Mathematical learning difficulties in Grade 1: The role and interrelatedness of cognitive processing, perceptual skills and numerical abilities

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Mathematical learning difficulties in Grade 1: The role and interrelatedness of cognitive processing, perceptual skills and numerical abilities

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BLOEMFONTEIN

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DECLARATION

I declare that the thesis hereby submitted by me for the MEd degree at the University of the Free State are my own independent work and have not previously been submitted by me at another university or faculty. I furthermore cede copyright of these articles in favour of the University of the Free State.

Lizelle Jacoba Eksteen
June 2014

DEDICATION

The hardest arithmetic to master is that

which enables us to count our blessings.

~ Eric Hoffer ~

I dedicate this dissertation to:

My parents, Hennie and Bea Eksteen,
my sister, Nelrize, and my brother, Chris,
who believed in me and constantly encouraged me.
Thank you for your love, empathy and support throughout this process.

I dedicate the knowledge and experience I have gained in conducting this reasearch
to all children with barriers to learning,
in particular children with mathematics deficits.

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- My thanks are also due to the Faculty of Education at the University of the Free State and the National Research Foundation (NRF) for financial assistance to undertake the study (NRF bursary grant number: 87728).
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- Most importantly, to my Saviour, thank you for being my pillar of strength. This thesis bears testimony to your unconditional love.

GENERAL ORIENTATION

The research submitted for examination was completed in accordance with Regulation G7.5.4.1 of the discipline Psychology of Education, Faculty of Education, University of the Free State. This regulation stipulates that a thesis can also entail the submission of two related publishable articles (in article format) for examination. The candidate therefore submits two related articles to fulfil the requirements of the qualification Magister Educationis (MEd) in Psychology of Education.

As indicated on the title page the registered title of this thesis is as follows:

Mathematical learning difficulties (MLD) in Grade 1: The role and interrelatedness of cognitive processing, perceptual skills and numerical abilities

The thesis consists of two related articles, namely one theoretical paper, entitled:

The role and interrelatedness of cognitive processes, perceptual development and early numeracy skills in early mathematical development

and one empirical article, entitled:

Mathematics proficiency among Grade 1 learners: The development and implementation of a numerical intervention programme based on the response to intervention (RtI) approach

A summary of both articles is included, explaining the conclusions drawn by the researcher upon completion of the investigation.

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ARTICLE 1

THE ROLE AND INTERRELATEDNESS OF COGNITIVE PROCESSES, PERCEPTUAL DEVELOPMENT AND EARLY NUMERACY SKILLS IN EARLY MATHEMATICS DEVELOPMENT

ABSTRACT

The development and application of mathematical competence have become a strategic educational objective for all learners in South Africa (South Africa 2010:3-6; South Africa 2011c:5; South Africa 2011a:2). The researcher believes that early preventive intervention for at-risk learners in the key aspects of mathematical proficiency might prevent serious deficits in mathematical competence. By the time learners enter formal schooling certain skills must have been acquired so that they will be at risk to develop delays with regard to basic mathematical skills and concepts. Their ability to acquire basic math concepts depend upon the interplay between cognitive, perceptual and other developmental factors. The purpose of this article is to identify the key issues related to mathematical learning disability (MLD) and afterwards to utilise the knowledge to draw up an intervention programme for mathematics that will be implemented within a response to intervention (RtI) framework.

Keywords: *mathematical learning disability, working memory, early numeracy skills, perceptual development, preventive intervention, response to intervention (RtI) framework*

1. Introduction

Mathematics is a pervasive component of everyday life and serves society and its individuals at many levels. It is extensively applied in diverse fields, for example, from measuring temperature, time and distance to pursuing a career in engineering (Schoenfeld, 2002:14; Tai, Qi Liu, Maltese & Fan, 2006:1143). Mathematical proficiency involves the ability to express oneself effectively in quantitative terms (Simmons, Willis & Adams 2011:141). It requires an understanding of numerical concepts and operations, and includes the ability to use this understanding in flexible ways to make mathematical judgements and develop useful strategies for handling numbers and operations. Mathematics therefore develops mental processes that enhance critical thinking, accuracy and problem-solving skills (South Africa 2011b:8).

Good mathematical skills involve competence in and an understanding of the numerical system. When analysed, these mathematical skills can be broken down into specific interdependent lower-order and higher-order skills. A child must first master lower-order skills, such as judging relative quantity and one-to-one correspondence, before more complex skills can follow as a certain level of developmental maturity is required for successful knowledge construction to take place (Bobis 2008:4-5). Preschool children develop mathematical skills through everyday encounters with quantitative information and numerical experiences. They master logical relationships, perform number operations and acquire mathematical concepts through the informal exploration of their own surroundings and by observing adults using numbers and quantities (Noël & Rousselle 2007:362).

The new Curriculum and Assessment Policy Statement (CAPS) for mathematics in the Foundation Phase forms the nexus between a learner's preschool mathematical experiences and the abstract mathematics of later grades. It comprises five content areas (number, operations and relationships, patterns, functions and algebra, space and shape and measurement) specifically designed to contribute to the acquisition of mathematics skills and concepts (South Africa 2011b:9).

During the first years of schooling (Grade 1-3) great emphasis is laid on the development of basic number knowledge including number identification, counting, magnitude comprehension and simple addition and subtraction calculations. A child's mathematical and perceptual skills develop simultaneously (Davis, Cannistraci & Rodgers 2009:2470; Noël 2006:365; Simmons et al. 2011:139-140), which indicates that various mathematical and developmental skills might be interdependent. Furthermore, certain cognitive processes (e.g. working memory) support mathematic cognition (Geary, Baily & Hoard 2009:277; Mazzocco & Myers 2003:219). The CAPS curriculum for mathematics in the Foundation Phase (South Africa 2011b:11) as well as Duncan (in Geary, Bailey, Littlefield, Wood, Hoard & Nugent 2009:265) highlight the importance of fundamental mathematical competence for later educational achievement. For this reason all skills, concepts and processes involved in mathematics development must be considered when investigating the factors that contribute to mathematics learning difficulties. Clearly, learners experiencing barriers in mathematics need some form of preventive intervention to minimise the possibility of failure which might one day amount to interminable struggles in the workplace and in meeting the demands of life.

2. Theoretical framework

The transformative paradigm provides a suitable theoretical frame of reference to explore the philosophical assumptions and guide methodological choices when investigating the factors involved in early mathematics development. In this paradigm knowledge is not regarded as neutral but is shaped by human curiosity and social relationships (Mertens 2010:12). The researcher's understanding of reality is influenced by the belief that knowledge is socially constructed. This study is grounded in two theories because of the significant relationship between intrinsic and extrinsic factors within an individual during the learning process.

Firstly, the researcher believes that the potential to develop proficiency or a deficit in mathematics exists within each individual and that this potential is influenced by biological, cognitive and behavioural factors. The causal model of Morton and Frith was consequently adopted. It concerns itself with three levels of development – biological, cognitive and behavioural – and their influence on each other (Frith 1992:13; Frith 1998:191). Although an interplay exists between the levels and they can therefore not be fully separated, the researcher chose to focus specifically on the cognitive and developmental (e.g. perceptual) factors that influence a child of school-going age as a means of identifying a learner's current level of acquisition and determining their true learning potential.

The second theory, cognitive dissonance, was first investigated by Leon Festinger in 1957. It is a psychological phenomenon which refers to the discomfort felt at a discrepancy between what you already know or believe, and new information or interpretation (Daryl 1967:184). Learning is thus the function of cognitive dissonance in teacher-learner interactions. The educator is the channel through which constant distance is created between knowledge and the desire to learn more. The quality of instruction is just as significant as the learner's input. A child who struggles would most likely place more value on acquiring a specific skill than the learner who finds no challenge in obtaining it.

3. The research problem and questions

After a thorough examination of the South African education system, questions were raised whether early childhood education cultivates competent learners for further studies at tertiary institutions. Mrs Angie Motshekga, Minister of Basic Education, pledged to ensure that South Africa's children all receive high-quality education at school level by signing a Delivery Agreement on 29 October 2010. On that occasion she said, "Our children and young people need to be better prepared by their schools to read, write, think critically and solve numerical problems" (South Africa 2010:3-6). Certain strategies were implemented to strengthen the foundational skills of literacy and numeracy (South Africa 2011c:5). For example, six million learners (Grade 1-6) across the country wrote the Annual National Assessments in 2011 to place an objective lens on the benchmark of achievement levels (South Africa 2011a:2). The results showed a very low overall performance with average scores of 30 per cent and lower in mathematics in each grade. The main problem identified was learners' inability to apply basic numeracy skills to execute mathematical operations. They also demonstrated serious conceptual shortcomings in the domains of fractions, patterns and mathematical functions, data handling and measurement (South Africa 2011c:18-19). These tests are now conducted yearly to monitor learners' progress and provide appropriate support to assist learners in need. The Free State Department of Education, for example, published a Mathematics Annual National Assessments (ANA) support handbook (2013) to provide guidance to Foundation Phase educators. Annual feedback on the ANA tests provides important pointers, confirms the shortcomings and presents a picture about the state of mathematics learning in the country. It also emphasises the importance of and need to develop early numeracy skills and concepts.

In view of the results of the Annual National Assessments, the following questions will guide the research:

- What role do cognitive processes such as working memory and executive functioning play in the mathematical development of children?
- What perceptual skills are important prerequisites for or predictors of mathematical development and proficiency?
- Why are number sense and counting considered important developmental milestones in the process of mathematics development?

- What are the characteristics of learners with mathematical learning disability (MLD)[1] and what difficulties do these learners encounter in the different skill areas of mathematics?
- Does the response to intervention (RtI) approach present a framework for preventive intervention to foundation-phase learners who experience mathematical barriers to learning?

4. The aims of the research

South Africa adopted an inclusive education system based on the *Education White Paper 6, Special Needs Education and Training System* (South Africa 2001), indicating that all schools should foster inclusion as an approach to address learning barriers. The researcher will investigate the RtI approach as a possible framework for providing support to learners with barriers and examine how early intervention could subsequently bring about mathematical proficiency. The researcher aims to investigate the role that cognitive processes play in mathematics and how perceptual and other developmental skills are interlinked with mathematics proficiency. The researcher will furthermore place an investigative lens on early mathematics skills, such as number sense and counting, and identify the characteristics and causes of MLD.

5. The research design and research methodology

In this theoretical article, the researcher will review previous research reported in journal articles, the policy documents of the Department of Basic Education, internet resources and newspaper articles to investigate the interplay of the various factors involved in the mathematical development of learners. In addition, the underlying numerical, perceptual and cognitive skills directly related to learners with MLD will be studied to outline the types of problems these learners experience. A brief discussion of the RtI framework as a possible intervention strategy is also presented. The RtI model and intervention programme will be discussed in detail in Article 2.

[1] In the South African educational context the term “learning impairment” is generally used. However, since this manuscript will be submitted to an international journal for review, the researcher opted for the term “mathematical learning disability”, which is a more general and more familiar term used in international publications.

6. Literature review

Entry-level mathematical skills are believed to be the best predictor of successful mathematics development throughout schooling (Geary 2011:252). Researchers agree that mathematics proficiency does not merely entail number recognition, but also refers to perceiving numbers and magnitude. Certain aspects should be taken into account when investigating the foundations of number processing and mathematics comprehension. Working memory is part of a range of cognitive systems that contribute to learning across all academic domains (Geary, Bailey, Littlefield et al. 2009:414). It is known to "control" specific mathematical skills, such as counting and arithmetic fact retrieval (Alloway & Passolunghi 2011:133-134; Geary, Bailey, Littlefield et al. 2009:266; Holmes & Adams 2006:339-342; Mazzocco & Myers 2003: 143; Noël & Rousselle 2007:363). In this article the researcher will utilise Baddeley's multiple-component model of working memory as a theoretical lens to explore the role of different memory components and the ways children acquire core mathematical skills (Baddeley & Logie 1999). Core developmental skills that lay the foundation for early number sense (Geary, Bailey & Hoard 2009:266) as well as cognitive process deficits observed among children with mathematics difficulty (Mazzocco & Myers 2003:223-225) should be taken into consideration. Finding equitable groundwork for core mathematical learning might contribute to the early identification and treatment of learners at risk of low achievement in mathematics (Passolunghi, Mammarella & Altoè 2008:231). It is therefore essential to determine the role and possible transfer of cognitive and perceptual skills. Although many other intrinsic and extrinsic factors support mathematics development this article will focus on the specific components highlighted here. In the following sections the influence of these components – cognitive processing, perceptual development and numerical abilities – will be discussed in detail.

