency. It is argued that problem-solving is constructed by learners themselves. Team members are there to support learners to develop a range of ideas in order to make sense of their world and thereby control the complexities within it.

On the demonstration of *kgati* (refer to Picture 4.6 in Appendix: F3), one learner, Tshepo commented:

> Tshepo: *As you are swinging the rope, I realised that the loop is facing downwards at times is facing upwards. That is an interesting observation*.

Mpho argued:

> Mpho: *Yes, you are right Tshepo. That can assist us to remember the characteristics of quadratic graphs. Remember the standard form is:* $(x) = ax^2 + bx + c$ *. Lets us note the following cases*:

- As the loop faces upwards, it's like the player is put inside the loop, then the curve is full(that is it contains something), hence we realise that when $a > 0$, then parabola faces upwards
- At the same time, as the loop faces downwards, it's like it is emptying the contents inside (that is, there is nothing in the loop), thus we conclude by saying that$a < 0$, then when drawing the parabola it has to face downwards.

Ms Mokoena with a smile, responded in this way:

> Ms Mokoena: *We are grateful that parents you are part of us in the lesson presentations. The demonstration you did, really helped our learners to understand the characteristics of the quadratic functions. Thanks Mpho and your members, which is a wonderful work indeed*.

Mr Talana added in this fashion:

> *We are really pleased about the demonstration by parents and remarkable conclusion deduced. This shows that you have internalised the problem-solving using the indigenous games. What else can you say about the gradient of the parabola?*

Thato responded:

Thank you sir for the comments. From what Mpho's group stated, it can be seen that: if the coefficient of x^2 is positive ($a > 0$), the graph faces upwards, we realise that the gradient varies from negative to positive. And in the same way, if the coefficient of x^2 negative ($a < 0$), the graph faces downwards, we saw that the gradient varies from positive to negative.

The extract above demonstrated that learners were empowered to take charge of their own learning. The use of indigenous games demonstrated by parents helped learners to discover basic and key features of the parabola. Again, the navigational and social capital possessed by learners made it easy for them to navigate these characteristics by visualising the play of *kgati*. The interactions that existed between learners helped them to learn from their peers. The use of indigenous games in the teaching of problem-solving motivated learners to think critically, and also gave them hope that they had a brighter future in problem-solving.

The part of the excerpt '….*the demonstration parents did, really helped our learners to understand the characteristics of the quadratic functions…',* echoed that the school community had opened arms for parents to help in the teaching of problem-solving. This also shows the essence of social justice, that is, the resources of teaching problem-solving can be acquired from parents too.

In conclusion, it can be observed that the evidence and the literature reviewed are in agreement. Averill et al. (2009:175) and Mncube et al. (2011:210) contend it is important that teachers and parents do scaffolding in teaching of problem-solving. This approach helped learners to unravel the mathematical content in the indigenous games on their own. In addition, Lemmer (2011:95) and Rivera-Figueroa, Carlos Ponce-Campuzano (2011:285) assert that teachers need to work with families on the teaching of problem-solving, rather than usurping powers bestowed upon parents with the intention of excluding them in the learning-teaching sessions. Figure 4.3.5.2 (a) below shows how learners were able to conceptualise the mathematical concepts such as positive and negative gradients of a parabola, through the involvement of parents by demonstrating the play of *kgati*. It amplified for learners why the conclusion was drawn that when the parabola faces upwards, the gradient varies from negative to positive, and vice versa.

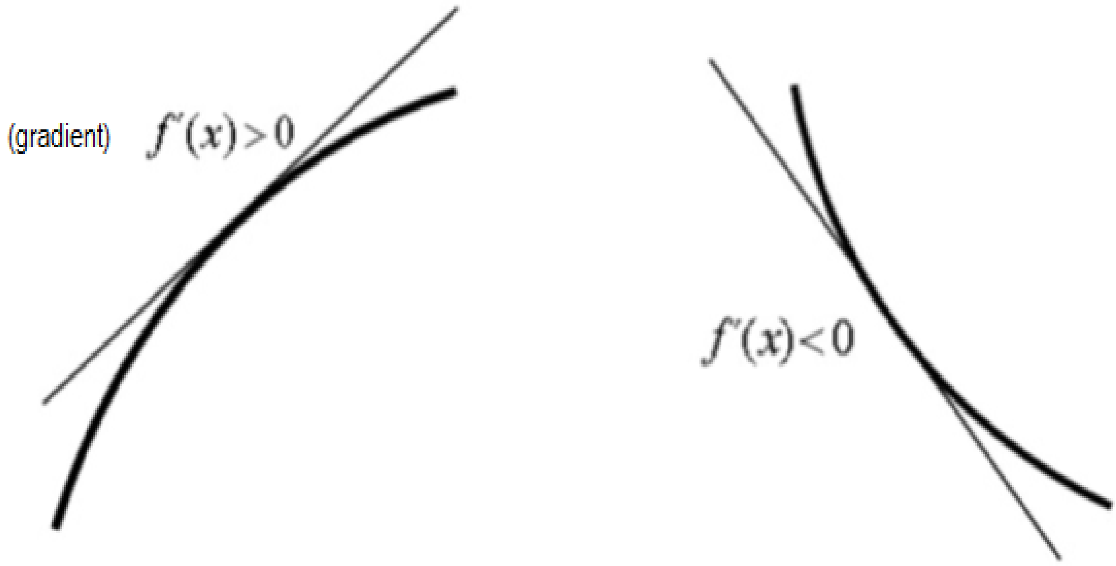


Fig.4.7: Mathematical concepts showed by the play of *Kgati*

4.3.5.3 Class interaction

According to Anthony and Walshaw (2009:154,156) quoting Setati and Adler (2001), code-switching, which involves the teacher substituting a home language word for a mathematical word, has been shown to enhance learners' understanding, especially when teachers are able to use it to capture the specific gradations of mathematical language. In the same way, Cao et al. (2006:87) argue that the involvement of parents in the teaching of problem-solving has a significant impact on learners' attitudes towards problem-solving, and learners' achievement in problem-solving. Also, it assists in making connections across mathematical topics, and is important for developing conceptual understanding of problem-solving. When learners find they can use problem-solving tools as a tool for solving significant problems in their everyday lives they begin to view mathematics as relevant and interesting.

As showed in section 4.3.5.2 above, the class interaction was energetic, where learners were taking centre stage in drawing reasonable conclusions.

Mr Talana wanted learners to further interpret the movement of kgati:

I am quite intrigued by the contributions you made in section 4.3.5.2 above. Referring to Fig 4.8 below and Picture 4.6 in Appendix F3, *Can we have other groups to make more contributions?*

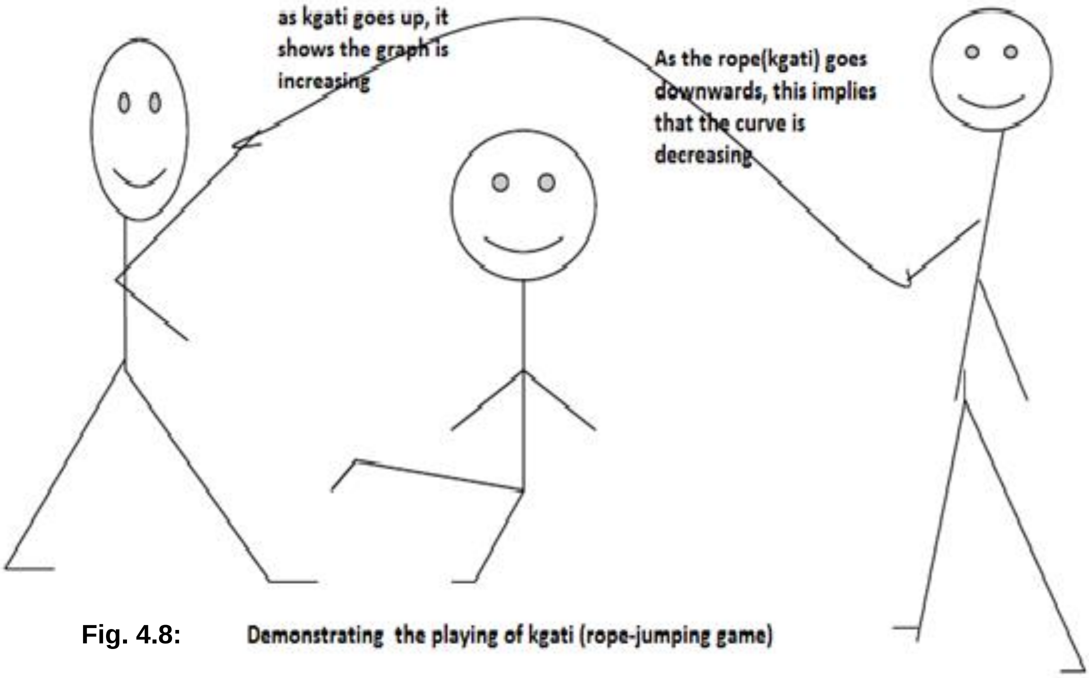


Fig. 4.8: **Demonstrating the playing of kgati (rope-jumping game)**

Learners from group B, represented by Napo, made their contributions in this way:

> Napo: *First we want to thank our parents who are with us today, to demonstrate the play of kgati. Your contributions of the indigenous games, really opened our way of thinking about problem-solving. Adding to what our groups shared with ours. As group B, we made the following observations: if the coefficient of x^2 is positive ($a > 0$), the graph faces upwards, the gradient varies from negative to zero and to positive. Also, if the coefficient of x^2 negative ($a < 0$), the graph faces downwards, the gradient varies from positive to zero and to negative.*

Puleng asked the question for clarification:

> *Hmmm…., Napo, why are you saying that the gradient varies from negative to zero and positive? According to me, if a =0, then we have a linear graph.*

Napo argued:

Ok, I understand your questions, but let us not confuse shape of the graph and the gradient of the graph.

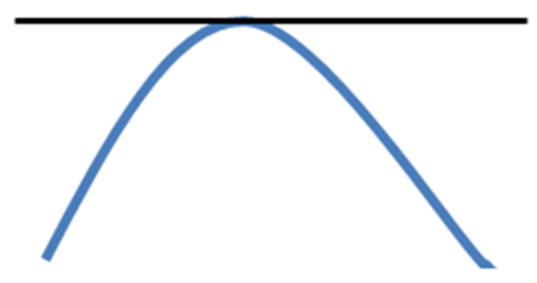
Dineo interjected, and responded in this manner:

You are quite right, Napo, as the rope/graph changes its shape is illustrated by the value of a. So what Puleng is raising is quite interesting, which can be demonstrated by tightening up and holding the rope straight. This shape resemble a straight line, that is where $a = 0$.

Napo, still on the floor, nodded his head, confirming what Dineo had raised, and responded to Puleng's question:

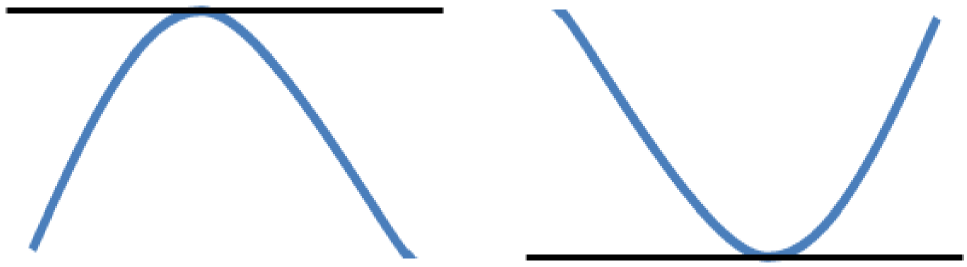
Thank you Dineo for the good explanations you made to clear confusion. Then to respond to the question asked. If you look diagram below, it demonstrates the turning point of the rope/graph, and straight line drawn indicates the gradient is zero. Thus, our argument above showed that if the graph faces downwards, then the gradient varies from positive to zero and to negative, and vice versa.

Napo illustrated on the board as to why the gradient is zero



Mr Talana commented as follows:

Thanks so much for the inspiring observations you made, and the excellent manner of clarifying issues that might be confusing. In closing the presentations, let me complement what you shared with us in this manner: the diagram on the board shown by Napo.



and the Fig. 4.8 above are key in assisting us to understand that: for maximum turning point the gradient varies from positive to zero, and to negative. Likewise, at the minimum turning point the gradient varies from positive to zero, and to negative. It is important that these characteristics will assist you to understand mathematical content as you progress into next grades.

The class interactions above showed the power of the analysis of indigenous games in understanding mathematical concepts and skills. The extract demonstrates that learners are fully equipped with problem-solving skills through the use of indigenous games. The phrase '….*let me complement what you shared with us in this manner… for maximum turning point the gradient varies from positive to zero, and to negative..'* , signifies that inputs learners shared with the class are respected and further expanded by their teachers, in explaining abstract concepts.

In addition, the excerpt '…*thanks so much for the inspiring observations you made, and the excellent manner of clarifying issues...*', is confirmation that teachers are acknowledging and respecting learners' free thinking and powerful comments made. This is an indication that teachers have high hopes and high expectations towards learners' performance on problem-solving.

On the other hand, the excerpt '... *demonstrated by tightening up and holding the rope straight. This shape resembles a straight line, that is, where* a *= 0*….' denotes that learners' navigational capital and linguistic capital helped them to figure out that holding the rope tight demonstrated the linear function. Once the linear function is horizontally drawn its gradient is zero. The language used to explain the concept of the gradient at the turning point as zero was at the level of grade 10 learners (that, is simple to comprehend). The extract '….*why are you saying that the gradient varies from negative to zero and positive? ….according to me, if* a *=0, then we have a linear graph...',* showed that the questions and explanations made by learners had a high level of social capital. The manner in which questions were asked and statements made showed how learners networked and learnt from others.

In conclusion, it can be noted that evidence and literature reviewed do correlate. Lemmer (2011:101) and Nyama (2010:24), quoting Du Toit, Froneman and Maree (2002:156), contend that by involving the parents an extra resource is created in the learning and teaching of problem-solving. Lemmer (2011:102) added that in volunteering projects the parents' role metamorphosed from conventional 'audience' or school visitor to classroom aides. Lemmer (2011) quoted one teacher describing how a poorly educated grandfather helped in the teaching and learning, '…*is telling African folklore for these learners. As a result the listening skills of the learners were improved.'* The same argument is noted with the framework of teaching of problem-solving using indigenous games. One learners remarked as follows: '... *we want to thank our parents who are with us today, to demonstrate the play of kgati. Your contributions of the indigenous games, really opened new ways of thinking about problem-solving…'..*As Epstein and Sheldon (2006:126) allude, this is evidence that parental involvement is ultimately aimed at improving children's learning in the problem-solving skills.

Mncube (2010239) and Tladi (2012:318) argue that the involvement of parents as partners in the teaching of problem-solving promotes good relationships with them. Also, communicating with parents is an easier way of informing them about the new developments in the Mathematics curriculum. This is a demonstration of the principles of social justice that the teaching and learning of problem-solving democratised. Parents are able to avail resources in the teaching and learning of problem-solving skills.

4.3.5.4 Assessment

According to Arora et al. (2009:24), Cau et al. (2006:88,89); Kellaghan (2009:120), Lemmer (2011:102), and Mncube (2010:242), parents' involvement in the teaching of problem-solving tends to develop into a high expectations of their children's performance and they spent more time helping, guiding and supervising the children's schoolwork at home. Learning and working on problem-solving at home was improved by parental interest in children's homework. The invigorated home–school partnership was summed up as follows: '*The children are the ones who are 'on top of the world' because they could see their parent and teachers with the same objective in mind'* (Lemmer, 2011:102).

On the research sites, during the parent-teacher meeting this is what transpired about the assessment in the teaching of problem-solving. One parent, Ratsomo, commented about assessment of problem-solving:

> *Ratsomo: ke leboha sebaka seo ke se nehwang ho hlahisa maikutlo aka ka tshebetso ya bana ba rona mona thutong ya mmetse. Ke fane ka mantswe bukeng tsa bana baka, ho ananela mosebetsi o mongata oo ba o etsang, le kamoo ba sebetsang hantle ka teng. Re lebelletse tse ntle hlahloong ya mahareng a selemo le ka Desember. Le rona batswadi re thuse ka ho etsa nako re tlo ba teng ka diphaposing le ho hlahloba mosebetsi wa bona lapeng mane.*
>
> (I want to thank the opportunity given to me. I am delighted about the quality of written work in their books. In my child's book, I expressed my appreciation about her performance and huge number of exercises on problem-solving. I expect good results in June and November / December examinations. We need to assist by availing ourselves in the teaching of problem-solving and checking their books at home.)

Mr Talana acknowledged the remarks of the parent, Mr Ratsomo:

> *Mr Talana: Le jwale ho a thabisa ho bona boikitlaetso ba batswadi ho hlahloba mosebetsi wa bana ha ba fihla hae, le ho etsa ditlhahiso, mme le boele le tekene. Hona ho kgathatsa bana ho etsa mosebetsi wa bona le ho teng ka diphaposing ka nako tsohle. Ban aba dotjhang diklas ha bay o.*
>
> (It's really encouraging to realise that parents have taken upon their shoulders to check all the assessment tasks given to learners, most of the parents do give comments and attach their signature. This motivates learners to work

consistently on their homeworks, tests, projects and so on. This approach has even made them to be in class all the time, no learners who are dodging classes.)

Mma Puleng added:

Tjhe, teng ho a iponahatsa hore bana ba rona ba natefelwa ke papadi tsena tsa setso ho ithuteng mmetse.

(It's true, it shows that they enjoy playing these indigenous games in the learning of problem-solving)

The extract showed the commitment that parents had towards the work of their children. Parents acknowledged the good work done on the problem-solving. They even felt empowered to take control on monitoring the written work of their children, even to avail themselves in the teaching of problem-solving. This is an indication that parents have hope and confidence about the framework of teaching problem-solving using indigenous games.

On the other hand, the part of the extract '….*this motivates learners to work consistently on their homeworks, tests, projects and so on. …this approach has even made them to be in class all the time, no learners who are dodging classes*….', showed that there was peace prevailing within the school environment, as no learners were absenting themselves in the learning of problem-solving. Order was being maintained in Mathematics classes. This further illustrated that the involvement of parents played a significant role in the teaching and learning of problem-solving. Learners were motivated to work consistently on their assessment tasks and encouraged by the comments by their parents. They held positive beliefs about the topic and were thus more likely to act in ways that enable or increase parent involvement than teachers who do not share similar beliefs (Hoover-Dempsey et al., 2002:345).

In conclusion, it is evident that literature reviewed concurred with the empirical data. Lemmer (2011:99) quoted one teacher in an informal settlement who was sceptical about the involvement of parents and doubted that it would yield good results. The framework of using indigenous games to teach problem-solving had significant impact on transforming the consciousness of parents and teachers. It also lifted professional self-esteem and engendered fresh hope that effective parent involvement was possible in the teaching of problem-solving which previously seemed unreachable. Jansen and Blank (2014:152) added that there is a myth that parents in disadvantaged schools only serve in SGBs and feeding schemes, and do not have skills to contribute to the teaching of problem-solving. They pointed out that this has been refuted by school principals who engaged under-educated (less well-educated) parents to participate in their children’s education. Parents were effectively engaged in small group discussions in class, role play, scenarios demonstrating the play of indigenous games and monitoring the learners’ work at home. In addition, Halpern (2005:142), the University of the Free State Report (2012:11) argue that human and social capitals were present in even the most disadvantaged parent and teachers to work together in achieving good results in the teaching of problem-solving.

4.3.6 Adequate content knowledge among teachers

According to D’Ambrosio (2009:90), the DoE (2003:9), Gerdes (2001:321) and Kemp (2006:128), teachers need to show good understanding of the Mathematics curriculum. It is constituted of various problem-solving skills which the teacher must have for full knowledge of its contents. D’Ambrosio (2009:94) and the DoE (2003:9) assert that mathematics is conceptualised as a human creation that deals with complex situations, whether natural, environmental or sociocultural, emotional facts and phenomena or unpredicted problems. Against this background, teachers will acquire strong content knowledge of problem-solving skills, and be able to demonstrate it in the teaching and learning of problem-solving skills.

In addition, Van De Walle et al. (2010:9) argue that content knowledge of mathematics and general pedagogy is one of the seven principles of mathematics teaching. It is very important for the teacher to demonstrate a high level of content knowledge as well as pedagogy of teaching the content. This is in line with what Pascuzzo (2013:233) refers to as ‘pedagogical content knowledge’, in which teachers must be competent. There must be profound, flexible, and adaptive content knowledge of problem-solving skills as well as a good pedagogical approach to the teaching of problem-solving.

4.3.6.1 Lesson plans show high standard

The lesson planning and class activities are not crafted by the teacher alone, but also involve learners, parents and other mathematics experts (Jansen & Blank, 2014:152; Kellaghan et al., 2009:119). This supports Muijis and Reynolds (2011:79) who contend that, for real in-depth learning, learners must be able to construct content knowledge on problem-solving skills. They can only do this if the teacher is clear on content knowledge on problem-solving skills.

On the research site this was clearly demonstrated, by using indigenous games to teach problem-solving skills. It was clear that teachers had a good understanding of the content knowledge on problem-solving skills extracted from the indigenous games. Learners mentioned the mathematical concepts and skills embedded in the indigenous games. The Mathematics teachers or mathematics experts with strong content knowledge helped in elucidating further and relating the connections with problem-solving skills. For instance, one group mentioned, Zanziba was considering the first two columns, in Table 4.9 above, that is '…..*throwing of the ghoen up and number of stones scooped out of the hole. Check here, as the number of throws increases, the number of stones scooped out of the hole decreases*…..' This is the observation the group made.

The teachers and mathematics experts provided more clarity on content knowledge of problem-solving skills, whereby they managed to link the practical observations with content knowledge on problem-solving skills. Thus, Mr Tshabane clarified by saying: '…..*that illustrate mathematics concepts such as ratios and proportion, and interesting what you told us now, is that as the number of throws increases, then the number of stones scooped out of the hole decreases, you have defined the mathematical concept called " indirect proportion".* This definition was coined by learners so that it was meaningful to them. Further explanation was made indicating that in some textbooks the definition of 'indirect proportion' is simply 'as the x values increases then the y values decreases or versa visa '. Some textbook definitions become incomprehensible for the majority of learners, as they are stated in an abstract way (refer to section 4.3.1 for more detailed is provided). The high pedagogical content knowledge of the teacher helps the team members to make precise connections between problem-solving skills and the structural nature of the indigenous game and the actual playing of the game.

The high level of content knowledge of teachers and mathematics experts is demonstrated by the questions raised by one parent, MmaSebolelo: '… *anyway let me ask this, will I be correct to say the 'indirect proportion' mentioned also signify 'demand and supply' scenario in Economics as the discipline?*' The Mathematics teachers showed understanding of content knowledge on problem-solving, and responded by giving an explanation that it is true that the demand and supply scenario also depicts the concept of indirect proportion or hyperbolic functions. Also it is testimony that they are able to acknowledge integration of problem-solving to other subjects, such as Economics. This implies that integration of problem-solving skills can help within the problem-solving skills themselves (refer to Figure 4.6) or across the subject.

The act of teachers to allow learners demonstrate their understanding on problem-solving shows that teachers empowered learners to have voices and gain confidence on problem-solving skills. What is exciting teachers did not just pour knowledge into the minds of learners, to expose them to how much they knew on problem-solving. The pedagogical content knowledge directed them to use teaching strategies, such as scaffolding, modelling and coaching (Muijs & Reynolds, 2011:79), appropriately, such that learners' competencies on problem-solving skills are not suppressed. Instead, it allowed teachers to use their great content knowledge on problem-solving by supporting and guiding learners only at certain intervals.

Through navigational capital, learners manoeuvred to come up with amusing conclusions or conjectures on problem-solving. The content knowledge portrayed by teachers helped them to validate the definitions and general formulae invented by learners. As Yosso (2005:78) suggests, linguistics have helped them to craft definitions, conjectures and general conclusions. Also learners were able to symbolise the general conclusions drawn, such as, $f(x) = -x + 10$ and $xy = k$.

In conclusion, it is evident that the findings of the empirical data correspond with the findings of the literature reviewed. According to Gerdes (2001:321), Hawera and Taylor (2011:341), and Nutti (2013:20) it is important for learners to do well in problem-solving skills. Moreover, learners are expected to integrate their everyday activities in problem-solving skills for better understanding. This was made possible by the pedagogical content knowledge of teachers in supporting and guiding the learning of problem-solving skills. They discourage the notion that teachers just pour information into the minds of learners as if their minds are empty vessels, as a way to illustrate that they are more knowledgeable than the learners.

This further supports the view of Altrichter (2005:5) and the Assembly of Alaska Native Educators (2002:9) that the mathematics classroom should not be viewed as a place in which teachers transfer knowledge on problem-solving skills into the minds of learners. Rather, the mathematics classroom can be considered as a laboratory in which learners refute and confirm the problem-solving formulae and definitions.

Noteworthy in the evidence was that teachers used their large content knowledge on problem-solving to guide learners to learn and discover problem-solving skills, formulae and conventions.

4.3.6.2 Class activities/tests

The classroom activities must allow learners to use their prior knowledge to execute the class tasks assigned to them. Godino et al. (2007:131) explain this process as the ostensive and non-ostensive facets. In this process the latter are modelled into the former (mathematical symbols) or visa versa. Using indigenous games to teach problem-solving skills illustrates onto-semiotic approach, when non-ostensive dimensions are modelled into ostensive facets.

On the research site, teachers demonstrated high quality knowledge on problem-solving skills by using ostensive and non-ostensive dimensions to design activities. These activities made learners figure out the problem-solving skills infused in the indigenous games. Through these activities learners were able to discover formulae and definitions. Also, they were engaged throughout the learning sessions.

In one of the class activities (refer to Table 4.16: Appendix F4), Mr Talana gave it to learners to work on:

> *Good morning class, today I want us to focus on this game morabaraba. Activity is designed as follows*…

When the activities are performed in those five groups (refer Pic. 4.7: Appendix F4), teachers, parents, experts of mathematics (district officials) or any other stakeholder present on the day is checking and monitoring the progress of small groups. Monitoring is normally in the form of facilitation of small group discussions, or scaffolding, reflecting, or any other appropriate strategy. These strategies ensure that learners are guided and supported whenever they need assistance. The small group sessions are followed by reporting to a larger team.

During reporting and reflections, Jane from group C, led the discussions:

Good morning class. We have responded to the investigation in this manner. We further decompose the structure in Table 4.7 above and come up with new figure (Figure 4.9 below) *is shown. We measured, discovered that these are squares, put together the form a little triangle in between these three squares. Area of the square shade in blue* is $25\, units\ squared$, *the one in red, its area is* $16\, units\ squared$, *lastly the one in yellow, its area is* $9\, units\ squared.$ *We further noted that the area of blue square is equal to the area of yellow square plus red square* $(25\, units\ squared = 9\, units\ squared + 16\, units\ squared.$

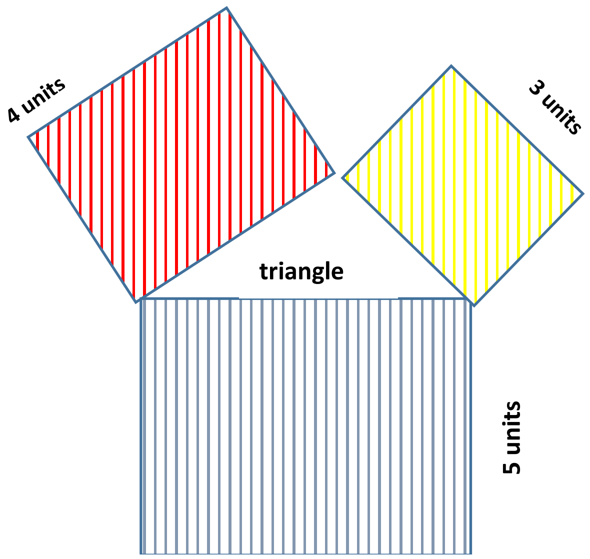


Figure 4.9

The broad content knowledge of the teachers and other experts helped greatly in putting certain issues on problem-solving skills into a clear perspective. It was said in earlier sections that is it important to teach problem-solving skills from learners' perspectives. One moves from the premises in which all learners understand the logic behind it. Ms. Mokoena gave her input as follows:

Good morning colleagues. Excellent work indeed. I am so delighted about the logical findings of Jane's group. The indigenous games you played you really learnt so much. Keep up, the good work. What you discovered is the fundamental of the so called theorem of Pythagoras. What is the name of the little triangle in between the squares?

One member of group A, Sello responded in this way

Thank you, mam; we are also delighted about interesting discoveries. The little triangle is the right-angled triangle.

Ms Mokoena: *Yes, you are quite right Sello and your group members. The longest side as you rightfully stated is five units. That longest side is called the hypotenuse.*

One learner, Dineo wanted to get clarification on the explanation of Ms Mokoena:

Can I find out is there a reason why it is called the hypotenuse?

Ms Mokoena: *Good questions, indeed, I do not have an idea, anyone to assist?*

Mr Talana: *Let's take that question as our homework for the next day.*

Ms Mokena: *On the your conclusion, that is* $25\,units\ squared = 9\,units\ squared + 16\,units\ squared$, *this called Pythagoras Theorem. This can be written as* $5^2 = 4^2 + 3^2$ *, then generally we have* $h^2 = x^2 + y^2$ *, given any right – angled triangle, with measurements of two dimensions. You can determine the other side, by using the discovery you saw.*

Dineo: *In our group, we used the general pattern* $t_n = n^2 + 4n + 4$, *to determine the area of three squares.*

Dineo' s calculations as shown of the board

- The first square (the red one);

$t_n = n^2 + 4n + 4 \quad n = 1\ first\ square,$

$t_1 = 1^2 + 4.1 + 4$

$t_1 = 9\ units\ squared$

Dineo: *For the second and third squares we used the same general formulas* $t_n = n^2 + 4n + 4$ *for second red square we used* $n = 2$ *and for third blue square we used* $n = 3$*.*

Ms Mokoena: *Oh, yes you are quite correct, that is great.*

Mr Talana commented in this way:

The observation you made are excellent in indeed. Thank Ms Mokoena, let me take the opportunity to make the team aware of these conditions regarding the theorem of Pythagoras. This theorem as you had observed, you only apply it when you have a right-angled triangle.

The activity (refer Table 4.16 in Appendix F4) given to learners showed the high level of content knowledge of problem-solving skills teachers have. Giving learners the theorem of Pythagoras defined as the hypotenuse squared is equal to the sum of the other two sides squared would have encouraged rote learning. The teachers let learners discover the results, as shown in the extract as '…*area of the square shade in blue is* $25\ units\ squared$*, the one in red, its area is* $16\ units\ squared$*, lastly the one in yellow, its area is* $9\ units\ squared$*. We further noted that the area of blue square is equal to the area of yellow square plus red square*…' The learners were empowered to have their voice and opinions in devising observations, making logically conclusions, and finally discovering the general formulae of problem-solving skills.

It is evident that learners were given hope to realise that their knowledge on problem-solving was equal to that of the experts in problem-solving skills. The teachers showed understanding and the power of pedagogical content knowledge. These translate into the learners showing a broader understanding of problem-solving.

Furthermore, The excerpts'… *we used the general pattern* $t_n = n^2 + 4n + 4$*, to determine the area of three squares*…' , showed that learners were displaying use of navigational capital, as they ventured into various ways of getting the area of square, other than the formula $= length \times breadth$. These various possibilities of using different formulae on problem-solving skills showed that learners were positively determined to work hard. This is a demonstration of their aspirational capital helping them to remain focused and determined.

In summary, it is evident that when using tangrams there are many geometric shapes of high order (Su et al., 2013:3) that can be deduced. It is in line with the empirical evidence above that in using indigenous game, *morabaraba*, there are quite remarkable observations, conjectures and general conclusions that can be drawn on problem-solving skills. The teachers allowed the learners to discover these conjectures on their own, whilst the teachers used their content knowledge effectively to guide and support learners in their endeavour. Teachers did not dominate the lesson activities by spoon-feeding learners with knowledge on problem-solving skills. This supports what Booker (2005:49), Godino et al. (2007:128) and Tennant (2003:3) alluded to as a sophisticated relationship of learners observed (extensive facet, Godino et al., 2007:132) when they are able to make general remarks (intensive facet, Godino et al., 2007:130, which is the abstraction process. Through the discovery of these formulae on problem-solving, even the so-called abstract formulae, these are now easy to comprehend.

4.3.7 High motivation among teachers

The effectiveness of teaching strategies, which made learners interact freely and enjoy the learning of problem-solving, has a significant impact on the motivation of teachers. The teacher becomes motivated to teach the actively engaging learners in learning problem-solving. Also, the involvement of parents in the teaching of problem-solving gives learners courage to work harder, knowing that parents are on their side, as partners in improving their performance in problem-solving. Teachers hope that their intervention in teaching problem-solving is consistent with how learners learn and that they benefit from it. Teachers tend to be encouraged to work with learners over long durations, as they derive pleasure in what they are doing. As a result, teachers develop trust towards their learners and are able to contribute positively in problem-solving. The learners' results on problem-solving are not theirs alone but the teachers also take ownership (Yildirim, 2012:165).

Thus, there is high content knowledge of the teacher exemplified by quality assessment tasks. The pedagogy knowledge, that is, skills and ability of teaching problem-solving efficiently, impact positively on the teachers' motivation to excel in his/her duties and responsibilities (Zollman et al., 2012:104).

4.3.7.1 Lesson planning

According to Phelps (2010:294), there must be a motive for designing a lesson plan. He defines motive as a wish to engage in a specific activity, such as designing a lesson plan to teach problem-solving. Reasons for engagement theory (Olatunde Yara et al., 2010:128) are that individuals should engage in certain tasks or events. This keeps the teacher motivated and focussed on achieving the lesson outcomes as stated in the plan.

Zollman, Tahernezhadi and Billman (2012:108) assert that the knowledge of and ability to teach the subject matter is important in designing a collective lesson plan. Such collective planning instils zeal and zest for the teachers and learners to do their best during lesson presentations. In the research site, the research team's use of the indigenous games to teach problem-solving skills made it necessary that the lesson planning be a collective effort of teachers, learners, parents and other participants from the education district offices (mathematics subject advisor) and other community members.

The Action Plan (Table 4.4: Appendix F1) below showed that the lesson planning was a collective of teachers, parents, learners and other stakeholders. In column 1, the team agreed on the indigenous game to be played. Table 4.15: Appendix F2), shows that the collective team was involved in deciding which indigenous games to play. These are indigenous games mentioned by the team to be played in the teaching of problem-solving skills. The teams were divided into smaller groups to brainstorm indigenous games to play. This suggested that the games be enjoyable to the team members. If the games were forced on them this might create a negative resistance from some members.

During this process all participants were excited at the thought of being fully engaged in the process, and this helped teachers not force people to do something they were not happy with. The willingness and desire to perform their roles in the lesson planning motivated learners, teachers and parents to be present when the indigenous games were demonstrated.

In columns 2, 3 and 4 the activities were collectively performed. Column 2 involved people leading in the game, whilst the next columns looked at monitoring and evaluation. All observers were given an observation sheet (refer to Table 4.14: Appendix F1) on which to mention all mathematical concepts and skills infused in the structural nature of the game and the actual playing of it. It was encouraging that the team members analysed *morabaraba*, extracting the problem-solving skills embedded in the indigenous game. This analysis made the classroom environment meaningful to the teaching of problem-solving skills. Learners easily related the problem-solving skills with their prior knowledge.

Table 4.14: Observation sheet in Appendix F1 and Figure 4.3 indicated the problem-solving skills learners and team members observed during the playing of *morabaraba*. One group pointed out the following mathematical concepts, as reflected in Figures 4.3 and 4.4:

> Thabang: *In our group we saw the following concepts: edges, vertices, faces, two-dimensional objects.*
>
> Ms Mokoena: *Thank you Thabang's group you really observed important issues, which constitute part of problem-solving skills, we have to learn in grade ten mathematics classroom. I am really happy to realise that you are on track.*

Another group, led by Thabiso, was given a chance to describe their reflection:

> Thabiso: *We imagined pulling morabaraba at the centre upwards, as demonstrated on the chalkboard [refer to* Figure 4.12 : Appendix F5], and at the same time visualising [Figure 4.13: Appendix F5] *the truncated one from the original figure*.
>
> Mr Talana: *This is exceptionally well. I am so delighted about the discoveries. In fact from the two-dimensional object, you came up with the three–dimensional figures.*

A learner, Dineo, from the floor posed the question:

> *These are quite interesting imagination you came up with, I guess Figure 4.3.7.1 (a) is the pyramid, but what is Figure 4.3.7.1 (b) called?*
>
> Ms Mokoena: *You are quite right, Figure 4.3.7.1 (b), the truncated figure is known as the frustum. This is important thing to realise that these are problem-solving skills in shape, space, measurement and trigonometry.*

The collective lesson planning in Action plan (Table 4.4: Appendix F1), revealed that the teachers empowered learners, parents and other stakeholders to own the process of the lesson planning. The Action Plan showed that all research team members were fully engaged with their roles to play clearly articulated. That removes the burden that all the work from lesson preparation to presentation is centred on the teacher. These innovations motivated teachers to excel in their daily facilitation sessions, and further motivate the teacher to have knowledge on content of problem-solving skills. As Mr Talana mentioned the name of Figure 4.13: Appendix F5 as '…*frustum*…'. The navigational capital made it possible for learners to imagine a pyramid and frustum.

As the excerpts '…*we saw edges, vertices, faces, two-dimensional objects*….' and further remarks by Dineo *'….I guess Figure 4.12 is the pyramid, but is Figure 4.13 called?*) demonstrated that learners showed high level of critical thinking, which is made possible by their aspirational and navigational capital. They managed to reconfigure different shapes, as shown in Figures 4.12 and 4.13 from the 2-dimensional object *morabaraba*, and conceptualised the 3-dimensional object such as pyramids and frustum. The comments from teachers, such as '….*I am so delighted about the discoveries. In fact from the two-dimensional object, you came up with the three-dimensional figures*…', is an indication that teachers are impressed and motivated about the high level of sophistication on problem-solving exposed by learners.

Teachers are giving learners and parents assurance that the prior knowledge they have is influential in understanding shape and space topics. A welcoming classroom environment made it possible for learners to speak their minds freely. Teachers created this environment by ensuring that learners took part in lesson planning and presentations. Teachers acknowledged learners' views, and showed them how they were relevant in the learning and teaching of problem-solving skills.

Finally, Hawera and Taylor (2011:343) reported that many children enjoyed the support of a variety of community members to fulfil their potential for learning problem-solving skills. The systems such as lesson planning and class activities, in which learners are fully engaged, motivated teachers and parents to do more in terms of teaching problem-solving skills. Teachers managed to link the problem-solving and those embedded in everyday practices in the communities, such as playing of indigenous games. In addition, Olatunde Yara and Omondi Otieno (2010:127) point out that the PDSI (Plan, Do, See, Improve) approach emphasises the need for the learners to carry out a well-planned learning activity that involves seeing for themselves and improving the activity even further for effective learning to take place. This correlates well with the evidence above that once learners are engaged from the onset stage of planning, they do activities largely by themselves, with only a little guide from teachers. During the reflection phase they are able to reflect and learn from others so as to improve their understanding of problem-solving skills.

The connection of problem-solving skills with the playing of indigenous games played an important role in motivating both learners and teachers. Learners were able to develop a broader view about the nature of problem-solving skills. It was evident that there was integration of real-life activities, various problem-solving skills such a trigonometry, shape and space. Also, these indigenous games had the ability to assist learners to capitalise on the potential of the problem-solving skills.

4.3.7.2 ***Class interaction is learner-centred***

The DoE (2003:2) and Olatunde Yara et al. (2010:127) argue that learners' participation from the start to the end of the lesson is crucial in the learning of problem-solving skills. The teaching activity should be learner-centred, based on experiments and improvisation during the course of the presentation. For instance, the teaching and learning of longitudes and latitudes in mathematics can be accompanied by improvising a metallic or plastic globe and using it to locate the position of an object along the equator. For learners to participate in the lesson duly, the teachers and parents must plan the lesson well. Similarly, the teacher must also evaluate every aspect of the lesson during teaching.

On the research site the team used indigenous games to teach problem-solving skills effectively. The learner participation was excellent, as learners, and other team members were engaged throughout the learning sessions. From the idea developed by learners on frustum, the teachers gave learners an activity that required them to apply their prior knowledge on *morabaraba*. The activity was structured in this way (refer to Table 4.16: Appendix F4).

On the Investigation 2, the teacher used the knowledge discovered by learners in section 4.3.7.1, when learners discovered that *morabaraba* can be changed into a 3-dimensional object, that is the cut pyramids known as frustum (refer to Figure 4.12 in Appendix F5 and Figure 4.13: Appendix F5).

This is what different groups identified from the diagrams. The Group A, led by Thabang said:

> *We tried to analyse the given scenarios in this fashion. We realised that the truncated pyramid (frustum), the bases are rectangular in shape. In Figure 4.13: Appendix F5, the lengths are* $\underline{a\ units}$ *and* $units.$ *Then the sides faces are trapezium in shape. Two heights are identified, as slant height and the straight height.*

The teacher commented positively, in this manner:

> Ms Mokoena: *Yes, your analysis is good, and the manner you interpreted the figures, it shows that you are on right track. Well done, go on with further details.*

The evidences above showed that the teacher is teaching problem-solving skills from the learners' perspectives. The designed INVESTIGATION 2 (Table 4.16: Appendix F4), is mainly based on what learners discovered (refer to Figures 4.12 and 4.13 in Appendix F5). The teacher took the discoveries of the learners, and introduced other topics such as determining the area, the total surface area and the volumes of the frustums. This revealed that learners are empowered on their inventions (that is frustum). This motivated the learners as they realised that the prior knowledge could be linked with problem-solving skills. As the teacher is empowering the learners, the teacher himself or herself felt motivated and encouraged by the high standard of the knowledge on problem-solving revealed by learners. The activities of this nature calls for equal involvement by and responsibility of the teachers, learners and other team members.

On the other hand, it is noted that the activities designed by the teacher are utilising and further unearthing the navigational capitals that learners possessed. The activities stimulated the interest of learners. The teaching strategy used in the teaching of problem-solving is learner-centred, which in itself helped to create the strong interaction and networking among team members. The power of the linguistic capital learners used is demonstrated by the phrase '…*pulling morabaraba at the centre upwards, as demonstrated on the chalkboard (refer to Figure 4.12: Appendix F5), and at the same time visualising Figure 4.13: Appendix F5 , the truncated one from the original figure….'*. the linguistic skills helped them to coin out the mathematical concept 'frustum' which then becomes easy to understand in formulating a definition.

In conclusion, in line with the evidences above, Petterson (2012:88) opines that there are four components to bear in mind when teaching problem-solving skills, which will interest the learners to master it and be more motivated. On their own learners need to experience the problem-solving skills, which in turn can be through the use of indigenous games. The prior knowledge they possess from home background should connect them to practical problem-solving skills imbued in indigenous games. If these components are in place, the teacher and parents (Averill et al., 2009:172,174) become motivated to assist learners to excel in problem-solving skills.

In addition, the classroom becomes interactive if learners are given space to experience by themselves why and how the problem-solving skills works. As in the evidence above, learners managed to craft mathematical concepts associated with the frustum. Attard (2012:10) supports the above evidence that most of the 'good' lessons in the teaching and learning of problem-solving skills includes physical activity, active learning situations involving concrete materials, and/or games.

The evidence showed that the imagination enabled the learners to make up their own problem-solving. The interesting and motivating factor is that these imagination problems were included as class activity, in which the teacher named some new concepts. Attard (2012:11) found that creativity made lesson presentation a favourite, as the learners were allowed to make links to their background information.

4.3.7.3 Assessment based on high order questioning

Campbell (2006:10,11), Griffin, League, Griffin, Bae and Griffin (2013:9) and Su et al. (2013:2,3) opine that for assessment to address high levels of cognitive thinking, citing The NCTM (2000), teaching of problem-solving skills has to focus on learners' conceptual understanding of problem-solving rather than procedural knowledge or rule-driven computation. In addition, the documents advocate a learner-centred approach that guides a discovery approach for learning problem-solving skills. Also, the DBE (2011:51, 53) argues that assessment should be used to understand and assist in the learners' development to improve the process of learning and teaching of the problem-solving skills. The high order assessment tasks are expected to show that the learner has acquired a conceptual understanding, and could involve making significant connections between different representations. Learners are expected to tackle not only familiar problems but also non-routine problems, which require higher order reasoning and processes. They need to show competencies on various cognitive level questions, varying from low order to high order questions.

On the other hand, the assessment in the teaching of problem-solving skills should promote mathematical processes, including but not limited to critical thinking, communications and logical reasoning. The problem-solving activities need to be enjoyable, with communication and interactions between teachers, learners and parents emphasised (Jansen, Blank, 2014:153; Haverhals, 2010:333,334; Williams et al., 2009:18)

At the research site, it was the prevailing spirit that assessment activities need to be on a high level. It was evident that the activities assigned to learners were of high order questioning, and mathematical processes were constantly addressed as the learners reflected. For instance, learners were given the class activities rephrased in this way (refer to Class Activity in Appendix F6). As they reflected on the activity, answers they shared included that of Group A, led by Mabine:

Mabine: *We structured our responses in this way*:

> $Given: f(x) = -x + 11\ .$
>
> *1st throw:* $f(1) = -1 + 11 = 10\ stones$
>
> *2nd throw:* $f(2) = -2 + 11 = 9\ stones$
>
> *3rd throw:* $f(3) = -3 + 11 = 8\ stones$

Others members from the team wanted to get clarity on the issues:

Jane: *Thanks for the presentation you shared with us. We had something different, especially the number of stones obtained, when the second, fourth and sixth throws are made. Like it is shown in the table below*, *the second throw there eight stones placed into the hole; the fourth throw there are six stones placed in a hole and the sixth throw there are two stones placed in the hole. We realised that to answer these three questions, we needed to describe the model that defined the throws of ghoen in placing stones in the hole. Hence, we concluded that the model is* $f(x) = m - x = -x + 10$, *is relevant in obtaining the second, fourth and sixth throws of placing the stones in the hole.*

Throwing the ghoen up	**Number of stones scooped out of the hole**	**Throw of the ghoen**	**Number of placed in the hole**
1st throw	$= 10 - (1 - 1) = 10 - 0 = 10$	2nd	$10 - 2$ =8
3rd throw	$= 10 - (3 - 1) = 10 - 2 = 8$	4th	$10 - 4$ =6
5th throw	$= 10 - (5 - 1) = 10 - 4 = 6$	6th	$10 - 6$ =4
7th Throw	$=10 - (7 - 1) = 10 - 6 = 4$	8th	$10 - 8$=2
9th Throw	$=10 - (9 - 1) = 10 - 8 = 2$	10th	$10 - 10$ =0
	$f(x) = -x + 11$		$f(x) = m - x = -x + 10$

The reflection session was concluded by Ms Mokoena with these remarks:

> *I want to thank all the groups that presented their responses*, *you have done us proud as the team .I am so encouraged with the high level of critical thinking you displayed in this tasks. Being able to realise that there are two functions, that is* $f(x) = -x + 11$ *and* $f(x) = -x + 10$ *to be considered for you to answer the questions well.*

The solutions displayed by Mabine's group were correctly performed. They were mechanically done, with no deep thinking. Relating the answers obtained to the actual play of the *diketo* solutions on second, fourth and sixth throws are not correct. The activity assessed learners so as to empower them to operate on a high level of critical thinking. It is clear that some learners, such as Mabine's group, required assistance with regard to exploring high level of thinking, as demonstrated by their workings '… *second throw:* $f(2) = -2 + 11 = 9\ stones\ldots$', practically, as second throw stones are placed in the hole. The intervention by means of the class activity helped them to venture with their navigational capital.

On the other hand, it was exciting to note that Jane's group denoted that learners were able to analyse, reflect and make sense of the problem before they could conclude that the answers were correct. This is an indication that learners were empowered to handle high level of questioning. Learners' interactions and networking were effective, since they realised that to respond well to the questions they needed to draw the table (as shown above) that demonstrated their reasoning, arguments and justifications. The linguistic capital enabled them to criticise the solutions they provided. Also, their linguistic and social capital ensured they recognised that the model used to describe the scooping of the stones out of the hole was mixed with the placing the stones in the hole, hence learners encountered difficulties with the interpretation of solutions to questions asked. The model describing the placing back of the stones into the hole was $f(x) = -x + 10$.

In addition, the comments of Ms Mokoena '…*I am so encouraged with the high level of critical thinking you displayed in this tasks*….', revealed that learners were doing well in problem-solving skills and operating at a high level of critical thinking. They were aware that for them to work out other problems they needed to derive the second model describing the number of stones to be placed back into the hole. The teachers were so encouraged and motivated by the level of commitment showed by learners in unravelling sophisticated problem-solving skills.

In conclusion, there is a correlation between the empirical data and the literature reviewed that teachers' attitude had an impact on the teaching and learning of problem-solving skills. When teachers are positively motivated, the assessment tasks given to learners are very interesting, meaningful and exciting. Hence, the learners are encouraged to explore and manipulate any problem-solving skills confidently (Alexander et al., 2005:16; Olatunde Yara et al., 2010:129).

Alexander et al. (2005:16) and Seah (2009:484) believe that using games in classrooms is a unique opportunity to incorporate problem-solving skills to enhance children's attitudes towards problem-solving skills and to improve their social skills. Learners are exposed to maximum learning, involvement and discussions in executing activities. It can be argued the use of indigenous games to teach problem-solving skills ensures that the educational imbalances of the past are redressed (DoE, 2003:8), and that equal educational opportunities in the teaching of problem-solving are provided for all sections of the population. The school mathematics curriculum embraces the home background of the child, (Jansen et al., 2014:154; Kellaghan et al., 2009:120), viewed as rich in the teaching and learning of problem-solving skills.

4.3.8 High expertise with regard to classroom practices

In achieving equity in the teaching and learning of problem-solving skills there must be learning resources for all learners, such as stationery, equipment and teaching aids, including chalkboard, rulers and protractor. Time allocated for the teaching and learning sessions and assessment activities are sufficient. Teaching and learning that nurtures problem-solving skills and independence allows time for thinking and encourages discussion among learners. Constant assessment of each learner's understanding through questioning, listening and observing enables fine-tuning of teaching (Association of Teachers of Mathematics, 2010:44; Olatunde Yara et al., 2010:129).

In the teaching and learning of problem-solving skills practical work is even more important because learners learn by doing. The problem-solving skills which are infused within the practical situations are made meaningful to learners to comprehend (Association of Teachers of Mathematics, 2010:44). The problem-solving skills which are filtered within the practical situation are made meaningful to learners to comprehend scientific practices and applications are thus rendered more meaningful. It is evident that an object handled impresses itself more firmly on the mind than one merely seen from a distance or in an illustration. Thus, practical work forms an important feature in any learning (Olatunde Yara et al., 2010:127).

4.3.8.1 Lesson planning has adequate resources and activities

In the lesson planning the teaching resources and learning activities must be properly arranged beforehand. This will help to stimulate all learners' active participation in their learning. The prepared materials and resources must be in a position to instil in learners' innovative and imaginative senses. This will help to make them actively involved in the teaching and learning of problem-solving skills. These resources and materials are supporting teaching and learning to ensure high interaction and communication among participants (Seay, 2009:484; Watson, Mason, 2007:205,210)

On the other hand, the pedagogical principles are important for learners to learn problem-solving skills. As a result, for the presentations to be successful the teacher is viewed as vital for initiating interaction between participants and striving for high quality in the conversation, i.e.. for initiating learners' process of constructing their own problem-solving knowledge (Hasson, 2010:186); Walshaw and Brown, 2012:185).

The research team used indigenous games to teach problem-solving skills, as it links a theoretical and practical part of mathematical concepts. The approach was learner-centred. Teachers and other members of the team were present in initiating and guiding the activities and discussions. The team members formulated the detailed

Plan of Action (refer to Table 4.4: Appendix F1), which ensured that the activities were executed as planned. Before the detailed schedule, normally the team meets two days beforehand. For instance, in the playing of *kgati*, members met to identify tools, equipment, resources and human resources to be used, or to participate in the game. As one parent commented:

> Me Motaung: *Ho ba tla re fumane hore ka sekolong ka mona le hloka eng bakeng sa papadi tsena tsa rona, jwale ka kgati, jwalo-jwalo. Re fumane hore tsena tsohle di fumaneha kae.*
>
> (Let us find out, whether all tools are here, like the skipping rope so that we make means as to how to secure equipment or tools that we don't have.)

As Pic. 4.6: Appendix F3 showed, phase 1 (playing of *kgati*) members agreed that the *kgati* (rope) was needed. As these games were also played during the Life Orientation physical period, the teacher in charge, Mr Maphodi commented:

> Mr Maphodi: *le se ke la kgathatseha, malebana le kgati tsohle di teng mona sekolong. Mohlomong re ka fana ka mabitso a batho batla bapala papadi eo*.
>
> (All resources for *kgati* are available here at school. Maybe we can have names of people to lead us in the play)

In the same meeting, the team members identified players for the game and volunteers were welcomed. The second column and second row (refer to Pic. 4.6: Appendix F3) indicated the players to lead in the playing of *kgati* and their roles and responsibilities. In column three and column four, other team members were observing the play of *kgati* (shown by Pic. 4.6: Appendix F3) and at the same time noting down their comments (sample of comments refer to Pic. 4.7: Appendix F4). One learner, Njabulo made the following observation:

> Njabula: *ha le letsoho la mme Motaung le phahama ho bonahala hore kgati e phahamela hodimo, moo le letsoho la hae le theohelang fatshe, le kgati e theohela fatshe*.
>
> (As Mrs. Motaung's hand goes up it shows the rope is going up, and as the hand goes down, it illustrates that the rope goes down).

On the other hand, teachers, experts of the subject and parents were monitoring the progress in small groups. As they monitor progress they do not give answers as such, but rather they are encouraged to stimulate the debate or discussions. To work within the limited time schedule, phases 1 and 2 were covered the previously day in the afternoon for one hour, then the last three phases were covered during the actual Mathematics lesson.

The scenario painted above showed that the lesson planning was a joint effort among team members. As Pic. 4.6: Appendix F3 revealed, all members had specified roles and responsibilities to play. All team members, especially parents and learners, are empowered to take part in the education of their children. On the other hand, learners are empowered to take lead in the class activities. Through the observation process teachers are empowering learners to use the analytical skills of identifying mathematical concepts and skills embedded in *kgati*. Since learners have a great wealth of navigational capital they were able to detect easily the mathematical concepts build in the game. This is evident by Njabulo's comments that '…*hand goes up, it shows the rope is going up*.' This demonstrates the properties of functions. As the graphs goes up this shows that at that given interval the graph is increasing, but as the hand goes dow, this signifies that the graph is decreasing at that given interval. The linguistic capital and social capital helped the groups benefit from one another as they shared information. The richness of the linguistic capital helped them to express themselves freely in an eloquent manner.

In conclusion, the evidence provided above validates Gadino et al.'s (2007:133) view of the onto-semiotic approach as demonstrating a close relationship between ostensive and non-ostensive dimensions. The theoretical issues are clearly demonstrated by practical issues for learners to understand problem-solving skills easily. As the *kgati* game was played it was clear that as the hand goes up the rope also goes up, which shows the theoretical properties of functions. At the given intervals the functions is increasing or decreasing. This demonstration helped learners to concretise the idea of increasing and decreasing functions.

4.4 CONDITIONS FOR THE COMPONENTS OF THE EMERGING FRAMEWORK TO WORK EFFECTIVELY IN THE TEACHING OF PROBLEM-SOLVING SKILLS

In the previous section the eight illustrative components of the emerging framework to teach problem-solving skills using indigenous games were extensively discussed. In this section, each component identified will be discussed with the contextual factors corresponding to this component of the framework to teach problem-solving skills using indigenous games. The circumstantial factors ensured that the components in the teaching of problem-solving skills using indigenous games were achieved as anticipated. Contextual factors impacting on good practices in the teaching of problem-solving skills, such as the subject matter presented to learners, were easily understood. The discussion will indicate the conducive conditions to which learners were exposed, and that made them understand the subject content more easily. For each of these components in the teaching of problem-solving skills, the appropriate factors will be discussed. As Rosand, Millar, Ipe and Healey (2008:6) and UNHCR (2009:214) have stated, conditions conducive to the teaching of problem-solving skills are not uniform.

In discussing these circumstantial factors, reference will be made to various theories, policies, legislative imperatives and previous research findings based on literature reviewed that encourage conducive conditions in the teaching and learning of problem-solving skills. The suggested contextual factors will be checked against the ones that emanated from the empirical data, taking various formats such as texts, spoken words, pictures and scenarios. In achieving the deeper meaning of textual forms, CDA will be used to analyse and interpret the data. The study is couched in the community cultural wealth, which gives voice to the voiceless, and validating the wealth of knowledge they possess in the teaching of problem-solving skills. In conclusion, the empirical data will be checked for whether it correlates with or refutes the literature reviewed.

4.4.1 Conditions conducive to meaningful subject-matter to enhance the learning of problem-solving skills

For the conditions to be conducive to the emerging framework the classroom environment should represent the social background of the learners, in the sense that there should be a variety of stakeholders taking part in the teaching and learning of problem-solving skills. The classroom is not seen from the narrow perspectives of the teachers and the learners engaging on the subject matter, but as one that affords teachers, learners, officials in the education district (experts in mathematics), and parents an opportunity to display their wealth of knowledge which encapsulates the rich mathematical concepts. The close relationship between school community and the communities should be established as it is considered important for supporting children’s learning of problem-solving skills (Hāwera & Taylor, 2011:341). In addition, Altrichter (2005:10) suggests that classroom settings be in such a way that community members are used to influence the teaching and learning of problem-solving skills.

At the research site, the sustainable team was constituted that worked together in the teaching and learning of problem-solving skills. Among other things the focus of team was on ensuring the subject matter was made accessible to learners. By bringing in and relating the background experience of the learner into the classroom environment there were series of meetings conducted to get the support of interested members. One parent, Me Puleng showed her commitment to being part of the team:

> *Hona ho bontsha hore le batswadi re lokela ho nka karolo e kgolo thutong ya bana ba rona ya mmetse. Haholo-holo ka diphaposi mane moo thuso e kgolo e batlehang teng.*
>
> (It is clear that as parents we need to play a major role in the teaching and learning of problem-solving skills, especially in the classrooms where the major assistance is necessary).

The team members worked as a collective in lesson preparations. This was supported by the Plan of Action (refer to Table 4.4: Appendix F1), which was a collective plan and agreement that guided the team in the teaching of problem-solving skills. It ensures that the planning is done well in advance of the class. All the necessary resources, human resources, instruments and tools are in place before the class starts. The Plan showed the names of people who led the play of indigenous game, for instance the plan in column 2 (below) showed the playing of *kgati* was to be led by Dineo, Puleng, Me Motaung, Thabiso and Jane. It also clarified the roles to be played by other team members. The plan is designed in such a way that all phases (refer to column 1 in the table below), that is phase 1 – phase 5, allowed members to be actively involved. The classroom environment permitted the discussions and arguments to reign throughout the teaching and learning sessions.

One group, led by Thabo, asked the group which was presenting mathematical concepts observed or infused in *kgati* as follows:

> Thabo: *can you clarify for us, how do you see that the loop (curve) of kgati is increasing or decreasing? We didn't get you on that point.*

It was obvious that other small groups saw funny things, that is, mathematical concepts which did not make sense to them, but it was exciting to realise that there were other members of the team who extracted mathematical concepts embedded within *kgati*. Ntswaki's group responded to the question, making a representation on the chalkboard which resembled the playing of the game (refer to Figure 4.8):

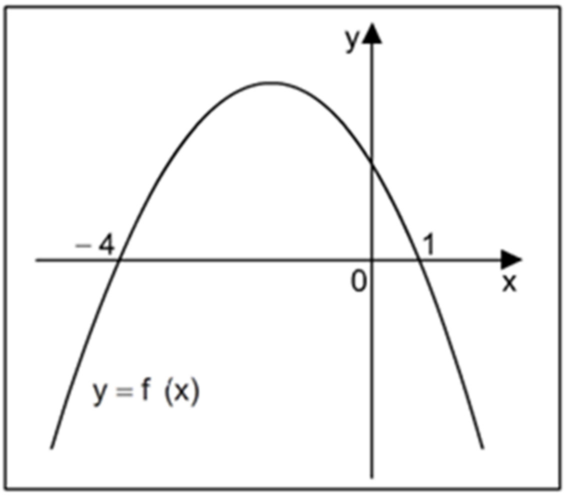
> *(See Figure 4.8) as Dineo 's hand goes up, even the kgati loop goes up, and also let us look at Puleng's hand, the hand goes down and the loop goes down too. Hence, we concluded that as the loop moves up, that indicates that the graph is increasing. The same way as the loop goes down that shows that the curve (graph) is decreasing*.

This demonstration showed that through learners' or team's observations there are mathematical concepts infused in the game. The raised questions or comments were not referred to the individual teacher, but the group presenting responded to the questions and comments. Other small groups were at liberty to make additions to the question raised. The team did not depend on the teachers or experts, but they were present to provide guidance and support if ever the focus slipped. The team members were given latitude to comment or ask questions for clarification at any given point.

After all groups had commented, the teachers and subject advisors in Mathematics normally gave their overall opinions and observations about the class presentations. That was done to sum up the lesson for the day and reflecting with the whole team on whether the lesson objectives had been achieved. Ms Mokoena made a summary:

> *We had a fruitful discussion, we really made a huge progress. We covered the most crucial concepts, like increasing and decreasing functions. As the homework, could you individually work on this exercise [refer to the homework below], then we discuss it the next day.*

HOMEWORK EXERCISE



(a) The graph of $f(x)$ is increasing and decreasing at which values of x

(b) When the graph of $f(x)$ is increasing and decreasing, can you describe its steepness or the gradient.

The scenarios above demonstrated that team planning for lesson presentations made it possible to bring in the experiences and prior knowledge of the learners. This circumstantial factor helped in making sure that the framework of teaching problem-solving skills using indigenous games turned out well. Picture 4.6: Appendix F3 pointed out the specific role that each member had to play or perform. As a result there was a huge commitment and collaboration among team members, in making certain that the subject matter presented was easily comprehended. This argument is supported by the comment of Ma Puleng '…*we need to play a major role, in the teaching and learning of problem-solving skills…*'

The scenarios demonstrated that team members were empowered to make proper planning and execution. The Plan was flexible and gave learners freedom to perform their activities with confidence. Also, it made a space for intensive discussions on observations made with regard to the mathematical concepts or skills infused in the structural nature of the indigenous games and the actual playing of them. The excerpts '…*can you clarify for us, how do you see that the loop (curve) of kgati is increasing or decreasing. We didn't get you on that point*.' This accentuates that as the explanation is made it should be very clear, thus ensuring that the content is easily understood.

It can be noted that social, navigational and aspirational capital gives learners an edge to interact closely with one another. This gesture helped in making a connection between the subject matter and their experiences. The phrase '…*as Dineo's hand goes up, even the kgati loop goes up, and also let us look at Puleng's hand, the hand goes down* …', demonstrates the powerfulness of the navigational capital in helping them in visualising the connection between the movement of the *kgati* loop and increasing and decreasing functions concepts. The linguistic capital used in explaining the movement of *kgati* helped them to conceptualise the theoretical concepts such as increasing and decreasing functions. In brief these constitute the core ingredients that made the framework of teaching problem-solving skills using indigenous games to succeed successfully.

In summary, the above evidence confirmed findings of David and Tomaz (2012:413) that classroom activities do not develop in a linear or predetermined way. As an individual teacher one planned the lesson and presented in a linear way. Rather, the activities of the class had to be flexible and multidimensional for learners with different needs and learning style to make precise meaning. Constructivists theory argue that the lesson presentations should allow leaners to discover meanings (Muijis & Reynolds, 2010:83,84) on their own, whilst making sure that the mathematics classroom is conducive to effective learning and discovery. The evidence above showed that the mathematics classroom is conducive to effective learning of problem-solving skills. The team created a space such that learners can play indigenous games to assist them in understanding the so called 'abstract mathematical concept'. The classroom environment symbolised the home environment, in which learners are encouraged (Vankúš, 2008:105) to learn problem-solving from the demonstrations of indigenous games by their parents (Ashton, 2007:7).

4.4.2 Contextual factors making learner-centred method of teaching problem-solving skills conducive to the emerging framework

In the same way as above, for the conditions to be conducive to the emerging framework, the teaching and learning of problem-solving skills should be empowering to learners. That means it has to allow learners to discover and reflect on realistic experiences. The teaching and learning session needs to happen in the learners' terrain, full of play and enjoyment which will encourage learners to take an active part. This is line with how du Toit (2013:19) views learning, that is, as constructive, accumulative and goal-directed. The knowledge that the learners acquire from home background is important in understanding the subject matter in a classroom. It is important for the conditions in the Mathematics classroom to feature in the experiences and prior knowledge of the learner. This will result in the learner having an interest to learning and being an active participant in the learning process. In addition, Yosso (2005:79) argues teaching and learning of problem-solving need to tap into the wealth of knowledge that learners possess from their home background. He points out that this marginalised knowledge is unused and not recognised in the classroom environment, which made learners passive in the learning processes.

In operationalising the framework of teaching problem-solving skills using indigenous games, the team was careful to put the learner at the centre, whilst the other team members were at the periphery of the learners' terrain. Other members were in the zone of proximal development (ZPD) as Kozulin, Gindis, Ageyev and Miller (2003:39,46) called it, such that they were able to give assistance, guidance and coaching when necessary. Learners were given freedom to be engaged from the beginning of planning until the lesson was presented, and assessment with reflection on the lesson learnt. The indigenous games played by learners were not just imposed on them but they were given a chance to decide on the ones they preferred (refer to section 4.3, table 4.15: Appendix F2). This was done to make sure that they came up with what they liked most, and where learner participation was high the activity on extracting the mathematical concepts and skills embedded in the indigenous games was handled by learners (refer to Figure 4.4 above). Through these activities made the discovery that within *morabaraba* there are many concepts and skills which can be learnt, for instance probability, strategizing, patterns, geometric shapes, areas, and perimeters. Other than the normal formulae: $A = lb$ for area and $P = 2(l + b)$ for the perimeter, $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ can be used to determine the area of concentric rectangles (refer to Table 4.7 above) and $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$ for the perimeter of concentric rectangles (refer to Table 4.7 above).

On the other hand, it was exciting to realise that within one indigenous game there were many topics or mathematical concepts that could be learnt. It was even plausible for teachers to note that integration of mathematics with other subjects, and integration with real-life situations can happen spontaneously. These instances made learners understand why it is necessary to learn problem-solving skills, as there is relevance to their life. This was central to making the framework of teaching problem-solving skills using indigenous games work.

The framework of teaching problem-solving succeeded, because the team created a conducive environment in which learners were at the forefront of all activities. Learners were given freedom to exercise their choice and the indigenous games to be played in classes were democratically chosen, with all having a say. Their feelings and interests on indigenous games they preferred were not oppressed, nor were their new inventions suppressed. In some instances, learners defined functions as patterns, since they noted that there was a relationship of variables. New discoveries, such as the general formula for the area and perimeter (concentric rectangles) as $A_n = dn^3 + an^2 + bn$ and $P_n = an^2 + bn + c$ respectively were applauded and acknowledged by the team members. The validation of their new discoveries by the teachers and other experts was amplifying the navigational and aspirational capitals they were displaying. Teachers made assurance that they needed to maintain hope and be motivated in being active in all class activities. This helped them to get deeper understanding of problem-solving skills. The attitude and the behaviour of the learners and other team members helped in sustaining the framework to produce good results.

In summary, it was evident that creating the proper platform to learn problem-solving skills made it easy for learners to take centre stage. At the same time, other team members were only making sure that small groups were given proper guidance, not necessarily dominating the discussion. This supports what Muijs and Reynolds (2010:79, 80) have suggested, namely that for effective learning of problem-solving, teachers need to instigate learners in using their abilities. The modes include scaffolding, where learners are given assistance only when they need it. Assistance or support can be provided by asking learners thought-provoking questions. The team demonstrated the ability to use the various modes of questioning and teaching to arrive at their own conclusions.

4.4.3 High level of motivation and interest among learners

Although the cognitive domain is important for the learners to master the subject matter so that they are able to remember high order questions or relate the new knowledge with the prior knowledge, it is also important that affective domains are nurtured for better performance in problem-solving skills to be realised. Affective domains relate to the feelings, emotions, interests and enthusiasm that makes learners do something. They must have eagerness to learn problem-solving skills, which in turn better the performance. Teachers and parents, as external factors play an important role in making learners want to learn problem-solving, hence the motivation by parents and teachers helps learners have trust in themselves, self-confidence and high self-esteem, which ultimately builds into learners developing intrinsic motivation (Averill, 2013:105; Campbell, 2006:8). When learners are motivated they also show that they care about themselves and others (DoE, 2003:2)

In carrying out the framework of teaching problem-solving using indigenous games, there were circumstantial factors that anchored the framework and yielded the results. Teachers and parents were key to motivating learners. One parent, me Lebone admired the way the Ditlhare's group presented, she commented:

> Me Lebona: *Bana baka ke thabile jwang feela mokgwa oo le bontsitseng tsebo e batsi ka teng, malebana le thuto ya mmetse.. Ka morabaraba le bontsha hore le ka etsa di-shape tse ngata ha kalo. Bona tsela eo le behang di-shape tse ding ka teng le fumana hore, ho na le traegele, mme le kgona ho botsha ka kotloloho hore kamano ke e feng. Le bontsha hore tioreme ya Pythagoras e tholahala jwang*.
>
> (My dear learners, I am so delighted about the quality work you displayed in using *morabaraba* to arrange these shapes into different ways (refer to Fig. 4.9 above section 4.3), such that these squares form a right-angled triangle. You also demonstrated how the theorem of Pythagoras can be formulated.

The parents and teachers are crucial in making learners belief in themselves by acknowledging and validating their contributions. In most cases, their comments and contributions were taken to design the lesson for the day. It was exciting for the learners to be in a classroom environment that had the presence of parents, and playing of indigenous games. In the previous section (4.3) it was suggested that the first two phases must be played the previous day in the afternoon. Whatever learners do it was to their best level, as they did not want to disappoint their peers, parents, teachers or district officials. Even to play what they liked most, the indigenous games, was a motivating factor on their part. The classroom environment was a source of inspiration for them, and accolades were showered on them for the slightest action. Such gestures of appreciation contributed greatly to building their self-confidence and high self-esteem. They realised that they were being taken care of by their peers, loved by parents and given enthusiastic attention by teachers. It was exciting to note that every comment or question was met by remarks such as '…*thanks for the wonderful and exciting presentation*…'

This shows that whatever was done was firmly acknowledged and encouraged by the team members.

It was very encouraging for learners to realise that all mathematical concepts discussed in class were mentioned by them (refer to Fig. 4.12: Appendix F5 and Fig. 4.13: Appendix F5), and even the assessment tasks administered were strongly related or had very close connections with what they discussed in their small group. Members of group A, headed by Sifiso, out of excitement and pleasure commented as follows:

> To learn problem-solving skills using indigenous games is of great assistance to us as learners. Generally things and games we do at home are of importance in our learning. Most of the times things that we do at home there is no fear, we do them freely.

For instance, the team accepted definitions coined by learners on ‘independent, dependent variables’ and ‘functions’. They related that to the playing of *diketo*, in which they noted that the throwing of the *ghoen* up (as independent variable) had an impact on the number of stones scooped out of the hole or placed in it (as dependent variable).

These circumstantial factors helped the framework of teaching problem-solving to be sustained. The Mathematics classroom was conducive to learning without their being afraid to make mistakes or being embarrassed in front of their peers or parents. This is justified by Sifiso’s remarks that in Mathematics class there was no fear of being embarrassed, but rather whatever they did was validated and acknowledged by the team members. They would be free to venture into many possibilities of obtaining the answer. This complemented comments raised by the parent ‘…m*y dear learners I am so delighted about the quality work you displayed in using morabaraba to arrange these shapes into different ways.’*

The parents, teachers and district officials were impressed by the navigational skills portrayed by the learners. The prevailing conducive conditions kept learners focused and consistently working hard to learn problem-solving skills. Even the presence of parents in the classrooms enhanced the familial capital, as learners were sharing their inputs and contributions of extracted problem-solving skills from games. In the small groups, interactions and networking showed a wealth of social capital. If questions were posed to the small group presenting one sensed that they deserved an appropriate response. For instance, one parent, Mr Lekau asked: '*Thanks for the wonderful presentation. But I'm little bit lost here, how do the two formulas for the area of the rectangle differ, that is* $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ *and* $Area = l\, xb \ldots$*'.*

Once more, learners were empowered to value any inputs made by small groups as very important. They tended to understand their need to respect the contributions made by others and learn from them. The social capital helped them to share, interact and network with other members of the team. The team spirit empowered them to have hope as they worked harder and would do well in problem-solving skills, and also to take care of those who were struggling to understand. These were the contextual factors that helped the framework succeed.

In conclusion, these circumstantial factors boosted the morale of learners to view problem-solving skills in a positive way. This is supported by the DoE (2003:62), which stated that teachers need to be sensitive to the manner in which they deal with learners' attitude towards problem-solving skills. The manner in which the problem-solving skills is taught has to make certain that they do not instil a set of attitudes described as 'mathematics phobia', leading to arrogance and domination by boys in class. Through the study is it clear that the contributions and inputs are equally made by both genders.

According to the Alaskan Department of Education and Early Development (2013:3), the DoE (2003:62) and Hawera and Taylor (2011:341), learners appreciated that problem-solving skills originated in African cultures rather than Greek ones. It was also found that they acknowledged the high level of knowledge displayed by their parents, in elucidating the rhythms and patterns in indigenous games, which links well with the problem-solving skills taught in classes. For instance, the movement demonstrated by *kgati* showed the mathematical concepts such as waves, crest, amplitude, periodicity, increasing and decreasing functions.

4.4.4 Circumstantial factors for learners to discovery problem-solving skills formulae and processes

According to De Walle et al. (2010:10), the DoE (2002:20) and the DBE (2011:4,8), Mathematics classroom atmospheric conditions represent the social background of learners, stimulating their high level of critical thinking on the presence of stakeholders such as parents, district officials from curriculum section and sports sections, community leaders. Also, cultural practices such as indigenous games and cultural artefacts can enhance their problem-solving skills and ability to imagine and formulate abstract concepts. As Yosso (2005:69) asserts, these subaltern communities possess rich knowledge which is essential in the teaching of problem-solving skills.

According to Gumbo (2013:439), when learners' community and background, experiences are taken into consideration, learning of problem-solving skills is more effective. These constitute the building blocks for learners to understand the high order formulae and processes of problem-solving skills. The appropriate factors in the teaching and learning of problem skills should not be limited to 'how' but rather feature the 'when' and 'why' of problems styles. For instance, learners will also question conclusions such as "*any number raised to the power of zero is one*", the appropriate question being 'why is it so?' This supports Altrichter's (2005:5) claim that the mathematics classroom should be seen as a place or laboratory in which to refute, confirm or justify the problem-solving findings. These reigning conducive conditions will encourage learners to take responsibility for their own learning of formulating problem-solving skills formulae and processes. It also allows them to be active participants in the learning-teaching process (DoE, 2002:2).

The use of indigenous games in teaching problem-solving skills was appropriate in helping learners understand that problem-solving skills are a product of investigations by different cultures, and it is a purposeful activity in the context of understanding highly sophisticated formulae and processes in problem-solving skills. Referring to Table 4.11 above.

Group C, led by MmaMpho, extrapolated the following:

> MmaMpho: *If you extend Table 4.11: above including column 4, it is the number of throwing the ghoen up added to the number of stones placed into the whole. Look at the pattern The pattern starts at 11, 12,13,….. we established that the general pattern can be described by* $t_n = n + 10, where\ n = 1, 2, 3 \ldots 9$***.*** *and also number of throws of ghoen up plus the number of stones scooped out of the hole can be described by the same general linear pattern.*

Another member of group C, Teboho made some additions:

> *It was interesting to explore varies options used the data we derived from the play of diketo.*

The conducive conditions are such that learners are able to ask themselves challenging questions. The data derived from the play of indigenous game helped them explore various ways of making conjectures and conclusions.

Ms Mokoena commented:

> *Thanks so much for the exploration you showed to us, we really learnt a lot from your presentation. Further you showed us that* $n + 10$*, and (*$-n + 10$*) +*$2n$ *are equivalent expressions.*

The atmospheric situations in the class in which problem-solving skills are taught using indigenous games created an environment for learners learning activities and hands-on, thus stimulating discussions and debates.

Jane's group pointed out the following scenarios:

> *Jane: suppose from the given table above you are asked to solve equations of the nature: solve for* x *given that:* $2 + x = 11$*. Two options to follow:*
>
> *(a) You can use the table above to read the correct answer or*
>
> *(b) Solve the linear equations*

It can be noted that the unknown represent the number of stones to be placed into the hole.

The above extract indicated how the circumstantial factors can contribute towards learners achieving a high level of thinking in problem-solving skills. The use of indigenous games in teaching and learning problem-solving skills has empowered the deprived and disenfranchised learners, who were formerly seen as receivers of knowledge more than creators of problem-solving skills process and formulae. Learners are able to generate different formulae using the power of their linguistic capital. This is illustrated by the phrase '…*take note the general pattern followed by the throws is* $2n$ *and the pattern described by the number of stones placed into the hole is* $-n+10$ *, adding this together you get the same general formula* $t_n = n+10$*, the same as the one MmaMpho got it earlier…'.* This excerpt demonstrates that learners possessed navigational capital that aided them to manipulate and manoeuvre the data produced from the play of *diketo*. These further demonstrate the cultivation of self-reliance and energising spirit of the learners. In addition the phrase '…*asked to solve equations of the nature: solve for* x *given that:* $2+x=11$*. Two options to follow'* shows that learners have a driving force of being creative in formulating problem-solving skills formulae and equations. The aspirational capital gave them courage to take on challenging and sophisticated tasks, and the stamina to make sustainable efforts to demonstrate a high level of critical thinking instilled by the conducive conditions prevailing. The cohesion of group interaction and readiness to share and network are sustained by the social capital revealed by learners.

In conclusion, according to the New Zealand Ministry of Health (2004:16) and Rosand et al. (2008:6), I argue that there is high possibility that the stipulated conducive conditions will bear fruit in the implementation of the emerging framework. The reigning conditions enhanced the open communication, trust among learners and other team members.

The evidence provided above confirms findings of the DBE (2011:8); DoE(2002:11), and Mahlomaholo and Netshadama (2010:5), that learners understand problem-solving skills such as observing, representing and investigating patterns, and quantitative relationships in physical and social phenomena. Through this processes new problem-solving skills formulae, conjectures and insights are generated and formulated. As the evidence validates, the inductive approach is viewed as true knowledge that produces a closer look at the phenomena, with learners able to investigate the observations and make conclusions based on the specific pattern observed. The general formulae learners deduced signify the deductive approach, which complements well with the inductive. This is further supported by Godino et al.'s (2007:132) onto-semiotic approach and the intensive-extensive dual dimensions, in which learners move from concrete to abstract concepts.

4.4.5 Appropriate conditions for parents to be highly involved in the teaching of problem-solving skills

According to the DBE (2011:8), Hāwera and Taylor (2011:340), Naong and Morolong (2011:240) in South African Schools Act (SASA) of (1996), the teaching and learning of problem-solving skills are featured in the National Curriculum and by law parents need to participate in all school activities, not limited to school governance. Rather, parents must be visible and given an appropriate platform in the mathematics classroom environment. Hāwera and Taylor (2011:340) portray the prevalent thinking of Māori society in the teaching and learning of problem-solving skills is the engagement of all community members. They have a wealth of knowledge which is marginalised by the Western system of education, which can be transverse to their children as a way of advancing the social transformational agenda, and ensuring that problem-solving skills are accessible to all, irrespective of race, culture, gender and age. In addition, Māori society believes that their interaction with cultural activities and environment is crucial in the learning process of problem-solving skills.

Mahlomaholo and Netshadama (2010:4) caution against 'organic intellectuals', whether Mathematics teachers and other experts in the field (e.g., subject advisors)) advancing a transformational agenda without due care and respect, and imposing rather than allowing the parents and other subalterns to determine their own agenda. Meanwhile the organic intellectuals provide advocacy and advisory roles in the teaching and learning of problem-solving skills. This emerging framework of teaching and learning problem-solving skills using indigenous games is anchored and sustained by the collective participation of learners, teachers, parents, community members and community leaders. These groups were recognised as the custodians of cultural practices, among others including indigenous games.

One learner pointed out that since the interaction between the school, community members and departmental officials was constituted; the Mathematics classes were vibrant, with an atmosphere of friendliness and cooperation among team members. Sentle, one of the learners, remarked as follows:

> *Jo, monate o mokalo ka klaseng ya mmetse, re na le batswadi, marena, le ba bahlanka ba lefapha la thuto. Re a tsheha, re keketehe.* Parents felt that they are part of the team; the language they used showed the richness of problem-solving skills *Dipapadi tsena tsa setso tseo re di bapetseng hae, le batswadi ka kakaretso ba re bontsha tse ding tsa dipapadi tsa setso. Re bona kamano e kgolo pakeng tsa mmetse le tse ding tsa dipapadi tseo re di tsebang ho tloha hae. Matsatsing ana ke fumane ke natefelwa hoba ka klaseng tsa mmetse, hoba ke ithuta tse ngata tsatsi ka leng.*
>
> (Wow, it is so exciting to be in a mathematics class with parents, traditional leaders, and departmental officials from the Department of Education at district level. We enjoyed a lot together with them. There is a link between problem-solving skills and indigenous games that we played at home, even our parents showed us a lot on these indigenous games. These days we learnt a lot in problem-solving skills.)

On phases 2 and 4, when all team members, including parents, were sharing their observations on the mathematical concepts and skills infused in the indigenous game, one parent, Mr Karabo, commented:

> Karabo: *maemo a renang ka phaposing ya mmetse ke a kutlwano, tlhomphano le kananelo ho seo o se etsang*.

(The atmospheric conditions prevalent in problem-solving lessons, are that one of harmoniously working together, of respect and acknowledgement on what one is doing.)

Parents and community leaders in the team were very dedicated and committed to the teaching and learning of problem-solving skills. One parent, Mrs Dimpe happened to be absent for the class discussion, but sent a text message to the class leader through by mobile telephone:

Ke tla ba siyo klaseng ya aftenunu kajeno, feela diphehiso tsaka ke di qoqile le me Kganyetso, le tla di fumana ho yena

I will be absent for afternoon class today, my contributions for the today's lesson are discussed with Mrs Kganyetso, who will give you feedback).

For the emerging framework to work and be sustained in the teaching and learning of problem-solving skills, the contextual factors allowed the free interaction between school community and community members. This was amplified by Mr Karabo's remarks '…*maemo a renang ka phaposing ya mmetse ke a kutlwano, tlhomphano le kananelo ho seo o se etsang*…' ('…the atmospheric conditions prevalent in problem-solving lessons are that one of harmoniously working together, of respect and acknowledgement on what one is doing'). The excerpt shows that there is a safeguarded bondage between school, and other partners such as parents, traditional leaders and education officials. Each party can freely express their views or ideas without fear of being belittled. All team members acknowledged and valued the opinions and thoughts of all members. Parents are empowered to regain access in the teaching and learning of problem-solving skills. The marginalised knowledge, in the form of navigational capital, helped parents to make powerful contributions during the reflection stages.

In addition the phrase ('……Jo, monate o mokalo *ka klaseng ya mmetse…..')('…..*wow, it is so exciting to be in a mathematics class….') are indications that learners enjoy the instruction through the use of indigenous games, in which laughter, peace and freedom prevail. Although there was laughter and enjoyment in the teaching-learning sessions of problem-solving skills, the focus is maintained as learners were expected to demonstrate expected lesson outcomes. All parties collectively take charge in making sure that the agreed upon lesson outcomes are achieved, and share ownership of achieving the desired outcomes of gaining deeper understanding of problem-solving skills. The responsibility is no longer on the teacher alone to see to it that the lesson outcomes are realised. The power is evenly distributed to parents, community leaders, and education officials, rather than to the mathematics teachers.

The text message from parent, Mrs Dimpe, shows the seriousness and the value that parents place on the teaching and learning of problem-solving skills. This further epitomises the aspirational and social capital possessed by parents in sustaining hope to succeed in problem-solving, through the learning networks. Even when there are barriers which hinder the progress, for example, if one assigned a role and responsibility but under unforeseen circumstances one happened to be absent from classes, the inputs were forwarded to the group leader. These circumstantial factors were significant, and signs of respect for all team members and love for one's obligations to serve the nation. During the planning session and preparation a parent was assigned a task to execute, but under unforeseen conditions she was unable to attend the session.