7. The interplay of predictive factors for mathematics achievement

Achievement at school is determined by a child's ability to combine multiple skills that are not officially taught but are crucial to the coordination of behaviour and performance in class. Children are expected to pay attention and remember information while controlling their impulses to initiate correct responses. These related behaviours (self-control, attention and task completion) are referred to as the executive function or self-regulation (Assel, Landry, Swank, Smith & Steelman 2003:28) and represent attentional shifting, working memory and inhibitory control (Cameron, Brock, Murrah, Bell, Worzalla, Crissmer & Morrison 2012:1229). The capacity to plan and sequence behaviour (executive function) has been linked to both mathematics fact retrieval and mathematics calculation procedures (Assel et al. 2003:27; Mazzocco & Myers 2003:143). Other skills associated with mathematics, in particular counting and performing elementary arithmetic problems, are visual-spatial and fine motor skills (Son & Meisels 2006:756; Ziegler & Stoeger 2010:198).

7.1 Working memory

In the following paragraphs the researcher will apply the multi-component model of Baddeley and Logie (see Figure 1) to demonstrate how working memory influences mathematics skill and concept development.

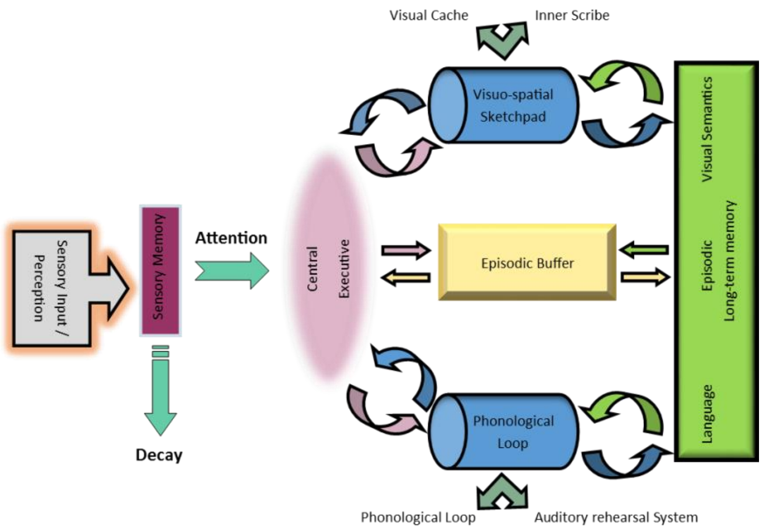

Figure 1. Working memory: The multi-component model (derived from Baddely & Logie, 1999).

Sensory memory acts as a mediator for the stimuli received through our senses. These stimuli are either perceived or discarded. When perceived, they are transferred into working memory via the process of attention (the ability to focus selectively on information). Working memory has limited retention and serves cognition as a temporary workspace where task-relevant information is being maintained, manipulated and processed. It has been referred to as “the brain’s Post-it note” (Mastin 2010) and also online cognition (Baddeley & Logie 1999:28). There are specialised components for different types of information. Working memory consists of a domain-general central executive system that regulates and coordinates three domain-specific subsystems: the phonological loop, the visuo-spatial sketchpad and the episodic buffer (Baddeley & Logie 1999:30). Working memory encodes its operations into long-term memory and also retrieves stored knowledge to manipulate and recombine it with new stimuli to deal with relevant tasks or problems. Mathematics performance is inevitably influenced by these components, depending on the type of mathematics task at hand (Holmes & Adams 2006:340; Passolunghi et al. 2008:232). What follows is a summary of these components and their role in mathematics learning.

7.1.1 The central executive system

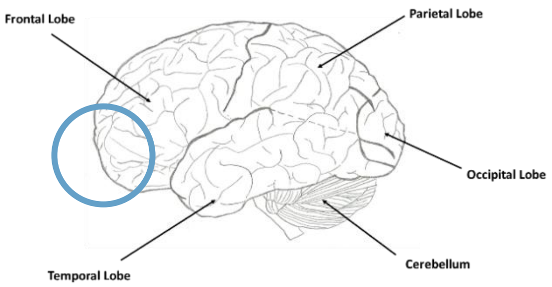

Figure 2. Areas of the brain associated with components of working memory (derived from Baddeley & Logie 1999:31, 53-55; Davis et al. 2009:2470-2471).

The central executive system (Figure 2) is associated with the frontal lobe (prefrontal cortex) area of the brain (Baddeley & Logie 1999:31). It is the domain-general component of working memory and is regarded as a predictor of children's accomplishments on broad measures of mathematical acquisition. The central executive system functions as a supervisory and attentional system which commands a number of tasks (Simmons et al. 2011:151; Baddeley & Logie, 1999:28) It is responsible for planning, inhibition and switching attention; it coordinates and manipulates information held in the two domain-specific slave systems (Holmes & Adams 2006:340); it controls the retrieval and encoding of representations within long-term memory and is important for success in understanding and applying strategies (Mazzocco & Myers 2003:245).

The central executive system is a credible indicator of maths performance in the first year of formal schooling (Alloway & Passolunghi 2011:134). Central executive resources are engaged in problems where procedural strategies are essential to accommodate various stages of the solution (Holmes & Adams 2006:343), e.g. multi-digit addition and multiplication. It is a significant predictor of addition accuracy when problems are visually presented (Simmons et al. 2011:151). Young children's arithmetical ability is supported by the central executive, in particular the application of algorithms and problem maintenance for calculation and estimation (Alloway & Passolunghi 2011:43; Baddeley & Logie 1999:134). As children grow older they start to rely on number knowledge and strategies to solve mathematical problems, which decreases the demands on the central executive system and employs other components of working memory during calculation.

7.1.2 The phonological loop

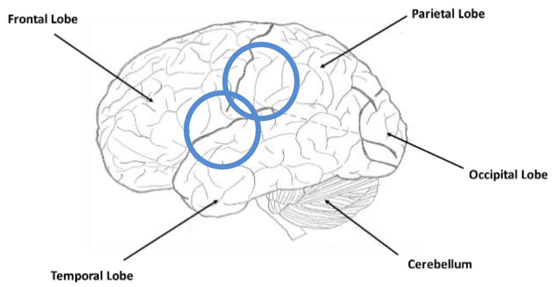

Figure 3. Areas of the brain associated with components of working memory (derived from Baddeley & Logie 1999:31, 53-55; Davis et al. 2009:2470-2471).

The phonological loop (Figure 3) is associated with the supra-marginal gyrus, situated in the lower part of the left hemispheric parietal lobe close to the junction with the upper part of the posterior temporal lobe (Baddeley & Logie 1999:54). The phonological loop is one of two domain-specific components of working memory (Holmes & Adams 2006:340). It is responsible for the temporary retention of auditory-verbal information (Simmons et al. 2011:140) and can be subdivided into a passive “inner ear”, which is known as the phonological store, and an active “inner voice” called the rehearsal system (Baddeley & Logie 1999:29). The phonological store produces verbal information in a phonological code, which deteriorates over time. The rehearsal system makes articulatory control processes available to prompt decaying representations in the phonological store. The binary function of these two sub-processes of storage and rehearsal affects immediate memory performance.

The phonological loop is responsible for number ranking, multiplication accuracy, solving single-digit problems and maintaining operand and interim results in multi-digit calculations (Alloway & Passolunghi 2011:133-136; Holmes & Adams 2006:340; Simmons et al. 2011:151). In contrast with the central executive system, the phonological loop is associated with mental arithmetic. Counting requires knowledge of number sequence, counting heuristics and keeping track of the running total (Holmes & Adams 2006:340). The phonological loop’s primary role is to encode and temporarily maintain the verbal codes that children (and adults) use for counting. It is also responsible for the acquisition of number facts which are stored in long-term memory as a plexus of verbally coded number facts (Baddeley & Logie, 1999:42; Holmes & Adams, 2006:340). This function comes into play when children start using direct fact retrieval as a solution strategy.

7.1.3 The visuo-spatial sketchpad

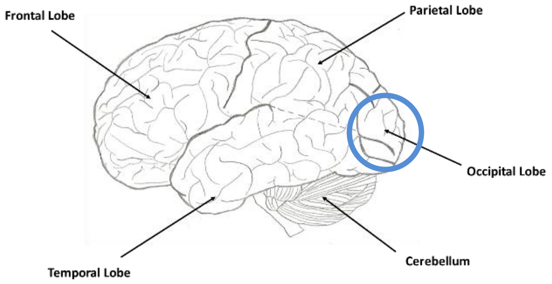

Figure 4. Area of the brain associated with components of working memory (derived from Baddeley & Logie 1999:31, 53-55; Davis et al. 2009:2470-2471).

The visuo-spatial sketchpad (Figure 4) temporarily mediates the storage of visual-spatial material (Alloway & Passolunghi 2011:133). It is a domain-specific system linked to the occipital lobe (visual area) and the inferior parietal lobe (spatial area) of the brain (Baddeley & Logie 1999:55; Davis et al. 2009:2471). The visual cache functions as an inner eye or visual imagery system (Baddeley & Logie 1999:42) which reserves visual patterns. The inner scribe is a spatially based rehearsal system responsible for the planning and representation of movement sequence (Baddeley & Logie 1999:29). The visuo-spatial sketchpad has been referred to as the "mental blackboard" of working memory (Alloway & Passolunghi 2011:133; Holmes & Adams 2006:341). It produces and manipulates visual-spatial codes along a mental number line during calculation. Children generate visual-spatial images of numbers to transcribe them, which evinces the visuo-spatial sketchpad as a significant predictor of number writing (Simmons et al. 2011:150). It supports number production and representation, and the symbolic magnitude appraisal for quantity discrimination (Alloway & Passolunghi 2011:133, 136; Davis et al. 2009:2475). The visuo-spatial sketchpad maintains a visuo-spatial framework that plays an essential role in problems that are represented visually, e.g. computation with blocks in the first year of formal schooling or multi-digit operations where place value and alignment in columns are imperative (Alloway & Passolunghi 2011:133; Simmons et al. 2011:151).

7.1.4 The episodic buffer

The episodic buffer is a recent addition to Baddeley's working memory model (Baddeley 2000:5). Its exact structure and function are yet to be elaborated on, but the available literature describes it as a temporary multi-modal store or workspace (Baddeley, Allen & Hitch 2011:1393; Gathercole, Pickering, Knight & Stegmann 2004:3; Henry 2010:1610). The episodic buffer is controlled by the central executive system and accessed through conscious awareness, which is the state of being aware of our current experience, both internal and external (Alkhalifa 2009:60; Baars 2002:47-50; Baddeley 2000:5). Its function can be described in terms of processing and storage, i.e. it accommodates the formation (processing) and maintenance (temporary storage) of data. The episodic buffer acts as a flexible interface for integrating information deriving from different sources and in different formats (Baddeley et al. 2011:3; Gathercole et al. 2004:1393). This means that it does not only temporarily store information in one modality (auditory, visual, spatial, kinaesthetic etc.), but allows various subsystems to interact even though they are based on different codes by joining information into unitary multi-modal representations. When taking these functions into consideration and placing the lens on mathematics, we can assume that the episodic buffer plays a pivotal role in problem-solving and calculation as it links up information from the visuo-spatial sketchpad, phonological loop, central executive and long-term memory during the unravelling processes to find the solution (Alloway & Passolunghi 2011:340; Holmes & Adams 2006:133; Simmons et al. 2011:140).

The researcher drew up a table (see Table 1) showing the sub-components of working memory according to Baddeley and Logie's model and the components of mathematics they support. It summarises the findings in this literature review on how working memory influences mathematics skill and concept development.

Table 1. Summary of the different components of working memory and how they serve mathematics.

Cognitive processing: Working Memory			
Central Executive	Phonological Loop	Visuo-spatial Sketchpad	Episodic Buffer
DOMAIN GENERAL supervisory system	DOMAIN SPECIFIC component Retention of auditory-verbal information	DOMAIN SPECIFIC component Retention of visual-spatial information	DOMAIN SPECIFIC component Links information for sub-systems and long-term memory into multi-modal codes
Commands planning and inhibition Selecting and monitoring information Focus and switching attention Multi-digit addition and multiplication. Complex mathematical problems requiring various solution stages and procedural strategies Number ranking and sequencing Aurally presented arithmetic (application of algorithms for calculation and estimation)	Counting Exact/mental arithmetic (add and subtract) Acquisition of number facts Number ranking Single-digit + and – problems Multiplication accuracy	Supports approximation, counting procedures and exact computation/ calculation Quantity discrimination Number line Number writing and symbolic magnitude judgement Written addition problems in symbolic form	Problem- solving

7.2 Perceptual development

Perception is a process that takes place between the brain and what it perceives from the outside world through the senses (Lewis 2001:275; Nel, Nel & Hugo 2013:143). The brain interprets and conceptualises data (impulses) from our senses to give meaning to what we experience. The main period for perceptual development is between the ages of 3 and 7, reaching adult levels around 12 years (Tsai, Wilson & Wu 2008). Perceptual skills serve as an impetus for learning. We can thus reason that young children need to develop and practise a certain set of prerequisite perceptual skills to equip them for specific academic abilities by the time they enter formal schooling. The researcher identified such skills from the available literature and found visual perception, motor development and spatial skills to be most important for mathematics. Several different aspects of these skills will be analysed and discussed in this section.

7.2.1 The relationship between visual-perceptual (motor and non-motor) skills and mathematics

Visual-perceptual ability is a mental process that enables us to perceive what we see with our eyes, i.e. to interpret cognitively what is seen (Assel et al. 2003:28; Barnhardt, Borsting, Deland, Pham & Vu 2005:138). These skills include visual memory, visual discrimination, visual sequencing, form constancy, visual position in space, visual figure-ground and visual closure. Visual-perceptual and motor skills cannot always be separated since motor responses are often required in visual-perceptual activities, for example, during a pencil-and-paper activity where the correct shape must be reproduced by means of drawing. One can therefore distinguish between non-motor visual-perceptual skills and visual-motor skills (Tsai et al. 2008:650; Brown, Rodger & Davis 2003:3). The interplay between non-motor visual-perceptual skills and mathematics is shown in Table 2.

Visual-motor skill is the "ability to integrate visual images … with the appropriate motor response" (Dankert, Davies & Gavin 2003:542; Barnhardt et al. 2005:138). Visual-motor deficits can have a negative impact on functional skill areas for learners (Brown et al. 2003:3), as this skill allows us to perform our daily activities. Visual-motor skills require fine manipulation of objects, for example to cut, draw and write, and are influenced by factors such as fine motor skills and eye-hand coordination (Dankert et al. 2003:542). This will be discussed in more detail under motor development (see 7.2.2).

Table 2. A summary of the importance of visual-perceptual skills in mathematics (Chabani, 2014; Cosford, 1982; Grové & Hauptfleisch, 1988; Nel, Nel, & Hugo, 2013).

VISUAL PERCEPTUAL SKILLS		
Sub-skill	**Definition/Notes**	**Influence on mathematics**
Visual memory	The ability to recall the visual appearance of an object, symbol or picture Forms the basis of all other visual-perceptual skills	Important for: mechanical mathematics, e.g. tables and arithmetic facts recalling the appearance of numbers, operational signs etc.
Visual discrimination	The ability to take note of detail, i.e. to see the similarities or differences between symbols and objects	Recognising the difference between for example a 2 and a 5 or similarities and patterns, e.g. ■ ▲ ● , … , … , …
Visual sequencing	The ability to see the order of numbers, digits or objects	To see the difference between 639 and 693 Remembering the sequence of steps in long division or multiplication
Visual form constancy	The ability to observe forms irrespective of colour, size or the angle from which it is viewed	**3**, 3, 3
Visual position in space	The ability to see that a symbol in a different position represents a different value or meaning, i.e. in the Western culture numbers are orientated in a left-to-right sequence.	Position in space, for example 6/9
Visual figure-ground	The ability to tell the difference between foreground and background	Isolate an object in the foreground, for example a worksheet containing: 3 5 9 +4 +2 +1 = = =
Visual closure	The ability to recognise the end of a sequence or a letter, number, picture etc. when part of it is missing	( )(•) (7)

7.2.2 The relationship between motor development and mathematics

Researchers agree that motor ability and cognitive development are interrelated (Diamond 2000:44; Westendorp, Hartman, Houwen, Smith & Visscher 2011:2773). One of the key features of motor development is that it involves interaction with the environment and as such reverberates at many different levels (Vereijken 2005). According to Piaget (Vickii 2013:6), infants actively explore their environment to construct knowledge. Numerous studies in neuropsychology (Diamond 2000:44, 50; Son & Meisels 2006:756; Westendorp et al. 2011:2773) and a review done by Diamond (2002) in Cameron et al. (2012:1231) on the relationship between motor skills and cognition, confirm that tasks which activate the prefrontal cortex – an area associated with attention and the executive function – also activate the brain area considered integral to motor processing, particularly the cerebellum. The interdependency of these two areas (the prefrontal cortex and cerebellum) in formal functioning leads to the conclusion that motor skills are an important predictor of achievement in general and specific cognitive abilities and that the development of children's motor skills is thus crucial to the construction of mathematics knowledge. Westendorp (2011:2774-2775) further expands on the topic of well-developed gross motor skills and their influence on cognitive functioning, saying that cognitive and motor functions have the same fundamental processes, for example sequences, monitoring and planning, and both seem to follow a similar developmental timetable. One can thus conclude that motor skills contribute to children's learning (Son & Meisels 2006:755) and that motor skill development serves as a verification of brain maturity (Luo, Jose, Huntsinger & Pigott 2007:597). As children grow older their motor skills develop into more sophisticated and coordinated actions while at the same time their brain functions become more mature. All motor skills start out as gross motor and then gradually develop into fine motor skills. Children initiate motor processes in academic tasks in much the same way as infants do to explore their environment, and most learning ventures in the classroom comprise perceptual and motor elements. Visual-motor skills, fine motor skills and eye-hand coordination (see 7.2.1) form the basis for gaining important functional skills such as cutting and pasting, using manipulatives, e.g. counters and number lines to do maths, holding and guiding pencils to produce legible handwriting, turning pages in a book and completing worksheets (Cameron et al. 2012:1230-1231; Ziegler & Stoeger 2010:198). Fine motor skills and eye-hand coordination can be paired together as one notion, since fine motor skills by definition involve small muscle movements that depend upon close eye-hand coordination, which in turn is controlled by vision (Luo et al. 2007:596). A certain amount of automaticity in coordinating fine motor skills frees up more space in the working memory, resulting in greater processing capacity to concentrate on problem-solving and more complex concepts (Cameron et al. 2012:1231; Luo et al. 2007:610).

From the discussion above it is evident that motor development plays an important role in academic learning. Rather than spend time on the symbolic representation of a number

("How do I make a number 9?" or "Which way does a number 5 face?") the child with advanced fine motor skills can devote the working memory to calculation instead of the mechanics of forming numerals. A child with advanced fine motor skills would use less time to complete assignments and would most likely be exposed to more learning experiences, i.e. proficiency in motor skills bolsters academic learning. For this reason the researcher finds it meaningful to include motor development, in particular fine motor skills and eye-hand coordination, as part of the intervention programme.

7.2.3 The relationship between spatial ability, visual imagery and mathematics

It has long been established within the field of mathematics that a child's *spatial ability* is pivotal to mathematics performance (Casey, Andrews, Schindler, Kersh, Samper & Copley 2008:270). It is an overarching concept that deals with the cognitive skills involved in understanding, manipulating, reorganising or interpreting relationships visually (Assel et al. 2003:28; Chabani 2014:8).

Spatial skills can be subdivided into visual-spatial skills and spatial orientation or "closure" (Kozhevnikov & Hegarty 2001:745).

Visual-spatial skills can be distinguished from spatial orientation by the fact that visual-spatial skills involve the rotation or transformation of visual material (Hegarty, Montello, Richardson, Ishikawa & Lovelace 2006:152-153; Kozhevnikov & Hegarty 2001:745). Children depend on visual-spatial representations when they first learn to count because of the great emphasis placed on concrete and pictorial representation. They often make use of their fingers to monitor which objects (e.g. blocks, beads) have already been counted (Cameron et al. 2012:28). Children's counting become automatised as they gradually move from using concrete aids to internalising or mentally representing the objects being counted. Visual-spatial skills are further significant when keeping track of number representation on a page and the order in which to write numbers, i.e. a child's ability to spatially present and interpret the meaning of numerical information (Cameron et al. 2012:28). The manifestation of an inability in this regard may include number rotation, misreading of arithmetical operation signs and difficulty with decimals (Geary 2011:253).

Spatial orientation is concerned with understanding the organisation of components within a visual representation (Kozhevnikov & Hegarty 2001:745-746). An example of a spatial orientation task would be Gestalt completion: it requires an individual to interpret the individual parts of an object as a whole.

Visual imagery is a cognitive process which matures in children between the ages of 8 and 11 years (Van Garderen & Montague 2003:246). Visual imagery or visual recall involves the construction of internal (mental) and external (e.g. pencil and paper) images and then using those images for mathematical discovery and understanding. Neurophysiological and neuro-imaging data provides evidence that the visual areas of the brain are composed of two functionally and anatomically interdependent visual and spatial components (Kozhevnikov, Hegarty & Mayer 2002:49; Van Garderen 2006:497). In Van Garderen and Montague (2003:497) and Van Garderen (2006:246) the researchers note Presmeg's taxonomy and identify five types of visual imagery (concrete, pattern, kinaesthetic, dynamic and memory of formulas). Pattern imagery is considered essential for mathematical problem-solving as it shows pure relationships depicted in a visual-spatial scheme. Pattern imagery involves the rational aspects of a problem and seems better suited to abstraction and generalisation.

The relationship between visual imagery, spatial ability and mathematical problem-solving can be illustrated by placing learners into two groups with regard to their preference for using visual imagery while solving mathematical problems: schematic and pictorial problem-solvers (Hegarty & Kozhevnikov 1999:684; Van Garderen & Montague 2003:247; Van Garderen 2006:496). Pictorial problem-solvers use the direct representation of an object described in a problem and include detail, e.g. colour, shape and size. Schematic problem-solvers use a representation of spatial relationships between objects, e.g. the location of the objects in space or the direction and speed of their movement, thus utilising their visual-spatial ability. Pictorial imagery is negatively associated with success in problem-solving as the "solver" often focuses their reasoning on irrelevant detail, making it difficult to formulate the necessary abstractions. Conversely, schematic imagery is classified as a more sophisticated type of imagery where concrete detail is disregarded and the focus is on the rational aspects of a problem.

One can thus hypothesise that mathematical tasks require spatial thinking (including visual-spatial skills and spatial orientation) and that visual imagery of mathematical concepts supports a child's intuitive understanding. Both are powerful tools in problem-solving (Blatto-Vallee, Kelly, Porter, Gaustad & Fonzi 2007:434; Hegarty & Kozhevnikov 1999:49). Marianne Frostig developed a standardised test in 1958-1963 to explore the development of children's visual perception between the ages of 3 and 9 years (Maslow, Frostig, Lefever & Whittlesey 1964:464). The test was later revised and is now known as the Developmental Test of Visual Perception – Second Edition (DTVP-2) (Mazzocco & Myers 2003:145). The DTVP-2 measures the five areas of visual perception and also includes visual-motor integration. The subtests are eye-hand coordination, copying, spatial relations, position in space, figure-ground, visual closure, visual motor speed and form constancy. Reliability and validity studies support the use of the test as the basis of support teaching programmes (Guntayoung & Chinchai 2013:5; Richmond & Holland 2011:36).

To conclude, this discussion has attempted to show that the acquisition of early mathematics skills and concepts is dependent on spatial ability, visual perception and motor development. For the reasons mentioned above these three components will be incorporated into the intervention programme.

7.3 Numerical abilities

Numerical ability comprises two essential competencies, namely number comprehension and counting efficiency (Passolunghi, Vercelloni & Schadee 2007:167). A correlation exists among rudimentary interrelated quantitative competencies and formal mathematics (Aunio 2006:1; Geary, Bailey & Hoard 2009:266; Geary 2011:253). The core competencies that define quantitative knowledge are essential for the identification of children's learning needs and the difficulties they might encounter (Jordan, Kaplan, Ramineni & Locuniak 2009:154). They also form the key components of preventive intervention programmes.

Case and his colleagues (Aunio 2006:4; Curtis, Okamoto & Weckbacher 2009:326) have suggested the existence of a cognitive developmental module – the "central conceptual structure for whole numbers" – to describe the maturation of children's mathematical reasoning and knowledge. As children mature, this structure allows for the interpretation of quantity and numbers in more superior and better equipped ways. It involves two phases: the first is a predimensional period where four- and five-year-olds have an independent global quantity and an initial counting schema. The second phase is the unidimensional stage, starting at about the sixth year, when children are able to join these two components to form a more sophisticated cognitive structure: the mental number line. In this stage enumeration to judge relative quantity, one-to-one correspondence and knowledge of written numerals, number words and cardinal set values develop. The central conceptual structure of whole numbers is parallel to other theoretical research which states that a critical component to formal mathematics is an evolved system for representing approximate quantities (Feigenson, Libertus, & Halberda 2013:74, 77; Geary, Bailey, Littlefield et al. 2009:266; Van de Walle 2006:119). Humans are born with a natural intuitive sense of numbers and quantities (Holmes & Adams 2006:340; Jordan et al. 2009:851) assigned to the approximate magnitude representational system that is anatomically located in the parietal lobe, a visual-spatial area in the brain (Geary 2011:252). Core quantitative skills, or pre-number abilities, form a skeletal framework for more advanced numerical development (Aunio 2006:10; Shood & Jitendra 2011:328) that derives from preschool and formal instruction (Berch 2005:154).