For the emerging framework to be sustained, the Australian Department of Education Employment Workplace Relations (ADEEWR) (2009:36); DoE (2003:9) and Kemp (2006:134) argue that the involvement of a third party, such as the parents, is crucial. The parents are viewed as a reservoir of knowledge of the workings of the ecosystems of their surroundings upon which they are dependent for a livelihood, welfare, wellness and sustenance. The evidence gathered justified the use of indigenous games as constituting the ecosystems of their surroundings. It signifies that knowledge on problem-solving does not resonate within social class, but rather all parties have an equal role to play in the teaching and learning problem-solving skills. It was evident in the research site that teachers respected and acknowledged the powerful knowledge on problem-solving skills which parents demonstrated.

Alexander and James (2005:16) point out that the engagement of a variety of the stakeholders, such as parents, is a way of enhancing and transforming children's attitudes towards problem-solving skills and also improves their social skills. In most cases, the participants were able to use their linguistic skills to give verbal descriptions of what they saw during the playing of indigenous games, after which they were able to translate the processes or their observations into mathematical language. This process illustrates the mathematical modelling which provides learners with the means to analyse, describe their world mathematically, and deepen their understanding of problem-solving, while adding to their mathematical tools for solving real-world problems (DoE, 2003:10).

4.4.6 Contextual factors that enhance content knowledge among teachers on problem-solving skills

The teacher has to possess the pedagogical content knowledge, which will help him or her to enable the learner to construct his or her own knowledge of problem-solving skills. The teacher has to demonstrate an ability to avoid the thinking of the learners being limited in problem-solving. Hence, it is important for the teacher to work collaboratively with other mathematics teachers and parents on lesson planning, presentation and assessment. This will help widen the learners' horizons on problem-solving skills. The conditions prevailing in such a class should allow learners to discover interesting and fascinating mathematical concepts, and thus provoke the thinking of learners.

Bansilal (2011:95), the DoE (2003:2), and Stott and Graven (2013:32-33) write about the space that needs to be created for the thinking of the learner, free from social boundaries or limitations. As the teacher has a relatively deep pedagogical content knowledge this will help learners to develop their zone of free movement (ZFM) and zone of promoted action (ZPA). The former, by its nature, is not fixed or limiting in the interaction between the learner and the environment, but the limited knowledge of the teacher can hamper the free thinking of the learner. The latter, the ZPA, refers to the set of activities, objects, or areas in the environment, in respect of which the learners' actions are promoted. It means that the ZPA offered by a teacher must engage with a learner's possibilities for development and promote actions that he or she believes are feasible within a given ZFM. The teacher's actions in the teaching of problem-solving skills have to enable learners to move across the boundaries of their existing ZFM, thereby changing it.

In the research site, the circumstantial factors compel learners to operate beyond the two zones. The teacher gave learners activities which did not restrict their critical thinking, for example derivations of area, total surface area and volumes of frustums (see section 4.3.7.1). In turn they came up with an interesting model (section 4.3.7.1, Fig. 4.12: Appendix F5 and Fig. 4.13: Appendix F5). The deep pedagogical content knowledge of the teacher contributed greatly in stretching the learners' horizons on problem-solving skills. One teacher commented as follows:

> Ms Mokoena: *Thank you my learners, with the three-dimensional models (frustum) you have obtained. Can you work out the volume and total surfaces areas of these structures with the polyhedral you have obtained?*

The teaching of problem-solving skills using indigenous games created conducive conditions for the learners to use their prior knowledge effectively, and gave teachers an edge in exploring the learners' capabilities to the full. These allowed learners to operate outside the zone of free movement and promoted action. The teacher also expected learners to perform the activity designed in this fashion:

> Ms Mokoena: (Referring to Table 4.9 above) *Deduce the relationship between the stones scooped out of the hole and the stones placed in the hole, if one throws the ghoen in the air.*

The activities designed for the class were all taken from what the learners generated. Group D, headed by Ntswaki, came up with the following argument:

> *We established the relationships, which is shown by the Table 4.12) below.*

Playing of diketo game at round two (seng 2)

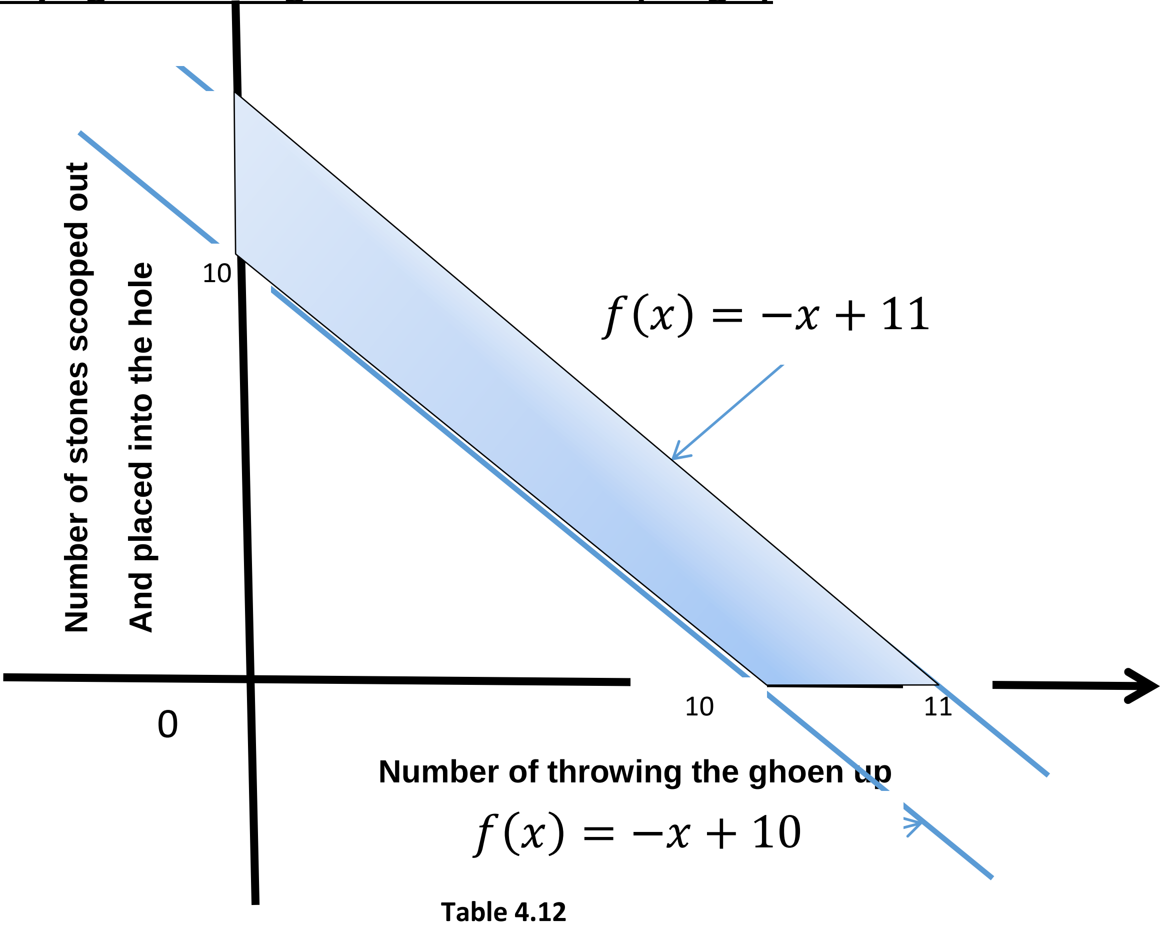


Table 4.12

One learner from group A, Zibanze applauded the comments as follows:

> Zibanze: *Wow, this is an interesting argument you are raising, let us look at this point, what type of figure is illustrated by that shaded region?*
>
> Ms Mokoena*: Yes, Yes, taking the argument further can you calculate the area and perimeter of the shaded region.*

The above scenarios showed that the contextual factors allowed teachers to provoke the thinking abilities of learners. The activity which was given to learners showed that teachers insisted on making learners operate outside the two zones, and was not determined by the teacher. Rather, the learners were given the freedom to operate outside the predetermined zones. This is also a demonstration of the wealth of navigational capital learners possess.

The extract '…..*taking the argument further can you calculate the area and perimeter of the shaded region?'* shows that teachers with a high level of content knowledge are able to design their own materials for activities. These are designed from the inputs and contributions shared by learners, which illustrate that they are empowered to be actively involved in the teaching and learning of problem-solving skills. In most cases these activities are generated by learners. In sustaining the emerging framework, the above discussions indicated the circumstantial factors that sustained the emerging framework and produced better results.

In conclusion, the cited evidences above coincided with Vithal and Skovsmose's (1997:145) view that problem-solving skills given to learners must be based on knowledge which learners bring from outside school, and in which problem-solving skills are developed from learners' own situations. Bowie's (2013:245) belief that all learners need to have access to problem-solving skills is in line with DBE (2011:9), for whom problem-solving activities are practised by all cultures. Hence, the use of indigenous games in the teaching and learning of problem-solving skills created an encouraging and conducive environment in which all learners could mirror themselves and their cultural practices, such as playing of indigenous games in mathematical content and skills.

4.4.7 **Conducive conditions for high motivation among teachers**

There are contextual factors that must prevail for teachers to demonstrate high levels of motivation in the teaching and learning of problem-solving skills. The teacher has to regard himself or herself as a lifelong learner in the teaching and learning sessions. This will help in allowing constructive debated and discussions to happen in the classroom. As a result, learners display their multiple intelligences or alternative approaches to problem-solving and there will be improved performance in problem-solving skills displayed by learners (Ebenso, Adeyemi, Adeyoke & Emmel, 2013:208,210; Sriraman & Lesh, 2007:59).

In addition, the DBE (2011:3) and Nyama (2010:35) stated that the teacher has to work cooperatively with all stakeholders in the teaching and learning of problem-solving skills as this will help the team members to have ownership in the planning and presentations of the lesson. There will be multiple ideas, sources and resources to be used in lesson presentation. The use of indigenous games and other activities is important in linking the real-life situations with problem-solving skills. This creates a platform for the teacher to use various teaching methods, which arouse learners' interest in the learning of problem-solving skills.

On the research site, the teacher did not play a dominant role in the lesson planning and presentations; rather they were collectively followed, which enabled learners to display high levels of understanding of problem-solving. This created a space for learners to interact meaningfully in class activities.

For instance, in section 4.3.3.1, these were conducive conditions reigning during the reflection. Briefly, this was echoed by Ms Mokoena:

> *It was really exciting and encouraging to listen to powerful presentation you did. Your presentation covered almost all what were supposed to do today. So many interesting points you raised and patterns derived from morabaraba. Points such as using the structural nature of morabaraba. Let us suppose that we have concentric rectangles of this nature* (Table 4.7, above).

This confirms that the contextual factors reigning in this class were excellent, as learners used indigenous games educationally to discover problem-solving skills on their own. This kind of situation motivated the teacher and the team members to prepare a favourable learning environment. It can be noted that when learners are motivated to learn problem-solving skills, it is automatically encouraging, and boosts the self-esteem of teachers to work harder in their duties of supporting learners properly to understand problem-solving. The teachers and team members continued to suggest alternative ideas and sources for lessons presented.

(Refer to section 4.3.3.1 on Mr Nhlapo's comments) These comments were phrased in a creative way, with learners expected to tap into their multiple intelligences on problem-solving skills. To briefly reiterate, Mr Nhlapo remarked:

Zanele and your group E, thanks for the demonstration you made. This is thought-provoking and quite interesting. Tell me then, eh, do you mean that functions are type of patterns?

The scenarios depicted above validate the effectiveness of contextual factors prevailing that helped the emerging framework to be implemented successfully. The comments quoted show they liberating learners from the fear of problem-solving skills. Learners are encouraged to use their navigational capital to overcome barriers to the learning of problem-solving. The high level of motivation shown by teachers and parents stimulated learners to maximise their learning efforts in learning abstract concepts in a meaningful way. The comments echoed by teachers and parents showed that learners' inputs are welcomed and acknowledged with high level of respect. For example, '…*Zanele and your group E, thanks for the demonstration you made. This is thought provoking and quite interesting*', shows that learners are given hope that their future in the learning of problem-solving skills is on the right track. The conditions are of the nature that teachers do not display arrogance or cite damning remarks that demotivate learners, rather the contextual factors advance the social and aspirational capitals for learners to network among themselves and maintain a positive attitude to future learning.

It can be concluded that there is a close correlation between the empirical data and literature reviewed. The Organisation for Economic Co-operation and Development (OECD) (2009:92) takes constructivist approach that learners are not passive recipients but active participants in the process of acquiring knowledge. The circumstantial factors in the teaching and learning environments dictate that mathematics teachers and the team members preferred to give learners the chance to develop on their own alternative ways to problem-solving skills, and allow learners to play an active role in instructional activities. The use of indigenous games in the teaching and learning of problem-solving skills develops the critical thinking reasoning processes and equally stresses the acquisition of specific mathematical content knowledge that the learner is expected to acquire.

For Ewing (2013:131), the funds knowledge or marginalised knowledge which are infused in the indigenous games are the building blocks for the development of schooled or academic mathematics concepts. As the teaching and learning of problem-solving skills portrayed the home environment, the conditions under which mathematical concepts and skills are taught appeared relaxed and enjoyable, which resembles those with which learners are familiar outside the classroom.

The contextual factors accentuated the essence of teaching and learning problem-solving skills using indigenous games to ensure that the rights of the children are protected, such that children gain access to problem-solving skills offered in higher grades or at higher education. This is in agreement with the United Nations (UN, 1989:8-9) statement on children's rights articulated in Articles 28, 30, and 31, namely that it is crucial that learning of problem-solving develop each child's personality, talents and to the fullest. Generally, this will translate into encouraging and motivating learners to respect their own and others' cultural practices.

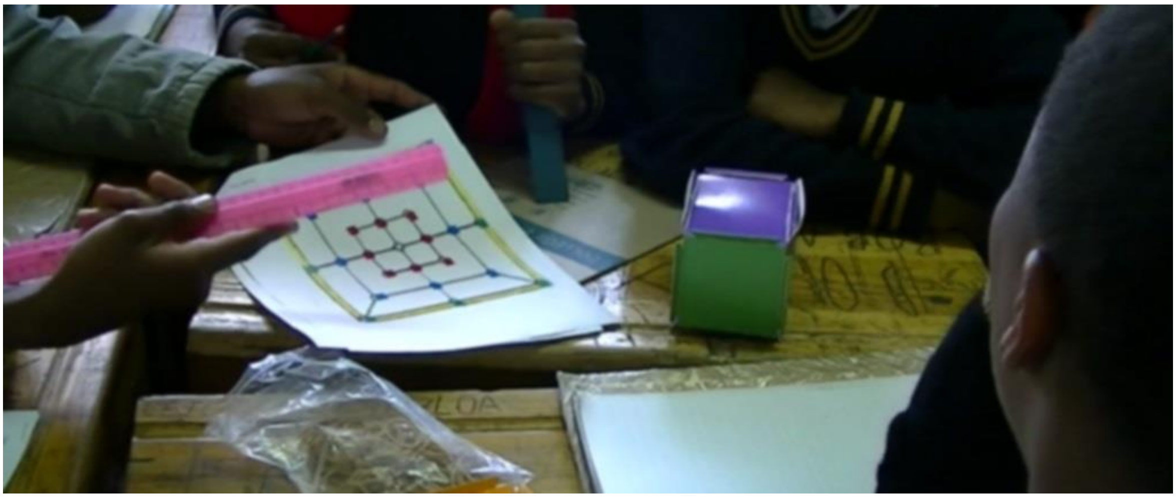
4.4.8 Circumstantial factors that enhance teachers' expertise with regard to classroom practices

According to the Association of Teachers of Mathematics (ATM) (2010:44), Harrington, Brasche (2011:23), in the classroom, learners, teachers and other partners should work cooperatively in the teaching and learning of problem-solving processes. This will help the learning environment to establish the conducive conditions that espouse the democratic values (Constitution of Republic of South Africa, 1996:3) to prevail. The contextual factors are anchored in human dignity, respect, equality and freedom that assist learners to contribute to knowledge creation in problem-solving skills. Also, they support learners, teachers and parents to work together in conjecturing and generalising the mathematical content extracted from indigenous games.

Averill et al. (2009:175) and Averill (2013:16) argue that proper factors in enhancing teachers' expertise in classroom practices provide learning space in which all learners are listened to (De Walle et al., 2010:47) with care, and the teacher values, notices, and recognises the thinking behind the comments made by learners. Also, mistakes and misconceptions are worked with positively, creating an atmosphere in which errors are viewed as valuable steps on the learning journey. There is praise and constructive comment that provides both feedback and '*feed-forward'.*

The learning space in the teaching and learning of problem-solving skills using indigenous games, provided learners with sufficient resources to analyse mathematical concepts in indigenous games. Learners were task-focused and actively communicated ideas with confidence among themselves.

The Picture4.4 indicated that the contextual factors in the classroom, afforded learners an opportunity to take the centre stage in doing class activity which involved the structural nature of *morabaraba*. In the activity in which they were engaged, they were responding to questions and the mathematical concepts and skills embedded within *morabaraba*.



Picture 4.4: Learners working on Morabaraba Activity

Group members debated the mathematical concepts observed in *morabaraba*:

> Leader of the group *(Letswalo): Rona re bona di-skwere, rektengle, ejese, le di- vetisese*.
>
> (What we observe in the board games are the squares, rectangles, edges, and vertices).

The circumstantial factors prevailing in the classroom showed the supportive environment, such that learners are self-disciplined. They also display high level of respect among their peers, teachers and other team members. One member from Letswalo's group made additions:

> Neo: *We also noted that the two-dimensional figure can be changed into three-dimensional figure (refer to Picture 4.4: above, Fig. 4.12 and Fig.4.13: Appendix F5), rectangular figure, frustums, where we can determine the edges, vertices, faces, total surface areas.*

The extracts and the Figure 4.4.8(a) above showed that the learning environment was conducive to learners sharing their observations freely with others. There is interactive thinking, in which learners' inputs and contributions are respected by other team members. The extract '…*the two-dimensional figure can be changed into three-dimensional figure….'* shows creativity and imagination in analysing the game with regard to problem-solving skills. The learners were given an opportunity to express their own ideas as they explored mathematical contents and skills.

In addition, the contextual factors allowed the home environment to be dominant in the classroom. Learners capitalise of the marginalised wealth of knowledge they possess. They navigated the *morabaraba* to come up with problem-solving content such as two-dimensional, three-dimensional, number of edges, and vertices in a rectangular figure or frustum. The social capital empowered them to interact collectively in learning the problem-solving skills.

In conclusion, the contextual factors prevailing made it possible for the emerging framework to be successfully implemented. There are close connections between empirical data and theoretical data. Akuno (2013:66) argues that democratisation of the creative processes of using indigenous games to teach problem-solving skills, indicating that every learner has something to contribute to (and to gain from) the learning experience. Each contribution is acknowledged and valued by the team members. This is clear that learning of problem-solving skills becomes a social and socialising event. In addition, Thomas and Brunsting (2010:3) assert that the prevailing contextual factors for emerging framework piqued learners' curiosity, actively engagement in learning problem-solving skills, and accommodate different styles of learners in the classroom.

On the other hand, it was found that the use of indigenous games to teach problem-solving skills helped the learners to master the subject matter well. In this framework, learners need to do more than watch, copy what the teacher is doing on the board, and keep pace with their teacher’s work. The circumstantial factors created a friendly learning space in which to make problem-solving skills their own creation (Anthony et al., 2009:151,152; Thomas & Brunsting, 2010:24).

4.5 Factors that threaten the implementation of the emerging framework

In section 4.3 components of the emerging framework to teach problem-solving skills using indigenous games were comprehensively examined. In this section, discussion will centre on threats to the implementation of the emerging framework and how they were circumvented. The components of the framework, such as methods to make teaching learner-centred, high level of motivation among learners and teachers, self-discovery of problem-solving formulae, good level of involvement by parents, adequate content knowledge among teachers, and high expertise with regard to classroom practices, will be discussed.

Akuno (2013:66), Mukhopadhyay (2013:94) and Lynn (2004:158,165) caution that other risks to the framework can be triggered by wrong perceptions. The cultural practices were discouraged by colonial education, the perception being that when engaged in games one is labelled 'lazy' or 'unintelligent.' In addition, Akuno (2013:67), Lynn (2006:117,118) and Morris et al. (2011:29,30) support the socio-cultural perspective on learning using indigenous games as an active and interactive process that occurs within a social context. From a cultural responsive perspective, teaching draws on culture as a basis for fostering problem-solving skills (PSS) achievement of marginalised learners. Teaching PSS using indigenous games emancipates teaching practices as the teachers are able to teach critically from the learners' perspectives. Indigenous games develop ideas, support exploration by learners to discovery mathematical content embedded within the games.

4.5.1 Risk factors that derail meaningful subject-matter in teaching of problem-solving skills

The emerging framework used indigenous games to teach problem-solving skills in a meaningful way. The discussion is detailed in Section 4.3.1. The use of indigenous games made learners active in the learning of problem-solving, as they realised that these were concepts with which they were familiar. Van De Walle et al. (2004:32) argue that for precise meaning of mathematical concepts the learners need to feel comfortable in trying out ideas and explaining their thinking.

The use of indigenous games made it possible to bring in the prior knowledge of the learners in contextualisation of problem-solving skills. As Yosso (2005:69) contends, the theory of community cultural wealth acknowledges and validates the marginalised knowledges of learners from their home environment. The linguistic and navigational capital that learners possess helped them to realise the close link of mathematical concepts and the actual play of the games or the structural nature of the game. This creates an environment conducive to contextualisation of problem-solving skills as central to developing learners' interest in acquiring problem-solving skills. For instance, in using *diketo*, learners observed that if one is throwing the *ghoen* in the air and scooping the stones out of the hole or replacing them there are independent and dependent variables identified. Also, it was meaningful for learners to craft their own meanings of independent and dependent variables.

It was gound that there were risks factors that threatened understanding of the subject matter. During the actual play of the indigenous games, one member of the SMT was concerned about the noise made by learners cheering up their team members for using indigenous games to teach problem-solving skills. One member of the school management team argued:

> Mr Tshiu: *Ke batla ke kgathatseha ka lerata le bang teng nako eo dipapadi tsena tsa setso di bapallwang, ho batla ho ba le lerata le lengata haholo ka diphaposing. Moo klase tse di ding di batla di sitiseha. Ke kopa tshebedisano mmoho tabeng eo. Mokgopi o ke o shebe se ka etswang malebana le taba eo.*
>
> (I am little bit concerned about the level of noise by the learners during the play of these indigenous games. In some instances it disturbs the classes in the same building. The team needs to think about that and look for alternatives to overcome the problem.)

It was a genuine concern raised by the members of the SMT, because when learners from other classes hear ululation and cheering it tends to disturb in their classes. On the other hand, the learning and teaching support material (LTSM) was not adequate. The materials helped parents and learners to decide whether the extracted concepts were mathematical concepts or not, as highlighted by one Life Orientation teacher, Mr Obang:

> *The problem I encountered is that am I good on these indigenous games, but the problem is identifying the certain mathematical concepts illustrated by the indigenous games. For instance, at one stage we mentioned the increasing and decreasing functions demonstrated by the movement of kgati*.

Ms Putsi, the subject advisor for Mathematics, responded at follows:

> *The argument raised by Mr Obang is possible to happen, that is why the group was constituted with people of various expertise and experiences. As you mentioned that you are good in indigenous games that is wonderful for the team. There are team members amongst us here who are good in certain aspects, such as their mathematical content, analysis of mathematical content, and representation of data in many forms.*

The response of Ms Putsi suggested a way forward on how the problem of addressing risks factors could be handled. The bringing together of people with different knowledge helped greatly in minimising the factors that might derail the emerging framework. As one parent indicated:

> *Me MaThabo: Nka thaba haholo ha re ka dumellana hore tshebetso ena ya rona mmoho, re sebedisa puo ya Sesotho hore re tsebe ho nka karolo ka ho phethala.dipuisanong.*(We will be pleased if we can be allowed to use the language we are familiar with, so that we can participate optimally in the discussions.

Ms Mokoena elaborated:

> *Yes, me MaThabo, that is the good point you are bringing to our attention. One parent did indicate the reason why he is always absent in the meetings or in class activities, it is because the teachers and Mathematics subject advisors dominate the discussions and, as parents, their roles are not clearly delineated.*

The team members played a very important part in the emerging framework. The Action Plan (refer to Table 4.4 in Appendix F1) was drawn up so as to make sure that all members have roles to play. The collective planning helped in making sure that indigenous games were not imposed on learners, rather they were given a chance to decide on the suitable games they wished to play. The remarks by Mr Tshiu showed that learners were excited when playing the indigenous games, to the extent that other classes were being disturbed. Therefore, in making sure that such social injustice, including disrespect for the teaching and learning in other classes, did not happen it was decided that the actual play of the indigenous games, that is phase one (refer to Section 4.3.1), take place in the playing field or, during bad weather, in the school hall. An alternative, as the traditional leader suggested, was to take learners for excursions to his workplace (*kgotla*), where they also formed part of the playing team and the analysis thereof. Other options were the afternoon sessions which accommodated parents working to be part of the team. In most cases, in phase 1 and 2, the actual playing of the games and reflections and analysis of observations were conducted, then the remaining three phases were performed in the classroom.

The extract '…*we will be pleased if we can be allowed to use the language we are familiar with, so that we can be able to participate optimally in the discussions…',* shows that it is the risk factor that can derail the emerging frameworks. In circumventing this it was decided that all members were free to express themselves in any language they felt comfortable or familiar with. This was a way of empowering learners and group members to have access to the understanding of problem-solving skills. It encouraged all members to take part in activities and discussions, as they were isolated, thus demonstrating the democratic values which suggest that no one be marginalised in terms of race, social class, gender or age.

Apparently there are parents who felt isolated: '…*one parent did indicate the reason, why he is always absent in the meetings or in class activities, it because the teachers and mathematics subject advisors dominate the discussions and their roles are not clearly delineated'.* This shows there were sessions which excluded the members unintentionally and that might weaken the emerging framework if not avoided. This was an indication that their linguistic capital was validated in the teaching and learning of problem-solving skills.

In conclusion, Jorgensen (2013:330) argues that most learners of the indigenous communities did not perform well in problem-solving skills because of the anti-democratic values, and mathematics is not learned in those contexts in which it is expected to be used by its learners (Berget, 2013:135). The discussions in class became dominated by the teachers and subject advisors, and portrayed problem-solving skills as the knowledge of the powerful, such that it took the form of abstract and impersonal discourses as they were not given the freedom to express themselves in the language with which they felt comfortable. There was evidence that parents in the team absented themselves because the discussion did not cater for them, so they felt isolated. In addition, Knijnik Wanderer and Oliveira (2005:101,103,105-106) emphasise that problem-solving skills should make knowledge comprehensible to all learners of all classes and social contexts. It was therefore important that in all discussions and deliberations taking place in class and in the field that they needed to be present. Variety of populations, notwithstanding their social position, should have access to the relationship between the practices of daily life and schooling (Berget, 2013:136).

4.5.2 Risks to the methods of teaching problem-solving skills

The methods used in the teaching and learning of problem-solving skills need to be learner-centred so that it is easy for the learners to comprehend the subject matter. The teaching methods used should empower learners to discover and reflect on realistic experiences (Kellaghan, 2009:1; Sillah, 2012:86), which confirms Mahlomaholo's (2010:16) view that classroom environments should capitalise on the learners' background knowledge, be always sustainable and empower learners in understanding problem-solving skills. Learners' backgrounds allow them to explore and investigate mathematical concepts on their own, using their rich wealth of knowledge (Yosso, 2005:70). On the other hand, the facilitation styles should be liberating (Lynn, 2006:118), making it possible for learners to contextualise the problem-solving skills within their daily lives. The facilitation styles by the teacher need to be diverse, allowing learners to explore various possibilities in understanding problem-solving skills.

On the research site, these approaches were followed, as the teachers strived to teach problem-solving from learners' perspectives. The team used indigenous games which most of the learners enjoy. One Life Orientation teacher remarked on the timeframe:

> Ms Sekoane: *Colleagues are we not taking too much time on one indigenous game? Are we going to be able to finish them as listed on our action plan? I think we need to speed up the progress.*

There were different views on the matter. Ms Mokoena, the mathematics teacher responded as follows:

> *Colleagues, the approach of using indigenous games to teach problem-solving skills is not about finishing the stipulated indigenous games. Rather is about how much work on problem-solving skills do learners learn. The other point to be noted is the coverage of mathematical concepts. There are many activities which were done to enhance learners understanding on problem-solving skills. Learners showed deep understanding on problem-solving skills.*

Mr Debako added to the argument raised by Ms Mokoena:

> *Yes, I agree Ms Sekoane, that as the team we need to be aware of those deterrent factors. But on the other hand, let us check the mathematical concepts which we dealt with in diketo (coordination game) or extracted within the games, for instance: patterns, relationship between ghoen thrown up and stones scooped out of the hole, independent and dependent variables, linear functions, factors and non-factors and many more.*

Furthermore, one parent, Mma Sebolelo raised the concern about certain small groups as follows:

> Mma Sebolelo: Ke betla ke kgathatseha, ke elellwa hore ha bana bana ba fana ka fitibeke, eba ngwana a le mong repotang ka dinako tsohle.

(I am a little bit concerned about certain small groups, it's one learner who gives feedback to a larger group, let us look at that and try to encourage them to rotate their roles.

Mr Talana added the inputs in the discussions in this manner:

O nepile me MmaSebolelo. ntho eo re ka etsang ke hore , ha rona di-fasilitheta tsa di dihlopa re sebetsa le bana bana, re ba elelliswe hore ho bohlokwa hore ba fapanyetsane mesebetsi ya bona, jwaleka , moeteli-pele(wa sehlopha, mongodi, mmuelli wa sehlopha.

(You are correct, me Mma Sebolelo, every time group's composition or task changes new roles need to be assigned to other members. So different learners can be accustomed to various roles, such as group leader, reporter, scribe, etcetera. The other point, as we facilitate in various groups, we need to advise them to rotate the roles. I hope that will solve the problem.)

In ensuring that all learners are effectively involved in their small groups, one does not have to give the correct answer, but rather provoke their thinking by posing challenging questions. Mr Mokoena observed that there were few individuals who were a bit quiet for a while in the small group discussions. He strategically engaged them by saying:

Re lokela ho nka karolo koala, ha re ke re utlweng hore Tefo le Puleng hore maikutlo a bona a reng?, le bo pattern ho kae hape mona diketong.(*All of us need to take part in this discussions. Let us give Tefo and Puleng a chance to tell us where they observe patterns in diketo*.)

Puleng responded:

Nna ke bona phetapheto ha o akgela keto hodimo le fatshe.

(*I repeated the movement as you throw the ghoen up and down*.)

To make certain that all learners were on the same level of understanding on problem-solving skills, each group facilitator (other teachers, subject advisors, parents) has to make certain that every learner has a chance to air his or her views or inputs. Those few struggling individuals were helped by other members of the group and group facilitators. In some instances, groups or individual learners volunteered to give feedback to a larger group.

The excerpt '…*colleagues are we not taking too much time on one indigenous games? Are we going to be able to finish them as listed on our action plan? I think we need to speed up the progress'* shows that the team members need to be careful of hindrances that might derail the emerging framework to attain the expected results of enhancing the performance of learners on problem-solving skills. Use of indigenous games is a way to reach the target or ultimate goal. The indigenous games are strategically used to stimulate the learners' interest in problem-solving skills and to tap into the rich wealth of knowledge which they have from the home background. This knowledge will empower learners to associate the prior knowledge with the new knowledge they have to learn.

The responses by Ms Mokoena and Mr Debako reminded the team to keep focussed on the target, and not to be derailed. Response to the report '…*there are many activities which were done to enhance learners understanding on problem-solving skills. Learners showed deep understanding of problem-solving skills*' justified that although it appeared that few indigenous games were played, much on problem-solving skills had been done (refer to section 4.3.1 and Fig.4.4). Also, it demonstrates that whatever activities were performed, learners needed to benefit by better understanding of problem-solving skills. They needed to be empowered to have easy access to problem-solving skills rather than focusing on the quantity of indigenous games performed. In mastering more of the problem-solving skills extracted from the indigenous games they had hope that they could do well in problem-solving skills, because of the navigational and aspirational capital that helped them maintain hope and navigate through understanding the abstract concepts.

The expression of concern about one learner providing feedback to a larger group and rotation of roles shows the importance of equity among group members in terms of participation and division of labour in their small groups. Nobody should be marginalised in their working, as that would have a negative impact on their performance on problem-solving skills. The group members should make sure that they interact and learn from one another by capitalising on the social capital they acquired from the home environment. Also, the linguistic capital helped them to visualise the pattern demonstrated by the play of *kgati* and *diketo*. Eventually learners were able to conceptualise the definitions of concepts such as functions and relationships. Phrases such as '…*let us give Tefo and Puleng a chance to tell us where do they observe pattern in diketo*…', exemplify the hope that they can still do well in problem-solving skills. Considering Puleng's response '….*repeated movement of throwing the ghoen up and down*', it reveals the practicality of diketo, helping Puleng to visualise the pattern in context and so coin the definition of pattern correctly. The imagination of Puleng is made possible by her navigational.

In conclusion, it is noted that there are correlations between the theory and evidence provided above. According to Vankúš (2008:105), learners use their background knowledge, abilities and experience to visualise problem-solving skills embedded within indigenous games. Also, to develop learners' memory, imagination and critical thinking, learners need not be given correct answers all the time, rather guidance on how to work out the given activity. The limitations which occurred in the teaching of problem-solving skills using indigenous games (Nutti, 2013:25) indicates that the teachers can act as potential agents by adopting Bishop's six universal activities, that is counting, locating, measuring, designing, playing and explaining. The team succeeded in applying designs of lesson plans together. The playing of indigenous games was used to extract mathematical concepts embedded in them, and these activities were performed through giving explanations and discussions that occurred in small groups.

4.5.3 Risks to high level of motivation among learners

Anthony et al. (2009:154) and Averill et al. (2009:173) argue that learners come to Mathematics classes being very motivated by the richness of their social environment. The home background is replete with verbal encouragement, and interesting and thought-provoking ideas, either from peers or parents. In line with Haverhals and Roscoe (2010:339), the use of cultural practices (such as indigenous games) in the teaching of problem-solving has many benefits for learners, teachers and parents. Learners are engaged in something with which they are familiar, hence they are excited in taking part. Also, the benefits include the perfecting of problem-solving skills, laying a solid base for better comprehension, assisting learners to make mathematical correlations, and emphasising the cooperation between problem-solving skills and society.

Subsequently, Noyes et al. (2011:180,181) argue that the teaching of problem-solving skills suggests that the teacher use various modes of teaching that will motivate learners to be interested in learning problem-solving skills. We can set free learners from the island of problem-solving skills and r transpose them on the mainland of life that encompasses problem-solving skills that is open, alive, full of excitement, and every time thought-provoking (Haverhals & Roscoe, 2010:339).

On the research site, it was interesting to observe that learners were motivated by the plying of indigenous games in the teaching of problem-solving. One district official in the Sports section, Mr Tshimomg argued as follows:

> *It is good that you are so excited in the field. But you happened to forget to note important mathematical concepts or skills you observed as you play. Remember as you play, you need to be vigilant of the mathematical concepts or any skills, values and attitudes which are revealed during the play of the indigenous games.*

Mr Talana advised the team members and learners as follows:

> *Yes, Mr Tshimong I fully agree with you. It is then important that as we play to jot down the skills or processes that are illustrated in the play before you forget. During the reflection period you can be able to refer back to your observation sheet. Also, as you work in pairs or small groups, please decide on someone who will be a scriber, reporter, group leader, as that will assist to be more focussed on our activities.*

One parent, Me Sefako added to the deliberations in this way:

As we play, let us reflect in pairs or small group, as to what have we learnt in the playing of indigenous games before we report to a larger group.

The above cited threats need to be taken in consideration during implementation. The extract '…*remember as you play, you need to be vigilant of the mathematical concepts or any skills, values and attitudes which are revealed during the play of the indigenous games'* shows that as much as learners enjoyed the playing of indigenous games no team members should forget that the main aim was for them to understand the problem-solving skills. In so doing that would allow them to be free from bondage of ignorance about the mathematical concepts and skills infused in indigenous games. In turn, learners would develop positive attitudes towards the learning of problem-solving, which would give them hope that they had a brighter future in learning and mastering problem-solving skills. In addition, the aspirational capital gives them courage to survive the hardships that might be obstacles to the learning of problem-solving skills.

The use of observation sheets, which learners are advised to use consistently not sporadically, was advocated: '... *it is then important that as we play to jot down the skills or processes that are illustrated in the play before you forget.'* This emphasises the importance of writing down their inputs on the observation sheet, and will allow groups to move faster when they reflect on what they have learnt in terms of the structural nature of the game and the actual playing of it. Also, the remarks of Me Sefako: '… *let us reflect in pairs or small group, as to what have we learnt in the playing of indigenous games before we report to a larger group',* emphasised the importance of working in pairs and small groups to report back on mathematical concepts embedded within indigenous games. This helped peer-teaching to take place before the larger group was engaged. The peer-teaching was facilitated by teachers, mathematics experts (subject advisors) and parents. During the peer-teaching, learners saw themselves as equal partners empowering one another. This is the kind of environment in which all feel free to add their voices or seek clarity on matters which are not easy to follow. In addition, it can be observed that learners use their social and navigational capitals to interact and network with their peers, as they manoeuvre and analyse the indigenous games to extract mathematical concepts and skills.

In conclusion, there is a link between the empirical data and theoretical data with regard to the factors that threaten the emerging framework. This is illustrated by Jones, Jones and Vermette (2011:845) and Cao et al. (2006:86), that it is motivating for learners to realise that there is great value in using indigenous games to learn problem-solving. Learners tend to have empathy to see the world from different viewpoints, as they enhance their understanding of problem-solving skills whilst having fun in playing the indigenous games. This is supported by Andersson and Seah's (2013:194) view that learners tend to value the use of indigenous games to learn problem-solving skills, motivating them to use their cognitive skills and emotional dispositions. The regular feedback learners received in small groups and larger groups motivated them to work well in understanding the problem-solving skills.

4.5.4 Factors that threaten self-discovery of problem-solving skills formulae and processes

In learning problem-solving skills with understanding, the teachers have to ensure that the home culture or environmental background is incorporated. This supplemented what Hodkinson (2005:110) and Kilpatrick (2009:112) view as the construction of learning, focusing on ways in which individual learners construct their own understandings of problem-solving skills. Also, teachers have to discharge different methods of teaching to elucidate pre-existing understandings or misunderstandings. Problem-solving skills as part of broader mathematics involves discovering, where learners refine and test conjectures so as to arrive at the correct conclusions. Further, the efforts are aimed at making learners more actively involved in learning the problem-solving skills by making the content more meaningful and engaging them in investigative work. As Haverhals and Roscoe (2010:333) argue, the self-discovery can help learners to view problem-solving differently, that is in an appreciative manner rather than perceiving problem-solving skills as closed, dead and emotionless.

On the research site, learners were given the worksheet No. 2 in Section 4.3.2.1 to answer the questions. One learner, Thabiso, in group A, asked the questions as follows:

> Tell me, in answering number (a) *'In the 2nd, 12th , 16th throw-ins, how many stones will be pushed into the hole?*', what formula do we have to apply in answering the questions.

In responding to the question, the facilitator (the subject advisor: Mathematics) who was in the group advised the group members as follows:

> Ms Putsi: *At this stage it is too early to think of the formula, how about if we look at other alternatives of answering the question. We are not that much interested in the product rather the process of getting the answer is very key.*

One of the group members, Lindo responded:

> *But, mam the formula is the easiest and quickest way of getting the answer.*
>
> Ms. Putsi: *Yes, in a mathematics classes or lessons we try by all means that you be problem solvers, and above that you be able to derive your own methods of getting the answers. Generally mathematics is not about using formulas to get the answers, but be able to reason out, as to how did you get the answer.*

Ms Putsi's comment was to discourage learners from thinking of formulas when answering the questions, and instead look at the conceptual meaning to deduce the formula from their observations.

During the feedback session (section 4.3.2.1) in responding to questions of worksheet No. 2, Dineo's group was presenting their possible answers. Group D, member Pule, wanted clarity:

> *Tell me, will I be wrong if I use the formula:* $\boldsymbol{f(m) = -\frac{m}{2} + 10}$, *and the answer as 9 stones without dwelling into the table you did, before showing us the answer?*

Dineo responded:

> *Yes, it correct, but I think you need to show all the details of your work, so that all of us can be able to comprehend what you are doing. I don't know, I might be wrong with my explanation, can other members assist to respond to the comment.*

Ms Mokoena, with a smile on her face, added a few comments on the discussions:

> *Thanks for response you made Dineo, and your members. You are quite correct. As you perform your calculations it is important to show all the detailed steps. Actually we are not only looking at the final product, but the key thing is the manner in which you get your answer, is very important.*

Mr Talana commented that these were important pitfalls that needed to be taken into considerations when learning problem-solving skills. He added in the discussion the following points:

> *On the other hand, one other aspect to note, by the time presenters give their possible solutions, I realised that there are certain members who are copying them down, apparently with no understanding. Our word of advice is that let us pay attention on what they are saying, for us to ask questions or comment appropriately. Surely we will give time to synchronise the responses given.*

The question: '….*what formula do we have to apply in answering the questions…?*', is an indication that some learners wanted to recall formulae and take shortcuts. This is a dangerous approach as it denies and suppresses learners' critical thinking in learning problem-solving skills. The navigational skills they possess are also suppressed, as it depowered them from dwelling on creative processes to navigating towards the possible answer. Rather, learners are inspired to think of creative methods to arrive at the answer: '…*it is too early to think of the formula, how about if we look at other alternatives of answering g the question'.*

As learners work on problem-solving, they should not limit their thinking to formulae or focusing on the correct answer. Rather, they need to be engaged on divergent ways and processes of attaining the answer. The approach of looking at the process of getting the answer will enhance their understanding of problem-solving skills. In the process of showing the details of their calculations or interpreting their steps, learners learn more about their own strategies to be employed in getting the convincing responses.