Numerical magnitude comprehension is a primary element of *basic number sense* (Feigenson et al. 2013:328), comprising lower-order number skills in infants. It is characterised by a non-verbal and implicit understanding of numerocity (counting arrays by estimation), ordinality (basic perception of more and less), counting (enumeration of sets up to three) and simple arithmetic (increase or decrease in quantity of small sets) (Aunio 2006:10; Feigenson et al. 2013:74). The quality of representations by the core system of approximate number magnitude is linked to mathematics ability by the time children enter preschool (Feigenson et al. 2013:76). Learners begin to integrate their conceptual comprehension of counting with quantity to form a mental number line which enables them to engage in more advanced number, counting and arithmetic skills (Aunio 2006:4; Berch 2005:154; Shood & Jitendra 2011:154). Associative and conceptual learning are interdependent, allowing children to progress gradually from primary quantitative development to more sophisticated maths comprehension (Bryant, Bryant, Gersten, Scammacca, Funk, Winter, Shi & Pool 2008:48). The development of basic number sense is influenced by the ability to compare, classify and understand one-to-one correspondence, to comprehend the concept of number magnitude, and to understand that magnitude relates to counting sequence and place value (Bryant, Bryant, Gersten, Scammacca, Funk, et al. 2008:48; Geary, Bailey, Littlefield et al. 2009:413).

Learners with adequate number perception gradually develop flexibility with numbers and their relationships as they become aware that numbers can be operated on, compared and used for communication. This flexibility characterises the trademark for *higher order number sense* (Shood & Jitendra 2011:323; Van de Walle 2006:119). Its components at the beginning of formal schooling include counting, number knowledge (number discrimination), number transformation (addition and subtraction), estimation and number patterns (Berch 2005:145; Jordan et al. 2009:852). These components relate to the National Curriculum and Assessment Policy Statement (South Africa 2011b:8-10) for entry-level mathematic skills and have been validated by research as important for mathematical concept development in young children. Children's number sense continues to evolve as they devise multiple ways of thinking about numbers, use numbers as referents, develop accurate perceptions about the effects of operations on numbers, discover flexible methods of computing and making estimates involving large numbers (Van de Walle 2006:119). The analysis that follows discusses the key components of early mathematics competency.

7.3.1 Entry-level mathematical skills and concepts

Seriation, classification, counting knowledge (procedural and conceptual) and magnitude comparison are preliminary arithmetic abilities known to be criteria for the early detection of children with arithmetic disabilities (Stock, Desoete & Roeyers 2010:253). Arithmetic is defined by the South African Concise Oxford English Dictionary (2002) as "the branch of mathematics concerned with the properties and manipulation of numbers" and "the use of numbers in counting and calculation". Desoete, Stock, Schepens, Boeyens and Roeyers (2009) describe how the work of Piaget and other Piagetian researchers led to the conclusion that seriation and classification are important for understanding numbers and are related to arithmetical achievement. They also explain that later research (post-Piagetian) includes initial counting and number magnitude as milestones in the development of early arithmetic. Desoete and her colleagues adapted the model from Dumont (1994) to explain how counting predicts ordinality and cardinality as outcomes of seriation and classification (see Figure 5). Early arithmetic is needed to diversify linguistic and numerical knowledge into math equations and algorithms; to understand mathematical concepts and operations; and to select applicable strategies for computation and problem-solving (Desoete et al. 2009:253-254). By continually executing arithmetic calculations a learner stores basic number facts in the long-term memory, resulting in direct fact retrieval.

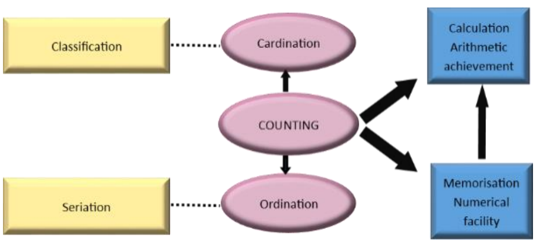

Figure 5. Desoete et al. 2009:254. Adapted from Dumont, J. J. (1994). A model illustrating the role of counting in facilitating arithmetic achievement and memorisation.

7.3.2 Seriation and classification

Seriation is the logical ability to sort a number of objects in sequence based on their differences in one or more dimensions, e.g. from smallest to largest. Classification on the other hand is the ability to sort objects based on their similarities in one or more dimension (Desoete et al. 2009:253; Kivona & Bharagava 2002; Stock et al. 2010:251). Children grasp the inclusion principle when they gradually coordinate seriation and classification and start to comprehend that numbers are series containing each other. The conservation principle follows after the inclusion principle when children understand that the number of objects in a collection only changes when objects are added or removed (Stock et al. 2010:251). These four logical abilities are classified as Piagetian-type tasks and according to Stock et al. (2010) account for about half of the variance in the arithmetic skill of children at ages 6 and 7 (Desoete et al. 2009:261*)*.

7.3.3 Counting

Counting outlines the learner's growing number awareness and forms an essential pathway to obtaining more knowledge about abstract numbers and simple arithmetic. It entails the production of whole number-words and connects number sequence with one-to-one correspondence in the sets being counted (Aunio 2006:3; Berch 2005:154). Children learn how to count by memorisation, reciting the correct string of number words while pointing at objects. Learners count by rote before they grasp that the last counted word indicates the amount of the set and how numbers are related to each other (Van de Walle 2003:117). Aunio (2006:3-4) categorises counting into six different phases. A primary understanding of amounts becomes apparent at around 2 years of age. The acoustic and asynchronic phases follow next, where children (3 years) are able to say number words but not in the correct order and later (4 years) use the correct order and point to objects. Six months later, during the synchronic phase, children use rote counting to point at or move objects correctly. Resultative counting emerges in children at 5 years of age. In this phase children understand that countable objects should be marked once and grasp the cardinality principle. They also understand that number words represent a growing series of magnitude and use them correctly, starting at one. During the last phase, the shortened counting phase, children 6 years of age can count upward starting at a specific number, e.g. five.

Desoete et al. (2009:253-254) and Stock et al. (2010:251-252) explain that counting knowledge consists of procedural and conceptual aspects. Procedural counting predicts arithmetical problem-solving and involves the ability to perform a counting task effectively to calculate the number of objects in a collection. Conceptual counting is associated with a child's understanding of why a procedure works or whether a procedure is admissible. In Stock et al. (2010:251) conceptual counting exhibits essential counting principles and aligns itself with the five widely cited counting principles of Gelman and Gallistel (1978):

- One-to-one principle: designating one counting word to each object counted
- Stable-order principle: knowing that the list of number words is always in the same repeatable order
- Cardinality principle: the number word assigned to the final object in a set represents the number of objects in that set
- Order-irrelevance principle: the order in which objects in a set are counted is irrelevant as long as every object is counted once and only once
- Abstraction principle: all the above principles can be applied to any unique collection or set of objects, e.g. when counting a set of black dots the same principles apply as to when counting a set of red blocks

7.3.4 Arithmetic

By school-going age most children have integrated their number knowledge and counting abilities and can employ number words and symbols (Arabic) to solve formal addition and subtraction problems by using a range of strategies. Typical approaches to calculate answers are counting with or without fingers and using the minus and sum procedures (Geary 2011:254). The former procedure involves starting at the larger-valued addend and then counting the value of the smaller addend, while the latter involves counting both addends starting at one. Children constantly switch between less and more sophisticated strategies depending on the complexity level of the problem. Counting provides the foundation for the development of arithmetic skills (Geary 1993:347). The continued use of counting results in long-term-memory-based representations for direct arithmetic fact retrieval and the reconstruction of the answers based on the retrieval of a partial sum (Desoete et al. 2009:254). The number of times a child uses counting to find a solution to a math problem increases the probability of storing an arithmetic fact in the long-term memory as an association is made between the problem and the answer (Geary 1993:347).

In summary it can be concluded that mathematics knowledge has a hierarchical nature, gradually progressing from rudimentary non-verbal to a systematic and symbolic understanding. The key components of early mathematics competency were discussed in detail as well as the developmental sequence in which children learn these concepts and skills. This analysis provides the groundwork for the planning of intervention strategies to guide children’s experiences in the domain of proximal development in the empirical research to follow.

8. The manifestation of mathematics learning disability

Several studies (both population-based and on a smaller scale) have found that 3 to 6 per cent of the school population experiences difficulty in at least one area of mathematics and will be diagnosed with a mathematics learning disability (MLD) before they complete high school (Bryant, Bryant, Gersten, Scammacca & Chavez 2008:20; Fuchs, Fuchs, Compton, Bryant, Hamlett & Seethaler 2007:311; Geary 1993:345; Geary 2011:251; Geary, Bailey, Littlefield et al. 2009:411; Geary, Bailey & Hoard 2009:265). Different terms for MLD are used in the literature, such as dyscalculia or mathematics disability, but for the purpose of this article the term mathematics learning disability will be used. The DSM-V of the American Psychiatric Association defines MLD as a below-expected performance in mathematics (comprehension of quantities, numerical symbols or basic arithmetic operations), given an individual's age, intelligence and years of education, which in adults significantly interferes with their daily activities (Geary 2011:251).

The best predictor of mathematics achievement throughout schooling is entry-level math skills; this implies that a lack of number sense is a fundamental deficit of children with MLD. In this view, MLD occurs when the basic numerocity process (intuitive understanding of the exact representation of small quantities and the approximate representation of magnitude) fails to develop normally and influences the capacity for learning number concepts and arithmetic in school (Bryant, Bryant, Gersten, Scammacca & Chavez 2008:21; Geary, Bailey & Hoard 2009:256; Geary 2011:257). The most frequent problems reported by researchers into MLD are the use of immature counting strategies when solving arithmetic problems and difficulty in understanding counting concepts (Geary, Bailey, Littlefield et al. 2009:143; Geary 2011:255; Mazzocco & Myers 2003:143). Furthermore, MLD children have a lower performance level on verbal short-term-memory tasks, have developmental delays in procedural skills, experience difficulty in acquiring and retrieving basic arithmetical facts, and frequently commit computational errors (Holmes & Adams 2006:340; Noël & Rousselle 2007:363).

In the previous section (see 7), the predictive factors for mathematics achievement were divided into cognitive processing, perceptual development (see 7.1) and numerical abilities (see 7.2); for logical reasons the manifestation of MLD will be categorised in the same way.

8.1 Mathematical learning disability as a result of impaired cognitive function and perceptual delays

The reason for difficulty in performing specific types of mathematical skills or tasks is often impaired cognitive function or an inability in the brain to receive and process information related to mathematics.

8.1.1 Impaired cognitive function

Memory connects thoughts, impressions and experiences. It is dependent on cognitive abilities and uses several brain systems. The researcher was specifically interested in working memory and the role it plays in the storage and retrieval of mathematical information (arithmetic facts) from long-term memory. When information in working memory decays too fast it leads to unreliable and incomplete representations of arithmetic facts in the long-term (semantic) memory (Noël & Rousselle 2007:363). A possible explanation for this impairment is that when the answer to a specific problem is available after computation the problem is no longer active in the working memory and the association cannot be stored. This problem may also cause procedural errors and thus incorrect associations in the long-term memory.

Another function of working memory is attentional resource distribution during problem-solving (Mazzocco & Myers 2003:247). Children with poor working memory resources may not have the answer to a specific arithmetic fact readily available for retrieval from the long-term memory and would most likely resort to immature counting strategies (such as finger counting or using the sum instead of minus counting) to keep track of number processing and to decrease the load on the working memory. Dependence on slower procedures for problem-solving, in turn, affects processing speed as it takes longer to count than to retrieve the answer from the long-term memory (Noël & Rousselle 2007:365). All three components of working memory are implied in the delayed learning progress of children with MLD (Geary 2011:256).

A weak central executive system will result in complications with computation procedures, organisational skills, attention, mathematics fact retrieval, applying and switching between learned solution strategies, and inhibiting irrelevant information from, for example, word sums (Holmes & Adams 2006:343; Mazzocco & Myers 2003:220). The interrelationship between the central executive function, mathematics ability and reading ability has also been reported. Children with reading disabilities seem to have difficulty with auditory memory, which results in problems with word retrieval, phonological decoding and weak associations in long-term memory – much the same characteristics as MLD children with direct arithmetic fact retrieval (Geary 1993:355; Geary 2011:256; Mazzocco & Myers 2003:220). Children with MLD only outperform their peers with MLD and reading difficulties in areas of math that involve a language component, e.g. word sums and counting.

Phonological processing was found to be responsible for computational skills development (Noël & Rousselle 2007:264). Children with MLD make frequent errors when they use counting (verbal codes) to solve simple addition and subtraction problems as they have difficulty keeping number words in mind while performing other tasks such as monitoring the next step of the solution strategy (Geary 2011:257; Mazzocco & Myers 2003:340). One contribution of the visuo-spatial working memory to MLD is poor performance in numerical processing (Geary 2011:257). MLD children are slower in number reading, number comparison and number sequencing (Noël & Rousselle 2007:366). Poor working memory seems to be persistent and is associated with developmental delays, which not many MLD children outgrow.

8.1.2 Perceptual delays

Perceptual developmental delays are responsible for difficulty in number combinations, determining distances, form constancy, number sequencing and copying from books, worksheets and writing boards (Brown et al. 2003:3; Grové & Hauptfleisch 1988:248-249). Numbers in answers are frequently either reversed or omitted. A learner will find it challenging to distinguish between shapes, sizes and colour or note the relationship between the symbol and the number (Kulp, Earley, Mitchell, Timmerman, Frasco & Geier 2004:44). Difficulty to distinguish between arithmetic signs and remember the sequence of steps in long division or multiplication has also been reported (Kulp et al. 2004:45). Visual-spatial deficits affect functional skills, e.g. the misinterpretation or misalignment of numerical information, number omission or rotation and conceptual understanding of number representations, such as that the 1 in the number 19 represents a total value of 10 (Barnhardt et al. 2005:138; Kulp et al. 2004:44).

8.2 Mathematical learning disability as a result of functional numerical deficits

Number sense is a core intuitive skill that forms the cornerstone for the development of mathematics proficiency. Children with a weak number sense will experience difficulty in counting objects, number sequencing, quantity discrimination and counting on from a given number (Chard, Clarke, Baker, Otterstedt, Braun & Katz 2005:4).

Furthermore, the procedural skills of children with MLD are characterised by lengthy problem-solving and frequent errors when calculating simple arithmetic and word sums (Bryant, Bryant, Gersten, Scammacca & Chavez 2008:21; Geary 1993:348; Geary, Bailey, Littlefield et al. 2009:255). Geary and his colleagues suggest that the early procedural deficits of MLD children indicate a delay in the acquisition of conceptual skills and counting concepts. Contrary to memory-retrieval deficits, procedural deficits seem to disappear generally by the end of Grade 2. Counting knowledge and principles are crucial to the development of computational skills. MLD children typically undercount when using the minus procedure (Geary, Bailey, Littlefield et al. 2009:355). They may, for example, count the three number words correctly, "4, 5, 6…" for the problem 4 + 3 = ?, but may not use 4 as the cardinal value of the first addend, resulting in miscalculation. Subtraction errors may include subtracting the larger number from the smaller one. Their application of elementary strategies such as finger counting and the use of sum procedures seem to persist over time. They seldom rely on memory-based processes or more efficient strategies to solve problems, irrespective of the complexity level (Geary & Hoard 2002:99-100).

Thus, from a cognitive and numerical perspective, we can differentiate three types of lower-order deficits in MLD children: semantic memory and procedural and visuo-spatial deficits (Geary 1993:223; Geary 2011:348; Holmes & Adams 2006:340). These subtypes can guide identification; are necessary for intervention strategies; and help to distinguish between a mathematics deficit and a delay. Semantic memory deficits co-occur with language and reading difficulties and are associated with poor arithmetic fact retrieval from long-term memory. Procedural deficits involve developmentally immature strategies, frequent computational errors and a delay in learning arithmetic concepts. Lastly, visuo-spatial deficits are characterised by sign confusion, number omission or rotation and the misinterpretation of spatially represented information, for example graphs or geometric shapes. A list of mathematics difficulties (reported by teachers as math weaknesses) was compiled by Bryant and Bryant in their research (Bryant & Bryant 2008:5). The list is contained in Table 3 to serve as a summary of the characteristics of children with MLD. It is clear that these learners have difficulty with many different skill areas of mathematics.

Table 3. A summary of the characteristics of children with MLD
(adapted from Bryant & Bryant 2008:5)

Ranked mathematics difficulties exhibited by a children with learning disabilities and math weaknesses
• Does not recognise operator signs (e.g. + and –)
• Omits digits on left or right side of a number
• Exhibits left-right disorientation of numbers
• Cannot copy numbers accurately
• Starts the calculation from the wrong place
• Writes numbers illegibly
• Does not remember number words or digits
• Reverses numbers in problems
• Experiences difficulties in the spatial arrangement of numbers
• Makes errors when reading Arabic numbers aloud
• Skips rows of columns when calculating (i.e. loses his or her place)
• Misaligns horizontal numbers in large numbers
• Misplaces digits in multi-digit numbers
• Fails to read accurately the correct value of multi-digit numbers because of their order and spacing
• Fails to carry (i.e. regroup) numbers when appropriate
• Disregards decimals
• Misaligns vertical number columns
• Orders and spaces numbers inaccurately in multiplication and division
• Calculates poorly when the order of digit presentation is altered
• Reaches “unreasonable” answers
• Counts on fingers
• Makes “borrowing” (i.e. regrouping, renaming) errors
• Takes a long time to complete calculations
• Cannot recall number facts automatically
• Fails to verify answers and settles for first answer
• Has difficulty with the language of math
• Has difficulty with multi-step problems

To summarise the discussion (see 7 and 8) of the various contributing factors and the characteristics and manifestation of MLD in learners, a graphical representation from Geary and Hoard (2002:102) is included as Figure 6. In contrast to normally developing learners, many MLD learners have a delay in grasping counting concepts and gaining knowledge of the different features of arithmetic skill development. The critical determinant of a learner’s mathematical success is how soon they progress from using counting-based procedures to memory-based processes. As previously discussed (see 7), many different factors contribute to an MLD learner's slow progression in this regard. The graphical summary (see Figure 6) illustrates the expected direction of normal strategy development as related to reaction time studies and the amount of working memory resources required for strategy use (Geary & Hoard 2002:101).

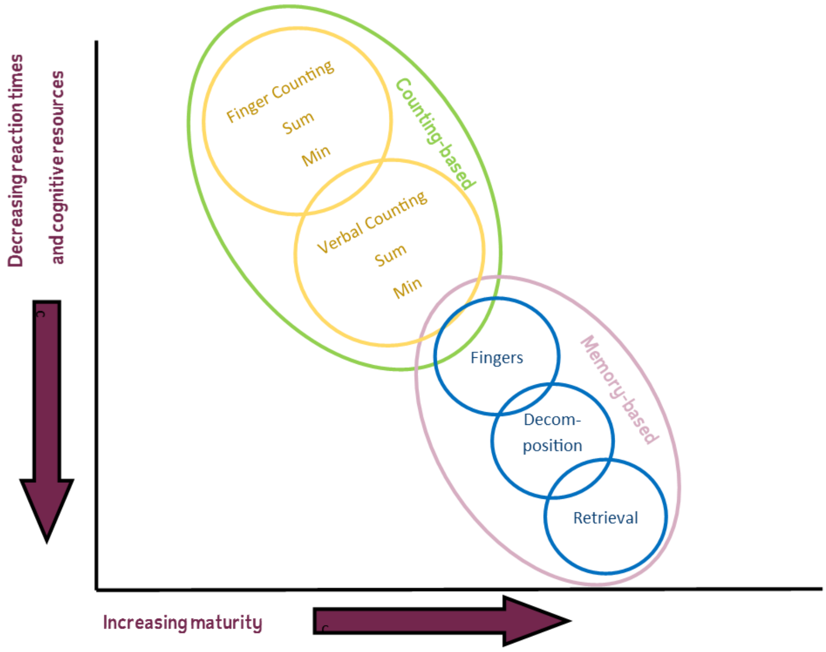

Figure 6. Graphical representation of developmental changes in the mix of strategies used to solve simple arithmetic problems (Geary & Hoard 2002:101).

The transition to memory-based processes has several advantages (Geary & Hoard 2002:101-102). It results in the quick solution of problems and reduces the demands on the working memory associated with solving these problems. It furthermore leads to automatic arithmetic fact retrieval, which, along with the accompanying reduction in working memory demands, seems to make complex problem-solving less error inclined. The intervention programme should thus include all the contributing factors which will advance MLD learners' mathematical competencies to the extent where the transition from counting-based procedures to memory-based processes is possible.

9. Intervention strategies for mathematics difficulty within an RtI framework

Inclusive education is a worldwide philosophy. The essence of inclusive education is that every child has the fundamental right to education and that all children, even learners with special needs, must have access to regular schools (Nel et al. 2013:6-8). The implications of inclusive education are that individual differences and learning needs must be catered for by planning and providing support to individuals. In many countries, including South Africa, the main approach to identifying children with learning disabilities (also mathematics learning disability) involved recording a discrepancy between intelligence capacity and academic achievement (Bryant & Bryant 2008:4; Fuchs & Fuchs 2007:14; Moors, Weisenburgh-Snyder & Robbins 2010:224). This approach is often referred to as the “wait-to-fail” model, where identification occurs only after failure. This is clearly ineffective and deprives these children of intervention at the time when it is critically necessary (Fuchs & Fuchs 2007:14; Moors et al. 2010:224; Semrud-Clikeman 2005:565). After 1994 reforms were needed to amend the South African school system and the methods to identify and support learners with disabilities and impairment. The Education White Paper 6 was accepted in 2001 as the policy for building an inclusive education and training system where the needs of all learners, including those with learning difficulties, can be catered for (South Africa 2001). The main focus of this policy is to integrate special and ordinary education (Nel et al. 2013:6). From the researcher’s point of view, the response to intervention (RtI) approach can be adopted to design additional support and provide multi-level teaching in order to address different learning needs. This model is based on prevention. The RtI approach imitates a public health framework which invests in preventing the number of new cases rather than waiting for a specific problem to occur and then following up with less than ideal treatments (Chard et al. 2005:3; Fletcher & Vaughn 2009:31). This approach is characterised by intervention in growing levels of intensity with quality general education as the starting point. It involves the early screening of all learners, intervention and progress monitoring (Bryant, Bryant, Gersten, Scammacca, Funk, Winter et al. 2008:49; Bryant, Bryant, Gersten, Scammacca & Chavez 2008:21; Gersten et al. 2009:4; Semrud-Clikeman 2005:31). Most RtI approaches consist of a three-tiered prevention system and are conceptualised as a triangle (Fletcher & Vaughn 2009:31; Fuchs, Fuchs, Compton et al. 2007:312; Gersten, Beckmann, Clarke, Foegen, Marsh, Star &Wetzel 2009:6 Moors et al. 2010:244) as shown in Figure 7.

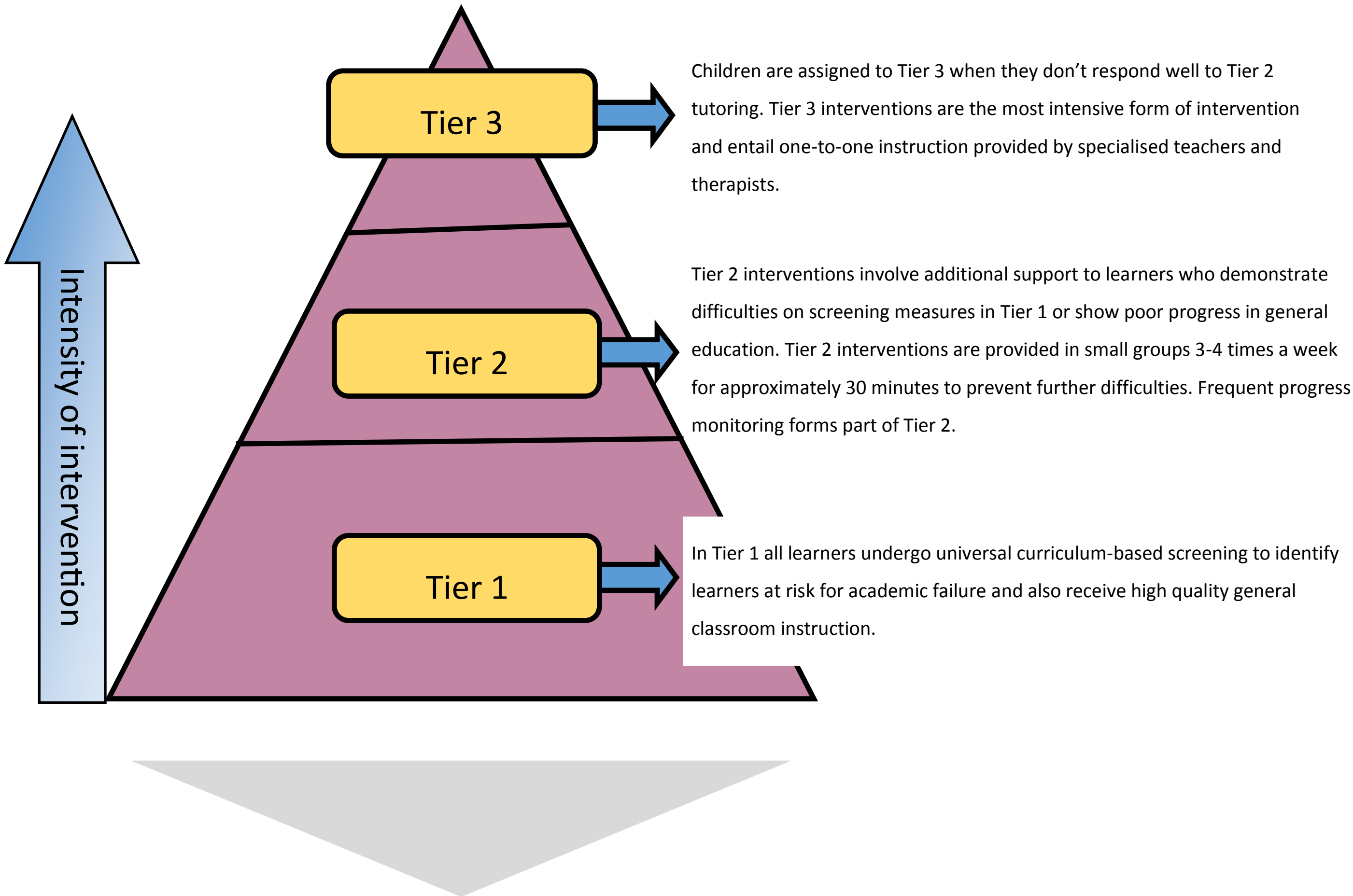

Figure 7. The response to intervention (RtI) approach as a three-tiered prevention system.