In addition, the extract '*… by the time presenters give their possible solutions, I realised that there are certain members who are copying them down, apparently with no understanding'* indicates that there are certain learners who do not listen as the presenter is providing feedback. That act prevented them from staying focussed on the activity, and they did not contribute fruitfully to the deliberations. The relationship between the presenter and other team members should encourage social capital to surface easily, with learners interacting, networking, and learning from others, without stumbling blocks.

Lastly, it can be concluded that the literature reviewed concurs with the evidence cited above. According to Hodkinson (2005:114) and Kilpatrick (2009:113), for the emerging framework to succeed the following needs to be accentuated in the learning of problem-solving skills: the use of indigenous games helped learners to have conceptual understanding. This refers to the comprehension of mathematical concepts, rather than memorisation of formulae in arriving at the correct answers (Su, Choi, Lee & Choi, 2013:2). This is further supported by the DBE (2013:127), which claims that learners are still copying the quadratic formula incorrectly from the information sheet. Either they have copied it erroneously or they have learnt it inaccurately and not used the formula sheet. Once the learners know the process of obtaining the answer, if they have used the incorrect formula it is easy for them to detect the mistakes made.

Learners gained confidence in procedural fluency and adaptive reasoning. They were able to carry out processes and procedures in a flexible, accurate, and efficient way. The navigational capital enhanced their strategic competences, as they were able to formulate general conclusion and represent mathematical concepts in various ways, that is visualise patterns numerically and represent them on graphs. In addition, learners developed adaptive reasoning and productive disposition, as they demonstrated a capacity for logical thought, reflection, explanation and justification on problem-solving skills. During the feedback sessions, as they present their arguments there were juxtapositions made to elucidate their assertions. Ultimately, they tend to see problem-solving skills as sensible, useful and worthwhile in their daily lives.

4.5.5 Factors threatening the involvement of parents

Escobar (2010:4) and Montero (2008:662) argue that the sustainability of the teaching and learning of problem-solving skills requires parents to play the major role. Parents are seen as powerful human beings, as builders and carriers of marginalised knowledge that encapsulates cultural practices, such as indigenous games. Because of deeper knowledge on cultural practices they are then essential in the emerging framework that uses indigenous games to teach problem-solving skills. Parents are relevant partners in the teaching of problem-solving using indigenous games, since they play the crucial role in the home environment of the learners. This is supported by Hodkinson (2005:114), who argues that within the home environment the parent plays a prominent role. The learning occurring is informally through participating in everyday life, in the family, local community and at leisure. Hence, Escobar (2010:4) and Montero (2008:662) ague that it is impossible to solve the problems of the communities if such members of the community (including parents) are not part of the framework to be developed to enhance learner performance in problem-solving.

In addition, Budd (2005:7) in Freira, 1971:1-5), asserts that parents and other community members need to be invited to participate in developing the framework of teaching problem-solving skills effectively using indigenous games. Parents have deeper knowledge on critical analysis of their cultural production, such as indigenous games, music and art. The processes of teaching and learning problem-solving skills should allow parents’ voices, actions, power and interests to be expressed freely. It would be unfair to oppress them, as Assembly of Alaska Native Educators (1998:22) and Montero (2008:669) contend, as the participatory nature of the framework proposed that parents be incorporated in the project of teaching problem-solving skills using indigenous games. The parents' presence is a sign of validating and respecting the knowledge (Yosso, 2005:70) they possess. Also, the framework afforded an opportunity to those oppressed by unfair living conditions to teach problem-solving and help them to exert their rights to transform the teaching of problem-solving skills, engaged as full citizens in society. The importance of parents in the teaching and learning of problem-solving is further accentuated by the DBE (2013:11) through congratulating all the hardworking parents, and communities for their obligation and dedication in espousing the class of 2013 to obtain the highest pass rate since the dawn of democracy in the Republic of South Africa.

The emerging framework of teaching problem-solving skills using the indigenous games did encounter threats to its implementation, but there were measures which were put in place to circumvent them.

One parent raised a concern of this nature:

> *Mma Tumelo: rona re iphumana le boemomg boo re sa kgoneng ho tla mona sekolong ka mabaka a mosebetsi le ho kgolla bana di-grant.Jwale re kopa hore mesuwe e utlwise maemo a rona.*
>
> (At times we are confronted by the situations where we were unable to attend the sessions because of working, and at times we had to collect the grant monies for our children. We believe that you will bear with us.)

Mr Tshabane responded to the comment in this manner:

> Mme Mma Tumelo taba ya hao e ya utlwahala haholo. Taba ya bohlokwa ke hore thuto ya ngwana ha e ya phethala bosiong ba motswadi. Jwale re tlo sebetsa ka tsela e tjena. Re kopa batswadi ba bontshe dipapadi tseo batla nka karolo e kgolo re tsebe ho etsa time table. Taba e nngwe ke hore phase 1 le 2, ke hore ho bapala ha papadi ya setso le ho fana ka tlaleho ho tla etsahala ka di-afternoons kapa ka diwikente. Moo boholo ba lona le tla kgona bo teng.(*Me Mma Tumelo your point is understandable. The important thing is, the parent's role is very important in the education of the*

> *child. As the Plan of Action is drawn, you will be requested to indicate session(s) you wanted to attend, and other thing is the actual playing of the indigenous games (phase one) and reflection sessions (phase two) are done in the afternoons and weekend to accommodate other parents who are tight up in certain matters to be available in large numbers.)*

Mr Kopano, the parent component, nodded his head in acknowledging the arrangements:

> Ke lebohe sebaka seo ke se nehwang. Lebitsong batswadi ba teng kwano le batla ho leboha ditlhlophiso tsena tse fihletsweng. Le rona re rata ka hohle-hohle hoba teng ha tsena di etsahala moo re sebedisang dipapadi tsa setso ho ruta mmetse.
>
> (*I want to thank the opportunity given to me. On behalf of parents we are pleased to be part of these arrangements. This will assist us a lot to avail ourselves in all activities where the teaching of problem-solving skills using indigenous games is demonstrated.)*

The other crucial point was raised by Mma Sebolelo about the reporting sessions in classes:

> Taba yaka ke ena, ke thabela ho nka karolo ho dihlotshwana tse nyenayne ka phaposing. Feela e ka re re qhelelwa thoko, ha mokgopi o kopane oohle moo ho fanwang ka diphehiso tsa sehlopha ka seng.
>
> (*My point is, we are delighted to be part of the small group discussions. But my concern is during the feedback sessions to a large group, we are side-lined, as no inputs are given by parents*.)

Mr Rantsho, the district official in Sports section in charge of indigenous music and games, responded to the concern:

> Karolo eo o e bapalang dihlotshwaneng ke e kgolo haholo, moo o hlahlellang bana bana ka tsebo eo ya hao ya dipapadi tsa setso. Jwale ka nako eo ba fanang ka tlaleho ho klass kaofela re lakatsa ho bona bona bana ba nka karolo e kgolo haholo, hore re bone hore ba utlwisisitse seo o

> neng o ba hlahlella sona mmoho le dintho tse ding tsa sehlopha.Re hatella taba ena ya OBE, e reng ngwana o lokela ho nka karolo e kgolo ho ithuteng ha bona. Rona re le batswadi re bapala karolo ya tsa boeletsi ha bona ba sebetsang ka dihlotshwana tsa bona.
>
> (*As parents, you are playing an important role, where you are monitoring and facilitating the discussion sessions of groups, and sharing your expertise on the playing of the indigenous games. During the feedback sessions we want to see learners taking an active and major role, as a way of assessing their understanding of mathematical concepts extracted from the indigenous games. Hence the focus is on them, not that you are marginalised. This is the application of OBE, where learner-centredness is emphasised. Parents and teachers play the advisory and facilitating roles in small groupings. As we draw conclusions at the end of the lesson, please feel free to contribute*.)

The school community is willing to work with parents in the teaching and learning of problem-solving. To make sure that the voices of parents are not oppressed because some are working an arrangement is made for the phase 1 and 2 to happen in the afternoon sessions and weekends. This is an indication that parents are not marginalised in the education of their children. The obstacles that hinder the presence of parents in the teaching and learning of problem-solving are collectively removed for the benefit of learners.

Parents feel that their role is appreciated in the teaching of problem-solving, as validated by the excerpts *'….we are pleased to be part of these arrangements. This will assist us a lot to avail ourselves in all activities where the teaching of problem-solving skills using indigenous games is demonstrated.'* The excerpt showed that parents are given hope and respect that their participation is acknowledged in the teaching and learning of problem-solving. There is equity, in the sense that parents are not excluded from the teaching of problem-solving in terms of their level of mathematical competence.

Also, the extract '... *But my concern is during the feedback sessions to a large group, we are side-lined, as no inputs are given by parents'* revealed that they were not marginalised, but the intention was to give learners centre stage in showing their competencies in problem-solving skills. The approach used by the schools or mathematics practitioners and teachers is to ensure that the navigational skills of learners is unearthed in the learning of problem-solving. This is also a demonstration of social capital, from which all members of the team benefited, and gained valuable knowledge and skills through the interactions occurring in the class. The phrase '...*this is the application of outcomes-based education*' made parents feel that they were not marginalised in the teaching and learning of problem-solving. The important aim is to give learners a chance to voice their arguments, and also to empower them to demonstrate their aspirational capital. Learners need to be able to elucidate their arguments when asked to do so. Parents are encouraged to feel free in making additions and comments during the large group sessions. This approach OBE applied to the teaching and learning of problem-solving skills helped individual learners, parents and all other members to acquire self-confidence.

Lastly, as Montero (2008; 664) indicated, the involvement of parents in the teaching and learning of problem-solving is an effective way of dealing with the problems of understanding mathematical concepts. Their expressions, actions, power, and interests vented in the processes of teaching and learning problem-solving skills are acknowledged and validated. In addition, the participation of community members (especially the inclusion of parents) has a ripple effect, such that the team collaboratively developed the participatory methods aimed at respecting and hearing those oppressed, and to engage them as full citizens in society in the teaching and learning of problem-solving skills. This is demonstrating a prosperous praxis (theory informing practice, practice well-informed by theory) with a divergent pattern (Budd, 2005:7, Montero, 2008:669).

Hodkinson (2005:114) argues that this is the orderly devolution to the subatern community and parents as the product of learning problem-solving is socially enacted. That is, these products of learning and teaching problem-solving are constructed through practices within the classroom, which entail the interaction between learners, mathematics teachers, Life Orientation teachers, district officials, parents and community leaders.

4.5.6 Factors threatening the adequate content knowledge of teachers

According to the DBE (2013:10) and DoE (2003:7), it is expected that Mathematics educators will demonstrate sound knowledge and skills in the teaching of problem-solving skills. Largely topics in Geometry and Data Handling can be presented eloquently and practically by using promptly accessible resources drawn from the learners' milieu, rich in mathematical concepts. It is recommended that teachers need to tap into the learners' background to concretise mathematical concepts which are too abstract to easily comprehend. It was reported by Moloi and Chetty (2011:2) that the mathematical performance of South African learners over the two SACMEQ initiative projects was not substantial. Teachers are encouraged to use varies strategies to teach problem-solving skills effectively. As a result, the research study is developing a framework for using indigenous games to teach problem-solving, to make a significant impact in understanding abstract mathematical concepts. One of its aims is to use the learners' environment to teach and learn mathematical concepts. Educators need to expose learners to extensive applications and high order questions embroiling tangible or concrete and abstract problem-solving skills.

Provasnik, Kastberg, Ferraro, Lemanski, Roey, and Jenkins (2012:4) indicate that the content knowledge of teachers is important, in that when learners are engaged in the TIMSS assessment they will be well-prepared, taking into account that it is organised around two facets: (1) a content feature stipulating the subject matter to be assessed; and (2) a cognitive dimension detailing the cognitive or thinking processes to be assessed. The content areas include number, algebra, geometric shapes and measures, data display and learners' mathematical thinking processes such as knowing, applying and reasoning.

The principals the research schools wanted to find out about were the materials that will guide the teachers in using the indigenous games to teach problem-solving.

> Mr Lekena: *I am impressed about the approach you are using of teaching problem-solving using indigenous games. Where can we get the materials*

that can be used by the newly appointed teachers or to train other Mathematics teachers to use the emerging framework?

Ms Mokoena responded:

Thanks very much, Mr Lekena for the thought provoking questions. This approach needs someone who has a high knowledge of the mathematical content, and high level of analysis skill, that is being able to link mathematical content. This is further complemented by the knowledge possessed by parents, and community leaders on indigenous games.

In addition to Me Mokoena' comments, Mr Talana said:

I just want to comment on these issues. The teacher's content knowledge and experience is crucial, and the knowledge brought diversified members of the team. Other stakeholders such as Life Orientation teachers, district officials from curriculum section and Sports section, parents etcetera, as the recordings are done for all activities done and presented can be collated and produce a document which can be disseminated to schools.

Ms Putsi commented as follows:

I learnt a lot from the team. During the content workshops, the very same team and learners can be used to demonstrate how the approach can be applied in a real mathematics workshop.

Mr Lefumo, one of the HOD Languages of the research schools, wanted clarification:

As you are using the indigenous games to teach problem-solving, can the learners go through the approach being prepared enough to sit for final year mathematics examinations or even to compete at the regional or national levels (SACMEQ) and international levels, that is TIMSS or PISA assessments.

Mr Talana argued strongly that:

> *The mathematical contents addressed using indigenous games are from the grade 10 mathematics: Curriculum Assessment and Policy Statement (CAPS).The mathematical concepts tackled in our sessions are the same throughout the globe. Learners can be able to compete at regional assessment and in assessments at international levels. The approach we are using it develops mathematical processes and assist learners to comprehend the mathematical concepts which are presented in an abstract manner. For instance in section 4.3.1.3, learners were able to formulate the formulas such as:* $A_n = (0,1)n^3 + (0,9)n^2 + (0,4)$ *, which can be used to calculate the area of concentric rectangles. Learners demonstrated that without using the normal formula:* $Area = l \times b$*, there is an alternative way of getting the answer. This is evident that the use of the indigenous games in teaching problem-solving skills potentiality of developing high level of thinking and understanding among learners. What was also encouraging is that they (all team members) realised that there are lot of mathematical concepts to learn from the indigenous games.*

The extract '… *I am impressed about the approach you are using of teaching problem-solving using indigenous games. Where can we get the materials that can be used by the newly appointed teachers or to train other mathematics teachers to use the emerging framework*?' suggests that the principal, Mr Lekena was happy about the approach of using indigenous games to teach problem-solving, but he had some concerns as to how the approach would be disseminated to the newly appointed teachers, so that they can use it in their teaching. This extract talks to the social justice, on how the resources and information on using indigenous games to teach problem-solving skills can be disseminated to the newly appointed Mathematics teachers. The emerging framework needs to be beneficial to the whole society. The non-existence of the materials in the teaching of problem-solving skills can be viewed as disempowering to the new crop of teachers, even to the larger community that are not part of the team.

The comments of Ms Mokoena pointed out that there was hope that the newly appointed teachers would easily participate well in the approach of using indigenous games to teach problem-solving skills, because of the high content knowledge on mathematical content. The navigational and socials capital possessed by members of communities would allow them to interact with the others to figure out the mathematical concepts embedded within the indigenous games. The emerging framework is knowledge-generating intervention, in which the engagement with other team members would elicit the required problem-solving skills.

The excerpt on the importance of teacher's content knowledge and experience*,* indicated that the team members had given others a hope that, with the knowledge on problem-solving skills possessed, they could empower and emancipate others on the mathematical concepts that can be easily accessible through the use of indigenous games. The excerpts further demonstrated the social capital team members possessed, making it possible for them to interact and share the information on the teaching of problem-solving using indigenous games. The linguistic capital helped team members to freely express their thoughts in explaining the problem-solving skills encapsulated in indigenous games. This is further echoed by the phrase '…….*I learnt a lot from the team. During the content workshops, the very same team and learners can be used to demonstrate how the approach can be applied in a real mathematics workshop*….', as the team members empowered other people well, as all were motivated, encouraged and rejuvenated in adopting the approach of teaching problem-solving skills using indigenous games.

On the other hand, the excerpt on the global uniformity of mathematical concepts gives hope that there is a future for the learners who went through the approach of using indigenous games to teach problem-solving skills. Such learners will be able to compete at national levels, that is grade 12 Mathematics examinations, and international levels as they are conversant with the problem-solving skills.

In summary, it can be pointed out that the observations made by the literature reviews are in agreement with empirical data. This is demonstrated by Norton, Irvin (2007:558) and Ross (2013:5), that teachers and researchers are encouraged to look for multiple resources in the teaching of problem-solving skills that can be used to concretise the problem-solving skills. The use of concrete materials reflected in indigenous games and learners' discussions helps them to make sense of abstract mathematical concepts.

Mukhopadhyay (2013:94) and Wong and Lipka (2011:822) argue that the use of cultural games in the teaching of problem-solving helps learners to understand the mathematical content. The playing of indigenous games relates to their daily life experiences, although its use in teaching problem-solving was discouraged by the colonial education authorities in favour of *ludo* and *snakes-and-ladders*, albeit teachers were encouraged to be creative in the teaching of problem-solving skills. In the same way, Gay (2010:68) argued care be taken of teachers with low performance in the content delivery of problem-solving skills, that is their quality of teaching problem-solving is not up to the required level, rather they attribute learners' failure to comprehend problem-solving to poor home environment. Wong and Lipka (2011:823) added that learners taught using the cultural activities to improve their conceptual understanding of the targeted mathematical content. It is therefore imperative that the assessment developers and teachers understand the cultural activities and mathematics embedded within them.

4.5.7 Threats to motivation of teachers in the teaching of problem-solving

Haverhals et al. (2010:333) and Noyes and Sealey (2011:188) assert that the teaching of problem-solving should not be about transmitting knowledge into the minds of learners, because that leaves learners as passive receptors of knowledge. Rather, to increase motivation among teachers and learners, interest should be generated by using creative ways of teaching problem-solving skills. The teachers need to explore some of the complex and interactive ways in which learning and teaching of problem-solving is managed, and view learners' results in problem-solving skills as theirs too, as that will stimulate teachers to instil the culture of performativity in the learning of problem-solving. This will come about when teachers are confident in the teaching and learning of problem-solving. In addition, the DoE (2003:41) pointed out that those teachers who are motivated in the teaching of problem-solving do embrace a participatory model in their teaching. They involve stakeholders, such as their peers in school and neighbouring schools, or officials from the district or circuit offices, being present when they make a class presentation so as to get feedback for future improvement.

At the initial stage of the research project, some mathematics and Life Orientation teachers felt that they were being overloaded with work unnecessarily.

Ms Temane commented:

> Ms Temane: *This is the waste of time; I still have Geography which I had to teach in grade eleven. I have to prepare the teaching aids that will be appropriate for teaching problem-solving skills. Really where will I get such time, and also be part of the team as well.at times as teachers we are overloaded with so many things.*

Ms Putsi argued:

> Ms Putsi: *Don't get discouraged mam; we are here to assist one another to make this venture possible. The advantage is that your learners will be able to realise that integration does not only occur within the subjects, but across the subjects including Geography and Life Orientation, that will help them to have a bigger picture on problem-solving skills. Also you will assist us to prepare the resources to be needed, the same resources or facilities used to play the particular indigenous game in your Life Orientation class. If you are committed with other classes, you can borrow us the equipments for playing the particular indigenous games, and when you are free in the afternoons or weekends you can join us. Apart from the equipments you will also assist the team with your expertise on indigenous games.*

MePuleng, one of the parents suggested:

> *E bile titjhere haeba bothata ba di sebediswa tsa ho bapala papadi tsena tsa setso bo le teng, re ka bua le bana bana hore batle le tsona mahae. Hobane bona baeka ba di bapala papadi tsena hae, ho mme batla thabela ha di sebediswa thutong ena ya mmetse.*

(If there is a problem about the resources and aids to be used in the playing of indigenous games, we should request these learners to improvise and bring them along from home. The very same indigenous games are played by most of them at their respective residential places)

Ms Mothoalo, the grade 10 Mathematics teacher, who was, teaching other subjects and Life Sciences in grade 10 and 11, commented:

Ke mametse ka hloko, jwale le re bakela mathata le stress, rona ba rutang diklase tse ngata tsa mmetse le Life Sciences, re etse ne neng mosebetsi was CASS o batlwang ke di-HOD le di LFs. Ba tlabe ba batla di-lesson plans tsa motho ka mong jwale rona re tla di nka kae. Le work schedule ke ena ka mona le yona. O tla be o bolellwa hore o morao ka yona.

(I listen to the discussions attentively, now you are causing us problems and stressful work. Bear in mind that we are teaching many classes, that is Mathematics and Life Sciences, how do you expect me to cope with HoDs and LFs' work. I will have to administer CASS components as per policy, compile lesson plans and I also had to ensure that the work schedule is updated)

To which Mr Ditshibana responded:

I think we are at the advantage position, where we can directly interact with our LFs and HoDs to get their pleasure about the matter at hand. I had these few comments to say, if we engaged on collective teaching of problem-solving that will assist us to learn from one another. The lesson planning will be there as evidence that the lesson did take place. You also mentioned that the lesson was facilitated by the following individuals, you then mention them. The discovery tasks that are performed by learners will be used as part of their continuous assessment activities.

Ms Putsi added:

You are quite right Mr Ditshibana. As subject advisors we are motivated if we realise that the quality of work on problem-solving is of acceptable

> *standard. The approach of teaching problem-solving skills using the indigenous games is of great importance because as teachers you cover a lot of work within the limited period of time. Integration of mathematical concepts happens spontaneously, and at the same time learners are able to validate their work as they work in groups, which is really encouraging and motivating move demonstrated by learners.*

Mr Talana said:

> *The involvement of various stakeholders such as district officials will assist us a lot. Can I also be clarified on the matter, the June examinations are common, does that mean that I had to conduct extra classes to cover those topics which were not covered during the class presentations? As I realised that the manner in which work schedule is followed is not in a linear way, that is, covering the term one, then term two in that order. Won't it be an extra load for the grade ten Mathematics teachers?*

The Further Education and Training (FET) subject advisor for Mathematics responded as follows:

> Mr Rasekhoro: *Thank you for the opportunity. It must be clear to colleagues that we are not teaching for the sake of covering the work as laid down in the work schedule; rather we want to make sure that learners do comprehend the mathematical content easily. When integrating the mathematical content with the ones of other terms that will assist learners to have broader understanding on problem-solving. I believe within the short space of time, you would have covered a lot.*

The comment about: '*waste of time,* I *still have Geography which I had to teach in grade 11. I have to prepare the teaching aids that will be appropriate for teaching problem-solving skills. Really where will I get such time, and also be part of the team as well, at times as teachers we are overloaded with so many things*…..', demonstrated that some teachers in the team felt that they were oppressed, as they were given too many classes to teach and were expected to play a major part in the approach of teaching problem-solving using indigenous games. The manner in which they were overworked and their relationships with their respective HODs and LFs (subject advisors) would thus be strained. This is amplified by the phrase '….*we are teaching many classes, that is Mathematics and Life Sciences, how do you expect me to cope with HoDs and LFs' work? I will have to administer CASS components as per policy, compile lesson plans and I also had to ensure that the work schedule is updated*…'. Being involved in the project of using indigenous games to teach problem-solving seemed to discourage them.

The misunderstanding that existed in the team was diffused through the interaction they had with other members of the team. The stressful teachers were given hope and assurance that their involvement would benefit learners and themselves as well. This argument is supported by the excerpt: *'….the advantage is that your learners will be able to realise that integration does not only occur within the subjects, but across the subjects including Geography and Life Orientation, that will help them to have a bigger picture on problem-solving skills…..'*. The aspirational capital they possessed could help them maintain hope, if they perceived the stumbling blocks. Their involvement would broaden their knowledge and they would learn from their colleagues. They could also help learners to see the interrelationships between mathematical concepts and other subjects, in the long run forming empowering experiences that would help them to juggle the many duties assigned to them.

On the other hand, the excerpt '…*does that mean that I had to conduct extra classes to cover those topics which were not covered during the class presentations. As I realised that the manner in which work schedule is followed is not in a linear way, that is, covering the term one, then term two in that order. Won't it be an extra load for the grade ten Mathematics teachers…',* signifies unfairness for grade 10 Mathematics teachers, as they would have to conduct extra classes to cover the work which was not covered in the teaching of problem-solving using indigenous games. The extra workload given to teachers was perceived as threatening their rest time.

The comments of Mr Rasekhoro '……*when integrating the mathematical content with the ones of other terms that will assist learners to have broader understanding on problem-solving. I believe within the short space of time, you would have covered a lot…'*, suggests that such problems mentioned by grade 10

Mathematics teachers could be overcome. Mathematical concepts can be integrated, for instance in section 4.3.3, where the pattern ($f(x) = -x + 11$) was developed when playing *diketo*. This pattern can be used to draw a linear function, and geometrically it can be used to calculate the area and perimeter of a triangle. In addition, the comments pointed out that involvement in the teaching problem-solving using indigenous games can be empowering to teachers. Learners can also see the interrelationships between various mathematical concepts, which help them to acquire deeper understanding of problem-solving skills. The social capital possessed by the team members can be of help in making sure that members network and learn from others on how to manage the stressful situations prevailing in the teaching and learning of Mathematics.

In conclusion, it is evident that there is a correlation between the empirical data and literature reviewed. According to Yildirim (2012:150), the learning strategy used by the teachers, such as using indigenous games to teach problem-solving, promotes learning and achievement of problem-solving skills among learners. In turn, learners are likely to be more motivated to learn and more engaged in problem-solving skills. This also enhances teacher–learner relationships, which is useful for learners' learning the problem-solving skills. Studies have equally given an account that learning strategy plays an extrapolative role in achievement in problem-solving. Likewise, the research team used the indigenous games to teach problem-solving.

The conducive classroom environment created by the teacher has either positive or negative influence on the affective and cognitive outcomes of learners. Also, motivated learners participate more in educational activities, so modifying instructional activities to enhance learners' motivational beliefs would be useful in creating effective learning ecologies. Olatunde Yara et al. (2010:128) concur with the empirical data that teachers' motivational behaviours can influence learners' achievement in problem-solving, and also teachers' positive support improves learners' motivational belief. As a result, learners are more likely to be interested in learning abstract mathematical concepts, and excel in classroom activities. On the research sites it was clear that if the teacher was demotivated in the teaching of problem-solving it would influence the learners negatively. The teaching conducted made learners mere passive recipient of knowledge, as there were no learners' activities designed.

4.5.8 Factors threatening the classroom practices

Teachers are central to creating conducive learning environment for learners to learn problem-solving skills effectively. That can include having clear learning objectives. The instructional discourse models upon which to create interesting conversations would succour learners to cultivate perspicacity that they can learn problem-solving skills (Montero, 2008:662). The classroom practices should not be constituted by a corpus of unconnected imperatives and symbolic manipulations but preferably be an inquiry based upon developing mathematical ideas. The data on class activities can expand their discernments that the learning of problem-solving skills can be fun. The nature of the instructional pedagogical discourse exercised by teachers should be collaborative, so the learners can work together as a team. In particular, the usage of concrete materials, games and explicit language should fortify instruction and learning in order to cultivate learners' positive perceptions of problem-solving skills (Anthony et al., 2009:154; Averill, 2013:15; Tamsin, 2013:491).

Anthony et al. (2009:154,155) and Campbell (2006:10,12) argue that it is important in the process of learning problem-solving skills for learners to identify their mistakes and reflect on their learning. The classroom practices must move away from the traditional practice of talk and chalk from the front of the class and the negative perception that mathematical concepts are dull, boring, and difficult and a compilation of rules that commonly make not much sense.

In section 4.5.7 above it was suggested that the teacher can cover much mathematical content when using indigenous games. However, one needs to be careful not to confuse learners in the presentation.

Learners in group D, represented by Alice, asked the teacher and the team in this manner:

Sir we are confused and lost, we followed the activity of deriving the patterns, but tell me how you move to drawing of the graphs and calculations of the area and perimeter in the preceding worksheets.

Mpho also raised her hand:

Yes, Alice I also struggled to link the graphs to the geometric figures drawn.

Ms Mokoena responded:

Thanks for the questions asked. I think we were too quick to move on the drawing and calculations of the area without emphasising the crucial issues on developing the patterns. Ok, let's concentrate on these facts today. You are expected to know the following:

(a) From the play of diketo, you need to know the mathematical concepts embedded in.
(b) Be able to identify all the mathematical concepts.
(c) Be able to derive various patterns from the play.

Mr Rasekhoro added to these comments:

What I realised in the groups some were still hooked up in the identifying the mathematical concepts embedded in the play of diketo. during the phase three (the actual presentation), let worksheet we construct focus on only one mathematical concepts (say the patterns) so that the next day we move on the next one which was identified in phase two (reflection). We will consider that not to cram too much content in a short space of time. We will give you enough chance to digest the contents covered.

Mr Talana added:

What we can do is to link the previous lesson with the current one, so as to show them a trend that these concepts do link or relate. Although the discussion in groups is a good idea, we will also give class assessment tasks, where we can also pick up the urgent issues that need clarification. We will also focus on in-class assessment. Whatever good points you are *raising during the group discussions, you need to have eloquent way of putting them down.*

The extract '…..*sir we are confused and lost, we followed the activity of deriving the patterns, but tell me how you move to drawing of the graphs and calculations of the area and perimeter in the preceding worksheets'*, shows that the teachers had assumed too much power in designing the activities, to the extent that learners were being given too many activities at the same time. It thus becomes difficult for learners to comprehend and see the relationships between the mathematical concepts.

In helping the learners to comprehend, it was decided that the learning outcomes needed to be clear, so as to give freedom to explore the activities in depth, before moving on to the next activity of problem-solving. This argument is justified by the excerpt '…*Ok, let concentrate on these facts today. You are expected to know the following: (a) From the play of diketo, you need to distinguish the mathematical concepts embedded in and (b) be proficient to identify all the mathematical concepts. (c) Be able to derive various patterns from the play…'.* The outline gives hope to learners that they will be able to master these mathematical concepts if the objectives of the lesson are clear. The aspirational capital allowed them to stay focused, and maintain hope, even amidst difficulties in which mathematical concepts were crammed within a short space of time.

On the other hand, the extract '…*we will consider that not to cram too much content in a short space of time. We will give you enough chance to digest the contents covered*…', showed that the work given to learners was given in small chunks so that they are able to comprehend the work. As the excerpt '…*what we can do is to link the previous lesson with the current one,'* also suggests that there is continuity between the previous lesson and the new knowledge to be learnt. The knowledge of learners on problem-solving will not be suppressed, rather it will be acknowledged and validated.

At the same time, the group work was encouraged, with more emphasis on written work in the classroom. This argument is justified by the phrase *'….also focus on in-class assessment*…..', to ensure that equity between the ideas presentation during group discussions and the work written down by the individual learners. This was so as to pay attention to powerless and disenfranchised individual problems on problem-solving skills. This calls for equal involvement of written and oral work of all individual learners.

In concluding, it is evident that there is a correlation between the literature reviewed and empirical data. Jones et al. (2011:847) and Kilpatrick (2009:161) posit learning targets as important in guiding and reaching the learning objectives of the lesson in the teaching and learning of problem-solving. In the teaching and learning environment, if the learning objectives are clear learners take an active role in class participations and deliberations. In such a classroom environment, there are instructional practices, programmes and materials that are effective for improving the learning of problem-solving. Hasson (2010:172) argues that in a problem-solving classroom practices all parties (that is learners, teachers and parents) are actively engaged in the construction of knowledges on problem-solving.

Also, D’Ambrosio (2009:93) and Budd (2005:22) argue that in the teaching and learning environment it is the obligation and responsibility of teachers to clear any confusion that exists between the oral problem-solving skills and written work on problem-solving. It is crucial that in the teaching and learning of problem-solving, the learning environment created is a form of ‘open classroom’ practice, in which the learners have to work together in demonstrating the competency in problem-solving skills. The learning and teaching of problem-solving skills is multi-dimensional, that is the teacher offers valid conditions for learners to learn problem-solving effective, the learners take responsibility for construction knowledge (Yosso, 2005:69), and the specific mathematical content is presented and jointly constructed in the classroom.

4.6 RESULTS OF USE OF INDIGENOUS GAMES TO TEACH PROBLEM-SOLVING

The previous sections 4.2, 4.3, 4.4 and 4.5 were in the process of developing the framework of using indigenous games to teach problem-solving skills effectively. These sections were respectively as follows: the challenges which existed in the teaching of problem-solving; the components to the strategy seen as good practices in the teaching of problem-solving; and conducive conditions for the emerging framework to teach problem-solving to work effectively. Finally, the factors that threatened the implementation of the emerging framework to teach problem-solving were fully discussed, providing ways to overcome the risk factors.

In this section indicators of success will be discussed, shedding light on the successes of the emerging framework to teach problem-solving skills. As Maxwell, Chahine (2013:62) and Sterenberg (2013:18) argue, a framework to use indigenous games in the teaching of problem-solving skills provides teachers, learners and district officials with the necessary tools to maintain relevance in the diverse and constantly evolving landscape of a Mathematics classroom. In addition, Tamsin and Deb (2013:493) assert that without a framework to use indigenous games to teach problem-solving skills school mathematics remains the 'secret weapon' of Western imperialism.

The indicators of success will be fully discussed under sub-topics, the opening paragraph of which will explain the good practices as espoused in the policies, legislative imperatives, research reports and literature review. The extracts by team members, including learners, teachers, parents, district officials, community leaders, is evidence of effective use of indigenous games to teach problem-solving. These extracts will be interpreted using CDA and tenets of community cultural wealth theory. Finally, a conclusion will be drawn, looking at the similarities and differences between empirical data and literature reviewed.

4.6.1 The mathematical content is easily accessible to learners

According to the DBE (2013a:6; 2013b:125) and the Singapore Ministry of Education (2013:8), all learners must have access to basic education, especially in the learning of problem-solving skills. It is inciting to realise that the substantive number of learners with mathematical skills and competencies surfacing from the schooling has grown extensively, especially in light of wider public concern about the declining mathematical skills of school-going individuals. They argue that care and attention should be given to the design of problem-solving learning activities, building confidence of learners and developing appreciation for the subject. The learning of activities, and assessment tasks should be fun, meaningful and relevant to the background of learners. In addition, Jorgensen, Lowrie (2013:130), Tamsin, Deb (2013: 484) contend that the learning environments in problem-solving skills should engage learners, while also providing deep learning opportunities.

On the research site, the learners were given the following class activity (see class activity 1 in Appendix F7.1 and F7.2) to work on:

One of the learners commented:

> Dineo: *I am able to understand the quadratic functions, especially the decreasing and increasing functions. I relate them to the playing of kgati which demonstrate practicality of decreasing and increasing functions*.

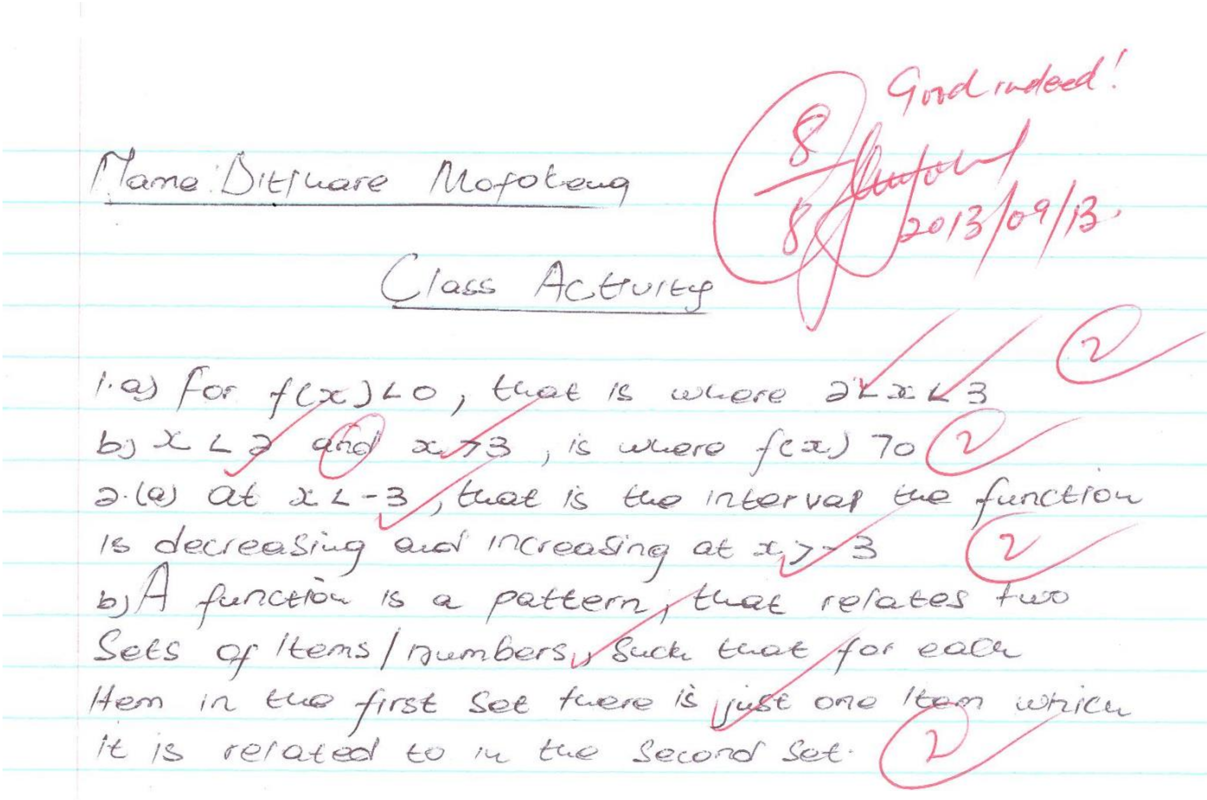
Thabiso supported the argument raised by Dineo:

> *Yes, Dineo you are quite right. The indigenous game, such as diketo and kgati, made it possible for us to understand the terminology used, and also put the the wording in a mathematical language or mathematics symbolic form. For instance,* $f(x) > 0$*, is the functions which is positive.*

Ms Mokoena, after marking the activity, explained:

> *My dear learners I am so impressed about your performance on the work I gave you. You performed extremely well. These are type of questions which learners experienced difficulties with.*

In Figure 4.10 (Ditlhare's Answer sheet) are the solutions given by Ditlhare, in responding to the class activity:


Name: Ditlhare Mofokeng

Class Activity

1.a) For f(x)<0, that is where 2<x<3
b) x<2 and x>3, is where f(x)>0
2.(a) at x<-3, that is the interval the function is decreasing and increasing at x>-3
b) A function is a pattern, that relates two Sets of Items/numbers, Such that for each Item in the first Set there is just one Item which it is related to in the Second Set.

8/8 Good indeed! 2013/09/13.

Fig: 4.10): Ditlhare's Answer sheet

The extract '…*I am able to understand the quadratic functions, especially the decreasing and increasing functions'* signifies that learners are empowered through the use of the indigenous games to teach problem-solving to master the interpretation of quadratic functions (parabola). As a result, learners have gained hope for a brighter future in the learning of problem-solving skills. As their teacher positively commented '…*these are type of questions which learners experienced difficulties with*', showing that they learnt these problem-solving skills with fun by using the indigenous games of which they were fond. This approach helped them to concretise the abstract concepts, which were difficult to make sense of.

On the other hand, the words '….*made it possible for us to understand the terminology used, and also put the wording in a mathematical language or mathematics symbolic form….',* demonstrate that the linguistic capital they possessed helped them to understand the terminology used, and enabled them to understand mathematical language easily and translate mathematical sentences into symbolic form, such as $f(x) > 0$, easily understood as increasing function. The excellence displayed by the learners in mastering problem-solving skills is justified by the statement '….*you performed extremely well*…' and the learners' answer sheet (Figure 4.10 above).

In conclusion, it can be noted that before the inception of this framework of using indigenous games to teach problem-solving, learners had difficulties in understanding these abstract mathematical concepts. This is in agreement with the DBE's (2013:131) claim that learners struggle to conceptualise the definition of a function and relationship. This was evident in learners' lack of clarity about terminology, such as 'one-to-one' and 'one-to-many'. The majority of learners lost many marks because of wrong comprehension of functions. Further, it was suggested that teachers needed to come up with strategies that would help learners to comprehend problem-solving skills more easily. Teachers were even advised to spend more time explaining mathematical concepts, such as $f(x) < 0$,$f(x) > g(x)$,$f(x).\, g(x) < 0$, and much more mathematical content.

It is evident that the use of the framework to use indigenous games to teach problem-solving has demystified the notion that mathematical content and concepts are too abstract to comprehend. Rather, the framework to use indigenous games to teach problem-solving has made learning more fun and interesting. This is supported by Bintz, Moore, Wright and Dempsey (2011:69), the DoE (2003:13), Fyhn, Eira and Sriraman (2011:185) and Warren and Miller (2013:151), who state that learners are capable of learning problem-solving, given that their background knowledge and home experiences are part of the classroom environment in the teaching and learning of problem-solving. Eventually, learners should understand and access various problem-solving skills easily.

4.6.2 The method of teaching and learning problem-solving skills is learner-centred

Teaching problem-solving skills using indigenous games puts the learners at the centre of teaching and learning sessions, where most of the activities are performed by learners. This is amplified by Taylor (2009:269), for whom learner-centred approaches are based on an emancipatory vision in which learners take charge of their own learning. Direct interventions by the educators and mathematics practitioners are regarded as interferences in a natural process in which learners are active, creative and self-regulated. On the other hand, teachers and other stakeholders act as facilitators in the teaching and learning sessions (DoE, 2003:2; Sillah, 2012:86; Tamsin & Deb, 2013:483). The learner-centred method allows the learning facilitators to teach problem-solving skills from the learners' perspectives. In this case, teaching becomes empowering to learners and allows them to discover and reflect on realistic experiences.

On the research sites, the framework of teaching problem-solving skills using indigenous games was consistently used. Learners in their small groups were given the below-mentioned worksheet 4 (refer to Appendix F8) to perform:

Mr Talana requested all the groups to round off their discussions and prepare to give feedback to the whole class. This is what transpired during the reporting session:

> Mr Talana: *Before we start with reporting on the activity, how did you find the activity?*

To which they responded:

> Class (responding as a group): *It was very easy, sir.*

Ms Mokoena commented:

> *It's really good news to hear that that the activity was easy to perform. According to you what made such impressive performance in your mathematics assessment and class activities.*

Dipalesa, a learner in the group, responded:

> *To be quite frank we struggled a lot the first two month of the year on problem-solving. But since, the advent of the framework to use indigenous games to learn problem-solving skills, we enjoyed a lot to be in your mathematics class. The framework made it possible us to concretise mathematical content.*

Thabiso supported the sentiments raised as follows:

The approach of using the indigenous games to teach problem-solving helped us to make contributions as we work in class or in our small groups. For instance, the activity you gave as now, we explained it in differently ways, because we understood what we are doing. It can be related to previous indigenous games we played.