Assessment serves a dual purpose across the RtI board. It has to reliably indicate learners at risk and serves as a progress-monitoring tool during and after intervention (Anderson, Lai, Alonzo & Tindal 2011:16). Many studies report oral counting, number identification, quantity discrimination, missing numbers and basic addition/subtraction combinations as measures of mathematics proficiency (Bryant, Bryant, Gersten, Scammacca, Funk, Winter et al. 2008:48; Chard et al. 2005:3; Crawford & Ketterlin-Geller 2008:7; Holmes & Adams 2006:266; Moors et al. 2010:231). Logic would then suggest that these skills (number knowledge) form an integrated part of the intervention programme for Grade 1 learners receiving Tier 2 intervention. The same applies to visual-spatial skills, eye-hand coordination and fine motor skills, and the subcomponents of working memory which form part of the non-numerical abilities for mathematics proficiency. Next systematic teaching and the concrete-representational-abstract model of instruction are highlighted as important tools to promote successful conceptual learning (Bryant, Bryant, Gersten, Scammacca & Chavez 2008:22;

Crawford & Ketterlin-Geller 2008:6; Fuchs, Fuchs & Hollenbeck 2007:15). Instruction is broken down into manageable units that are sequentially scaffolded with concrete representation of the mathematical concepts as the starting point (Feigenson et al. 2013:330). Learners then gradually progress to more abstract representations as they become familiar with the newly taught skills (Geary 2011:258).

In an RtI framework the primary goal would be to provide a platform from which math proficiency is cultivated on the basis of prevention and prior to the onset of substantial academic deficits. Within the hierarchical nature and scope of South African education and mathematics teaching the RtI approach presents an excellent framework for preventive intervention, where the first year of formal schooling should be regarded as a critical year for children to build on their early number sense and numeration in order to acquire new math skills and concepts. In the empirical article the researcher will elaborate on how all the elements of the RtI approach discussed in the previous paragraphs were incorporated during Tier 2 preventive intervention.

10. Conclusion

Assessment studies in recent years about the state of education in South Africa have shown that the number of learners functioning at adequate levels in mathematics is unacceptably low (Nel et al. 2013:235; South Africa 2011c:18-19). The Education White Paper 6 states that every child has the right to education and access to a regular school (South Africa 2001). This implies that children with special needs must also be accommodated through planning and the provision of individual support. In order to become proficient and master the content of the mathematics CAPS curriculum learners must be able to recognise, comprehend and apply basic skills and concepts. Although the visions and targets for improving learner performance in mathematics have been set (South Africa 2011c:18-19; South Africa 2001), empirical research is necessary to support and improve the situation in South Africa (Simkins 2013:4). The researcher has highlighted the importance of early quality intervention that might potentially produce learners that are not challenged in their efforts to learn mathematics for future scholastic success and broader life outcomes. In this theoretical article an investigation of the interplay between various factors concerning mathematical proficiency was based on the three research questions.

- The first part of the article was devoted to the cognitive processes involved. Learning takes place when new information is linked to existing knowledge and previous experiences are stored in the long-term memory. The subcomponents of the working memory (central executive system, phonological loop, visuo-spatial sketchpad and episodic buffer) have been identified as being imperative in mathematics development. A summary (see Table 1) was made of each subcomponent and its unique contribution to mathematical proficiency. The central executive system is engaged in problems where procedural strategies are essential; it also controls direct fact retrieval and encoding from long-term memory. The phonological loop is responsible for number sequencing, multiplication accuracy and mental arithmetic. It is also responsible for the acquisition of number facts. The visuo-spatial sketchpad supports number production and representation, and symbolic magnitude appraisal for quantity discrimination. The episodic buffer plays a pivotal role in problem-solving and calculation as it links up information from the visuo-spatial sketchpad, phonological loop, central executive system and long-term memory during the unravelling process to find the solution to a mathematics problem.

- Furthermore, certain perceptual skills were found to influence mathematics performance (see 7.2). The main period for perceptual development is between the ages of 3 and 7, reaching adult levels around 12 years (Tsai, Wilson & Wu 2008). Perceptual skills serve as an impetus for learning. The reciprocity between visual-perceptual skills, spatial ability, motor development, visual imagery and mathematics was discussed. It was found that when these skills and abilities become automatised through practice the likelihood of more learning experiences is increased as space is freed up in the working memory, resulting in a greater processing capacity to acquire new mathematics skills and concepts in a shorter period of time.
- Basic number sense is defined as the ability to classify and compare one-to-one correspondence, comprehend number magnitude and understand that magnitude relates to counting sequence and place value. Number sense is considered a developmental milestone for early arithmetic and forms the skeletal framework in which basic numerical abilities (see 7.2) develop into more advanced mathematical competence. A child’s number sense is in continuous progression as they devise multiple ways of thinking and using numbers as referents.
- Counting skills can be categorised into six developmental phases (Aunio 2006:3-4) where primary understanding of amounts becomes apparent around the age of 2 years. Gelman and Gallistel identified five conceptual counting principles that are associated

with a child's understanding of how and why a procedure works or whether a procedure is admissible (Stock et al. 2010:251). By school-going age most children have integrated their basic number sense and counting abilities and are able to employ these skills to develop and store arithmetic facts in the long-term memory.

- Many MLD learners have a delay in grasping counting concepts and gaining knowledge on different features of arithmetic skill development. The critical determinant of a learner's mathematical success is how soon they progress from using counting-based procedures to memory-based processes. In MLD learners there seems to be a delay in this transition. From both a cognitive and a numerical perspective we can differentiate three types of mathematics learning disabilities: semantic, procedural and visuo-spatial deficits (see 8). These subtypes guide the identification of learners at risk, help to distinguish between a mathematical deficit or delay and are necessary to plan intervention strategies within a response to intervention (RtI) framework.

The researcher is of the opinion that the RtI approach can be adopted to design additional support and provide multi-level teaching to address different learning needs in South Africa's inclusive education system. This model is often described as preventive in nature where investments are made to minimise the number of new cases rather than wait for a specific learning problem to occur. During the empirical research an intervention programme will be implemented which includes all the various factors to promote mathematical proficiency from an early age. The success of this multi-tiered intervention model will be investigated as it will form the platform from which intervention will take place.

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ARTICLE 2

MATHEMATICS PROFICIENCY AMONG GRADE 1 LEARNERS: THE DEVELOPMENT AND IMPLEMENTATION OF A NUMERICAL INTERVENTION PROGRAMME BASED ON THE RESPONSE TO INTERVENTION (RtI) APPROACH

ABSTRACT

Many factors contribute to and predict success in mathematics. In the South African education system a disturbing number of learners enter formal schooling without the necessary competence in the prerequisite skills, which negatively influences their learning experiences. This disadvantage makes them vulnerable and places them at risk to develop a mathematics learning disability (MLD). This study examined the effects of a six-month preventive intervention period for at-risk Grade 1 learners. The intervention purposefully targeted the skills identified in the literature as important for mathematics development. It involved both developmental and numerical skills.

The results demonstrated that Grade 1 learners in the experimental group (N = 36) significantly outperformed those in the control group (N = 35) with regard to perceptual and development skills, including numeracy abilities. Pearson correlation analyses yielded the most significant relationships between numeracy skills (+ and –) and the following ASB subtests, namely Visual Perception (r = 0.61); Spatial Abilities (r = 0.66); Numerical Abilities (r = 0.65); Gestalt (r = 0.61) and Visual Memory (r = 0.62). As we move forward in understanding the nuances of implementing a response to intervention approach to support Mathematics learning disability, assessing the effectiveness of strategies to improve learners' mathematical skills is essential. Furthermore, results from this study demonstrated high effect sizes, confirming the value and contribution this research may have in directing South African and international research with regard to supporting learners with mathematical learning disabilities. Recommendations were made for future research and the implications for best instructional practice were outlined.

Keywords: *Mathematics learning disability (MLD), response to intervention (RtI) framework, scaffolding techniques, concrete-representational-abstract (CRA) activities, number sense*

1. Introduction

The primary objective of the South African education system is to provide quality education for all learners that will enable them to achieve their full potential (Prinsloo 2001:344). Education is a constitutional right and underpins the belief that it could provide for and sustain learning for all children (South Africa 1996a: sec. 29:1). Consequently, the fundamental aim of the South African Education Policy White Paper 6 is to cater for the needs of all learners, including learners with barriers to learning (South Africa 2001). Schools and educators are therefore liable and responsible for the mathematical proficiency of learners. An all-important educational goal would thus be to ensure that early mathematics instruction meets the needs of all learners with or at risk for mathematics learning disability (MLD).

One of the core difficulties that learners with MLD tend to experience is number sense development. As outlined by Berch (in Prinsloo 2001:344 and Van Nes & de Lange 2007:213), "processing number sense ostensibly permits one to achieve anything from understanding the meaning of numbers to developing strategies for solving complex math problems". Learners with MLD are moreover consistently weak at retrieving arithmetical facts from long-term memory (Dowker 2005:324) and have trouble with conceptually and procedurally understanding mathematics problems (Doabler & Fien 2013:277).

The aim of this study is to investigate the importance of identifying early signs and predictors of MLD to alleviate and possibly prevent the effects of MLD in later grades. An effort to meet at-risk learners' mathematical needs requires an integrated system, such as the response to intervention (RtI) approach, for the early identification of MLD, the implementation of effective intervention and ongoing progress monitoring (Ketterlin-Geller, Chard & Fien 2008:33). In agreement with many authors, the researcher holds the view that it might be advantageous to investigate the possibility of preventive intervention for school beginners before the onset of mathematics failure (Bryant & Bryant 2008:3; Bryant, Bryant, Gersten, Scammacca & Chavez 2008:20; Dowker 2005:330; Geary, Baily & Hoard 2009:265). In the literature review in Article 1 (paragraph 6) certain contributing factors and skills were identified that might affect mathematics acquisition. It is imperative to consider these skills and factors when drawing up an intervention programme as learners should be guided through effective scaffolding techniques to acquire and expand their mathematics knowledge (Feigenson, Libertus & Halberda 2013:330). The researcher used the literature to design a well-balanced mathematics intervention programme and will apply it to assist learners in dealing with their mathematics difficulties.

2. Theoretical framework

The researcher extended the causal model of Morton and Frith (Frith 1992:13; Frith 1998:191) as the theoretical scope for this study. Morton and Frith are both well-established researchers in the field of cognition and development. The causal model recognises both extrinsic and intrinsic barriers to learning. Morton challenges researchers to include cognition as a further source in the causal modelling approach to understanding developmental disorders (Korell, Marino & Ferraro 2010:325). He proposes a diagrammatic model (see Figure 1) to delineate the relationships between biological processes and cognition, which in turn lead to behaviour. Environmental influences are also included at all levels (Frith 1998:193).

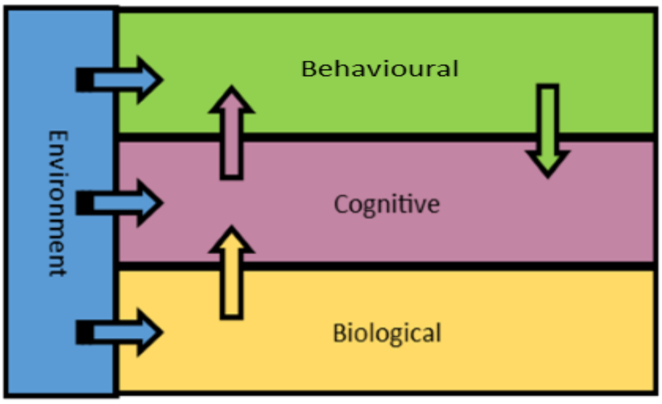

Figure 1. Diagrammatic representation of the causal model of Morton and Frith.

The causal model seems to be an excellent tool for examining childhood development and understanding the causes of barriers to learning, as well as the diagnoses and modes of intervention. With regard to this study and MLD, this means that any of these contexts may potentially contribute to mathematics difficulties. These may for example include environmental circumstances which influence prior knowledge and skills, biological heredity, a combination of other learning disabilities with MLD, insufficient time to tackle barriers to learning and support educators who have not received adequate training to meet the needs of learners. Although the researcher acknowledges all intrinsic and extrinsic factors that influence the development of barriers to learning, the focus will specifically be on the cognitive and developmental influences on mathematical proficiency. The only behavioural and environmental factor that will be taken into account is the effect of intervention on the mathematics performance of at-risk or MLD learners. The application and usefulness of this causal model will be demonstrated in more detail in section 6.3 (Developing the intervention programme).