Dineo interjected:

You are quite right Thabiso, take for instance, number (c) of the activity: we discovered that the pattern is $+x = 10\ or\ y = -x + 10$ *. This pattern described the playing of diketo game, where one is in round two. But practically, negative numbers cannot be included in the table, as there are no negative throws of the ghoen. There are some limitations to be considered.*

Pulane argued:

I agree with you Dineo that is correct. Looking at the model: $y = -x + 10$*, that describe the play of diketo, where one is in round two. Then, this implies that in round two, stones to be placed in the hole take even numbers between 10 and 0. Following the rules of the games, it is not correct to place odd number of stones in the hole.*

Mr Talana responded to the arguments raised:

I [am] so impressed about the level of debate and engagement we have. I am so excited about discussion, I can realise that even people had started to feedback about their findings from the activity. This proves that you did enjoy and learn a lot from the piece of work. And further relating the activity with the indigenous games we played last week. You are able to relate the real situations to abstract mathematical concepts we do in class. I think my colleagues will agree with me that you demonstrated high level of critical thinking and good communicative skills of problem-solving concepts.

The abovementioned conversations illustrated that learners were actively involved in classroom discussions and showed high levels of interpreting problem-solving. This is justified by the extract '...*number (c) of the activity, we discovered that the* *pattern:* $y + x = 10 \ or \ y = -x + 10$ *describes the playing of diketo game, where one is in round two. But practically, negative numbers cannot be included in the table, as there are no negative throws of the ghoen*'. Learners were so empowered to think critically about problem-solving skills that they were able to critique the mathematics activity to see its relevance to practical life situations. This was made possible by their navigational and social capital, which enabled them to collaboratively reflect critically on the class activity. They were further cooperating with their peers and other team members in problem-solving skills in an open and supportive way to achieve the shared lesson outcomes. Also, the familial capital demonstrated that no one was left out, but rather all team members were embraced in the process of teaching and learning problem-solving skills. Thus, their learning of problem-solving skills was strengthened.

Also, in saying '...*the approach of using the indigenous games to teach problem-solving helped us to make contributions as we work in class or in our small groups. For instance the activity you gave as now, we explained it in differently ways, because we understood what we are doing. It can be related to previous indigenous games we played*', learners were pointing out that the framework to use indigenous games to teach problem-solving had created a sustainable environment to learn freely, without fear of being oppressed by the powerful discussions of teachers. Their linguistic capital helped to make meaningful contributions, such as '……*where we play indigenous games and relate them to the mathematical content help us a lot*.' The use of 'we' demonstrated that the high level of social capital and the manner in which learners interact in a Mathematics class formed a togetherness in approaching problem-solving activities.

The excerpt '…*but since, the advent of the framework to use indigenous games to learn problem-solving skills, we enjoyed a lot to be in your mathematics class. The framework made it possible us to concretise mathematical content*….', showed that their aspirational capital helped them to maintain hope to excel in problem-solving. Learners further attested that they enjoyed working on problem-solving activities, even feeling delighted to be in a Mathematics classroom.

In concluding the section, it is crucial to indicate that the literature reviewed concurs with the empirical data. As Jorgensen and Lowrie (2013:130) argue, the use of indigenous games in the teaching of problem-solving resulted into unintended learning outcomes. The learning outcomes showed ripple effects, such as learners being committed to their school and good performances in problem-solving, increasing the chance of employment and life opportunities to succeed (Tamsin & Deb, 2013:483). In addition, the Constitution of the Republic of South Africa Chapter 2: Bill of Rights (1996:4) states that everyone, including learners, has inherent dignity that must be respected and protected by others in expressing his or her views. The team members on the research sites have created an environment conducive to use of indigenous games to teach and learn problem-solving effectively.

4.6.3 High motivation among learners

According to Averill (2013:105) and Waege (2009:85), motivation is a potential to direct behaviour. As in the definition, learners' motivation towards problem-solving may be manifested in cognition, emotion and/or behaviour. For example, learners' motivation to obtain a good grade in problem-solving may be expressed in happiness if learners scores high on any assessment tasks. It may also be manifested in studying for a test (behaviour) and in new conceptual learning (cognition) when studying for the test. Therefore, it is imperative that in the teaching of problem-solving skills, teachers attend to specific and holistic problem-solving needs of their learners, and problem-solving needs that happen to be marginalised, such as navigational and aspirational skills (Yosso,2005:69), which happen to be side-lined in the teaching of problem-solving.

Swanepoel and Gebrekal (2010:403,413) and Waege (2009:88) demonstrate that pupils' motivation cannot directly be observed, or thus measured, and it needs to be recreated through interpretation of the observable variables, such as focus on learning and understanding mathematical concepts as well as on getting correct responses; excitement in captivating in problem-solving activities, which relates to learners' intrinsic motivation towards problem-solving; related positive (or negative) feelings about problem-solving; and keenness to manage threats and to handle challenging tasks or activties on problem-solving as well as self-confidence of learners towards handling exercises on problem-solving.

The worksheet above in section 4.6.2, was given to learners in class, after which they spoke to their teachers in rephrasing the question differently. Group A, represented by Wetsi suggested:

> *Sir, how about we redesign the worksheet you gave us in different way. I suggest that the question be phrased as follows: describe the practical (real-life) scenarios defined by the models and critically analyse them.*

Other group members listened with interest to the suggestions made by group A. Tlala from group D responded:

> *Hrrrr, Wetsi, that is a challenging question that one, but the way I see it, yes it is doable*.

Ms Mokoena (with a smile on her face), responded:

> *Ok , Group A with your powerful suggestions. Tlala agreed with them that is possible to work them out. What do others think or say?*

Tshidi, from Group C commented as follows:

> *Yes, mam, we can give it a try. I concur with latter speakers that it is a doable sum. In short, previously, we developed model or patterns from scenarios, then in this case we have to check where does this model feature and why do we say so, hence we have to justify our arguments. At far it seemed to be a daunting task, but Dineo's previous comments or explanation partly alluded to the question.*

Mr Tlalana, with happiness, expressed his feelings:

> *I am overwhelmed; what an activity to do! Thank you very much for the contributions you made. Yes, I fully agree with you that lets give it a try. I spoke to my colleagues; we are in agreement that go attempt this as your homework today.*

The class, laughing, heard representative from group B, Ditlhare responded:

> *Sir, are you simplify saying that we are able to creating problem-solving activities?*

Ms Mokoena responded:

> *Oh yes. I realise that you are capable of designing learning materials, which is an excellent move, in mastering problem-solving.*

In the context of these conversations, the extract '…*Sir, how about we redesign the worksheet you gave us in different way? I suggest that the question be phrased as follows: describe the practical (real life) scenarios defined by the models and critically analyse them*….' denoted that learners had developed intrinsic motivation and a high standard of performance in problem-solving. It also showed confidence they had developed in it. This was echoed by the words: '…*I realise that you are capable of designing learning materials, which is an excellent move, in mastering problem-solving…',* showing that teachers had relinquished the power to learners, such that there were no unequal relations of power with regard to the knowledge of problem-solving.

On the other hand, the phrase '… *we redesign the worksheet you gave us in different way..',* signifies the navigational capital of learners, as they were able to tactically devise alternative ways of asking the question, the response to which requires a deeper understanding of problem-solving. Also, the excerpt '…*yes, mam, we can give it a try. I concur with latter speakers that it is a doable sum…*', shows that learners believed in a collective effort to work out the solutions to problem-solving. This further illustrated the inherent value of social capital, which made it possible to interact with their peers and arrive at convincing arguments.

The phrase '…*that is a challenging question that one, but the way I see it, yes it is doable…*', explicates the aspirational capital and resistance capital which helps them demonstrate the vigour, zeal and zest to obtain solutions to the problem, irrespective of barriers confronting them. The barriers or obstacles can be abstract mathematical content and lack of motivation to work on problem-solving. Although there are perceived challenges they were confident ain their determination to get the correct answer.

In conclusion, it has been observed that there is a close correlation between the empirical data and literature reviewed on motivation of learners in learning problem-solving skills. Waege (2009:90) and Williams and Forgasz (2009:96) argued that in motivating learners to be actively involved in the learning of problem-solving it is recommended teachers frequently ask them to explain their solutions and strategies. When learners are faced with problems with which they are unfamiliar they enjoy collaborating in finding the solutions. Also, the impact of quality teaching on problem-solving using the indigenous games had a profound impact on how learners learn and further created a sustainable learning environment in which learning is enhanced.

Hodkinson (2005:110) claimed if learning of problem-solving is properly enhanced it will result in learners being motivated to engage in problem-solving exercises. This also helped learners to construct their own understandings of problem-solving skills. Teachers can then easily adopt scaffolding upon pre-existing misunderstandings or understanding in comprehending the problem-solving skills.

4.6.4 Self-discovery of problem-solving skills formulae and processes

The framework for using indigenous games to teach problem-solving is effective in integrating ideas and mathematical concepts or processes. It also creates new possibilities of reaching at the answer to the problem to be solved. In such instances, learners thrive on inventing unusual solutions to problems and do so in an intuitive way. This argument is that learners do not arrive in a Mathematics classroom or lesson as empty vessels (Hattingh, deJong, 2009:38; Hodkinson, 2005:109; Tamsin Deb (2013:485), in agreement with Norton and Irvin's belief (2007:551,556) that the teaching and learning of problem-solving skills should be tied to contextual issues for learners to develop deeper understanding of abstract concepts. This allows learners to find alternative ways to the answer or discover intriguing solutions. It has been observed that cases of formal teaching of problem-solving will face barriers if not linked to concrete framework of using indigenous games.

In section 4.3.6.2 (see Fig. 4.9 above), it was exciting to become conscious that with the use of the *morabaraba* (one example of indigenous games), learners discovered the following conclusions, including a number of patterns:

$25\ units\ squared = 9\ units\ squared + 16\ units\ squared$, this is called Pythagoras Theorem. This can be written as $5^2 = 4^2 + 3^2$, then generally we have $h^2 = x^2 + y^2$,

- general pattern $t_n = n^2 + 4n + 4$, to determine the area of three squares.
- hence , the area the first square (the yellow one)(refer to Fig.4.9 above), is calculated as follows:

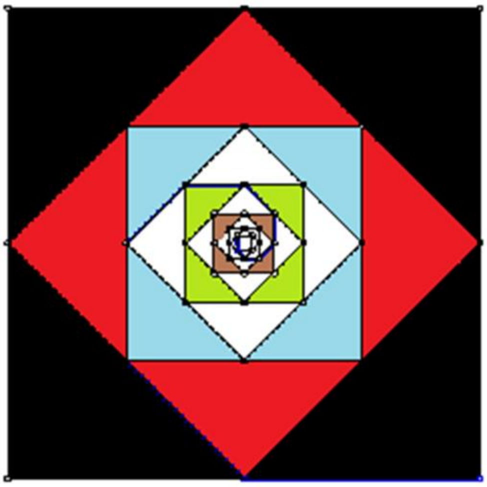
> - The first square (the red one);
>
> $t_n = n^2 + 4n + 4 \quad n = 1\ first\ square,$
>
> $t_1 = 1^2 + 4.1 + 4$
>
> $t_1 = 9\ units\ squared$

- For the 2nd and 3rd squares we used the same general formulas $t_n = n^2 + 4n + 4$, for 2nd red square (refer to Fig.4.9 above)we used $n = 2$ and for 3rd blue square(refer to Fig.4.9 above) we used $n = 3$.

The below-mentioned activity was given to learners to work on.

ACTIVITY 2:

-



Consider the above –mentioned diagram. Answer the following questions

(a) If the area of the largest square is $25cm^2$, what is the length of the side of the second largest square? Explain.

(b) Calculate the length of the side of the third largest square.

(c) Calculate the area of the fourth square

(d) Following the pattern, can you calculate the area of the 8th square. What conclusions can you draw

(e) Now write down the first 8 terms of the sequence that will consist of the perimeter of the square.

As the learners gave feedback, the following issues transpired in their discussions:

Group A, represented by Lehlohonolo gave their solutions in this way:

> *Good morning class, after a long discussions and disagreements in our groups, we finally arrived at this conclusions. The sides of the largest square is $5cm$, because the area is $25cm^2$.*

Lehlohonolo paused, giving a chance to other members to pose questions for clarifications:

Let me pause a bit for you to ask questions before I moved on. My group members will assist me to give answers.

Lekau: *Thanks Lehlohonolo and your group members, can you tell us why do you say the sides of square is five centimetres, how did you arrive there?*

Ntaoleng from Group A responded:

Remember the properties of the square. All sides are equal; therefore each side is five centimetres.

Thanks, Ntaoleng for the clarification you made. In the absence questions or comments, lets us carry on. Then the sides of the second largest square were structured as follows: since the corners of second square touches the middle point of each side of the first square (that is $\frac{5}{2}cm$ *long from the midpoint to the corner of the square).then, that brought us to conclude that the sides of the second square form the hypotenuse in relation the first square. Hence we used the Pythagoras formula to calculate the side of the second square:* $(\text{side of the second square})^2 = \left[\frac{5}{2}\right]^2 + \left[\frac{5}{2}\right]^2 = \frac{25}{4} + \frac{25}{4} = \frac{50}{4}$

$$\text{side of the second square} = \sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2}.$$

Lehlohonolo: *Let us consider (Table 4.13 below) for further calculations and clarifications*.

Lehlohonolo explained the calculations in the table for the class:

After we have done the table 4.13 below, we answered the remaining questions as follows:

Objects	Sides in cm	Area in cm^2
1st largest square	$5cm$	$25cm^2 = \frac{25}{1} = \frac{25}{2^0}$
2nd largest square	$\sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2}$	$\frac{50}{4}cm^2 = \frac{25}{2} = \frac{25}{2^1}$
3rd largest square	$\sqrt{\frac{100}{16}} = \frac{10}{4}$	$\frac{100}{16} = \frac{25}{4} = \frac{25}{2^2}$
4th largest square	$\sqrt{\frac{50}{16}} = \frac{5\sqrt{2}}{4}$	$\frac{50}{16} = \frac{25}{8} = \frac{25}{2^3}$
…………..	………………	………………..
………………	……………….	………………..
nth largest square		$t_n = \frac{25}{2^{n-1}}$

Table 4.13): Lehlohonolo's calculations

Lehlohonolo: *After we have done the table 4.13 above, we answered the remaining questions as follows*:

- (b) the length of the side of the third largest square is $\frac{10}{4}$
- (c) the area of the fourth square is $\frac{50}{16}$ or $\frac{25}{2^3}$
- (d) to calculate the area of the 8th square, we looked at the general pattern discovered as $t_n = \frac{25}{2^{n-1}}$, therefore the area of 8th square is $t_8 = \frac{25}{2^{8-1}} = \frac{25}{2^7} = \frac{25}{128}$

Lehlohonolo: *Thanks for listening to the long presentation. I hope we are ready to take questions or comments from the floor*.

Group C members commented as follows:

Pulane: *Wow, that par excellent, we are really impressed about the detailed explanation you did. This is really the demonstration that the use of the indigenous games to teach problem-solving has indeed made these mathematical concepts very easy*.

Mr Tshehlana: *I am impressed about the good work you have presented. I am convinced that the use of the framework to teach problem-solving has deepen your understanding. What is interesting about your discoveries there are so many mathematical concepts you have demonstrated*.

Matlau, from group A commented:

Yes , sir you are right, there are quite number of inventions you have shown us, namely: the ratio of area is constant(that is the ratio is $\frac{1}{2}$ *) and 1 can be written as* 2^0*(that is, 1=* 2^0*).*

Ms Mokoena added:

The approach of using indigenous games to teach problem-solving employed since the beginning of this semester has yielded quite inspiring results. The knowledge that you are producing is of high standard. Look at the pattern you developed ($t_n = \frac{25}{2^{n-1}}$*), that is where the ratio is constant, that pattern is known as the geometric pattern. Again, logically, from the pattern you developed, it becomes clear that any number to the power of zero is one (that is*$a^0 = 1\ , where\ a > 0$)

As demonstrated in section 4.3.6.2, as learners engaged on indigenous games, they managed to come up with many discoveries. Inventions such as '$25\ units\ squared = 9\ units\ squared + 16\ units\ squared$, this *called Pythagoras Theorem. This can be written as* $5^2 = 4^2 + 3^2$ *, then generally we have* $h^2 = x^2 + y^2$ *, and general pattern*$t_n = n^2 + 4n + 4$*, to determine the area of three squares…'*. These discoveries showed that learners were empowered on problem-solving by using indigenous games. The knowledge which they possessed from the home environment was not marginalised by teachers, rather it was unleashed to reveal the wealth of knowledge that learners had on problem-solving. It was used for the benefit of learners to understand problem-solving. Learners' active involvement in problem-solving through the use of indigenous games gave them hope and trust that their discoveries of general conclusions '………$h^2 = x^2 + y^2$, $t_n = n^2 + 4n + 4$ and $t_n = \frac{25}{2^{n-1}}$, *when the ratio is constant, the pattern is called the geometric pattern*…') would be of high quality and standard. It was pleasing for them to note that their teachers did not oppress their navigational capital, which helped them to explore and invent mathematical formulae. In addition, *the phrase '….we are really impressed about the detailed explanation you did*' signified that the linguistic capital armed them with social skills that made it possible to communicate their findings (see the learners' workings above) fluently during the feedback sessions.

The extract '……..*the approach of using indigenous games to teach problem-solving employed since the beginning of this semester has yielded quite inspiring results. The knowledge that you are producing is of high standard. Look at the pattern you developed ($t_n = \frac{25}{2^{n-1}}$), that is where the ratio is constant, that pattern is known as the geometric pattern*………', explains that the framework to use indigenous games in the teaching and learning problem-solving had contributed significantly to enhancing the learners' conceptual understanding on mathematical content. Also, the navigational capital made it possible for learners to conceptualise their thinking, and ultimately write their thinking processes symbolically (that is , $t_n = \frac{25}{2^{n-1}}$). In the same way, the excerpts '… *then, that brought us to conclude that the sides of the second square form the hypotenuse in relation to the first square. Hence we used the Pythagoras formula to calculate the side of the second square:* $(side\ of\ the\ second\ square)^2 = \left[\frac{5}{2}\right]^2 + \left[\frac{5}{2}\right]^2 = \frac{25}{4} + \frac{25}{4} = \frac{50}{4}$ $side\ of\ the\ second\ square = \sqrt{\frac{50}{4}} = \frac{5\sqrt{2}}{2}$ *…..',* denote that the linguistic capital helped them to express their thinking verbally and symbolically.

In conclusion, it has been noted that to a large extent the empirical data confirms the literature reviewed. It has been observed that learners can do well in problem-solving if given opportunities to integrate their everyday activities into problem-solving learning in schools (Nutti, 2013:20). The use of the framework to teach problem-solving solving demonstrated that learners need not be seen as empty vessels to be fed with knowledge (Hodkinson, 2005:110), that is to be transmitted with problem-solving skills. Rather, Waege (2009:90) contends that the teaching approach must give more space to learners to satisfy their needs for competence and autonomy in the learning of problem-solving. This will help them to develop mathematical processes such as creativity, critical thinking and reasoning skills needed in the learning of problem-solving. Also, the instructional activities used in the classroom should be the ones that support and amplify the development of problem-solving skills.

During the feedback sessions, learners were very active in showing the mastery of problem-solving. They construct meanings of mathematical concepts, and formulate general conclusions on their own, as they negotiate interpretations and solutions. Likewise, Ewing (2013:135) and Norton, Irvin (2007:552) argue that learners find generalisations within the elements of problem-solving. It was possible for learners to make generalisations because they had a foundation of numeracy and arithmetic.

4.6.5 Good level of parental involvement

The framework to use indigenous games to teach problem-solving has had a great impact in motivating parents to take an active role in the classrooms. In all the sessions of teaching problem-solving using indigenous games, there has been a representative from the parent component. This supports the view of Nutti (2013:57), Tamsin (2013:485), Webb (2013:89) and Zhongjun, Bishop and Forgasz (2006:88), that in most of the sessions conducted, parents acted as role models in the teaching of problem-solving, as well as expectancy socialisers. As role models, parents exhibit behaviour that learners imitate and later adopt as part of their own repertoire. As socialisers, parents influence learners' achievement through expectations, and communicate their prospects in the messages they give regarding their beliefs about their children’s abilities, and the intricacy of several tasks.

Tamsin and Deb (2013:492) added that the involvement of parents in the teaching of problem-solving is regraded as having a considerable impression on learners’ attitude and learners’ achievement in problem-solving. This is an important collaboration between teachers, community members, parents and learners, in which different voices, perspectives on the learning and teaching of problem-solving can be heard and validated. Hāwera and Taylor (2011:341) found that the use of indigenous games in the teaching of problem-solving has reaffirmed the great responsibility parents are confronted with in assisting their children to master problem-solving. Again, the parents viewed the framework as a way of restoring the cultural dignity of children, hence it is important for learners to appreciate that they possess a long and rich problem-solving heritage and can be mathematically capable.

One parent, Rantja commented:

> *Ke rata ho bolella mokgopi ona hore sena seo re se etsang mona ka ho bapala papadi tsena tsa setso, le ho analaesa mmetse o teng ho tsona. ke mosebetsi o motle ka tsela e makatsang. Le rona re se re kgona ho thusa bana lapeng mane ka mmetse. Ke maobanyana mona bana baka lapeng mane ba ne ba bapala papadi ena ya malepa, ba etsa heke ya pele, ya bobedi jwalojwalo. Jwale ke shebile sena seo ba se bapalang ka tjhesehelo e kgolo. Ke a ba botsa hore ke disheipi di feng tseo ba di bonang. Yaba ba makala ke ho botsa hore, monnamoholo mmetse o tsebella kae? Ba nkaraba ka hore re bona sekwere, rekthengele*
>
> (I want to tell the group that what we are doing here, playing these various indigenous games and analysing them, it is an exciting thing indeed. I am now able to assist my children with some mathematical content embedded within the indigenous games. One day they were playing string games**,** I was so interested to view what they are doing, as they were constructing gate one, two and so on. I asked them what geometric figures are portrayed when constructing these gates. Before responding to my question they laughed at me, asking me as to who stimulated my knowledge on problem-

solving skills. After a while they responded by saying that they identified the square, rectangle as displayed by the string game (refer to Fig.4.6.5(a).)

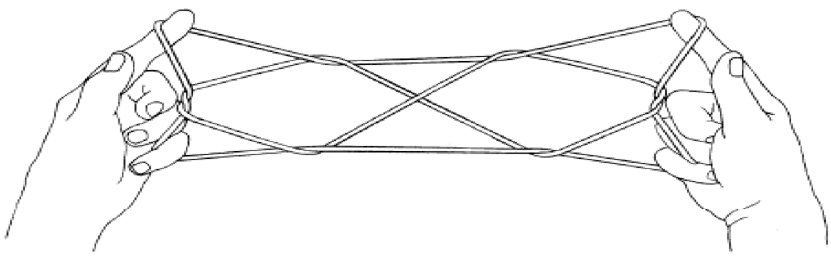


Fig. 4.11: String game demonstrating gate 2 (Wirt & Mitchell, 2000:125).

The official in the Sports section, Ms Ntwa commented on the parent in this manner:

Ms Ntwa: *Ho a thabisa ka nnete ho elellwa ka moo batswadi ba etsang mosebetsi o motle ka teng, moo ba thusang bana malapeng a bona ka ho ruta mmetse ka tshebediso ya dipapadi tsa setso. Hape o ka ba bontsha kamano e teng pakeng sheipi ya mahakore .a mane le le traengle, hore ke pethene efeng e etsahalang.*

(It is really quite pleasing to realise that parents are taking an active role in the teaching of problem-solving using indigenous games at their respective homes. You can even show them the relationship between the relationship between the quadrilateral and the triangle, and note the pattern demonstrated by the two geometric figures).

Mma Sebolelo argued in this way:

Nna ke thabetse tshediso ena ya dipapadi tsa setso,ke shebile kamoo mora waka, Teboho a sebetsang hantle ka thutong ya mmetse. Moradi waka e moholo o etsa mphato wa leshome ffela o sebetsa hampe haholo thutong ena ya mmetse. Jwale ke botsa mesuwe ya hae hore bothata ke eng. Ke ba eletsa ka hore batle sekolong sena bat lo sheba kamoo mokgwa ona o sebetsang ka teng, hore bana ba rone tsebe ho atleha ha selemo se fela.

(I'm so excited about the framework of using indigenous games to teach problem-solving. It has worked a lot for my son, Teboho who is now doing well in problem-solving skills. My daughter at another school in grade Twelve, she is performing badly in problem-solving. I suggested to the mathematics teachers that they must consult with you, so that you showed them how to use indigenous games to teach problem-solving skills).

The sentiments were echoed by Mr Liholo, SGB chairperson in the parent meeting:

Re batla bo leboha boiketlaetso ba batswadi le matitjhere, bahlanka ba lefapha la thuto ho nka karolo e kgolo ho rutweng ha mmetse ka tshebediso ya dipapadi tsa setso. Ka ho qololeha batswadi ka ho ntsha thuto ena pele baneng ba bona malapeng. Ha o sheba matshwao a bana dithutong ena ya mmeste a ntlafetse hahol, ho qala ka diteko tysa kgwedi, hlahlobo tse mahareng a selemo, seka-makaolakgang le dihlahlobo tsa ha selemo se fela (refer to the attached marksheet below).

(We want to thank all stakeholders (that is parents, teachers and district officials) who took part in the teaching of problem using indigenous games. In particular the parents who even took initiatives of teaching problem-solving by using indigenous games to their children at home. Learners have done extremely well in the all the assessment tasks, including June examinations, trial and final examinations.) (refer to Table 4.18 in Appendix F12).

The extract '…..*I am now able to assist my children with some mathematical content embedded within the indigenous games. One day they were playing string games*….', showed that parents were so empowered in using indigenous games to teach problem-solving, to the extent of making their children conscious that as they play different types of indigenous games at home they needed to be aware of mathematical content embedded in them. The parents gave hope to their children that mathematical content is easy to understand as it is noticeable in all the indigenous games they play. Even the linguistic capital that they possess helped them to realise the geometric figures '….*they identified the square, rectangle as displayed by the string game*…' within the game they were playing. This further illustrated the familial capital, that is, the good values and behaviours that children need to demonstrate in learning the problem-solving skills.

In the same way, the excerpt '….. *I suggested to the mathematics teachers that they must consult with you, so that you showed them how to use indigenous games to teach problem-solving skills*', signifies that parents are agents of transformation. They encouraged other schools to improve learners' performance in mathematics by adopting the framework of using indigenous games to teach problem-solving skills. This further shows equity, that the parents have great influence in making changes in the teaching of problem-solving, rather than waiting for education officials to cascade the framework to other teachers and schools.

The extract '…*learners have done extremely well in the all the assessment tasks, including June examinations, trial and final examinations…'*, testifies that if parents take the major role in the education of their children, particularly in the teaching of problem-solving, the learners' performance in Mathematics is likely to improve tremendously. Parents are given respect and hope that the excluded knowledge on problem-solving is validated in Mathematics in the school curriculum by adopting the framework to teach problem-solving using indigenous games.

In concluding, the literature reviewed confirmed what was observed on the research site. Cao, Bishop and Forgasz (2006:86) assert that a significant number of researches have investigated the roles that parents play in children's learning of problem-solving. It has been suggested that parents' active participation has a major impact on learners' positive attitudes towards problem-solving skills, learners' achievement in problem-solving. The Assembly of Alaska Native Educators (2002:9) and the DBE (2011:5) have concluded that parents have powerful leverages in the implementation if they are actively included in the implementation framework of using indigenous games to teach problem-solving.

The teachers need to ensure that parents are provided with adequate information that will make them effective from the development phase of the framework to the implementation phase. This argument stated that formerly excluded communities or parents must form part of the school communities. We are cautioned not to use parents selectively, when circumstances favour the school communities, but they need to be involved throughout the school programmes. Similarly, Cao et al (2006:88) highlight the assistance parents can offer, that is, direct assistance, such as helping children with problem-solving difficulties and mathematics subject choice. Also, the indirect assistance, such as parental encouragement, parental expectation, and parental attitudes towards problem-solving were frequently identified as having a significant impact on learners' attitudes towards problem-solving, learners' participation in advanced level Mathematics and students' achievement in Mathematics.

4.6.6 Adequate content knowledge among teachers

According to Bernstein (2009:290), the underlying pedagogic theory is likely to be self-regulatory, where the boundaries between mathematical contents are open. The linkage between the idea and several contents must be systematically and coherently worked out. Sparrow and Hurst (2012:23), Taylor (2009:270) and Williams and Forgasz (2009:96) contend that Mathematics teachers should bring relevance to the type of knowledge formally collated in mathematics curriculum, and the tacit knowledge which children brought from home environment. The manner of approaching the mathematical content should be high quality and reachable by the learners. Teachers who mastered the pedagogical content knowledge make significant difference to learners' academic and personal development.

Thus, the use of the framework with indigenous games to teach problem-solving helped the research team simplify the mathematics content knowledge. Nutty (2013:57) added that the teaching of problem-solving needs to be blended with contextual factors, such as cultural practices and including indigenous games. This will help learners to grasp the mathematical content more easily.

Out of the indigenous games, such as *morabaraba* and *diketo*, teachers came up with the following practical activities (See ACTIVITY 3; Appendix F8):

The shapes 1-5 (in Appendix F9.1) shown are the structures of dismantled *morabaraba* shape, which were reorganised into new shapes. The new formulated shapes enabled the teachers to unpack the mathematical content and skills (as shown in the Table 4.17: Appendix F9.2):

Consequently, the Table 4.17 in Appendix F9.2 and shape 1- 5 in Appendix 9.1, were used by the teachers to teach problem-solving using indigenous games.

The above, table and shapes, (that is, Table 4.17 in Appendix F9.2 and shape 1- 5 in Appendix 9.1, showed how the teachers are empowered to reorganise the *morabaraba* figure to produce new resources or materials to be used in the teaching of problem-solving. They are able to come up with different shapes from this and other indigenous games to extract grade 10 Mathematics contents, and even beyond grade 10. The navigational and linguistic capital helped teachers to manoeuvre various indigenous games to illustrate the core mathematical content taught to learners. On the other hand, the linguistic capital gave them the ability to link the everyday language to mathematical content. The statement in Table 4.17 'the *relationship between the circumference and the diameter*', shows that using the proficiency of their linguistic ability, they are able to write the relationship as the ratio $(\frac{c}{d})$ or symbolically as $\frac{c}{d} = \pi$ (where π is the ratio).

Furthermore, the social capital that the team members (teachers, subject advisors, parents, and community leaders) displayed, made it possible for them to network and exchange knowledge on the teaching and learning of problem-solving.

In conclusion, it can be stated that there is a close link between the literature reviewed and the empirical data. Asante, Mereku (2012:23) and Meaney, Trinick, Fairhall (2013:235) showed how the use of indigenous games to teach problem-solving influenced the teachers' practices on handling mathematical content in simplified and concrete ways.

De Cruz and De Smedt (2013:4,5) argue that teachers' and other team members' mathematical content knowledge is equipped with specialised cognitive capacities that facilitate their understanding of it infused in the indigenous games. Also, De Cruz, De Smedt (2013:4,5) and Sun (2011:69) suggest that the acquisition of mathematical content can be characterised by the framework of using indigenous games to teach problem-solving skills

4.6.7 Motivation among teachers

Waege (2009:90) and Williams and Forgasz (2009:98,99) point out that intrinsic, altruistic and extrinsic motivations played a vital role in motivating teachers in their daily work. The greatest motivation come from intrinsic and altruistic factors, with the following motivational factors amongst the most important: (i) desire to work with learners in the learning of problem-solving, where learners are taught in such a way that they comprehend easily to concretise the mathematical content; (ii) supporting learners' growth, and development in learning problem-solving skills; and (iii) employing appropriate methods of learning problem-solving. As a result, the learners can finally make significant contributions to their societies in general, making teachers motivated that they have fulfilled their role of teaching problem-solving more effectively (Ebenso, Adeyemi, Adegoke & Emmel, 2012:208).

In the same way, the extrinsic rewards influencing the teachers' motivation can include appreciating the involvement of the teamwork in the teaching of problem-solving and job security, that is the financial matters, professional development and emotional support are well taken care of by the employer, and professional status of teaching, that is, the teaching approach of the teachers using indigenous games can also motivate learners to work harder. Even the involvement of others stakeholders (parents, district officials) in the teaching and learning of problem-solving has a great influence on motivation teachers to perform excellently in the teaching of problem-solving.

Learners were asked to give parents or guardians assessment task books for checks and comments. Figure 4.14 in Appendix F10 is a sample of how some parents commented.

These comments by parent, Mokwena, '… *ke thabetse mosebetsi o motle oo ke o bonang, mme ke kgotsofetse haholo* (I am happy and satisfied about outstanding achievement) showed that parents were satisfied about outstanding performance of their children in problem-solving. Parents achieved trust and confidence in the approach used by the team in the teaching of problem-solving using the indigenous games. At the same time, the learners' outstanding performance gave the team members and teachers encouragement to work harder in helping learners to understanding the problem-solving.

The extracted version of the Mathematics school visitation report (below, Fig. 4.15 Appendix F11), exemplifies that the district officials (subject Advisors: Curriculum Section) are encouraged by the good work displayed in the teaching and assessment of problem-solving. The teachers are motivated by the comments in the mathematics school visitation report '…*there is a huge improvement in learners' performance, especially the topics which gave learners difficulties* …', which illustrates that teachers and team are not oppressed in trying the approaches that can improve the learners' performance in problem-solving. Further, these comments demonstrate that teachers and other team members are given freedom in practising the best practices, such as the framework to teach problem-solving using indigenous games that improve learners' performance. The aspirational capital possessed by the teachers showed that assisting them to maintain hope and confidence in the framework they put in practice would produce good results. This is evident by the reports compiled by the education district officials.

The attached marksheets (refer Table 4.18 in Appendix F12) amplified that the framework to use indigenous games to teach problem-solving was effective. All learners did well in all the assessment tasks administered, with class averages for all the tasks at above 50% throughout the year. The learners' marks shown in Table 4.18 in Appendix F12 reflected that they had been empowered to master problem-solving skills, in turn motivating teachers to consistently work hard and improve on their teaching methods.

In conclusion, it has been noticed that the empirical data coincided with theoretical data. Exploring indigenous knowledge systems (IKS) via the use of indigenous games to teach problem-solving makes learning relevant to making mathematical concepts more meaningful to learners. Activities should be of a nature that will engage learners in hands-on explorations for meaningful and deep learning to occur. In current society it is vital for educators not only to engage learners in learning but also to ensure that the learning of problem-solving is applicable to their lived experiences and background situations. Teacher education must attend to the constantly changing environment that teachers must manoeuvre in order to maintain their relevance and that of the content they are to deliver in the classrooms (Grootenboer, Sullivan, 2013:169; Maxwell and Chahine, 2013:63,68;

Thornton, Wendy, Debbie, 2011:235; Waege, 2009:90). Thus, the above factors, if taken care of, motivated teachers to play their role of teaching problem-solving effectively.

4.6.8 High level of expertise with regard to classroom practices

The use of a framework to teach problem-solving skills using indigenous games created a conducive environment for the teachers and learners to interact freely. Taylor (2009:269) argues that the learning environment should be such that learners take control of their own learning (Hāwera, Taylor, 2011:341), and teachers and other team members are seen as guides and facilitators in the teaching and learning environment. Also, Ewing (2013:131-132), Harrington and Brasche (2011:23) and Hāwera, Taylor (2011:340) regard the process of learning as owned and framed by the community. The process of learning the problem-solving skills allows the access of parents and community members in the classroom environment. Their relationships with teachers and learners ensure that problem-solving emerges in a context related to the experiences and background of learners.

The use of indigenous games to teach problem-solving skills encouraged large numbers of learners to attend the indigenous games competitions and festivals and field excursions. The traditional leader extend the invitation as follows:

> *Morena Letsoku: Tjhabana saka, ha re kopaneng ka Moqepelo ona o tlang mane kgotla ho tlo ithabisa le ho ithuta moholo ka dipapadi tsena tsa setso. Re na le baeti ho tswa metsaneng e mabapi*.
>
> (Team members, let us meet the next Saturday at *kgotla* to enjoy ourselves playing the indigenous games, and learn from others. We will be hosting visitors from afar and the places surrounding our village.)

Ms Mokoena responded:

> *We welcome the invitation, we will come being prepared to take active role. Let us bring writing materials so that we can be able to jot down*

> *mathematical skills and contents that can be experienced, so that we can unpack the mathematical contents infused in such games.*

Life orientation teachers, Mr Mazibuko, commented:

> *It is interesting to note that our learners really enjoyed to participate in the indigenous games. It is a good exercise as they have developed high level of analysis, and formulating exciting conjectures through the playing or observing these types of indigenous game.*

Dineo, with great excitement, commented:

> *We really appreciate the field excursions of this nature, where we are going to experience the practicality of mathematical concepts*.

The extract '…..*let us meet the next Saturday at kgotla to enjoy ourselves on playing the indigenous games, and learn from others….'*, showed that the teaching and learning of problem-solving skills is not restricted to being within the classroom walls, but extends beyond. The team members are empowered to realise that the mathematical resources to teach problem-solving can even be accessible at playing grounds, kgotla. The phrase '….*Let us bring writing materials so that we can be able to jot down mathematical skills and contents that can be experienced*' exemplifies that cultural activities (such as indigenous games) are rich in mathematical content, with learners reminded to bring along their writing material so that they note down the mathematical contents or skills demonstrated by the play of these indigenous games. This also implies that navigational and social capital are essential resources to be used to unpack the mathematical content infused in these games, and also social capital that will allow them to interact and learn from others.

The excerpts '…..*really enjoyed to participate in the indigenous games….. as they have developed high level of analysis, and formulating conjectures….*', illustrated that learners enjoyed attending such festivals on indigenous games, and at the same time they are used for learning purposes, that is, in understanding the abstractness of mathematical contents and processes elicited by the indigenous games. A festival of indigenous games is an environment which is free and unstructured; however it provides a rich context in which the problem-solving skills can be learnt. Dineo’s remarks echoed the sentiments that the play of indigenous games frees them to realise that mathematical contents can be derived from what they are familiar with. Also, it gives them hope and confidence that they can manage to do well in problem-solving.

In conclusion, the evidence above supports literature reviewed. For instance, the theory of Dienes (Sriraman and English, 2005: 258-259) on the learning of problem-solving, postulates that indigenous games permit learners to experiment with parameters and variations in order to begin analysing the mathematical content infused in the games.

The Association of Teachers of Mathematics (2010:44) and Baturo, Norton, Cooper (2004:87,88) acknowledges the active participation of learners in constructing their own knowledge from background environment through interaction with physical reality and through social interaction with peers and teachers and parents. In the classroom practice, it is vital to note the role of the teacher is not simply to facilitate learning by helping learners to move through their Zone of Proximal Development (ZPD) (Cripps & Clark, 2012:6), instead to make the transition from being unable to complete a task to being able to do so. This suggests that the power to learn problem-solving rests with learners, while teachers has to take a subordinate role. The teacher has a duty to seek ways to empower learners in problem –solving. Among others, teachers be able to engaged in collaborative activity, in which they are given a chance to discuss the mathematical contents immersed in the indigenous games.

More importantly, teachers used a wide range of teaching strategies, such as the strategy to teach problem-solving using indigenous games and to still interest to learn problem solving. Also, stimulate all pupils’ active participation in their learning together with innovative and imaginative resources, and including practical activities or hands-on activities (that is, using the indigenous games to learn mathematical content). Also, where appropriate, the team used the outdoors (Attard & Northcole, 2012:32) and environment (festivals on indigenous games).

As a result, the framework to teach problem-solving using indigenous games helped them to realise mathematical thinking they were capable of and to build upon this. Thus, it is necessary to know the mathematical landscape in which the learners are occupied, that is the nature of community mathematics to which the children are and have been subjected to.

CHAPTER 5

PRESENTATION AND DISCUSSION OF THE FRAMEWORK TO USE INDIGENOUS GAMES TO TEACH PROBLEM-SOLVING

5.1. INTRODUCTION

This chapter presents the framework of using indigenous games to teach problem-solving in a grade 10 Mathematics classrooms, describing how it was conceptualised from the onset, through the planning phase and up to the implementation.

5.2 PREPARATION PHASE

The preparation phase began with the researcher fulfilling the requirements of the University, through the supervision cohort project (Burnett, 1999:48; Nkoane, 2013:393). Staff of the faculty were invited with other PhD and master's students to listen to a presentation in which assistance was given to shaping the title of the study and meeting the requirements of the Committees for Title Registration and for Ethics, prior to approval by the Faculty Board. The study follows a postpositivist approach, in which it maintains that human beings are considered and so there are multiple ways of knowing (McGregor et al, 2010:423), underpinned by community cultural wealth, in which the knowledge of other students and staff members is acknowledged as true knowledge.

This first stage of preparation was important in making the students feel less isolated. The model gave them an opportunity to communicate their research with their promoter, mentors and fellow students. As a result they gained greater breadth of knowledge about the research. As the study took place in the community and addressed the daily lives of people it was important to take ownership of the study (Lavalle, 2009:27) and be an ambassador for the research to nearby communities.

5.2.1 The study coordinator

As noted above, it is the responsibility of the researcher or study coordinator to ensure the processes and procedures of the University are adhered to. After the Ethics Committees and Committee for Title Registration, and Faculty Board approved the study the researcher requested permission from the Department of Basic Education (DBE) to conduct the study in the chosen school so as to develop the framework of using indigenous games to teach problem-solving.

The researcher met potential participants to explain the framework for using indigenous games to teach problem-solving. To establish trust (Lavallee, 2009:26) the participants were consulted on a one-to-one basis, and I introduced myself to all of them (Absolon, 2010:75). The communities and schools did not feel comfortable sharing their knowledge or disclosing information to people who might benefit for their own cause, unless the study was to be beneficial to participants (McGregor et al., 2010:424). In order to diffuse the power relations the researcher would not dominate the discussions, but rather their interpretations would be acknowledged. In turn, the researcher acquired much knowledge from the participants consulted (Absolon, 2010:78; Lavallee, 2009:26).

During the meetings clarification was made of the study's title and objectives with emphasis on why the community, schools and higher institutions were seen as potential partners in the teaching and learning of problem-solving (Ferreira et al., 2012:50).

5.2.2 School community

According to Lavallee (2009:23) it is important that the researcher interacts with the participants before the actual implementation of the framework, and in this case did so with the SMT, SGB, Mathematics and Life Orientation teachers, interested teachers and the grade 10 learners. Meetings helped to ensure that the researcher and the participants understand the objectives of the study and could work together. This is in line with Meko's (2013:199) claim that preparation meetings allow the approached participants to think critically and honestly about their contribution in finding solutions to the identified problem.

Potential participants' voices were listened to (McGregor et al, 2010:425), and issues resolved. The researcher did not use his/her powers to dictate what they were expected to do in the study. The school community made contributions that revolved around the five objectives and various groupings hinted at the problems they faced in the teaching and learning of problem-solving. At the same time, they were also suggesting possible solutions. In the deliberations one teacher argued:

> *…re batla lefapha le lona le be teng*
>
> (…we want the departmental officials to be part of this research).

I realised that some of the solutions would come from them. Also, they alluded to the conducive conditions expected to prevail for the framework to work effectively, possible threats and how to counter risks. One member of the school community said:

> *Di LFs ha di notla sekolong mona hoba mosebetsi wa bona o mongata…, or re tla arenja matsatsi ao ba leng free ka ona*.
>
> (LF [learning facilitators] will not come to school here because they are overloaded with schools they service. Otherwise we arrange with them to come when they are free.

This is an indication that the information discussed was helpful as it mapped a way forward for the study. The manner in which the objectives were structured allowed the participants (school community) to fully participate, rather than be limited or isolated (Alaranta, 2006:2; McGregor et al., 2010:423,424).

The atmosphere in the meetings was one of respect for the SMT, SGB, and teachers, and acknowledgement of their importance to the study. The researcher did not use his/her powers to dehumanise them (Lybeck, 2010:95) or blame them for the poor performance of learners in problem-solving. The mode of discussion was how we together could develop a framework for using indigenous games to help us to teach and learn problem-solving effectively.

On the other hand, it was crucial that they be given a detailed presentation on developing the framework, rather than impose it on them simply because approval has been received from the DoE. It was detailed in that I explained the need for developing the framework together and their experiences were validated in the discussions. It would not be an individual effort of the researcher or demonstration of superior knowledge on mathematical content. The approach of the presentation was such that we needed one another in the research. The methods of presentation promoted the democratic values and social justice, and the participants' feelings and ideas would not be supressed.

5.2.3 Community members

After the meetings with the school community, the SMT decided to take the lead by inviting the general body of parents. The meetings concluded that it was important they be included (Lavallee, 2009:26). This study integrates cultural practices (indigenous games) in the mathematical content presented in Mathematics classes (Brown, 2008:2). The inclusion of community members was crucial as most parents, community leaders and traditional leaders operate outside the main school curriculum or their knowledge is marginalised (Racette, 2008:56) from the mainstream.

In the meetings with the community members the researcher elucidates what it would mean to be part of the study. The other school members assisted in clarifying questions posed by parents and elaborated on the objectives. As Lavallee (2009:28) noted, there would be switching and overlapping of roles between the researcher and participants, as the latter take ownership of the research. The word 'research' was hyphenated as 're-search', meaning to do again or look twice (Mahlomaholo, 2012 in the presentation to the PhD and masters cohort group; Racette, 2008:64, citing Absolon & Willet, 2005). It was important that we looked back together, to identify the challenges and chart a way forward to craft the components of the framework.

It was explained that I would not exploit them and any benefit from the study would not be at their expense. Rather the study aimed at benefitting all the research participants. It was to be guided by the democratic principles of respect, relevance, reciprocity and responsibility (McGregor et al., 2010:424; Racette, 2008:64; Wane, 2009:160). It was further explained to the community members that their participation would ensure that the knowledge was not suppressed or marginalised. Again, Stinson et al. (2012:44) contend that community participation in the study be perceived as a struggle to revitalise and protect cultural tradition (such as the playing of indigenous games), not to be marginalised or isolated from the mainstream Mathematics curriculum. There is much wealth of mathematical content within the indigenous games (Lynn, 2004:154; Yosso, 2005:69).

Their presence was validated when they expressed themselves freely in crafting their framework. Also, their participation would help to alleviate the misrepresentations of community's indigenous games through intent and/or misunderstanding. The presentation stimulated the parents and community leaders to be part of the study. They understood that the marginalised knowledge they had would be valuable in improving the learners' understanding of problem-solving skills, and further intensify the teaching methods used by teachers. The community members began to realise that their contribution would be significant in improving the socio-economic status of the subaltern children. The study is seen as emancipating human beings (Barker, 2012:202) from social stereotypes.

Thus, the scholar-warrior notion (Leonard, 2008:155; Racette, 2008) of thoughts and actions was combined to develop the framework of using indigenous games to teach problem-solving. This notion is characterised by cooperative learning between learners, teachers and community at large, as the actions showed by the playing of indigenous games is translated into understanding abstract mathematical concepts.

5.2.4 Education district officials

The researcher scheduled a meeting with departmental officials in the curriculum section (dealing with Mathematics matters from grades 4 to 12) and officials responsible for the Sport section were consulted. With regard to the Sport section, only officials responsible for indigenous games and indigenous music were met.

After informal presentations all district officials agreed to participate in the study. They were encouraged by assurances that it was to embrace education, mathematics and cultural wealth and practices. It would accentuate problem-solving as a human endeavour, that is, one that through the continuing inventiveness of human thought and actions continues to create new aspects of problem-solving that have been recreated through social interactions over centuries of human existence (DoE, 2003:62; Vilela, 2010:346).

The district officials perceived the study as forging links between officials in the General Education and Training band (GET), Further Education and Training band (FET), official in the Sports section, academics in Higher Education Institutions (HEIs) of learning, and the community at large. It would explore ways to give back or plough back resources to the young leaders of tomorrow, by collectively developing the framework to enhance learner performance. The research was strengthened when all key sectors, namely teachers, learners, parents, education officials and community leaders agreed to drive the process. This supports Racette (2008:63), for whom a study should demonstrate that participants are no longer subjects of objective study, but rather they create content knowledge through interacting.

5.2.5 Ethical considerations

For the research participants to take part freely in the framework it was important that they knew the parameters of the study. According to Chilisa (2012:86), ethical issues in the research include codes of conduct that are concerned with protection of the research participants from physical, mental or psychological harm. Hence, it was crucial that the Ethics Committee of the University approve the study before it was conducted.

The participants were informed about the research paradigm, that is, the post-positivist approach. It would acknowledge their role, such that as human beings they would not be controlled as objects of the study, but rather as active participants in the process (McGregor, 2010:425). This would enable them to display their real selves. Such emotions and feelings were respected in the process of the study. They were made aware that PAR was the method to be used in the study, as it emphasises equality among team members, with no one seeking to dominate or suppress others during interactions. This confirms Racett's (2008:62) view that study ethics need to be grounded in participants' worldviews, privileging their marginalised knowledge (Yosso, 2005:69) and language. Hence, they were allowed to express their ideas in any language with which they felt comfortable.

It was important to share this information with all research participants, for them to understand the expectations of the study. It would enable them to make informed decisions on whether to be part of the study. It was explained how the data was to be generated, with instruments such as video camera and audio-tape to record activities and deliberations during the class interactions and meetings. All the information captured was to be kept confidential and not disclosed to outsiders of the study. Underage learners who agreed to take part in the study were given consent forms on which to record signed permission from parents or guardians. It was clarified that this was a voluntary activity, and if one wished to withdraw from the study at any time they were at liberty to do so, without recrimination. That nobody was forced to participate in the study was made clear from the outset.

5.3 PLANNING PHASE

Immediately after the informal discussions with possible participants they were all invited to attend the formal planning meeting. Also, the suggested agenda was distributed, so as to allow inputs to be added before the date of the meetings. To allow flexibility, even on the day of the meeting, additions were taken or considered. Before the actual implementation of the framework, the team members agreed on the series of meetings to be held to feature and streamline the activities to be conducted, so as to operationalise the objectives of the study. During the planning meetings, activities to be conducted were discussed, and the roles to be played by different participants in executing the activities decided upon. Timelines for the completion of activities were set forth.

5.3.1 Formulation of action plan

In operationalising the objectives of the study, the team members planned to have the set of activities to be executed in five phases (refer to section 3.3 and Table 4.4 for detailed elucidation). For each phase, there were team members responsible for carrying out the tasks, whilst other members were responsible for the monitoring and evaluation, using the observation tool to record all their observations and experiences whilst the playing of specified indigenous games was in progress. The activities conducted maximised the participation of all participants (learners, teachers, parents, education officials and community leaders). This agrees with Chilisa (2012:254), who contends that an action plan gives space and voices to the marginalised or supressed groups. The second phase was dedicated to the reflection on the lesson learnt in phase one, that is the playing of the indigenous games. In this phase, participants were placed in small groups to reflect on mathematical content displayed or visualised within the indigenous game.

Phase three was planned in such a way that it was conducted in a lesson the following day. This allowed the person tasked with lesson presentation to prepare well, based on the reflections or discussions transpired in phase two. The activities designed would take into consideration the prior knowledge of learners, so as to generate mathematical content on their own, and to decide on hands-on problem-solving activities that would allow learners to contextualise mathematical concepts or mathematical content using the indigenous games played. In this phase learners would be given class activities to be performed in small groups, in which parents and teachers would act as facilitators or scaffolding where necessary (Muijs & Reynolds, 2011:79). The planned problem-solving activities were performed in such a way that learners would discover mathematical definitions, and mathematical concepts on their own, still using their knowledge or lived experiences of indigenous games. For instance, the during the play of *diketo* (when the player is on round one), learners were able to realise that the relationship between the throwing of the *ghoen* into the air, scooping the small stones out of the hole and replacing them in the hole illustrates a particular pattern, such as a linear one. These activities allowed learners and other participants to add their voices (McGregor, 2010:423), which made them achieve in-depth understanding of mathematical content, rather than simply memorising. They became active participants, rather than objects to be filled with information by their teacher.

In phase four, the knowledge generated within their small groups was now shared with the larger group. As they gave feedback to the larger group of participants they were to expect contributions from the floor in the form of questions, clarifications and additions on the activity under discussion. This was in keeping with claims by Barker (2012:12) and Sudersan (1998:260) that a human being-based research has to allow the critical voices of the participants. In this way, their minds are liberated and freed from oppression. Ultimately this reveals the beauty and social justice built into the topic.

Phase five was dedicated to the facilitator, who introduced the lesson to give a summary based on feedback from different groups, and corrected misconceptions that learners might have experienced. This was a demonstration that knowledge is perspectival, in that it allows multiple interpretations (Barker, 2012:20). Finally, learners were given assessment tasks to be performed either in class or at home.

5.4 IMPLEMENTATION PHASE

This is the very important phase that actualises or puts into practice the action plan which was jointly crafted by the participants in section 5.3. It gave guidelines to be followed when implementing the framework of using indigenous games to teach problem-solving. Discussion was held on the need to implement the framework, also focusing on the components of that made it implementable and the conducive conditions which made it successful. Risks and threats framework, and how the contextual factors were circumvented, were addressed, and indicators of success in the implementation of the framework discussed.

5.4.1 JUSTIFYING THE NEED TO IMPLEMENT THE FRAMEWORK

Before the framework was conceptualised, it was observed that teachers transmitted mathematical content into the heads of learners. The teacher was seen as a powerful person with the authority to stand in front of the class and tell learners what to do in problem-solving. As Matthews et al. (2005:2) argue, it is unwise to perceive the minds of learners as empty vessels that must be filled with mathematical content. The teacher-centred approach to problem-solving made it difficult for the learners to comprehend the presented subject matter. In essence their critical thinking had been suppressed, as they were expected to know mathematical definitions and formulae by heart. The teachers were teaching with a deductive approach, in which learners had to know general conclusions (such as definitions and formulae (e.g.,$y = mx + c$). They were not afforded an opportunity to conceptualise them and teaching methods used were strange to their way of living or playing. There are contextual scenarios with which learners are familiar from the home environment, but the familial and social capital (Lynn, 2004:154; Solórzano & Yosso, 2002:37, 2005:78) was not being considered.

Learners were being drilled to master mathematical formulae, with most of the questions prompting regurgitation. Once a formula was remembered the learners merely substituted figures to get the final product. At times they wanted to know why certain definitions (e.g., $a^0 = 1$) or mathematical concepts were phrased in an abstract way, (e.g., why is $a^0 = 1$?), and in responding the teacher would confidently respond by endowing a shallow reasoning that $a^0 = 1$ is the definition). Such a leaner is not sufficiently answered. Apparently, the teacher was adopting the positivist approach by which they maintained there is one way of getting the answer. He or she did not allow multiple interpretations to arrive at the truth. This type of reasoning shows that the teachers had limited content knowledge, in most cases displayed by oppressing the critical reasoning and divergent thinking of learners.

Once learners realised that their questions were not being adequately responded to they tended to memorise such abstract mathematical concepts. Also, the method of teaching was foreign to their daily life experiences. As a result their performance in problem-solving was declining. In most cases, it was rare for teachers to acknowledge the huge wealth of navigational capital (Yosso, 2005:79) that learners own, and which remain unused in the teaching and learning of problem-solving. Unconsciously, they utilise the cultural wealth they possess to resort to negative actions or behaviours towards problem-solving. They tend to generate ‘don’t care’ attitudes towards their work on problem-solving, for example, being demotivated to engage in activities. As a result, they began to think that problem-solving was for the ‘powerful people’ and ‘chosen few’ in their society (Nasir, Hand, & Taylor, 2008:226). Ultimately, they marginalised themselves or suppressed their desire to access mathematical content.

The situation is worse for subaltern parents, as they have been completely excluded from the teaching of problem-solving. It was generally inferred that the knowledge they possess would not allow them to understand the process, forgetting that their navigational capital (Yosso, 2005:79) made it possible for them, for example, to count their herd of cattle through one-to-one mapping, that is, by associating one cow to one stone.

In summary, developing the framework to use indigenous games to teach problem-solving is a way of using resources at the disposal of all learners, to access abstract mathematical concepts.

5.4.2 Components of the framework to teach problem-solving

The components of the framework were designed to respond to the challenges identified above and introduce equal access to the learning of problem-solving, that is, no learner is viewed from a deficit perspective, and their lived experiences are used to tap into hidden or abstract mathematical content. The majority of learners are exposed to indigenous games from the home environment, that is, one that is advantageous to them. The framework acknowledges the contextual environment with which they are familiar and integrates the playing of indigenous games (AU Report, 2014:1) with the learning of mathematical content.

Regarding the teaching method used by teachers to make learners access subject matter easily it is evident that teachers’ use of the ‘talk and chalk’ method made it difficult for learners to comprehend mathematical content. The framework would encourage them to be active participants in the learning and teaching process. During the playing of indigenous games, they could record their observations with regard to the structural nature of the indigenous game, mathematical skills, language used in the play or any observations worth noting that transpire. Learners discovered that the indigenous games are imbued with mathematical concepts and content. The framework allows for a learner-centred approach and integration of mathematical content to the context of playing the indigenous games (DoE, 2003:2, 62). Linking the playing with constructive learning of mathematical content was extracted from the indigenous games (refer to Fig. 4.4 and Fig.4.5 for examples of extracted mathematical content and mathematical processes).This was then used as a basis for designing more class activities that would help learners to discover other mathematical content, as prescribed by the Mathematics Curriculum Assessment and Policy Statement (CAPS).

Learners recorded the number of times the *ghoen* is thrown into the air, the number of stones scooped out of the hole and number of stones replaced in the hole. They discovered that the relationship between the throwing of a *ghoen* and stones placed in the hole or scooped out of the hole formed a particular pattern, that is, a linear relationship (refer to Table 4.9), which is described by general formulas such as symbolically as $f(x) = -x + 11$ (for scooping stones out of the hole) and $f(x) = -x + 10$ (for placing back the stones in the hole). They understood that there are two variables involved, that is throwing of the *ghoen* into the air and scooping the stones out of the hole or placing the stones in the hole. They even coined their own definitions of independent variables (throwing of *ghoen* in the air, which is symbolically denoted as (x)) and dependent variables (either scooping stones out of the hole or replacing them in the hole (symbolically shown as x). From the observations of the patterns illustrated by *diketo*, learners were able to inductively reason that the general formula can be expressed as $f(x) = -\frac{x}{2} + \frac{21}{2}$ (this is true, when playing round one).

In addition, it became apparent that subject matter is meaningful, looking at the recording done by learner in the Table 4.8, comparing the first and second columns readings or visualising the play of *diketo*. They realised that, as the number of throws of the *ghoen* in the air increases, the number of stones scooped out of the hole decreases. Likewise, when the number of throws of the *ghoen* into the air increases, the number of stones placed in the hole decreases. This is an effective way of defining the indirect proportion. The framework validated the multiple viewpoints of arriving at the truth (Barker, 2012:201,202; McGregor, 2010:423). Again, learners discovered and coined their own definitions of implicit and explicit conditions.

Subsequently, learners become so motivated to play indigenous games that they were able to extract mathematical formulae and definitions on their own. That also boosted their confidence in realising that their home background was powerful in contextualising and simplifying the abstractness of mathematical content. Even teachers became motivated (Moller, 2009:16, in African Union report) to teach problem-solving, as learners were now performing well. Learners led class discussions, no longer looking for the correct answers from their teachers. They were emancipated (Barker, 2012:202) by a realisation that they possessed great wealth of knowledge on problem-solving. It was evident that they had reached a new level of analysis, due to their linguistic capital and navigational capital.

Parents play a crucial role in the teaching and learning of problem-solving. Their navigational and aspirational capital (Yosso, 2005:78) helped them to pose critical questions (Barker, 2012:201) and made contributions that helped learners to obtain deeper meaning on problem-solving. The manner in which parents described or demonstrated the playing of indigenous games helped learners to formulate definitions of mathematical concepts, rather than memorising the ones in the textbooks or given by their teachers. For instance, in one session parents were demonstrating the play of *kgati*, when some learners discovered that the movement described by the loop of *kgati* exemplifies the parabolic function (refer to Fig. 4.8). As the hand of the player went up the loop of *kgati* was moving upwards, which showed the increasing function; also as the hand of the player went down, the loop of *kgati* went downwards, denoting the decreasing function.

The linguistic, aspirational and navigational capitals are imbued with many mathematical concepts, of which parents were not even aware. On the parents' side they also felt rejuvenated and emancipated in realising that problem-solving is a human endeavour (DoE,2003:62), which validates their cultural practices. Also, they realised that the marginalised knowledge of people of colour is legitimate, appropriate and critical (Solórzano et al., 2002:37).

Such a realisation by the teachers (Nasir et al., 2008:208,209) of the huge wealth of human resources, notably the involvement of parents, that is unused, can be used to improve their practices. The presence of parents, district officials and community leaders improved the discipline of learners in classes. Learners became cooperative in their small groups, and even gave feedback to the larger groups. The teacher used the feedback to 'feed-forward' (Kelley et al., 2011:89) for the next class presentations.

5.4.3 Conditions for the framework

In this section the circumstantial factors will be discussed. These are conducive conditions which made it possible for the framework to be successfully implemented. It must be noted that the contextual factors were not perceived as a uniform approach designed for all (Rosand et al., 2008:6; UNHCR, 2009:214), but rather as appropriate for the particular conditions in this study. Hence, the conducive conditions for meaningful subject-matter and learner-centred approach to work for the framework, the following were considered. The atmospheric conditions in the classroom represented social backgrounds of learners. The indigenous games used were listed by learners themselves, hence they were actively involved in extracting the mathematical concepts embedded within the games. Spontaneously, learners worked collaboratively on class activities. The teaching-learning process created is not only controlled by a powerful authority, in the form of the teacher, but rather all the participants take charge of teaching practices. At some stage the discussions are led by parents or certain group of learners. The learning environment is friendly to all research participants and no one is marginalised because of his or her level of knowledge or experience. There is a healthy interaction between learners themselves, learners and learning facilitators, such as teachers, district officials, and parents. Learners do not expect to be given correct answers by the teachers, as they critically engage one another in groups (Nasir et al., 2008:201,202,210). The interactions between participants were of respect and validation of the knowledge shared. Interactions (Nasir et al., 2008:202), characterised by robust debate, such as:

> *Can you please justify the argument you put forth or what evidence can you provide to substantiate your viewpoint*?

This demonstrated that all responses were interrogated before being accepted as valid. Unresolved issues or disagreements are resolved by learning facilitators, that is, teachers, district officials and parents (refer to chapter 4 for various meaningful definitions and mathematical formulae formulated by learners), and with verification by subject specialist to clear up misconceptions that might have been created in the process.

There is a close relationships between various stakeholders, such as school community, education district officials, civil society members and representative from institutions of higher learning. Nobody shows excessive power or blames others for not teaching problem-solving effectively, but rather all are engaged in developing a framework to help learners demonstrate a higher level of understanding of problem-solving. Also, the conducive conditions were such that all participants capacitated, empowered and emancipated one another. In some instances, the district officials (Mathematics subject advisors) argued that some of the findings of the study would be cascaded to other schools so as to ensure that learner performance in problem-solving was significantly enhanced.

It was ensured that teaching and learning happened within the learners' terrain, one replete with play, and at the same time instil a high order of learning and enjoyment (Nasir, Hand & Taylor, 2008:187). Such landscapes motivated learners to play an active role in assigned activities, whilst teachers and other stakeholders felt encouraged by the dedication and commitment towards their work.

5.4.4 Threats to the framework

This section reports on a discussion of the risks factors which might upset the framework, and how they were circumvented. During the pre-planning sessions, there were individuals who felt that being involved in a study was a futile exercise as the research coordinator would be incentivised at their expense, and they would not benefit from the exercise. Such criticism did not demotivate the team members from contributing towards nation building. It was argued that looking at the departmental results for analysis of Mathematics at exit points, that is, grade 6, 9, and 12, there was a crisis, as learners performed poorly in problem-solving. This created false perceptions that problem-solving was for certain hegemonic groups, not for learners in rural areas or those from poor socio-economic backgrounds (Nasir et al., 2008:210). The research coordinator and other research participants stated that, as responsible citizens (DoE,2003:2), we were bound to take a lead in addressing the crisis in problem-solving for the future leaders of the country, not to put blame on learners and teachers for poor performance in problem-solving skills. Irrespective of whether the education received is informal or formal, all are capable of collectively executing the tasks to develop a framework to teach problem-solving in rural schools, in a way that all different stakeholders will use a wealth of resources they possess from their social backgrounds.

Looking into the teaching and learning situations, the following risks factors were identified. The classroom space was too small to cater for playing of some of indigenous games, and the level of noise was too high as team members were applauding up their members. It was decided that the playground be used for stage one and stage two, that is, where the actual playing of the game was taking place and reflection on the lessons learnt from it. All learners would thus have ample space to explore the activity. Stage one was so exciting to learners that they neglected to record their observations on the prescribed sheets. They took more time to report their findings during the feedback sessions. They were advised that as they did applaud and make critical observations they needed to record their observations on the sheet, so as to speed up the group discussions and consolidate their findings for reporting to a larger group.

In larger groups they needed to be ready to clarify and defend their arguments. In times of bad weather we used the school hall for the actual playing. At times, so as to have ample time for the stage one and stage two, afternoons were used, then in the next day's lesson stages three to five were completed in class. The afternoon was ideal for other research participants, especially those engaged with other activities during the day. Other team members, for instance, the HoDs for Mathematics and Life Orientation, the Life Orientation teachers and the deputy principal had other classes to teach, so the afternoon and weekend sessions were ideal for them to attend full sessions.

The time on tasks, when learners worked on activities in class to discover mathematical concepts on their own, or in their small groups, was given little time. This was because the presenter for the day took more time to introduce the lesson or activities. Some presenters resorted to a ‘talk and chalk’ method (teacher-centred approach), with little or no involvement of learners. Others were quick to give learners responses to the activities without being given a chance to interrogate the activities. Learners were denied an opportunity to invent mathematical definitions and conjectures on their own. These instances were caused by their strong convictions that the teacher, seen as the powerful person in problem-solving, had to transmit knowledge into the minds of learners. Teachers and other research participants tended to use the traditional way of teaching problem-solving, which is oppressive and perpetuates inequality (Lynn, 2006:18; Yosso, 2002:102).

During the feedback sessions such cases were corrected to ensure that learners had enough time to interrogate the subject matter on their own. The presenter of the lesson had to briefly introduce it and devote more time to learners’ activities. Further, the instructions for the activities had to be clear and easy to follow. As a result, time was not wasted on explaining the activities to learners. The rotational systems, that is when presenters of the lesson had to rotate their roles of presenting, enabled others to emulate the good practices of engaging learners through the teaching and learning sessions. To present the lesson, participants had to work in pairs or small teams to ensure that thorough preparations for the class activities were in order and clear for execution by learners. The involvement of research participants, especially the presence of subject specialists, assisted in demonstrating good teaching practices. Also, parents provided support by monitoring the effectiveness of the groups, thus ensuring that learners did not digress in their discussions to matters outside the assigned activities.

On the other hand, at times parents felt excluded from the discussions as English as a medium of instruction. Subaltern parents felt marginalised, as English was seen as the language of accessing learning of problem-solving (Chicks, 2002:470,471; Hornberger, 2005:3). It was agreed that no one be prevented from using any official languages, either in a formal or informal setting (Chick, 2002:470; Constitution of Republic South Africa Act 200 of 1996:14-15; Hornberger, 2005:6). The use of a home or mother tongue helped learners obtain a deeper understanding of mathematical concepts (Bush et al., 2010:164; Wong & Lipka, 2011:823), whilst code-switching was used to accommodate all participants.

Some teachers complained of the heavy workload because they were teaching two or three subjects with many learners. They indicated that they spent too much time, including afternoons and weekends, attending to certain important sessions, though the subject advisors for Mathematics and Sports section and SMT unanimously agreed that the work planned and presented collectively, as well as the assessment tasks performed in the study, could be used as evidence to file in their portfolios. They did not have to redo their individual work for portfolio filling purposes. The departmental guidelines required work schedules or pacesetters to be followed accordingly, that is, in term one all the work be taught. The same applied to terms two up to term four. Through the assistance of subject advisors, the provincial and district education departments agreed that the work covered in the study be done in an integrated fashion. For example, shapes and space topics could be presented simultaneously with algebra and functions within one activity of indigenous games. Much mathematical content cut across main topics, such as algebra, trigonometry, geometry, data-handling and probability.

5.4.5 Evidence of the framework

In this section indicators of success for the framework are discussed. This is a demonstration that the framework for using indigenous games to teach problem-solving produced fruitful results.

The teaching method, that is, the learner-centred approach used by the teachers, enabled learners to master problem-solving relatively easily. As learners interacted with other participants of the research in playing indigenous games, they successfully discovered many mathematical concepts, definitions and formulae. For example, other than using the normal area formula of rectangle, that is, $Area = length\ by\ breadth\ or\ A = lb,$ through the analysis and interpretation of the board game, they used $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ (where A is the area of the rectangles and n is the number of rectangles), as the formula to calculate the area. Likewise, $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$ (where P indicates the perimeter of the rectangles and n indicates the number of rectangles), as the formula to work out the perimeter of a rectangle. This is an indication that learners developed a high level of understanding of mathematical content areas, rather than mere memorisation of formulae with no understanding.

In most cases, the discussions in class were led by learners in small groups, when they even asked teachers difficult questions. Of interest was they did not wait for the corrections or answers from the teacher, rather they searched for information in various books and accessed the Internet for information. The following day they led deliberations on unfinished work for which they had not arrived at a conclusive answer (Thomas & Brunsting, 2010:10). Learners reconfigured *morabaraba* (refer to Fig. 4.12 and Fig. 4.13) from 2D objects to 3D objects, when they realised that with two-dimensional objects they were able to find the area and perimeters, whereas with three-dimensional space they were able to calculate the volume and the total surface area. As in Bloom's taxonomy of learning, they operated at higher level of comprehension, application, analysis, synthesis and evaluation. Learners developed critical and divergent thinking (Haylock, 2013:7; Shatzer, 2008:652) in that they believe that the presentations made must be supported by proper motivations as to why such agreement was reached.

That learners did not rest until they had responses to unfinished matter in class or unresolved problem discussed the previous lesson indicates that they were highly confident about their understanding of the subject matter. The next day they took the lead in obtaining their findings on what they discovered on the unresolved activity of the previous lessons. Learners' performances significantly improved (refer to Table 4.18: Grade 10 mark sheets), which also inspired teachers to do their utmost in assisting them. The subject advisors drew up very impressive school visit reports (refer to fig. 4.15) about the overall performance of teachers and learners in their effective way of teaching and learning problem-solving. The reports complimented the teachers on the all-inclusive nature of their classroom practices.

The district officials encouraged teachers from underperforming schools in problem-solving to visit the school conducting the study to learn comprehensive and effective way of teaching problem-solving, and to observe how subaltern parents supported learners in their classrooms. It was realised that parents from this school conducting the research had developed a greater appreciation in the schooling of its children. In all the parental and class-teacher meetings, parents were present. Their contributions were valuable, as they advised other parents on how to support learners in their respective subjects, especially problem-solving, in line with the lens of community cultural wealth (Lynn, 2004:153,154; Yosso, 2005:69). Parents demonstrated to others that their capital wealth was of help to learners in their problem-solving. Of particular significance, parents in the research team were able to articulate their stance using the theoretical framework.

5.5 CONCLUSION

This chapter has outlined how the framework can be implemented successfully. It has considered key elements under each phase to be addressed. It was important that the successful implementation of the framework begin with the preparation phase, in which all possible participants met and presented the rationale for the study. In some cases the informal meetings were on a one-to-one basis, in others with certain groups of communities, or various sections in the DoE, for deeper understanding of the study. Misconceptions that existed during these meetings were clarified successfully. This was done in a respectful way, such that possible participants were not coerced to be part of the research.

The planning phase and the implementation phases motivated the research participants to realise that this study was being conducted by them and for their own benefit. All their inputs and contributions would be considered in building a sustainable framework. At certain intervals the feedback sessions were held so as to feed-forward processes and further check as to whether the objectives were still being followed, as agreed upon. This motivated participants to come forward with constructive ideas, whilst also being mindful of demerits that might derail the framework. Finally, the actual implementation phase was operationalized successfully. The processes put in place in previous phases yielded a successful product. The framework was trialled and tested. As Sudersan (1998:256) states, the means are as important as the ends.

The next chapter will focus on the findings of the study and make recommendations for the framework for using indigenous games to teach problem-solving efficiently.

CHAPTER 6

FINDINGS, CONCLUSIONS AND RECOMMENDATIONS FOR THE FRAMEWORK TO TEACH PROBLEM-SOLVING

6.1 INTRODUCTION

This study aims to enhance the teaching of problem-solving to grade 10 learners through the use of indigenous games. This was done by developing the framework of teaching problem-solving, as presented in the previous chapter. This chapter presents a synopsis of previous chapters by indicating how the study contributed to the body of knowledge. This is done by presenting the findings, and is followed by recommendations. Limitations of the study are acknowledged, that is, the gaps that can be attended to by future researchers conducting studies or compiling research articles. Finally a conclusion is drawn.

6.2 THE AIM OF THE STUDY

As shown in the previous chapters, this chapter reiterates the aim of the study as enhancing the teaching of problem-solving skills to grade 10 learners by using indigenous games. It follows that the main question is phrased as: *how can we enhance problem-solving skills in a grade 10 Mathematics classroom using indigenous games*? In responding to the research question, the aim of the study was categorised into five objectives, which are responding to the research question. The first objective was to identify challenges to the teaching of problem-solving. The second focussed on the components of the framework to address the identified needs. The third focussed on the conducive conditions that make the components to the framework work effectively, and the fourth examines the contextual factors that could derail the framework, and so devise means to circumvent them. Finally, we consider the indicators of success to the frameworks, that is, what information can be provided as evidence that it is efficient.

Through literature reviewed and empirical data generated there were concepts developed, that is the constructs which are used to address the five objectives of the study.

6.3 THE NEED FOR DEVELOPING THE FRAMEWORK OF USING INDIGENOUS GAMES TO TEACH PROBLEM-SOLVING

The needs assessments made collectively by the researcher and research participants through a literature review and empirical data confirmed that there are challenges to the teaching of problem-solving. Thus, based on that the findings, recommendations are made.

6.3.1 Subject content too abstract in the teaching and learning of problem solving

Bothe the literature reviewed (2.5.1.2) and data generated found that in most cases teachers are used to teacher-centred methods in the teaching of problem-solving, that is, the teachers are more active than the learners. Leaners merely absorbed information presented by the teachers (refer to sections 2.5.1.1 and 4.2.1). Although the DBE (2011:3,4) encourages teachers to engage learners in lesson presentations, some claimed that this was not practical as the content to be covered on the work schedule was too great, and learners were reluctant to take a lead in class presentations (2.5.1.1.2, and 4.2.1.2). The teacher-centred method is used to teach the raw subject matter as it is presented in the work schedule, with no creativity included to make learners active participants in the learning process. The subject matter was being presented in an abstract manner which made it difficult for the learners to comprehend. Teachers took a deductive approach, with learners presented merely with a general formula but no opportunity to draw their own conclusions. For instance, a teacher demonstrated in a lesson presentation:

> *In this particular sequence the last term will be defined as* t_n, *which simplify equal to* $a + (n-1)d \ldots$

The teacher merely transferred the general formula ($a + (n - 1)d$) into the minds of learners without showing how or why it came to be in that fashion (see 4.2.1.2 and 2.5.1.1.2). This resulted in learners digesting and regurgitating with no understanding. They then wrote incorrect formulae when making calculations in exercises of problem-solving. Although in the final examinations learners were given a formula sheet, some still chose incorrect formulae to determine the solutions .This is a clear indication that they did not understand these formulae, of which the methods used by teachers encouraged learning of general formulae and mathematical definitions, with no understanding. As a result, they 'consumed' with no interrogation of the subject matter presented. Further it was found that the teaching method gave teacher an upper hand or powerful authority, which in turn suppressed the creativity of the learners and discouraged them from asking questions (refer to 4.2.1.2).

In conclusion, the teacher-centred method made learners passive recipients of the subject matter. As the teacher was rushing to complete a work schedule, no creativity was used to engage learners. They were given raw subject matter as from the CAPS document, and the presented formulae were too abstract for learners to comprehend. The teacher-centred approaches gave teachers unnecessary power to dominate the class presentations in the teaching of problem-solving. As a result, learners felt intimidated and ultimately withdrew from the centre of teaching and learning to the periphery.

The type of questions asked in the process of teaching and learning sessions were of low order, for example, 'do you understand?', to which learners responded with either 'Yes' or No' responses. The '*how*' and '*why*' questions which come spontaneously when the learning environment is learner-centred were not used.

6.3.1.1 Recommendations

The study recommends that teachers be creative in the teaching of learning of problem-solving. There is a wealth of marginalised knowledge which is imbued with much mathematical content. The background environment of the learners is rich in mathematical content, which can help them to become actively involved in the learning of problem-solving, as they are familiar with it. The use of indigenous games can help them to exchange roles easily, and identify the mathematical content of the games. At one stage there can be a lesson presenter and critical audiences (active learners). Through the observations of play, they can detect patterns and find the relationships that enable them to make general mathematical formulae (more details refer to 4.2.5).

6.3.2 Lack of motivation among teachers and learners

The study found that a teacher-centred approach is top-down or autocratic, consequently learners feel demotivated and the subject matter presented is difficult to comprehend. They withdraw from classroom discussions and if they do ask questions they receive unsatisfactorily answers. For example (4.2), one learner asked the teacher:

> '…*but sir, I do not understand, why are you saying* two *to the power of zero is one?'*

To which the teacher replied:

> …*yes my child, is it because* a *to the power of zero is one (that is,*$a^o = 1$)*, that is ,any number to the power of zero is equal to* 1*, it is the definition to be used, which you have to know by heart*…'

This kind of response is not appropriate to the questions asked. It indicated that the learners were expected to memorise the formulae, and understanding immaterial. These frustrations or unclear explanations made leaners lose interest in learning problem-solving. Further evidence of their being demotivated by the unclear and abstract approach to problem-solving (4.2.1.2), after a response by the teacher one whispered to her friend:

> ...*eish, o a bora, o tla qeta neng, wena o utlwisisa?*

> *(... he is boring us, when is he going to finish his presentation, or are you following what he is doing?).*

This negative attitude displayed by learners revealed a serious problem in understanding the subject matter as presented. Immediately, Learners lose interest in problem-solving, and do not focus on class presentation. Hence, their performance in assessment tasks tend to be very low.

The study found lack of participation of learners in class also frustrated some teachers (4.2.4 and 2.5.1.7). Although some were under the impression that when learners are quiet they were following the presentation they realise after tests and examinations that the learners had not understood, as they failed in problem-solving questions. The teachers thus in turn became demotivated, leading to a vicious circle.

The conclusion drawn here is that the passiveness of the learners in class presentations and discussions following a teacher-centred approach has caused learners to develop a negative attitude towards problem-solving. Also, the poor performance of learners dissuaded them from paying attention in lessons. The ‘talk and chalk’ method (2.5.1.7.2) contributed to poor performance of learners in problem-solving and made them react negatively to problem-solving.

6.3.2.1 Recommendations

The study recommends that it is important for teachers to ensure that the teaching and learning of problem-solving fully engages learners. At times the explanations or answering can be thrown back to the class so that the passive ones can interact with others. Although the work schedule showed the mathematical content to be treated, it is the responsibilities of the teacher to create activities that motivate and stimulate interest of learners in learning problem-solving. In addition, teachers do not have to think of expensive resources to be bought for the class, rather the marginalised knowledge of learners can be tapped into when teaching problem-solving. It was found that the framework of using indigenous games proved to be efficient in instilling enthusiasm for and knowledge of problem-solving.

6.3.3 Non-involvement of parents

The study found that the teaching and learning of problem-solving has excluded parental involvement. Teachers preferred to plan alone, not considering parents as valuable assets in creating practical scenarios in class. Even in the planning and presentation of the lessons other colleagues were excluded. The knowledge that parents possessed is discredited in the teaching of problem-solving, but surprisingly, in appointing teachers for teaching post, parents are included in the interviewing panels. Their expertise and knowledge is welcomed.

On the other hand, even parents accepted being alienated from the teaching of problem-solving (4.2.5.1), as evident in remarks made by one of the parents:

> *...a ko mphetise mona ngwanaka, jwale rona boholo ba batswadi ha re sa fihlela boemo bo hodimo ba dithuto tseo tsa lona, tsa boemo bo phahameng, re tla kgona jwang ho nka karolo projekteng na.*
>
> *(…explain this to me my child, the majority of us, as parents, did not study up to your level of education. How is it going to be possible for us to take part in this study?)*

This supports the argument that parents accept the excuse to be sidelined in the teaching and learning of problem-solving. In the final analysis, it was found that parents are blamed for the non-support of their learners. They are labelled as 'uninterested' in their children's welfare and blamed for not placing high value on problem-solving (see 2.5.1.5).

The conclusion drawn from this is that, before the study, the school and parents were working in isolation. Parents did not know the happenings in the classrooms, nor how they could help to better the learning problem-solving. The marginalised knowledge possessed by parents was only utilised at home to bring up the children, but not in the school environment. This broadens the gap between school community and subaltern parents. Parents are used selectively when circumstances suits the school community.

A parent remarked as follows:

> *'…le rutwa eng sekolong ha le sa tsebe letho tje?*

(*What is that you are taught at school, you look so ignorant of many things?'*).

Neither party communicated properly about the teaching and learning in school or the wealth that remains untapped in the process. In essence, both parties need one another in integrating mathematical content and cultural practices, that is, integrating theory and practice.

6.3.3.1 Recommendations

The study recommends that the two parties, that is, school community (teachers, education officials) and subaltern communities (parents, community leaders), work together in the teaching and learning of problem-solving. It is evident that parents possess rich community capital, whether aspirational, navigational, familial and/or social, which can be used to promote creativity in the teaching of problem-solving. The study further recommends that teachers and parents use the indigenous games to teach problem-solving, because all these rich capital and mathematical contents are infused in indigenous games (see 2.5.2 and 4.3).

Just as many parents were capacitated and empowered before in the governance and finances of schools, so they should be in the teaching and learning of problem-solving. If made aware of the huge wealth of knowledge they possess, with regard to playing of indigenous games, and capitals, they can be effectively use it in teaching problem-solving, for the benefit of improving the learners' performance in mathematical content.

6.4 COMPONENTS OF THE FRAMEWORK

In this section the components of the framework are discussed, in response to the challenges identified in 6.3, crafted as collective contributions of all research participants.

6.4.1 Meaningful subject matter

The study found that the teachers integrated the mathematical content with the prior knowledge of the learners, which is the playing of indigenous games. Teachers used those games played by learners and parents to extract mathematical content embedded in them (4.3.1). It was an interactive session in which all learners worked together to play the indigenous games, whilst other participants used the observation sheet (Table 4.14 in Appendix F1 and Fig. 4.3 to visualise the mathematical skills and content demonstrated. They discovered many mathematical concepts infused in the playing of indigenous games.

As the DoE (2003:9) argues, problem-solving is a human activity practised by all cultures. The study found many mathematical concepts and much content (4.3.2.1), as revealed by the one of the participants:

> *…re ile ra bapala diketo hape hore re arabe ditsopo. Ra fumana hore ka kgetlo la bodedi majwe a kenang ka mokoting ke 9, 12th ke majwe a 4, and 16th ke ketwana tse 2.Potso ya bobedi, hore o qete seng one o lokela ho ba le makgetlo a 19.*
>
> *(…to answer the questions, we demonstrated by playing diketo again. Through the play of diketo we deduce the following: for second throw, there are nine stones to be pushed into the whole, twelfth throw there are four stones and sixteenth throw there are two stones. The patterns can be followed in that way. To complete round one (seng one) there are nineteen throws to be made.)*

The study found that linguistic capital and social capital (Yosso, 2005:78) enabled learners to freely interact and learn from others (sections 2.5.2.2 and 4.3.1.2). It helped them to conceptualise that the play of *morabaraba* infused mathematical content such as probability concepts, algebra, including patterns and relationships, lines and polygons (refer to Fig.4.4 for more details).