3. The research problem and questions

The current state of mathematics performance among learners in South Africa is a major cause for concern. The Centre for Development and Enterprise (CDE) formulates practical policy proposals for addressing social and economic challenges. Two university-based experts, Nicholas Spaull and Charles Simkins, were commissioned to write research papers to the CDE on the state of schooling in South Africa as of early 2013. They concluded that the teaching of mathematics in South African schools is among the worst in the world (McCarthy & Oliphant 2013:3; Simkins 2013:4). In a Timms (Trends in International Mathematics and Science Study) study conducted in 2011 it was found that South African learners had the lowest performance of the 21 middle-income countries that participated (Simkins 2013:4). Spaull (2013:10) reports that South Africa's extremely high youth unemployment rate (50 per cent) is closely linked to the quality of schooling. Mathematics is not only a core requirement for higher education, but also for most modern knowledge-intensive work. The lack of knowledge regarding the teaching of mathematics may translate into inadequate support for at-risk learners. Clearly all-embracing interventions are needed to reconstruct perspectives and priorities for promoting mathematics proficiency across all borders of our current school system.

Given the above problem statement, the researcher formulated the following research questions:

- Can Grade 1 learners at risk for mathematics difficulties significantly improve their performance by participating in the intervention programme, which includes a combination of direct instruction and mathematical scaffolding techniques?

- Do significant relationships exist between certain perceptual skills (e.g. visual-spatial skills, visual-motor skills and fine motor skills) and Grade 1 learners' mathematics proficiency?

- Is the response to intervention (RtI) framework a suitable approach for improving mathematics proficiency?

4. The aims of the study

The main aim of this quasi-experimental study was to design and implement a numerical intervention programme for at-risk Grade 1 learners to address possible mathematical difficulties within an RtI framework. The programme included the different skills required for mathematical proficiency as well as developmental skills which proceed from and are associated with competence in and an understanding of the number system. After the intervention period of six months, re-assessment was to take place to determine whether the learners had significantly improved mathematical skills and concepts that would enable them to participate in the learning process without additional support.

5. Ethical procedures

As part of the process of obtaining informed consent, the Free State Department of Education (reference number: 16/4/1/01-2013) as well as the parents, headmaster and educators of the participating school were informed of the purpose of the study and approval was sought before commencement. At the same time, it was explained that participants' privacy and anonymity would be strictly protected and that non-participation entailed no disadvantage. Consent from the Ethics Committee of the Faculty of Education at the University of the Free State was also obtained (Ethical clearance number: UFS-EDU-2013-050).

Special care was taken to ensure that no participants with vulnerabilities were selected. Suitable candidates fell within the criteria for inclusion. The researcher was available to monitor participants and the study itself. As one participant moved outside the demographic area during the study, this participant no longer met the criteria for a suitable candidate and was not included in the study. Little could be done to compensate for this risk factor. The study did not collect identifying information of individual participants (e.g. name and address); participants' anonymity was therefore ensured. All data gathered from this study is treated as confidential. The researcher will securely store the data documents for a minimum of three years. If the data is not needed after this period, it will be destroyed. The study is of a quantitative nature and statistics will be available to stakeholders such as the Free State Department of Education and the Faculty of Education at the University of the Free State.

The research was done by the primary researcher under the direct supervision of the research supervisors assigned by the Faculty of Education, University of the Free State. All data was processed by a qualified data programmer.

6. The research methodology

6.1 The research design

The empirical investigation entailed a quasi-experimental, pre-test-post-test research design with matched experimental and control groups. The control group was selected in such a way that the dependent variables resembled those of the experimental group as closely as possible before commencement of the experimental intervention.

With reference to the formulated aim, the empirical study entailed the following:

- the identification and assessment (pre-testing) of Grade 1 participants (discussed below)
- the development and implementation of a numeracy intervention programme within the RtI framework
- the post-testing (mathematics scores) to determine the effect of the mathematics intervention programme on the performance in mathematics of Grade 1 children.

6.2 Participants and sampling design

A purposive sampling design was applied to select learners who could provide the necessary information regarding mathematical deficiencies. Grade 1 learners were selected from a mainstream school in the Motheo district in the Free State province. The experimental and control groups consisted of 60 Grade 1 learners selected from one multicultural, English-medium, Section 21[1] school. The same demographic area allowed control over variables such as language, educational environment and socio-economic factors. General educators from the school assisted in administering screening tests and identifying participants. The control group received classroom instruction in accordance with the guidelines of the Foundation Phase Curriculum and Assessment Policy Statement (CAPS) (South Africa 2011) and only took part in the pre- and post-testing. The experimental group received intervention by means of the pull-out system, where learners were withdrawn from class during the assigned periods.

In order to obtain suitable candidates for the experimental group, the following criteria were established:

- Participants had to be in Grade 1.

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[1] Section 21 schools receive partial funding from the department (subsidies) and are responsible for managing their own finances and maintenance. Parents of Section 21 schools are liable for school fees (South Africa 1996b).

- Participants (male and female) had to be 6 years old, turning 7 in the year when the study was conducted.
- Participants had to be English Second Language learners whose Language of Learning and Teaching (LOLT) was English.
- Participants had to be free of additional impairments, such as attention-deficit hyperactivity disorder (ADHD) or any other behavioural disorders.
- Participants had to qualify as learners in need of mathematics intervention; therefore they had to perform below average in the screening tests, i.e. an average 3 and 3 > in the ASB subtests, and below age level on the Ballard and Westwood Timed Arithmetic Tests.

General classroom educators (Grade 1) assisted in the identification of the participants. The selection involved administering certain measuring instruments, namely:

- ASB tests (Aptitude Test for School Beginners). These consist of eight subtests (perception, spatial, reasoning, numerical, gestalt, coordination, memory, and comprehension and listening skills).
- Ballard and Westwood Timed Arithmetic Tests. These two tests consist of a series of arithmetic sums (add and subtract) which have to be completed within a timeframe of one minute. They rely on the fast recall of basic number facts from long-term memory.
- Raven’s Coloured Progressive Matrices. This test is a non-verbal, “culture friendly” multiple-choice IQ test, designed for children between the ages of 5 and 11. In each test item the child is asked to identify the missing element that completes a pattern. John C. Raven originally developed the test in 1936.

These tests were used because they are regarded as reliable and valid; they can be applied for both the first assessment and the re-assessment after six months. As specific mathematics problems can be identified, the tests have diagnostic value as well. The study population comprised all Grade 1 learners, who had to complete permission forms before the researcher could conduct the first assessment. Of the 139 learners who received forms for permission to participate, 133 learners returned their forms and participated in the pretesting. The sample consisted of 60 at-risk learners with mathematics problems.

All the tests were administered and scored in accordance with the procedures outlined in the test manuals. The tests were marked and the results were verified by an independent marker.

Of the 131 learners who participated in the pre-tests, 60 met the criteria and were included in the final sample. An attempt was made to match the experimental (N = 30) and control groups (N = 30) by pairing the learners according to age, gender and pre-test scores (mathematics). With regard to gender, 14 boys and 16 girls were assigned to the experimental group, and 19 boys and 11 girls to the control group. The average age of learners in the experimental group was 6.6 years (SD = 0.30), compared to the control group, which was also 6.6 years (SD = 0.39). The means and standard deviations of learners in the control and experimental groups with regard to all the biographical data and test variables are presented in Table 1 and discussed in the section that deals with results and hypothesis testing (see section 6.6).

After the assessments were conducted, the experimental group received intervention by way of the pull-out system, where learners were withdrawn from class during the assigned periods (for six months). During the same period, learners in the control group received normal classroom instruction in accordance with the Curriculum and Assessment Policy Statement for the Foundation Phase.

In the following section the researcher will elaborate on the RtI framework that informed this intervention study, including the development and implementation of the numeracy intervention programme.

6.3 Developing the intervention programme

When setting objectives for early intervention in respect of mathematics skills, concepts and processes, one must be mindful of findings from the research literature regarding the characteristics of young learners that encounter problems with mathematics (see Article 1), for example, difficulties with number sense and arithmetic. The aim of the study is to investigate best practices and effective intervention. The intervention strategies are based on skills that precede and are linked to mathematical development, which, when implemented, might potentially decrease the number of learners experiencing failure. In this section the researcher will discuss the rationale for using the RtI framework and elaborate on what this approach entails, followed by a discussion of the development of the numeracy intervention programme, including the activities that were developed, the nature of instruction and the procedures that were followed during its implementation.

6.3.1 The response to intervention (RtI) approach

Current research in mathematics intervention emphasises the efficacy of a response to intervention (RtI) approach (Anderson, Lai, Alonzo & Tindal 2011:15; Crawford & Ketterlin-

Geller 2008:5; Fuchs, Fuchs & Hollenbeck 2007:13; Fuchs, Fuchs & Compton 2012:257; Moors, Weisenburgh-Snyder & Robbins 2010:226). The significance of this multi-tiered model lies in providing additional support to learners who are underperforming in a particular academic domain and need more intensive assistance to reach the desired outcomes (Ketterlin-Geller et al. 2008:34). Figure 2 gives a schematic representation of the RtI framework. The purpose of screening is to establish which learners are at risk of mathematics failure and which learners are functioning at the appropriate level for success so that instruction can be based on learners' needs (Ketterlin-Geller et al. 2008:34). The complexity of instruction increases in each layer of intervention, i.e. intervention takes place in growing levels of intensity starting with high-quality general instruction in Tier 1 (Fuchs, Fuchs, Compton, Bryant, Hamlett & Seethaler 2007:312; Rolfhus, Gersten, Clarke, Decker, Wilkins & Dimino 2012:1). Tier 1 is characterised by core classroom instruction that benefits all learners regardless of their mathematics proficiency. In Tier 2, at-risk learners who performed below their average peers in Tier 1 participate in supplemental small-group instruction. A second round of screening is done to determine whether learners have made adequate progress in Tier 2. Tier 3 provides intensive instruction, often by special education services, to learners who did not respond to Tier 1 and Tier 2 instruction (Gersten, Beckmann, Clarke Foegen, Marsh, Star & Wetzel 2009: 4-5; Rolfhus et al. 2012:2-3).

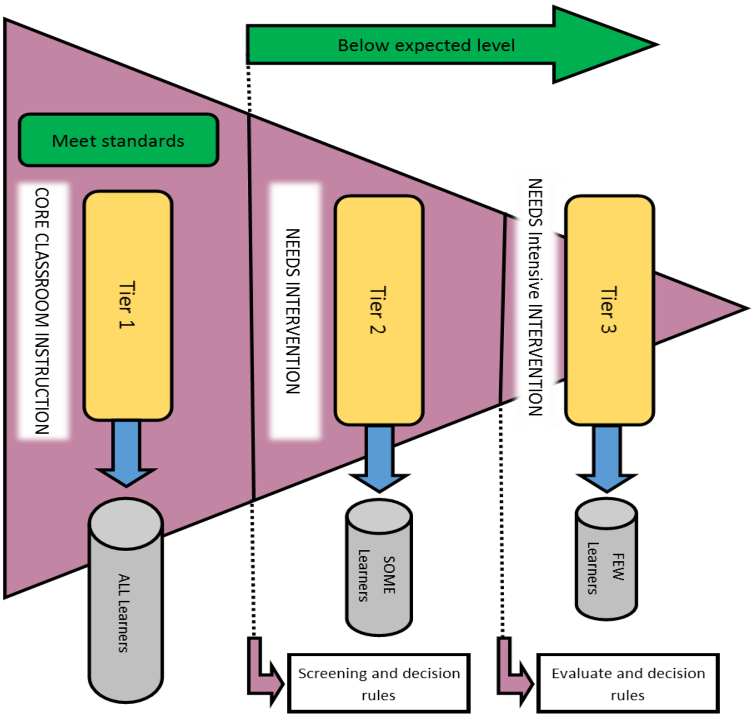

Figure 2. A schematic representation of the RtI framework.

Allsopp and his co-authors (in Allsopp, Mchatton & Farmer 2010:281-282) suggest five anchors for mathematics intervention within the RtI framework. These anchors are considered the axis from which the nature of instruction can be differentiated at each tier depending on the needs of learners and characteristics of the mathematics content, for example, more time will be spent on a specific component of mathematics in Tier 3 than in Tier 1 or 2.