In conclusion, contributions of the study to the body of knowledge are that the use of indigenous games in the teaching of problem-solving integrates the theory and practices. In developing mathematical definitions and general formulae, the participants analysed and interpreted the practical processes demonstrated by indigenous games (for more details refer to 2.5.2.2 and 4.3.2.1). Yosso's capital wealth (linguistic, social, navigational, familial, aspirational capitals), which is used constantly in the play of indigenous games, formed the basis of unearthing the mathematical concepts.

6.4.1.1 Recommendations

The study recommends that the framework for using indigenous games be used to discover mathematical concepts, definitions, mathematical content, conjectures and general conclusions which are hidden. The use of indigenous games stimulates the learners to freely use capital wealth to discover subject matter in a meaningful way. It also helps in integrating real-life activities with mathematical content, so that integration comes spontaneously, with no need to force it on the class.

6.4.2 Motivation among teachers and learners

The study found that learners were excited and encouraged to realise that the home knowledge (that is, the playing of indigenous games) was useful in their interactions in the learning of problem-solving (2.5.2.3 and 4.3.3). Before the introduction of the framework the learners and subaltern communities thought that mathematical content and concepts were foreign to the cultural practices, hence they developed negative attitudes towards teachers and also were repelled by the subject matter itself. Once exposed to the framework, however, they came to value their cultural practices as infused with mathematical content. This made leaners contribute positively in class presentations and ask constructive questions that showed deeper understanding (4.3.3.1). This implied that their self-esteem and confidence was being boosted (2.5.2.3). For instance, one group contributed positively in this fashion:

> *…. let me contribute by saying linear function (like* $f(m) = -\frac{m}{2} + 10$*), is pattern which has been generalised from the Table 4.8, then it is correct to say linear functions are special types of patterns. Then we can go on and*

deduce further, from morabaraba, we discovered that perimeter: $n^2 + (3,8)n + 0,8,$ *and area:* $A_n = (0,1)n^3 + (0,9)n^2 + (0,4)n$ *.Then,, it is clear that quadratic functions and cubic functions are another types of patterns*.'

Thus, learners demonstrated own-ability, assertiveness, leadership, trust and security (2.5.2.3.1), when interacting and networking with others, during the class presentation on problem-solving. Besides, it was a touching moment for teachers when they experienced once demotivated learners making huge strides and efforts in mastering problem-solving (2.5.2.7 and 4.3.7). One teacher commented:

> *This is exceptionally well. I am so delighted about the discoveries. In fact from the two-dimensional object, you came up with the three-dimensional figures.*

The study found that the use of indigenous games played a vital role in linking the subject matter of problem-solving to real-life situations. The diverse methods of teaching intensified learner engagement and maintained interest. It also served as a prime motivation for the teachers (2.5.2.7).

In conclusion, the study added much to the body of knowledge. The use of the framework made learners more self-driven in the learning of problem-solving. They displayed leadership roles when interacting in their groups, and became assertive when presenting their arguments. In essence, these are important skills which are needed in real world, and must be nurtured in the classrooms. Learners realised that all mathematical content in the CAPS document could be practically demonstrated in the framework, thus encouraging them to make an extra effort. They were delighted to realise that they had discovered frustum, which is the mathematical concepts for higher grades and beyond high school level, which is, also treated at tertiary level. They discovered mathematical concepts from grade 10 and beyond on their own, with teachers playing a facilitation role, or providing scaffolding only when necessary. A great wealth of mathematical knowledge was displayed by parents as they demonstrated certain indigenous games.

6.4.2.1 Recommendations

The researcher recommends the use of the framework in the teaching of problem-solving as it promotes self-esteem and confidence. Navigational and aspirational skills encourage learners to overcome the difficulties encountered in the process of learning. Once they are engaged in the framework of using indigenous games, the learning of problem-solving stimulates their thinking skills (critical analysis), and they want to explore more and venture into the unknown with confidence to succeed. For instances, in algebraic concepts, they understood why variables may be independent or dependent variables, and conditions implicit or explicit.

6.4.3 Involvement of parents

The study found that before the framework of using indigenous games to teach problem-solving was used parents and teachers had not been working together in teaching problem-solving. Parents expected teachers to ‘make miracles’ to ensure that the demotivated learners succeed in problem-solving. In most cases, discouraged learners tended to be disruptive in school and classes, so the presence of parents restored order during the teaching of problem-solving. This confirmed findings of Roux et al. (2009:391), that excellent mathematics teachers do not leave aside the parents and communities in the teaching and learning of problem-solving (2.5.2.5).

They study found that teachers are good in mathematical content (theory), and even parents and communities have excellent resources, such as indigenous games in which are embedded mathematical content (4.3.5.1). The resources parents used are helpful in concretising the mathematical content, as they practically demonstrate. Learners and teachers make deductions from the series of observations, for example discussing and demonstrating the play of *kgati*:

> *…as MmaTumelo and MmaPakiso (parents) are swinging the rope, I realised that the loop is facing downwards at times is facing upwards. That is an interesting observation, where one can conclude in this way:*

- *As the loop faces upwards, it's like the player is put inside the loop, then the curve is full (that is it contains something), hence we realise that when $a > 0$, then parabola faces upwards.*
- *At the same time, as the loop faces downwards, it's like it is emptying the contents inside (that is, there is nothing in the loop), thus we conclude by saying that $a < 0$, then when drawing the parabola it has to face downwards…*

These observations constituted the key features of parabolic functions and learners were excited to conceptualise the mathematical concepts. Thus, the subject matter was easily understood by learners and made meaningful.

In conclusion, the study contributed to the body of knowledge in that although the majority of parents did not have the teachers' formal education on pedagogical content on problem-solving, they had equally great community cultural wealth capital, which brought creativity to the teaching of problem-solving. Also, the huge wealth of knowledge on indigenous games was crucial in contextualising and concretising the abstractness of subject matter. All participants, whether teachers, learners, education officials, parents or other community members and leaders, understood the necessity for working collectively in the teaching and learning of problem-solving. This confirmed the post-positivist argument that human beings are central in research processes and there are many ways of knowing the truth.

6.4.3.1 Recommendations

The study recommends that in the teaching and learning of problem-solving using indigenous games parents need to be included as active participants helping to generate the mathematical content hidden in them. The involvement of parents helps to bring creativity to the teaching and learning of problem-solving. The parents' wealth of knowledge can be tapped into when teaching and learning problem-solving. To ensure that all participants are in unison, code-switching can be used. Hornberger (2005: 5, 6) citing US Supreme Court 1954: Syllabus[(e)] Title VI of the Civil Rights Act of 1964 and the Equal Opportunity Act of 1974 and the Constitution of Republic South Africa Act 200 of 1996:14-15, state that all official languages can be used in any informal and formal settings, with equal status. The study found that giving English prominence over other languages was viewed as another form of oppression. Again, it was viewed as hiding the access of problem-solving from subaltern learners, as their mother-tongue or home language was not English. Thus, the study recommends that problem-solving be taught in the language of the participants' choice.

6.5 CONDUCIVE CONDITIONS FOR THE FRAMEWORK

In this section, the circumstantial factors which the study contends make it possible for the framework of teaching problem-solving to be successfully implemented will be discussed.

6.5.1 Conditions for meaningful subject matter

The study found that the use of the framework to teach and learn problem-solving, replicated the home environment (refer to 2.5.3.1 and 4.4.1). Learners interacted and networked freely, at the same time learning and observing mathematical processes and content infused in the games. Even the language used was important to demonstrate mathematical content easily:

> *.... look at the figure 4.8 and Pic. 4.1 as Dineo 's hand goes up, even the kgati loop goes up, and also let us look at Puleng's hand, the hand goes down and the loop goes down too. Hence, we concluded in our discussions that as the loop moves up, that indicates that the graph is increasing. The same way as the loop goes down that shows that the curve [graph] is decreasing*.

This extract demonstrates that learners learn freely in a relaxed atmosphere, free from intimidation by a powerful teacher who has to transmit the correct information into the minds of learners. Also, free from the narrow perspectives of the classroom teaching is not done by powerful human beings who know all the correct answers. The study found that the classroom conditions that prevailed were the ones that accommodated all the participants as equals, sharing information and leaving even the teaching environment open for critical contributions. The contextual conditions which allowed the close relationships among the teachers, learners, parents, and education officials provided an opportunity to display their rich capital wealth, which encapsulates the mathematical concepts (refer to 2.5.3.1 and 4.4.1).

As shown above, the home environment being replicated into the classroom, through the play of indigenous games allowed learners to be engaged in cooperative learning. It was found that the teaching of problem-solving needs to be empowering to learners, such that it happens in the learners' terrain. This is full of play, enjoyment, and encourages learners to take an active part in the teaching and learning of problem-solving (2.5.3.2 and 4.4.2). These circumstantial factors allowed learners to open up to 'how, when and why' type of communications, thus enabling them to formulate mathematical definitions and draw general conclusions. That was possible after they observed processes and illustrations of indigenous games.

The conclusion drawn here is that the framework naturally empowers learners to learn from their peers. Teachers and parents also displayed supportive roles or stimulated discussions in small groups than providing correct answers. Through participants' interactions a relationship of trust and a culture of inquisitive minds to learn more was instilled (refer to 2.5.3.4 and 4.4.4.).

6.5.1.1 Recommendations

The study recommends that contextual factors be based on promotion of democratic principles and values, which ultimately allow peace, freedom and empowerment to prevail in the classroom, such that learners realise that the framework emancipates them to understand that cultural activities are rich in mathematical content. Learners easily identify and learn mathematical content, such as order of operations, cubic, hash functions and different geometric configurations (refer to 2.5.3.2 and 4.4.2), without the teachers having to transmit mathematical knowledge and content in their mind. The framework provides learners with the flexibility to coin general formulae and mathematical definitions in their own ways, thus showing deeper understanding.

6.5.2 Motivation among teachers and learners

The study found that the conducive conditions that motivated learners to do well in problem-solving were, among others, the teaching methodologies used by the teachers. These approaches encouraged learners to discover mathematical concepts on their own, through interacting with their peers. Teachers and parents played a modelling role, with learners not being treated as objects to be fed with information. Rather, they were acknowledged for their critical feedback, and accepted as valuable in mathematical knowledge discipline. Constructive feedback was given by their teachers during feedback sessions to larger groups. At times, when learners went off-track (Burnett, 1999:50) or showed misunderstanding of some mathematical concepts, teachers provided supportive guidance and facilitation to help them feel at ease in accessing proper knowledge of problem-solving. Learners were given more time on tasks, which made them more dedicated and committed towards their schoolwork (refer to 2.5.3.3 and 4.4.3).

The cooperation shown by learners in class encouraged teachers to perform to their maximum in executing their duties in class. Learners showed respect and tolerance as they discussed the class activities in their groups (refer to 2.5.3.7 and 4.4.3). The motivation of teachers and parents is demonstrated by the extract:

> *…bana baka ke thabile jwang feela mokgwa oo le bontsitseng tsebo e batsi ka teng, malebana le thuto ya mmetse.. Ka morabaraba le bontsha hore le ka etsa disheipi tse ngata ha kalo. Bona tsela eo le behang disheipi tse ding ka teng le fumane hore, ho na le traegele, mme le kgona ho botsha ka kotloloho hore kamano ke e feng. Le bontsha hore tioreme ya Pythagoras e tholahala jwang.*
>
> *(My dear learners I am so delighted about the quality work you displayed in using morabaraba to arrange these shapes into different ways [refer to Fig.4.9 in section 4.3], such that these squares form a*

> *right-angled triangle. You also demonstrated how the theorem of Pythagoras can be formulated*) [refer to section 4.4.3].

Section 4.4.7, adding to the above extract, found that teachers regard themselves as lifelong learners, and although they knew these mathematical concepts they allowed learners to discover them. They thus provided constructive feedback on their inventions, without negative attitude such as ‘this is obvious’, ‘the answer is very simple’, or ‘why are struggling to get the correct answer?’ The learners did not regard teachers as ‘too hard to please’ (Burnett.1999:47).

The conclusion drawn here is that the relaxed atmosphere in the teaching and learning of problem-solving encouraged learners to be creative in discovering the abundance of mathematical content using indigenous games. It was not a case of established master inducting the new ‘apprentice’ into the ‘mysteries of the craft’. The learner as academic apprentice learned by observing how the master conducted his or her teaching of problem-solving (Burnet, 1999:46; Nkoane, 2013:393,394). It is imperative that teachers and parents in their respective classes know their learners so that those who are bullied at school or home and those who experienced major loss in their lives (Burnet, 1999:47), such as death, can be given emotional support to learn problem-solving effectively. The relationship between learners, teachers and parents (all research participants) should be one of mutual trust and respect.

6.5.2.1 Recommendations

The study recommends the use of the framework in the teaching and learning of problem-solving, as it gives both parties, teachers and learners, freedom to interact and share information on problem-solving with an equal status. The created environment in the teaching of problem-solving is interactive among learners themselves and learners and teachers. Both groups are motivated to execute their duties in the teaching and learning of problem-solving. The contextual factors cited above stimulated the acquisition of knowledge on problem-solving, and encouraged teachers to serve as role models, not as masters who have powerful knowledge.

6.5.3 Conducive conditions for the involvement of parents

The study found that in the teaching of problem-solving parents must not be put at the periphery of the framework, but rather at the centre of teaching and learning of problem-solving. This confirms the viewpoint of the Australian Ministry of Education and the Republic of Rwanda Ministry of Education, that parents be viewed as a reservoir of knowledge that can be tapped on to assist with creativity in the learning of problem-solving. Parents are knowledgeable on indigenous games, which contain mathematical content. Section 2.5.3.5 and 4.4.5 demonstrated that the wealth of knowledge displayed by parents was acknowledged in the teaching of problem-solving. This was illustrated by one of the grade 10 learners:

> *Wow, it is so exciting to be in a Mathematics class with parents, traditional leaders, and departmental officials from the Department of Education at district level. We enjoyed a lot together with them. There is a link between problem-solving skills and indigenous games that we played at home, even our parents showed us a lot on these indigenous games. These days we learnt a lot in problem-solving skills…'* (extract from 4.4.5)

Other participants, such as teachers, education officials and learners were mindful not to oppress the parents by using only one powerful language (English), but code-switching was used to include parents in the discussions. In addition, teachers were careful not to impose matters on parents, but showed advocacy and an advisory role in the pre-planning, planning and implementation phases of the framework (2.5.3.5 and 4.4.5). The study found that such circumstantial factors made parents happy to be part of the research and felt accepted as equal partners in the teaching of problem-solving. This was revealed by the comments of one of the parents:

> *…maemo a renang ka phaposing ya mmetse ke a kutlwano, tlhomphano le kananelo ho seo o se etsang.*

(The atmospheric conditions prevalent in problem-solving lessons, are of harmoniously working together, of respect and acknowledgement on what one is doing… (extract from 4.4.5)

The conclusion drawn is that problem-solving is a humanistic endeavour, in which human participation and marginalised knowledge are acknowledged in the teaching of problem-solving, irrespective of the level of education one has reached. This argument is guided by Yosso's community cultural wealth theory, that posits a huge wealth of knowledge is ignored or not tapped into in teaching. Such excluded knowledge is rich in explaining mathematical content, and instils creativity for learners to access problem-solving with ease. In countries such as Australia and the Republic of Rwanda it was observed that marginalised knowledge is possessed by parents. In addition, Lynn (2004:156) and Yosso (2002:96) posited that we need to demystify the myth that the knowledge of indigenous games possessed by parents of colour exemplified lack of intelligence and laziness. These arguments were misleading and further hid the evidence that subaltern communities know problem-solving skills from birth. They also ignored parents of colour, labelling them as uninterested in the education of their children (Lynn, 2004:156; Yosso, 2002:96).

6.5.3.1 Recommendations

The study recommends that in developing the framework for using indigenous games to teach problem-solving, the inclusion of parents is crucial. The marginalised knowledge they possess is important in concretising the abstractness of mathematical content. Participants should use the language(s) with which they feel comfortable, not a hegemonic language that intentionally suppresses emotions and feelings. The Constitution supersedes oppressive laws or rules. Examinations and regulations should be modified to allow learners to show their creativity, through expressing their knowledge in any official language with which they feel comfortable.

6.6 THREATS TO THE FRAMEWORK

In this section, risk factors that have threatened the implementation of the framework are discussed, with ways in which the risk factors may be countered. Akuno (2013:66), Mukhopadhyay (2013:94) and Lynn (2004:158,165) warned that risk factors include wrong perceptions about indigenous games. Cultural practices were discouraged by the colonial education system, the perception being that those engaged in them were lazy and unintelligent.

There is a ripple effect from using the framework, as it spreads out to include other participants in the games and thus their understanding of the subject matter. Teaching using indigenous games is culturally responsive, as it draws culture as a basis for fostering the problem-solving skills (PSS) and achievement of marginalised learners. Teaching of PSS using indigenous games emancipates teaching practices as the teachers is able to teach critically from the learners' perspectives (Akuno, 2013:67; Lynn, 2006:117,118; Morris et al., 2011:29,30).

6.6.1 Subject matter in the teaching and learning of problem-solving

The study found that as learners were playing the indigenous games there were high levels of noise in nearby classes, such that learners could not concentrate (2.5.4.1). The team members decided to execute stages one and two in the playgrounds, that is the actual playing of the indigenous and the reflection on the lessons learnt. Part of stages two, and three to four were completed in class (refer to Table 4.4). It was found that the use of the framework stimulated the interest of learners in problem-solving. As they interacted they had fun and demonstrated creativity in extracting the mathematical content embedded within the games.

There were Insufficient materials for participants, especially some learners and newly appointed teachers, to cross-check or verify the correctness of the extracted mathematical concepts from indigenous games while practising either at home or after school hours. The materials developed by the teachers were authentic and reliable in terms of the appropriateness of the mathematical content. The designed materials during the preparation and feedback sessions were collected and distributed to learners and newly appointed teachers to familiarise themselves with the framework.

The study revealed that discussions on problem-solving were dominated by teachers and subject advisors. Learners tended to rely on one leader to give feedback to the larger groups. During the reflection meetings it was decided that learners would rotate roles as they interacted in their groups. It was suggested that when the larger groups posed the questions other members would help the reporter to answer them. That was viewed as detrimental to the framework because it sent the wrong interpretations that problem-solving was the preserve of powerful individuals. The same applied to teachers, who were advised not to dominate discussions but rather allow learners to take charge of problem-solving.

The conclusion drawn here is that all discussions and deliberations be recorded and stored for future use, and the designed actives be filed for the novice teachers to use them in implementing the framework in their respective classes. The participants of the study encouraged constant interactions among members, hence the small groups were effective in discovering mathematical concepts.

6.6.1.1 Recommendations

It was observed that risk factors persist in the framework, but to make it workable it is important that participants be engaged in looking at ways of overcoming the threats. They should allow the participants to reflect on various ways to help reduce them. Thus, the study recommends that critical reflections be instituted at every stage of implementation. Participants should be allowed to interrogate the framework and give critical inputs, such that when re-planning is done it can strengthen the implementation of the framework.

6.6.2 Motivation among teachers and learners

The study found that teachers felt that they were over-stretched in terms of workload. There was too much planning and administrative work in subjects they were teaching other than Mathematics. This made them lose focus and concentration on the work schedule. They delayed giving learners feedback on written projects and assignments (2.5.4.7 and 4.5.7):

> *Now you are causing us problems and stressful work. Bear in mind that we are teaching many classes, that is Mathematics and Life Sciences. How do you expect me to cope with HoDs' and LFs' work? I will have to administer CASS components as per policy, compile lesson plans and I also had to ensure that the work schedule is updated'* (extract in section 4.5.7).

It was then agreed that all facilitators in the teaching of problem-solving would work collaboratively in all areas, such as planning and administrative work. As a result, they would not have to perform individual planning, which would have consumed more of their time. The subject advisors of the respective subjects, that is, Mathematics and Life Orientation, were in agreement with this (4.5.7).

The conclusion drawn is that collaborative work should also be demonstrated in planning and administrative work. Instead of marking project books it was agreed that teachers and subject advisors would share the marking and administrative of CASS marks. It was found that much on the work schedule had been covered through the use of the framework, making made it easier for learners to prepare for common examinations administered in June and September.

6.6.2.1 Recommendations

The study recommends the use of the framework to teach problem-solving, as most of the work is carried out by learners. Teachers need to provide them with monitoring and support on the work they have finished or are busy with. Subject experts from the district offices were of great help to both teachers and learners, in terms of giving feedback to learners on time, and relieving teachers of some CASS work.

6.6.3 Non-involvement of parents

The use of English excluded parents from being active participants, thus some were not attending on regular basis (2.5.4.5 and 4.5.5). The study found that to resolve that problem code-switching could be employed to accommodate all participants, and they were free to express themselves in any of the official languages. To encourage attendance, activities were performed in the afternoons or over the weekends to allow working parents and those engaged with other activities during the weekend to be available.

The conclusion drawn is that at times parents felt excluded when learners were put at the centre, and wanted to give as many answers as possible. It was then agreed that contributions by parents and teachers would welcomed, but only at a later stage after learners had contributed. In the closing remarks or in summarising the lesson for the day, such parents' and teachers' contributions could be added. If given at the discussions sessions they should stimulate learners' thinking rather than transferring it to the learners' minds.

6.6.3.1 Recommendations

The study recommends that parents be brought in on some of the legislative imperatives. The New Curriculum expects learners to discover subject matter on their own, through being engaged on learner-centred activities. Teaching of problem-solving is not about giving learners correct answers but discovering fascinating mathematical ideas or concepts. Teachers and parents should help learners to clear misconceptions which might have existed in the process through scaffolding.

6.7 SUCCESSES

The successes indicated that the framework produced desired results, confirming that it was indeed useful to teach and learn problem-solving effectively.

6.7.1 Subject matter in the teaching and learning of problem-solving

The study found that learners were able to relate the subject matter with indigenous games on their own, that is, they managed to extract mathematical content from the indigenous games and also designed activities which further demonstrated the content embedded in them (2.5.5.1 and 4.6.1). This is evident from the words of one of the learners:

> *what we noted, looking at the function we deduced, that is* $f(m) = -\frac{m}{2} + 10$, *where* m *is number throw the ghoen up, thus* m *is referred to as the 'Independent variable'. On the other hand,* $f(m)$ *the number of stones placed into the hole, as follows* $f(m)$ *is described as 'dependent variable.' I will request my group members to make additions..'* (see 4.3.4.2).

The study also showed that the majority of learners performed well in the assessment tasks during the course of the year, June examinations and trial examinations (refer to grade 10 mark sheets: Table 4.18). In most cases, they showed high levels of understanding of mathematical concepts, as there was much interaction among learners and teachers in proposing possible ways of arriving at the conclusion.

It was found that the framework made it possible for the method used by the teacher to allow learners to interact and learn from others. Learners were given leeway to control class discussions. Teachers or subject specialists did not take a major lead in steering the discussions, but they supported the initiatives brought forth by the learners. Learners discovered various mathematical formulae with understanding, rather than memorising these general conclusions without understanding. One of the learners said:

> *I suggest that the question be phrased as follows: describe the practical (real life) scenarios defined by the models and critically analyse them*.

This is an indication that there was a high level of engagement on the subject matter. Apart from playing indigenous games, learners realised that there are real-life scenarios that can quite well demonstrate mathematical content. The finding here is that mathematical model or patterns are the results of observable and non-observable cases happening in daily life (2.5.5.4). This is illustrated by Pramling-Samuelsson’s (2008:637) recommendation that teachers also contribute to a challenging and rich environment. This includes using their own knowledge to create situations, tasks, and play milieus. The study found that learners too are in the position to create challenging and rich tasks that can be used in learning problem-solving. They are empowered to design activities which are of quality in teaching and learning problem-solving.

The conclusion drawn here is the use of the framework to teach problem-solving has instilled independent critical thinking in learners. The questions or class activities designed are of a very high order thinking. The subject advisor indicated that the material developed would be used to train other teachers to simplify the mathematical content for learners.

6.7.1.1 Recommendations

The study recommends that teachers teach from learners’ perspectives, that is, using the activities crafted by learners, rather than imposing activities which might scuttle critical thinking. That would show a radical shift from teacher-centred approach to learner-centred approach, which good practice emphasised. This will advance the agenda of sustainable learning of problem-solving.

6.7.2 Motivation among teachers and learners

The study found that learners were encouraged and confident to see their capability in producing high quality of work of problem-solving. The activities which were designed by learners were used in the teaching and learning of problem-solving. They were also motivated by the learning environment, which was welcoming and accommodative to their comments (2.5.5.3; 4.6.3). This is evident in the comments made by teachers, for example:

> *I am overwhelmed; what an activity to do! Thank you very much for the contributions you made. Yes, I fully agree with you that lets give it a try. I*

spoke to my colleagues; we are in agreement that go attempt this as your homework today.

This also showed that teachers were happy and encouraged by the high level of work by learners. The school visit reports by the subject advisors showed that they too were overwhelmed by the manner in which teachers, parents and learners were being involved in the teaching of problem-solving (more details 4.6.7).

The conclusion drawn here is that use of the framework to teach problem-solving created inspiring conditions that motivated learners to realise their potential in problem-solving (4.6.4). The work by learners and teachers has been acknowledged and validated by the education officials, as these developed materials are used in content training sessions.

6.7.2.1 Recommendations

The study recommends that the learning materials be written from learners' perspectives, using the materials to enhance the learning of problem-solving. With the understanding that the misconceptions which might have arisen would be included and how they were dispelled or clarified. That would serve as motivation to both teachers and learners to excel in mastering problem-solving. Teachers become motivated to work with parents and district officials in the teaching of problem-solving. It becomes easy for learners to be disciplined and inspired to impress the parents and district officials of their performance. As the district officials are working closer with teachers and learners they tend to understand the concerns and address them accordingly, unlike speculating on the solutions for the challenges that teachers and learners experienced. Hence, the study recommends that the classroom interactions of teachers, parents and subject advisors be constantly encouraged. Making school visits after weeks or months is a futile exercise, which does not bear useful results.

6.7.3 Involvement of parents

The study found that parents had extensive knowledge of indigenous games, which is useful to help learners to visualise mathematical concepts and for teachers to engage learners in meaningful activities on problem-solving. It was evident that the parents' involvement helped teachers greatly in understanding the behaviour of certain learners, and helping such struggling learners to succeed in problem-solving (refer to 2.5.5.5 and 4.6.5). As one parent commented:

> My daughter at another school in grade twelve, she is performing badly in problem-solving. I suggested to the mathematics teachers that they must consult with you, so that you showed them how to use indigenous games to teach problem-solving skills (more details 4.6.5).

In addition, the study found the partnership between teachers and parents to be of great help, as learners were given extra support on problem-solving. Parents developed great appreciation of the performance of their children in problem-solving.

The conclusion drawn here is that parents have a key role to play in the teaching of problem-solving, rather than simply being used selectively in certain school activities. Parents were able to assist neighbouring schools to enhance learners' performance in problem-solving by using an indigenous games model.

6.7.3.1 Recommendations

The study recommends that parents be seen as good partners in the teaching of problem-solving. The excluded knowledge that parents have in the playing of indigenous games is to be used to contextualise the teaching and learning of problem-solving. The language of interaction and learning can be the mother-tongue language, which allows parents to feel confident in sharing the knowledge they possess or preferably code-switching can be used to accommodate all research participants. Also, the study recommends that the marginalised knowledge of parents be used as it stimulates the critical thinking of learners. This concurs with Pramling-Samuelsson's view (2008:672) that learners create knowledge when they play and this gives them opportunities to explore many mathematical concepts.

6.8 LIMITATIONS OF THE STUDY

A limitation of the study is that it was conducted using one secondary school in the rural area of Thabo Mofutsanyana Education District in Free State Province. The study did not look at a reasonable sample that could be used to generalise the results, rather it was conducted to seek and provide in-depth data on teaching problem-solving using indigenous games. This agrees with Ryan's (2006:21) observation that qualitative studies generally deal with smaller numbers than quantitative research. It can be noted that the study of using indigenous games to teach problem-solving sought to interpret historically and culturally significant phenomena that would help learners to understand problem-solving more easily. In the same way it enabled teachers to teach problem-solving more effectively. The study was intended to help illuminate people's everyday lives.

Although learners and research participants mentioned the following indigenous games: *morabaraba* (board game), *diketo* (co-ordination game), *kgati* (rope-jumping game), *ncuva* (board game), *melamu* (stick-fighting game), *malepa* (string game), *ditshomo* (story-telling game), *dibeke* (*umabhorisha*), *Kho-kho*, and *jukskei* (target game) to be played, the study focussed only on *morabaraba, diketo, kgati, melamu,* and *malepa*, as there was great interest in these games. Apparently, these were the most preferred by the research participants. Future researchers can also look at the ones which were not played by the participants, for further identification of mathematical content.

The study was conducted in a school in which the language of learning and teaching (LOLT) was English, but as a way to accommodate all participants a compromise was reached in which participants agreed to use mother-tongue language or code-switching with fuller participation by all. As the study relied on people's words (Ryan, 2006:21) to generate primary data and valued participants' perspectives on their world, the use of mother tongue and code-switching helped it to reach its objectives. The use of mother-tongue was appropriate when the questions were posed in English by the teachers or other researcher participants. The conditions created were such that the subaltern parents were able to debate extensively with the other participants, such as district officials who seemed to have powerful authority in curricular matters.

6.9 CONCLUSION

The study was conducted to formulate a framework for using indigenous games to teach problem-solving in a grade 10 Mathematics class. The framework naturally stimulated learners to be active participants in learning problem-solving. Of significance is that the mathematical content and mathematical concepts extracted from these indigenous games did not focus on grade 10 mathematical content only, but cut across all grades, that is, high school problem-solving up to tertiary education. It was noted that this framework makes the learning of problem-solving sustainable over various phase of schooling. The research participants, especially parents, confirmed that learning and teaching of problem-solving does not end in the classrooms, as even in home environment they helped learners to enjoy the beauty of problem-solving. The marginalised knowledge, which is very rich in mathematical content, helped parents to play a pivotal role in the teaching of problem-solving.

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APPENDIX A1: Letter from the Supervisor to Head of School

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE
YUNIVESITHI YA FREISTATA

August 23, 2011

UFS·UV
EDUCATION
OPVOEDKUNDE

September 27, 2011

Dr MM Nkoane
Head: School of Education Studies
University of the Free State
BLOEMFONTEIN – 9301

Dear Sir

Registration of Mr Tshele Moloi into the PhD Programme (Study Code 7910 and Module Code DKT 900)

Mr Tshele Moloi's application complies with the requirements for admission into the PhD (Study Code 7910 and Module Code DKT 900) qualification. I thus request that his registration be processed accordingly.

Yours faithfully

MG MAHLOMAHOLO
(Professor and promotor)

Faculty of Education – Name of school / Fakulteit Opvoedkunde Naam van Skool
T: +27(0)51 401 0000
F: +27(0)00 000 0000
E: email@ufs.ac.za

205 Nelson Mandela Drive/Rylaan
Park West / Parkwes
Bloemfontein 9301
South Africa / Suid-Afrika

PO Box / Posbus 339 (25)
Bloemfontein 9300
South Africa / Suid-Afrika
www.ufs.ac.za

APPENDIX A2:Ethical Clearance Application

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA

UFS
UV

Faculty of Education
Ethics Office

Room 12
Winkie Direko Building
Faculty of Education
University of the Free State
P.O. Box 339
Bloemfontein 9300
South Africa

T: +27(0)51 401 9922
F: +27(0)51 401 2010

www.ufs.ac.za
BarclayA@ufs.ac.za

17 November 2011

<u>ETHICAL CLEARANCE APPLICATION:</u>

ENHANCING PROBLEM SOLVING SKILLS IN A GRADE 10 MATHEMATICS CLASSROOM USING INDIGENOUS GAMES.

Dear Mr TJ Moloi

With reference to your application for ethical clearance with the Faculty of Education, I am pleased to inform you on behalf of the Ethics Board of the faculty that you have been granted ethical clearance for your research.

Your ethical clearance number, to be used in all correspondence, is:

UFS-EDU-2011-0059

This ethical clearance number is valid for research conducted for three years from issuance (expiring end of 2014). Should you require more time to complete this research, please apply for an extension in writing.

We request that any changes that may take place during the course of your research project be submitted in writing to the ethics office to ensure we are kept up to date with your progress and any ethical implications that may arise.

Thank you for submitting this proposal for ethical clearance and we wish you every success with your research.

Yours sincerely,

Andrew Barclay
Faculty Ethics Officer

APPENDIX B1 : Permission letter from Free Department of Education

Department of Education
FREE STATE PROVINCE

Enquiries: LV Alexander
Reference: 16/4/1/41 - 2012

Tel: 051 404 9283
Fax: 086 6678 678
E-mail: research@edu.fs.gov.za

2012 – 11 - 06

Mr T. J. Moloi
1028 Riverside
PHUTHADITJHABA
9866

Dear Mr Moloi

REGISTRATION OF RESEARCH PROJECT

1. This letter is in reply to your application for the registration of your research project.

2. Research topic: **ENHANCING PROBLEM SOLVING SKILLS IN A GRADE 10 MATHEMATICS CLASSROOM USING INDIGENOUS GAMES.**

3 Your research project has been registered with the Free State Education Department.

4. Approval is granted under the following conditions:-

4.1 The name of participants involved remains confidential.

4.2 The questionnaires are completed and the **interviews are conducted outside normal tuition time**.

4.3 This letter is shown to all participating persons.

4.4 A bound copy of the report and a summary on a computer disc on this study is donated to the Free State Department of Education.

4.5 Findings and recommendations are presented to relevant officials in the Department.

5. The costs relating to all the conditions mentioned above are your own responsibility.

6. **You are requested to confirm acceptance of the above conditions in writing to:**

DIRECTOR: STRATEGIC PLANNING, POLICY AND RESEARCH,
Old CNA Building, Maitland Street OR Private Bag X20565, BLOEMFONTEIN, 9301

We wish you every success with your research.

Yours sincerely

MJ MOTHEBE
DIRECTOR: STRATEGIC PLANNING, POLICY AND RESEARCH

3irectorate: Strategic Planning, Policy & Research - Private Bag X20565, Bloemfontein, 9300 – Room 301, Old CNA building, Maitland Street, Bloemfontein 9300 - **Tel: 051 404 9283**/ Fax: **086 6678 678** **E-mail: research@edu.fs.gov.za**

www.fsdoe.fs.gov.za

education

Department of Education
FREE STATE PROVINCE

Tel: 051 404 9275
Fax: 051 404 9274

Enquiries: LV Alexander
Reference no. : 16/4/1/41 - 2012

2012 - 11 - 06

Mrs B. S. Tshabalala
Director: Thabo Mofutsanyana Education District
Private Bag X817
WITSIESHOEK
9870

Dear Mrs Tshabalala

NOTIFICATION OF A RESEARCH PROJECT IN YOUR DISTRICT

Please find attached copy of the letter giving **Mr Moloi** permission to conduct research in Thabo Mofutsanyana Education District.

Mr Moloi is a lecturer at the University of Free State – Qwa Qwa Campus and is studying for Doctor of Philosophy Degree in Curriculum Studies with the University of Free State.

Yours sincerely

MJ MOTHEBE
DIRECTOR: STRATEGIC PLANNING, POLICY AND RESEARCH

Directorate: Strategic Planning, Policy & Research; Old CNA Building, Maitland Street, Private Bag X20565, Bloemfontein, 9300 - **Tel: 051 404 9287** / 9275; **Fax: 051 404 9274 - E-mail: research@edu.fs.gov.za**

www.fs.gov.za

APPENDIX B2: letter from Free State Department of Education

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
QWAQWA CAMPUS/-KAMPUS

UFS
UV

Attention: Mr MJ Mothebe

The Director: Strategic Planning, Policy and Research

Private BagX 20565

Bloemfontein

9301

RE: ACCEPTING THE CONDITIONSIN APPROVAL LETTER

I want to thank the Free State Department of Education: Strategic Planning, Policy & Research Unit for allowing me to conduct the research project in Thabo Mofutsanyana education District schools. I accept the all conditions stated in the approval letter that I will abide by them.

Thank you. Hope you find this in order.

Yours Sincerely

Mr Moloi Tshele J

Ph.D student at Free State University Qwaqwa Campus

University of the Free State, Qwa Qwa Campus, Kestell Road, Phuthaditjaba
T: +27(0)58 718 5002 email: moloijt@qwa.ufs.ac.za Cell: 082 202 5870

APPENDIX C1: Letter of Accepting the terms and conditions of FSDoE

APPENDIX D1:

CONSENT BY EDUCATOR

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. Since your school is part of this study, you are requested to take part in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. You will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
082 202 5870 or 058 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you

____

TJ Moloi

Name ____

Signature ____

Date ____

Contact details ____

APPENDIX D2:

CONSENT BY PRINCIPAL

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. Since your school is part of this study, you are requested to take part in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. You will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
082 202 5870 or 058 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you

____

TJ Moloi

Name ____

Signature ____

Date ____

Contact details ____

APPENDIX D3:

<u>CONSENT BY SUBJECT ADVISOR : MATHEMATICS</u>

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. Since your school is part of this study, you are requested to take part in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. You will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
082 202 5870 or 058 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you

____

TJ Moloi

Name ____

Signature ____

Date ____

Contact details ____

APPENDIX D4:

CONSENT BY TRADITIONAL LEADERS

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. Since your school is part of this study, you are requested to take part in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. You will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
082 202 5870 or 058 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you

____

TJ Moloi

Name ____

Signature ____

Date ____

Contact details ____

APPENDIX D5:

CONSENT BY OFFICIAL: SPORTS SECTION

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. Since your school is part of this study, you are requested to take part in this research in order to give it credibility. Participation is not compulsory and if you decide not to participate, that will not be held against you. You will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
082 202 5870 or 058 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you

____

TJ Moloi

Name ____

Signature ____

Date ____

Contact details ____

APPENDIX D6

CONSENT FOR PARENT/LEGAL GUARDIAN

I am currently doing research with UFS on enhancing the teaching of problem solving at grade 10 mathematics class using indigenous games. As your child is a minor, you as the parent/legal guardian are kindly requested to provide permission for your child to be part of this research project. Your child will be interviewed and confidentiality, anonymity and legality issues about this project will be discussed with you, the teachers and principal as it is imperative that you fully understand the nature and purpose of this study. You are free to withdraw your child from this study at any stage.

This project will comply with the rules and regulations of conducting a research.

If you would like any additional information, you are welcome to contact me at
O82 202 5870 or 058- 718 5345
Email- moloijt@qwa.ufs.ac.za

If you would like to participate in this research, sign below by giving consent.

Thank you.

Name of learner

____

Grade

____

Signatureofparent/guardian

____

Date

____

APPENDIX D6

Foromo ya tumello ho ya ho motswadi/mohlokomedi

Ke etsa diphuputso le Univesithi ya Foreistata mabapi le ho rutwa ha thuto ya dipalo ka tshebediso ya papadi ya setsho. Ka ha ngwana wa hao a sa le monyane ka dilemo, o kopuwa ho mo dumella hore e be karolo ya projeke ena ya diphuputso. Ngwanahao o tla botswa dipotso bonngweng ba hao, ntle le he hore lebitso la hae le qohollwe ebile maemo a tsa se molao mabapi le projeke ena a tla buua le wena, mesuwe/mesuwetsana le mosuwehlooho. Ho bohlokwa ke hona hore o utlwisise maemo le sepheo sa diphuputso tsena. O na le bolokolohi ba ho hula ngwana wa hao neng kapa neng diphuputsong tsena.

Projeke ena e tla ikamahanya le melao yohle ya ho etsa diphuputso.

Ebang o batla dintlha tse ding ho feta mona, o ikopanya le nna
082 202 5870 or 058 – 718 5345
Email- moloijt@qwa.ufs.ac.za

Ebang o dumella ngwanahao hore a be seabo projeke ena, o ka tlatsa mona.

Ke a leboha.

Lebitso la ngwana	:...
Kereiti	:...
Boitshaino ba motswadi/mohlokomedi	:...
Letsatsi	:...

APPENDIX D7

INVITATION TO PARENTS TO TAKE PART IN RESEARCH

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

FACULTY OF EDUCATION (QWAQWA CAMPUS)

SCHOOL OF MNST

Private Bag X13 PHUTHADITJHABA 9866 E-mail:moloijt@qwa.ufs.ac.za

+27(0)58 718 5002 +27(0)58 718 5002 Cell: 082 202 5870

12 June 2012

Attention: Me Tsotetsi and Group Members
Tsheseng
Witsieshoek
9870

KOPO YA HO SEBETSA MMOHO DIPHUPUTSONG

Nna, Tshele Moloi, ke etsa diphuputso ka tshebediso ya dipapadi tsa setso ho ruta thuto ya dipalo(Using of the indigenous games in the teaching of Mathematics). Kopo yaka e ka mokgwa o latelang:

- Me Tsotetsi le moifo wa hae ebe karolo ya diphupusto tsena . Ho ba karolo ya diphupusto tsena ebe ka boithaopo. Le motho ya keneng mona diphupustong tsena ha batla ho itokolla ha ho phoso.
- Ho ditho tseo e tla ba karolo ya diphuputso tsena , ka nako tse tla bolelwa batla memelwa sekolong sa Manthatisi ho fana ka tsebo eo ba nang le yona ka dipapadi tsa setso.

Diphuputso tsena di batla ho fihlela tsena tse lateng:

- Bohlokoa bo teng ba hore bana ba rutwe ka dipalo ka tshebediso ya dipapadi tsa setso
- Ke dikarolwana difeng tse lokelwang ho elelwa hloko kapa ho latelwa ha ho sebedisa dipapadi tsa setso ho ruta Mathematics(Dipalo)
- Ke ditshitiso difeng tse teng tse lokelwaneng ho elwa hloko
- Ho beha tekong moralo wa ho ruta dipalo ka tshebediso ya papadi tsa setso.

Hape, ke ho bona diphupusto tsena di kgona ho bontsha nyallano thutong ya dipalo le setso sa rona. Re le setjhaba ka karetso re batla re shebela setso sa rona fatshe kapa re nyatsa setso sa rona. Sena se tla tlisa hore re le setjhaba re rate setso sa rona, hoba setso sena sa rona se ruile haholo ka dithuto tsena tse rutwang dikolong, haholo-holo thutong ena ya dipalo. Sena se tla etsa hore bana ba sebetse hantle dithutong ena ya dipalo. Mosebetsing yohle e teng mafapheng a mmuso kapa poraevete a batla bana ba nang le boitsebelo bo hodimo thutong ena ya dipalo.

Ke a leboha.

Ka Boikokobetso

Monghadi : Tshele Moloi

EDUCATION OPVOEDKUNDE UFS UV

APPENDIX D8

INVITATION TO TRADITIONAL COUNCIL TO TAKE PART IN RESEARCH

UNIVERSITY OF THE FREE STATE
UNIVERSITEIT VAN DIE VRYSTAAT
YUNIVESITHI YA FREISTATA
UFS UV

FACULTY OF EDUCATION (QWAQWA CAMPUS)

SCHOOL OF MNST

Private Bag X13 PHUTHADITJHABA 9866 E-mail:moloijt@qwa.ufs.ac.za

+27(0)58 718 5002 +27(0)58 718 5002 Cell: 082 202 5870

23 May 2012

Attention: Batlokoa Tribal Authority
Tsheseng
Witsieshoek
9870

KOPO HO LEKGOTLA LA MORABE

Nna, Tshele Moloi, ke etse diphuputso ka tshebediso ya dipapadi tsa setso ho ruta thuto ya dipalo(Using of the indigenous games in the teaching of Mathematics). Kopo yaka e ka mokgwa o latelang:

- Ke kopa hore ke fumane tumello ya hore ke etse puisana le ba bang ba ka pusong ya borena, hore ba mphe nalane ya dipapadi tsena tsa setso. Sepheo sa dipapadi tsena tse setso.
- Le hore ba ka pusong ya borena kapa setjhaba ho ba tla lakatsa ho ba karolo ya diphupusto tsena bo ka thabelwa. Ho ba karolo ya diphupusto tsena ebe ka boithaopo. Le motho ya keneng mona diphupustong tsena ha batla ho itokolla ha ho phoso.
- Ho ditho tseo e tla ba karolo ya diphuputso tsena , ka nako tse ding batla memelwa sekolong sa Manthatisi ho fana ka tsebo eo ba nang le yona ka dipapadi tsa setso.

Taba kgolo ya diphupusto tsena ke ho nyalana thuto ya dipalo le setso sa rona. Re setjhaba ka karetso re batla re shebela setso sa rona fatshe kapa re nyatsa setso sa rona. Sena se tla tlisa hore re le setjhaba re rate setso sa rona, hoba setso sena sa rona se ruile haholo ka dithuto tsena tse rutwang dikolong, haholo-holo thutong ena ya dipalo. Sena se tla etsa hore bana ba sebetse hantle dithutong ena ya dipalo. Mosebetsing yohle e teng mafapheng a mmuso kapa poraevete a batla bana ba nang le boitsebelo bo hodimo thutong ena ya dipalo.

Ke a leboha.

Ka Boikokobetso

Monghadi : Tshele Moloi

EDUCATION
OPVOEDKUNDE

APPENDIX D9

SAMPLE OF ATTENDANCE REGISTERS OF RESEARCH PARTICIPANTS

MEETING WITH THE FOCUS GROUP FOR THE RESEARCH

ATTENDANCE REGISTER

Venue: Makhathisi Date: 30 JULY 2012

Title	Surname & Initials	Contact number	Email address	signature
MR	Moloi T-J	0822025876	moloijt@ [illegible].ufs.ac.za	[signature]
MISS	Gatebe N.S			Gatebe N.G
Mr	Mit Mlogwa	0729729255		Mit Mlogwa
MR	MOEKETSI LJ	0727731064		[signature] 30/07/12
MR	Moloi M.A	0782398708		[signature]
MS	Khambule J	0125751761		[signature]

APPENDIX D9

SAMPLE OF ATTENDANCE REGISTERS OF RESEARCH PARTICIPANTS

FOCUS GROUP MEETING

ATTENDANCE REGISTER DATE: 16/08/2012

SURNAME	NAMES	SIGNATURE
RADEBE	PHILLEMON	M.P [illegible]
SOTHOANE	MALESHOANE	S.M. Sotloane
Manamang	Tsiu	[illegible]
Mofokeng	Motsamai	[illegible]
Moloi	Palesa	Smola
TSOTETSI	MOTONGWENYANA	[illegible]
Mahlaba	Maditaba	MAC Mahlaba
Motloung	MASENTLE	M.P. Motlo[illegible]
TSOTETSI	CONNY	[illegible]
MALAKOANE	MODIEH ROSELINE	M.R Malakoane
SEPOTO LEHLOHONOLO	LEHLOHONOLO	L.C [illegible]
Lakaje K	Kutlo B	K.B. [illegible]
Mokoena	Mamomi	m-T [illegible]
Sithole	Mpho	Sithole
Lebekhulu Betty	Bongani	Lebekhulu [illegible]
Ramosobela	Keneuwe	Ramosobela
Ramosobela	Lineo	Ramosebetso
Xaba	Nosizwe	XABA N.D
Nhlapo	Phozile	Nhlapo
Batebe	Nomasondo	Batebe
Motaung	Lehlohonolo	Motau[illegible]
[illegible]	[illegible]	[illegible]
Mosiuoa	Diphapang	[illegible] Mosiuoa
Molefe	Palesa Edith	EMolefe.
Motaung	Lerato Betty	[illegible] B Motaung
Moloi	Dinkeng	Moloi
Mbongo	Mmuso	Mbongo
KHABALALA	Mokgalaware	[illegible]
Selepe	Dumisani	[illegible]
Molefe Lefu	Lefu	[illegible]

APPENDIX E1: INFORMATION SESSIONS

INFORMATION SESSION 23 June 2012

Participant 1 : ho batswadi bohle ba le teng ka kopang ena, ke rata ho le amohela. Ke bontsha thabo yaka ka hore le be teng, re tsebe ho qoqa ka taba tse re amang. Re a tseba bohle hore ban aba lokela ho etsa thuto ena ya dipalo, hoba ke yona e hlokahalang dithutong tse hodimo.

Participant2: motsamaisi wa morero ona ke rata ho leboha haholo ka tsela eo o sebeditseng taba tsa hao, ka hore kopano ya mofuta ona e be teng. Re ikutlwa re tlela ke matla ha re elellwa hore bokamoso ba bana ba rona bo hodima rona, haholo jwang ho nka karolo thutong ya mmetse.

Participant1: re a leboha ka mantswe ao a kananelo.

Participant3: jwale rona le batswadi re tlo etsa jwang hore re ikakgela ka setotswana ho ruteng thuto ena ya mmetse, hoba le sekolong ha re ya fihla.

Participant 3: Ka tsebo eo o nang le yona ho tloha hae, tsebo ena ya dipapadi tsa setso, ke yona e tlo re thusa hore ban aba utlwisise thuto en aka bobatsi.

Participants 4: jwale ho hlakile hore mokgwa ona kapa tsela ena bana ba rutwang dipalo e lokelwa ho ntlafatswa. Tjhe, re sa re bana ha ba rutwe ka tsela e lokelang.

Participant 1: o nepile weso, ha re mona ho qhaqha tse entsweng ke batho ba bang, ka tsela yah ore ha ba tsebe seo ba se etsang. Feela nnete ke hore ha re shebang tseo eo re ka fetolang dintho molemong wa roneng koala.

Next information session 2 July 2012

Participant 1: dumela koala bana, mmoho le batswadi ba rona kajeno. Aktivithi eo ke faneng ka yona ha re etsang metsotso e mashome a mararo re ntano fana ka tlaleho ho yona.(Good morning my dear learners, as well as parents and colleagues who are in today.lets us see can't we finish this activity within the next thirty minutes, then we can start reporting)

Participant 1: re a leboha ka morolo wa lona ho etsa aktivithi eo, jwale ha re faneng ka sebaka ho grupu wane, hore e fan aka tlaleho ya bona, den, kamorao le ka botsa dipotso kapa le phehisa dipuisanong(thank you for your activeness displayed by various groups. Let us give a hearing to group one, so that they give us their feedback, then after presentations, they can take questions and comments from the floor.

Group 1*:* *rona ha re shebile straturale neitjha sa morabaraba,re bona r ektengele e nyane, ho latele e kgolwanyane, e kgolo,(lebella Fig. 4.2 ,Pic. 4.2 and Fig.4.3) jwalo-jwalo. rektengele tseo di entswe ka dilaene. Ha papadi e bapalwa re bona tsena, re beha dikgomo tsa rona ka ho fapanyetsana, o lokela ho nahana ka kelo hloko pele o beha kgomo ya hao, hore o tsebe o hlola enwa wa direng. O menahana mokgwa wa ho hlola papadi ena*(as we view the structural nature morabaraba (board game), we see rectangle of various sizes, the big one, the bigger one, and the biggest one (see *Fig. 4.2 ,Pic. 4.2 and Fig.4.3)* These rectangles are made out of lines .On the actual playing of the game, these are apparent; we play by giving chance to each opponent to place his/her token cow on the board, you have to think strategically before you place the token cow on the board, so as to maximise chances of winning the game, and also anticipating the movement that the opponent might take)

NEXT LESSON PRESENTATION 3 JULY2012

Participant 1; Thank you, let get feeback from group B(ke a leboha ha re f umaneng tlaheho ho tswa ho group B)

(Linda, the group leader from group B, presented solutions in this fashion:)

Linda: we really enjoyed to work on the activity. Lots of answers came out, but finally we agreed on one solution. We will present our solutions there after we can take questions and comments from the floor.

Linda: Question 1, the figures are squares, but after measurement were performed in question 2, we realised that the geometric figures showed are rectangles. Remember the properties of square and rectangles. That is what we provided as our motivation, and the table below:

Figures	$Length\ x\ Breadth$	$Perimeter$	$Area$
$figure1$,$where\ corners\ are\ in\ red$	$1,4\ cm\ x\ 1\ cm$	$4,8\ cm$	$1,4\ cm^2$
$figure1$,$where\ corners\ are\ in\ blue$	$2,6\ cm\ x\ 2\ cm$	$9,2\ cm$	$5,2\ cm^2$
$figure1$,$where\ corners\ are\ in\ green$	$4\ cm\ x\ 3\ cm$	$14\ cm$	$12\ cm^2$

Group B's table of answers

Participant from group D: Thank you group B, we really enjoyed your presentation, the excellent way you did the presentation. We hope our presentation will match yours

Group B: Generally the breadths increase by $1\ cm$ every time. The length of the rectangle with blue corners is $1,2\ cm$ more than the length of the rectangle with red corners. The rectangle with green corners is $1,4\ cm$ more than the length of the rectangle with blue corners. Generally the pattern followed by the lengths can be described as follows: $l_n = (0{,}1)n^2 + (0{,}9)n + 0{,}4$ (where l indicates the lengths of rectangles and n indicates the number of rectangles). The general pattern of perimeter and area of the rectangles can be illustrated as follows: $P_n = (0{,}2)n^2 + (3{,}8)n + 0{,}8$. (where P indicates the perimeter of the rectangles and n indicates the number of rectangles) and Area pattern is as follows: $A_n = (0{,}1)n^3 + (0{,}9)n^2 + (0{,}4)n$ (where A is the area of the rectangles and n is the number of rectangles).

APPENDIX F1

PLAN OF ACTION

Activities	Person responsible	Monitoring	Evaluation	Timeframe
Phase 1: Playing of Morabaraba game	Two pairs of learners and parents volunteered to play the game	Mathematics Teachers and other research participants	Observation sheet used to record observations	45 minutes
Phase 2: Reflection on the lesson learnt from playing the game	Grade 10 class and the focus group	Mathematics Teachers and other research participants	Discussion on mathematics concepts/skills/knowledge identified from the game	30 minutes
Phase 3: Presentation of the lesson :	Subject teachers: The Researcher or the grade 10 teachers or mathematics subject advisor	Mathematics Teachers and other research participants	Lesson plan is crafted based on the identified mathematical concept	60 minutes
Phase 4: reflection on the lesson presented	Grade 10 learners and focus group participants	Mathematics Teachers and other research participants		30 minutes
Phase 5 Assessment of the lesson	Teacher assessment, peer-assessment, and parent assessment		Give feedback on activities done during class, after class or at home	

Table 4.4

OBSERVATION SHEET

Activity: Morabaraba game

STRUCTURAL NATURE OF THE GAME

Mention any mathematical concepts/skills/knowledge that you observe

----

Any other observations you want to mention:

PLAYING OF THE GAME

Mention any mathematical concepts/skills/knowledge that you observe

----

Table: 4.14

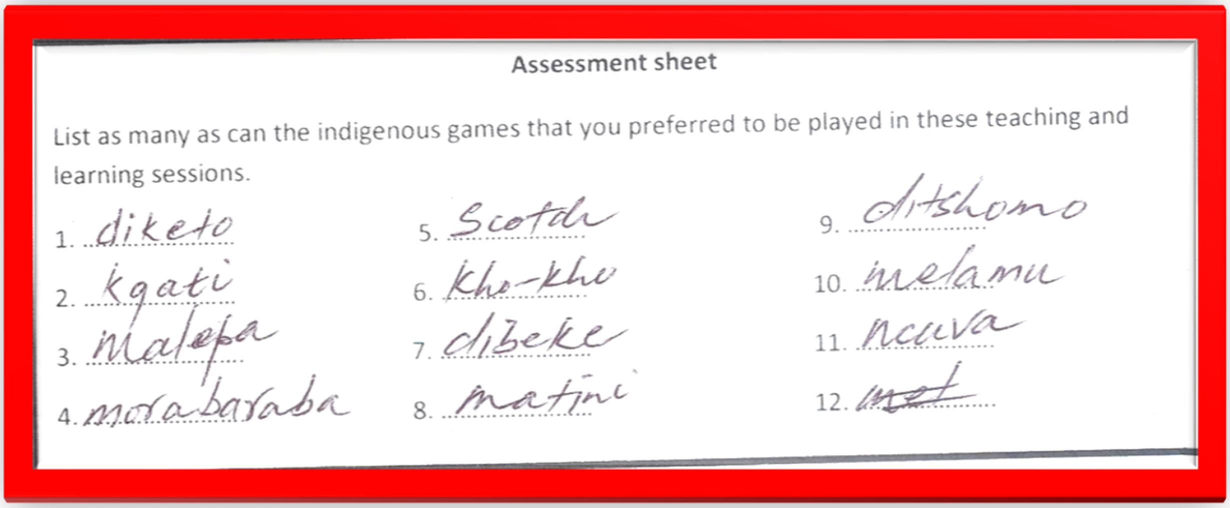
APPENDIX F2

Assessment sheet

List as many as can the indigenous games that you preferred to be played in these teaching and learning sessions.

1.	5.	9. ....
2.	6.	10. ....
3.	7.	11. ....
4.	8.	12. ....

Table: 4.15: Assessment sheet used to get the most preferred indigenous games



Assessment sheet

List as many as can the indigenous games that you preferred to be played in these teaching and learning sessions.

1. diketo	5. Scotch	9. ditshomo
2. kgati	6. Kho-kho	10. melamu
3. malepa	7. dibeke	11. ncuva
4. morabaraba	8. matini	12. ~~met~~

Table: 4.15

APPENDIX F3

Plc. 4.5: Group interactions on Investigation 1

Pic. 4.6: Parents demonstrating how to play Kgati

APPENDIX F4

WORKSHEET ON INVESTIGATION 2

Instructions:

1. In groups of five, each group is expected to have five members.
2. Perform the following investigation activity.

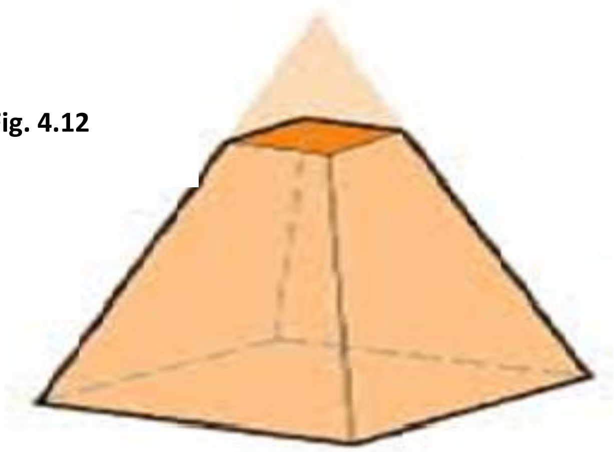
   Given shapes in Appendix F 9.1 is the decomposed structure of morabaraba. Decompose the structure further to establish the relationships and discover any patterns or general conclusions.

Table: 4.16

Pic. 4.7: Discussions in progress

APPENDIX F5

Fig. 4.12



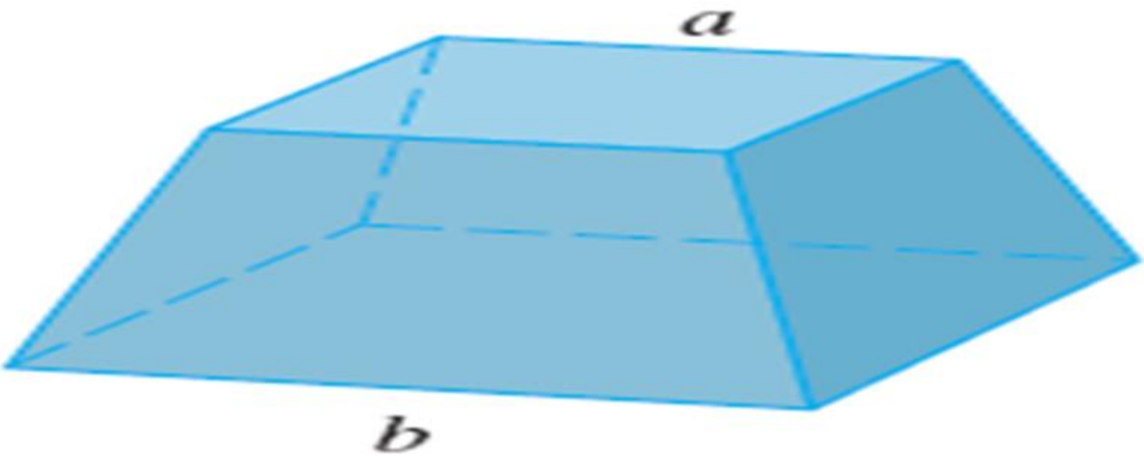


Fig. 4.13

APPENDIX F6

Class Activity: APPENDIX F6

Instructions:

- Work out this activity as the group and justify your calculations if possible.

Given the scenario:

Suppose there are 10 token stones used in the play of morabaraba, and the player is on round 2(seng 2). the number of stones scooped out of the hole is inversely proportional to the number of times the ghoen is thrown into the air, and the relationship is modelled by the $f(x) = -x + 11$ *.*

Find the number of stones scooped out of the hole, if the player is showing the 1^{st} *,*2^{nd} *,*3^{rd} *,*4^{th} *,*5^{th} *,*6^{th} *,*7^{th} *throws, before the ghoen fell out of his/her hand or he or she drops the ghoen.*

Class Activity

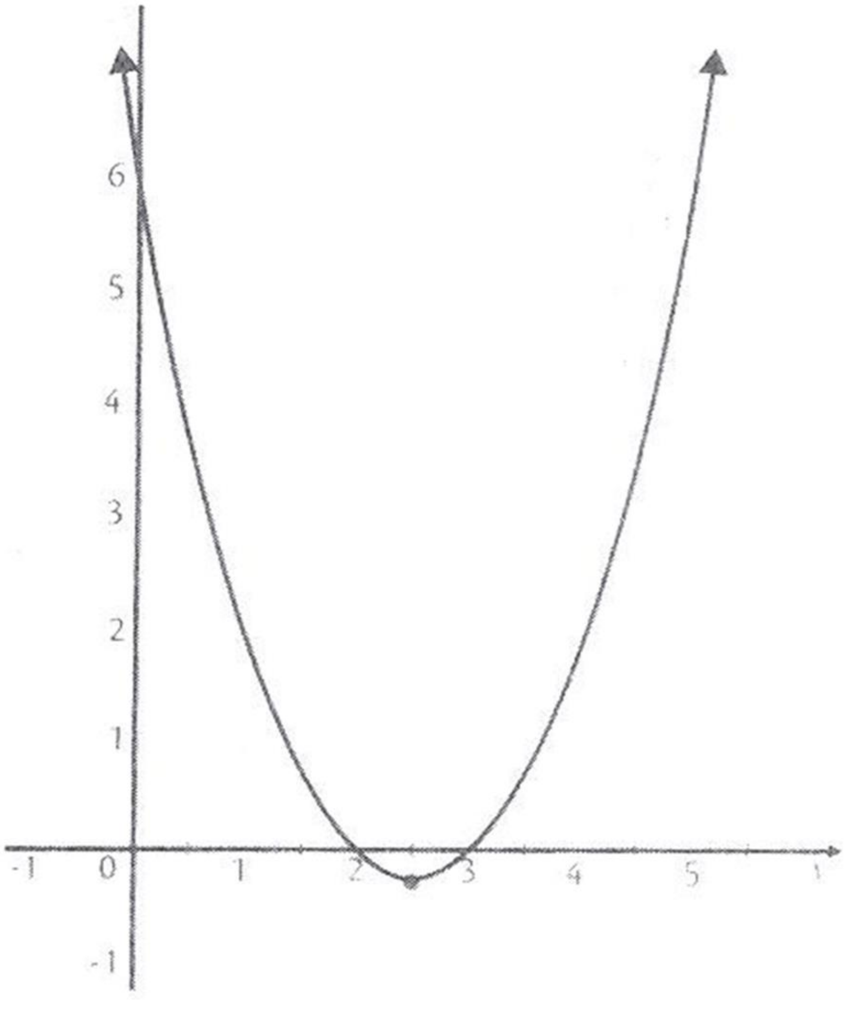
APPENDIX F7.1

Grade 10 Mathematics **Class Activity 1** **13 September 2013**

Instructions:

1. Answer ALL the questions.

2. Please work individually.



1. Given the graph of $f(x) = x^2 - 5x + 6$,

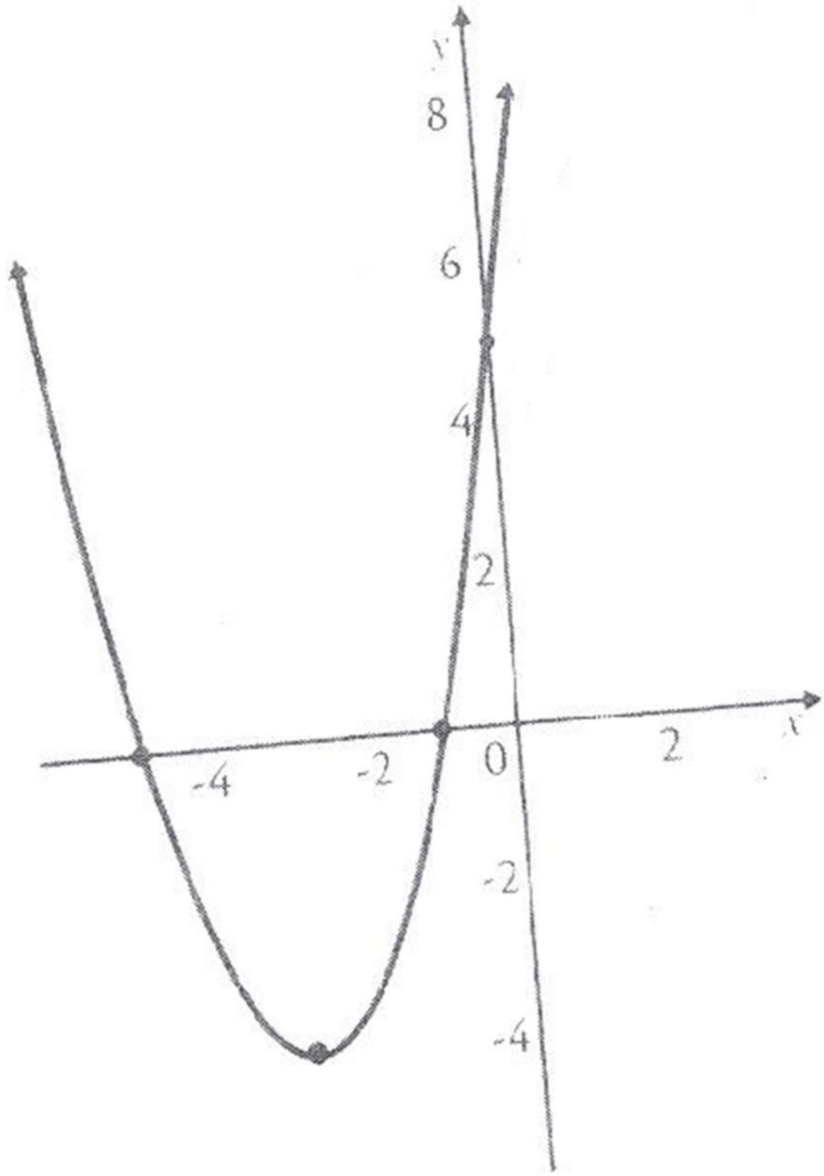
(a) For which values of x is $f(x) < 0$.

(b) Write down the values of x for which $f(x) > 0$.

APPENDIX F7.2

Class Activity (cont)

Question 2



Given the graph of $f(x) = x^2 + 6x + 5$,

2(a) Write down for which interval the function is decreasing and for which interval the function is increasing.

(b) In your own words, define the function

APPENDIX F8

WORKSHEET 4 (APPENDIX F8)

Instructions: Answer ALL the questions. Discuss this in pairs or in groups . Each group must have 5 members

1. **Describe each of the following tables in words and also formulate the patterns.**

(a)

(x)	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$f(x)$,	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**

(b)

(x)	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$h(x)$,	**-7**	**-5**	**-3**	**-1**	**1**	**3**	**5**	**7**

(c)

(x)	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$h(x)$,	**-7**	**12**	**11**	**10**	**9**	**8**	**7**	**6**

WORKSHEET ON INVESTIGATION

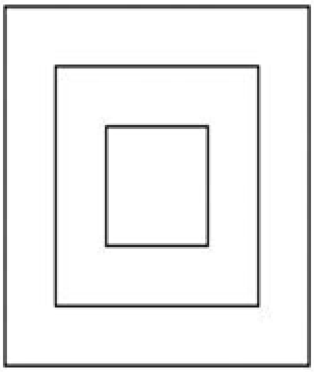
Instructions:

3. In groups of five, each group is expected to have five members.
4. Perform the following investigation activity.

   Given the shapes below are disentangled structures of morabaraba. Extricate the structure further to establish the relationships and discover any patterns or general conclusions.

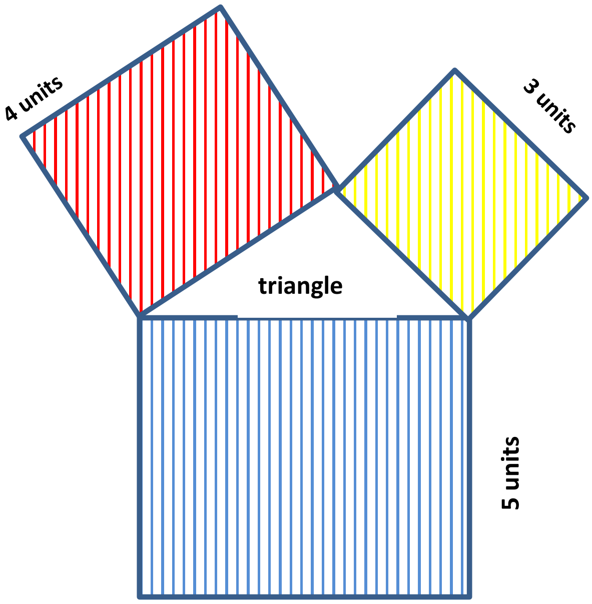
ACTIVITY 3:

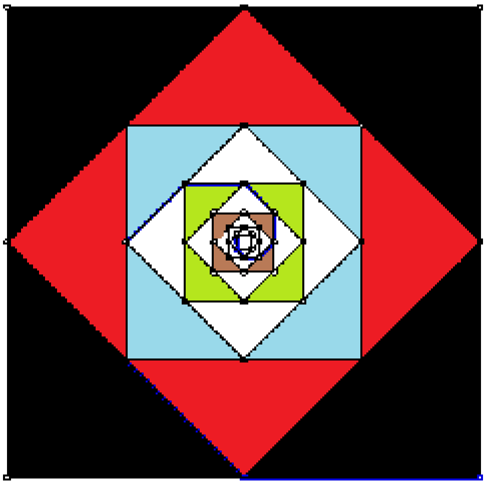
APPENDIX F 9.1



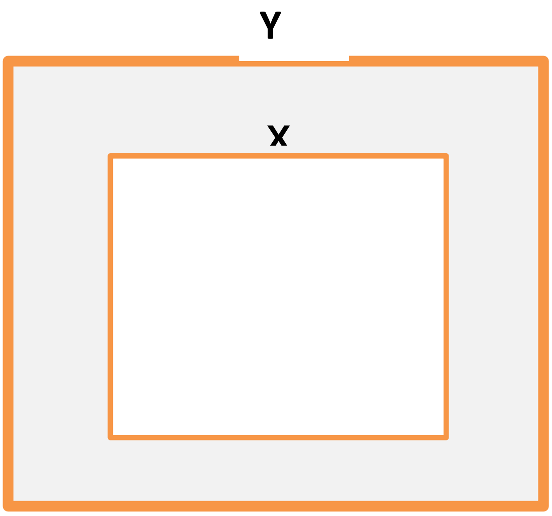
Shape 1

Shape 2

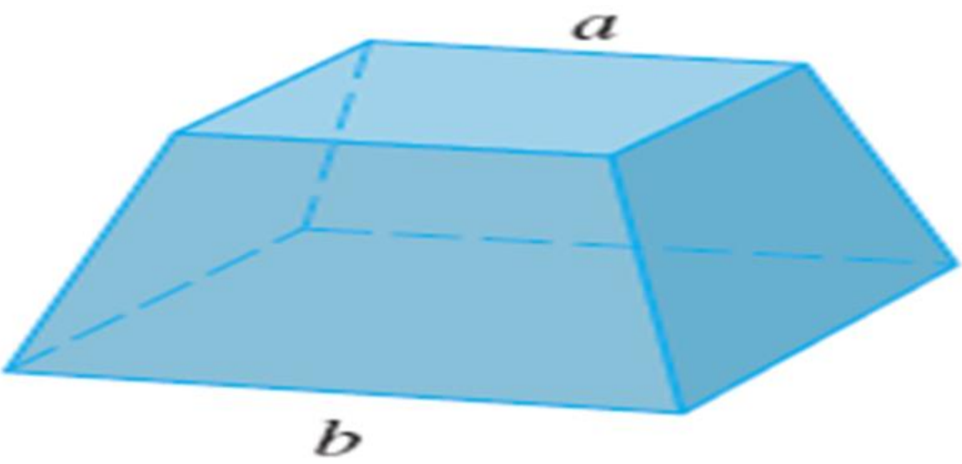




Shape 3



Shape 4



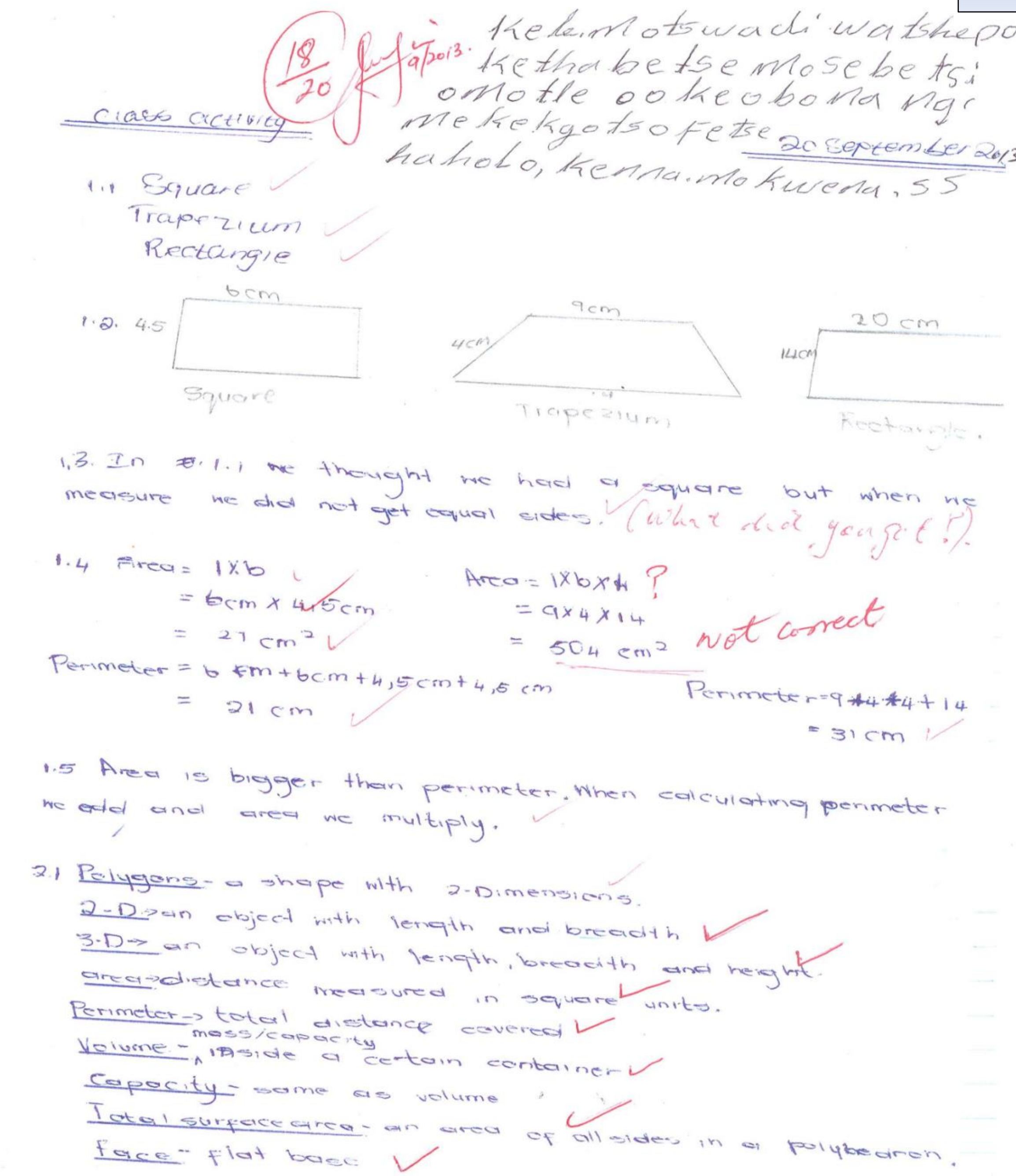
Shape 5

APPENDIX F9.2

Table 4.17: mathematical content/skills extracted from indigenous games

Shapes	**Mathematical content/skills**
Shape 1 (refer Appendix F 9.1)	Concentric rectangle, areas and perimeters of these shapes, the relationships between the areas and perimeter, measuring the sides of the rectangle
Shape 2 (refer Appendix F 9.1)	Right-angled triangle, squares, the relationship between the areas resulted into the discovery or understanding of Pythagoras theorem
Shape 3 (refer Appendix F 9.1)	Concentric squares, where the arrangements follow a geometric pattern, the exponents illustrates why $a^0 = 1$
Shape 4 (refer Appendix F 9.1)	Areas, perimeter of squares, the difference of squares(which is showed by the shaded portion, as $y^2 – x^2$)
Shape 5 (refer Appendix F 9.1)	3-D objects can be constructed from 2-D objects, volume of the frustum, and total surface area of the frustum.
Kgati (refer to Fig.4.8 and Pic.4.6 in APPENDIX F3)	Shows the quadratic functions, Maximum and minimum turning points, increasing and decreasing functions, positive and negative gradients
Diketo (refer to Pic. 4.3.2.1(a):	Demonstrates linear functions, linear equations, conceptualisations of independent and dependent variables, circles visualised in diketo can be arranged is such way that it illustrates the area of the circle, gradient = $\frac{change\ in\ y}{change\ in\ x}$
Malepa(string games)(refer to Fig.4.6.5(a) above.	Patterns, quadrilaterals, triangles, the relationship between the quadrilateral and triangles can be expressed as the linear pattern

The handwritten showed how the parent commented


18/20 9/2013.

Kekemotswadi watshepo ke thabetse mosebetsi o motle oo ke o bona ngwana ka ke kgotsofetse 20 September 2013

haholo, Kenna mokwena, 55

Class activity

1.1 Square
Trapezium
Rectangle

1.2. 4.5 6cm Square

4cm 9cm 14 Trapezium

14cm 20 cm Rectangle.

1.3. In 1.1 we thought we had a square but when we measure we did not get equal sides. (What did you get?)

1.4 Area = l x b
= 6cm x 4.5cm
= 27 cm²
Perimeter = 6cm + 6cm + 4,5cm + 4,5 cm
= 21 cm

Area = l x b x h ?
= 9 x 4 x 14
= 504 cm² not correct
Perimeter = 9 + 4 + 4 + 14
= 31 cm

1.5 Area is bigger than perimeter. When calculating perimeter we add and area we multiply.

2.1 Polygons - a shape with 2-Dimensions.
2-D → an object with length and breadth
3-D → an object with length, breadth and height.
area → distance measured in square units.
Perimeter → total distance covered
Volume - mass/capacity inside a certain container
Capacity - same as volume
Total surface area - an area of all sides in a polyhedron.
Face - flat base

Fig.4.14 : Parent's comments

APPENDIX F11

MATHEMATICS

SCHOOL VISIT REPORT TOOL 2013

education
Department of Education
FREE STATE PROVINCE

NAME OF OFFICIAL: PUTSI MJ

PURPOSE OF VISIT: Monitor the implementation of subject policy and to support the teaching and learning of Mathematics

FINDINGS and Recommendations:

1. **Monitoring and control by HOD**
   - Good Practices:
     - The HOD supports the implementation of the framework of using indigenous games to teach problem solving.
     - The HOD holds departmental meetings with his subordinates and resolutions that help educators keep focus are taken in the effective ways of teaching problem solving
     - The HOD monitors the teachers work as agreed in the departmental meeting and evidence of this appears in learner books

2. **Teaching and assessment**:
   - Good Practices:
     - Good quality tasks (across all the grades) are given to learners, and marking is also of a good quality. the tasks are also of high standard
     - Assessments are given frequently, and Assessments are planned for during the planning of lessons
     - All activities are learner-centred; the active role is done by learners.
   - Bad Practices:
     - No bad practices seen or heard of
   - Recommendations:
     - It must be pointed out that there is a huge improvement in learners' performance, especially the topics which gave learners difficulties. The learners' performance surpasses that one of term 1, especially in grade 10 there is a huge achievements, with regard to term-tests, June examinations and trial examinations.
     - The team of mathematics teachers and other stakeholders (including the Headmaster) are doing extremely well in Mathematics.
     - The lesson plannings are up to date, and showed that the tasks are learner-focused.

3. **Curriculum Coverage**: The teachers in grade 10 – 12 are on par with the work schedules, which is really impressive to note such a remarkable work

....	putsimj@gmail.com	17/10/2013
Signature: Subject Advisor	e-mail address	Date

Fig.4.15: Mathematics School Visitation Report

APPENDIX F12

MATHEMATICS

Grade 10 A marksheet 2013											
			50	50	50	50	100	100	100		
Sequence	Last Name	Initials	test 1	tes2	project	Investigation	June Exam	Trial exam	Nov/Dec Exam		
1	BUTHELEZI	TT	48	41	30	43	98	76	87		
2	DAKILE	SP	43	40	30	43	99	97	98		
3	DLAMINI	P	38	38	30	43	81	95	88		
4	DLAMINI	LM	41	40	31	44	84	96	90		
5	DLOMO	V	43	33	34	34	67	88	78		
6	HLONGWANE	TN	39	34	34	39	73	98	86		
7	KGALAPA	S	30	43	37	34	77	78	78		
8	KUBHEKA	ZR	30	43	30	43	86	90	88		
9	KUBHEKA	LC	35	45	29	30	75	87	81		
10	LETSIKA	PP	32	46	31	44	90	67	79		
11	MAHAMOTSA	SP	29	41	25	36	77	69	73		
12	MAHLASELA	SE	49	35	36	32	87	73	80		
13	MAHLASELA	ZS	34	39	29	41	80	78	79		
14	MAHLOHLA	TS	41	33	32	46	87	76	67		
15	MALAPE	SP	35	41	29	41	82	56	69		
16	MCHUNU	SC	31	44	28	40	84	88	86		
17	MHLATSI	LB	33	47	29	41	88	71	80		
18	MLABA	ZP	45	44	39	30	74	91	83		
19	MOELESO	KA	30	43	25	36	79	82	81		
20	MOFOKENG	PP	28	40	30	43	83	87	85		
21	MOFOKENG	LF	29	41	31	44	85	87	86		
22	MOKHESENG	BD	34	49	26	37	86	83	85		
23	MOKOENA	NP	32	46	32	46	92	99	96		
24	MOKOENA	T	30	43	33	47	90	72	81		
25	MOKOENA	BP	29	41	29	41	82	85	84		
26	MOLABA	NC	38	46	32	33	79	76	78		
27	MOLAPO	MR	24	34	33	47	81	65	73		
28	MOLOI	LA	19	27	32	46	73	67	70		
29	MOSIA	FF	34	49	29	41	90	65	78		
30	MOTAUNG	MJ	32	46	28	40	86	77	82		
31	MOTLOUNG	ZM	22	31	25	36	67	71	69		
32	MOTLOUNG	NP	33	47	45	39	86	63	75		
33	MOTLOUNG	T	33	47	36	41	56	58	57		
34	MOTSOENENG	LP	27	39	28	40	79	77	78		
35	MSIMANGO	TE	26	37	39	42	79	65	72		
36	MTHEMBU	ME	29	41	33	47	88	65	77		
37	MZOBE	M	28	40	29	41	81	71	76		
38	NDABA	NP	38	44	42	34	78	67	73		
39	NKGI	BP	34	49	30	43	92	69	81		
40	NTESO	NS	28	40	26	37	77	71	74		
41	NYEFOLO	ML	40	45	26	35	80	70	75		
42	SEFALI	ME	32	46	42	35	81	61	71		
43	SELEPE	LJ	26	37	30	43	80	71	76		
44	SELLO	P	33	47	36	41	88	66	77		
45	SELLO	TP	31	44	36	39	83	78	81		
46	SEMELA	N	31	44	32	46	90	87	89		
47	TAU	CS	39	35	26	37	72	67	70		
48	TISETSO	T	29	41	28	34	71	67	69		
49	TSELEDI	LK	26	37	39	31	68	69	69		
50	ZENGELE	S	35	45	41	47	92	80	86		

Class teacher: Mokoena MJ

29/11/2013

HOD: Mr Talana KB

2013/12/02

Table 4.18: **Grade 10 mark sheets**