The five anchors are as follows (partially adapted from Allsopp et al. 2010:282):

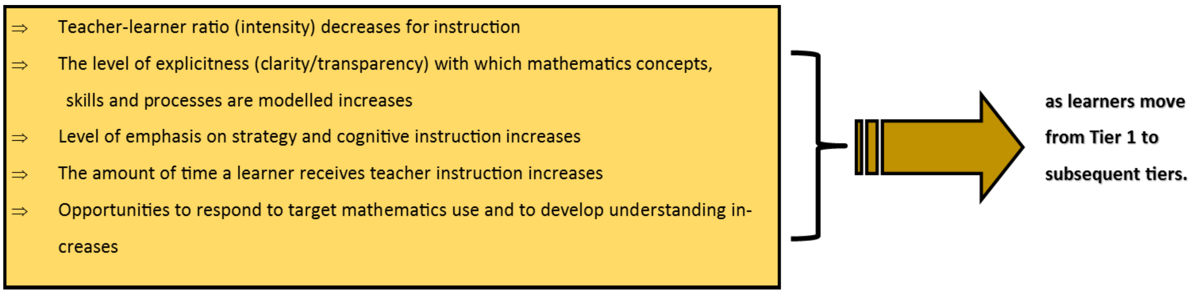

Figure 3. Summary of the five anchors for differentiation in tiered mathematics instruction.

In this study the RtI framework serves as the foundation for intervention with its primary function the prevention of future mathematical difficulties in Grade 1 learners. The researcher focused specifically on Tier 2 intervention for learners that had been identified as at risk. It is important to note at this point that Tier 2 intervention did not replace Tier 1 curriculum-level instruction but rather aimed at filling the gaps of learning for participants who performed below the expected level on the screening tests. A second round of screening took place to determine what the success rate of the programme was. Learners who still performed below the expected level and therefore qualified for Tier 3 intervention were incorporated into the support teaching programme for 2014, where the level of intervention is considered more intense. Learners who tested on level and could progress without additional support as they presented a good foundation for mathematical skills and concepts returned to Tier 1 general education.

What follows is a discussion of the nature of the instructional procedures and strategies that were adapted within the intervention programme during the period of six months.

6.3.2 The intervention programme

6.3.2.1 The nature of instruction

The researcher recognises the importance of effective instructional procedures found in the literature review (see Article 1) on the best practices underpinning mathematical development and preventive intervention. Preventive intervention is a proactive measure intended to keep a potential problem from becoming a disability (Ramey & Ramey 2010:489). The function of preventive intervention in this study is to reduce the prevalence of MLD. Its objective is to address the effects of existing risk factors (e.g. poor prerequisite skills), and it is aimed at learners who scored below the expected level on screening (pre-test) variables. Studies on effective instructional procedures have shown that explicit instruction through a range of scaffolding techniques, with concrete-representational-abstract (CRA) activities, immediate corrective feedback and the provision of a solid conceptual foundation, and effective procedural strategies will improve learner performances (Bryant, Bryant, Gersten, Scammacca & Chavez 2008:22; Crawford & Ketterlin-Geller 2008:6; Fuchs et al. 2007:15; Feigenson et al. 2013:33; Geary 2011:258). Explicit instruction adds value to teacher-learner interactions and increases the number of learning opportunities for at-risk or MLD learners (Doabler & Fien 2013:277). It involves the expanded use of a graduated or scaffolded instructional sequence that starts with the concrete manipulation of objects, proceeds to pictorial representations and lastly progresses to abstract symbols (Ketterlin-Geller et al. 2008:35). During the onset of teaching ample time is needed, which then slowly decreases as learners become more automatised in skills and concepts by means of practice and more experience. Graduated or scaffolded techniques may include opportunities for guided practice or the reteaching of a concept or procedure (Nel, Nel & Hugo 2013:63). Immediate corrective feedback assists learners in grasping new concepts and skills and prevents them from "learning" incorrect procedures by repeating a wrong technique. By using the minus procedure, learners will, for example, undercount by recounting the last number name of the first addend when adding the second addend, as they are not aware that the last number represents the cardinal value of the first set: 5 + 3 = 7, "five plus three are … five, six, seven … the answer is seven". To illustrate the researcher's viewpoint on the nature of instruction the causal model of Morton and Frith (see section 2) is applied.

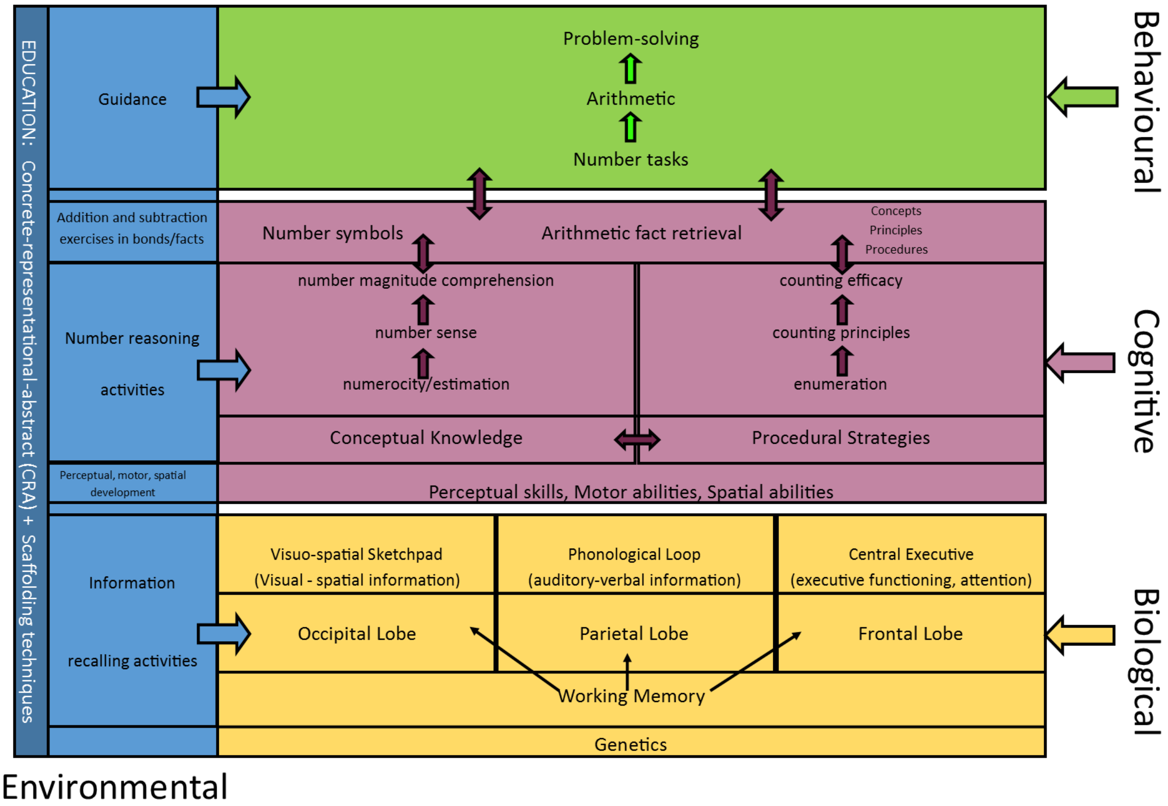

Figure 4. The causal model of Morton and Frith applied to the nature of intervention in the study.

By the time learners start formal schooling they might have a high probability for mathematics difficulties or even MLD due to a lack of informal experience with core mathematics skills (see Article 1 section 7.3) such as numerocity (counting arrays by estimation) and basic number sense. Difficulty in number sense impairs the acquisition of the procedural and conceptual aspects of mathematics (Doabler & Fien 2013:277-278). Learners who experience procedural delays will, among others, struggle to use effective counting strategies and recollect answers to basic combinations. Conceptual understanding happens when a learner grasps mathematical reasoning and interpretation (such as comparing number relationships of more and less). When the procedural and conceptual aspects of mathematics are impaired the acquisition and storage of arithmetical facts in long-term memory are hindered. A good conceptual understanding cuts down on memory load, which frees up available memory capacity to learn and store new information in long-term memory (Bryant, Bryant, Gersten, Scammacca, Funk, Winter, Shih & Pool 2008:50; Bryant, Bryant, Gersten, Scammacca & Chavez 2008:20). Therefore, impaired basic number sense and numerocity lead to poor basic counting strategies and principles, which in turn lead to poor conceptual and procedural understanding. This hinders the acquisition of arithmetic facts and more sophisticated knowledge of the size of numbers, number relationships and patterns.

From this discussion, it is evident that the nature of instruction in this study displays (a) explicit instruction; (b) scaffolding techniques; (c) CRA activities and (d) immediate corrective feedback; and also (e) fostered a solid conceptual foundation and effective procedural strategies.

6.3.2.2 The selection of components

The selection of components to be included in the intervention programme was built on the analysis of the particular sub-skills learners need for mathematics proficiency. It included, for example, counting and counting principles, arithmetic and perceptual skills. These components should not be viewed as all-inclusive, as they were selected from earlier studies based on their importance to mathematical development. See Appendix 2 for a description of these components, their definitions, how each relates to mathematics and example activities used in the intervention programme.

6.3.2.3 Procedure

Both the experimental and the control group received quality classroom instruction based on the prescribed Foundation Phase CAPS curriculum of the Free State Department of Education during the second and third school terms of 2013 (six months). In this period learners in the experimental group also received Tier 2 intervention for 45 minutes once per week. The intervention programme focused on strategies that would improve core mathematics skills of at-risk learners in a small group setting. The programme was embedded into the five anchors suggested by Allsopp and his colleagues in the RtI framework and included all the components in Appendix 2, which form part of well-developed core mathematics abilities. The intervention programme was furthermore divided into five subsections including counting skills and simple arithmetic, number identification, missing numbers, quantity discrimination and shapes. Each subsection consisted of four training sessions, three weeks of intervention lessons and then an extra session in the fourth week to revise all the work. The revision lesson allowed for extra practice in the newly acquired skills or concepts. The fourth session also gave an opportunity to compensate for any work learners had missed due to unforeseen circumstances such as in cases where it took longer for them to grasp a concept or where learners were not able to attend their session because of mandatory events taking place at school. Activities were scaffolded and structured in a concrete-representational-abstract manner. The components of the programme were arranged according to their difficulty level, e.g. the first few lessons comprised fewer activities and were more concretely orientated. For the purpose of clarification of the lesson structure and the nature of activities in which learners engaged, part of a lesson from the subsection *quantity discrimination* is discussed.

6.3.2.4 Example lesson on quantity discrimination

The researcher firstly revised concepts and skills covered in the previous session and then began the lesson by verbally explaining the concept of quantity discrimination using concrete aids as examples. Learners were then provided with a booklet containing worksheets to be answered in the remainder of the lesson. Both positive and immediate corrective feedback was given to learners. After learners had completed the first worksheet instructions were given for the next worksheet to be completed. This step was repeated until all the worksheets on quantity discrimination in this section were completed. In the next two sessions more activities on quantity discrimination were completed to ensure mastery of concepts and skills.

The final 20 minutes of each lesson consisted of perceptual training including concrete, semi-concrete and abstract activities. Perceptual skills were selected to complement each subsection of the intervention programme. In this lesson, for example, visual form constancy and visual discrimination were included. Aspects of motor and spatial skills are needed to perform numerous functional tasks such as cutting, pasting and drawing. These skills form an integral part of acquiring academic abilities and for this reason the researcher deemed it important to include motor activities, specifically fine motor skills and eye-hand coordination as well as spatial activities (visual-spatial, visual-motor and gestalt) into all the intervention sessions.

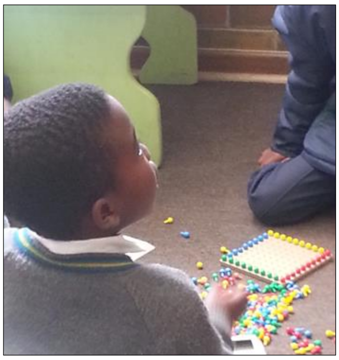

Figure 5A **Figure 5B** **Figure 5C**

In Figure 5A, 5B and 5C learners in the experimental group use various activities to develop their mathematical and perceptual skills.

6.4 The research hypothesis

The following research hypotheses were formulated for this intervention study.

Research hypothesis 1

The mathematics performance of Grade 1 learners who were exposed to the response to intervention (RtI) mathematics programme was better than that of learners who were not exposed to the programme.

In statistical terms, this research hypothesis may be represented as follows:

H_0: $\mu 1 = \mu 2$

H_1: $\mu 1 > \mu 2$

where $\mu 1$ = average numeracy scores of the population of Grade 1 learners in the experimental group

$\mu 2$ = average numeracy scores of the population of Grade 1 learners in the control group

A directional alternative hypothesis was formulated since it was expected that the mathematics performance of learners who completed the RtI mathematics programme (experimental groups) would be better than the performance of learners who were not involved in the programme, based on the reassessments.

Research hypothesis 2

There is a positive correlation between the subtests of the ASB test[2] and the mathematical abilities[3] of Grade 1 learners in this study.

In statistical terms, this research hypothesis is represented as follows:

H_0: $p = 0$

H_1: $p \neq 0$

where p denotes the correlation coefficient in the population.

Since previous research investigating the possible relationships between different aspects of perceptual and developmental skills and their relationship with mathematical abilities yielded inconsistent results, the researcher in this study formulated a non-directional alternative hypothesis.

6.5 Statistical analyses

With regard to Hypothesis 1, a quantitative approach was applied to analyse and interpret numerical data collected from the pre-tests and post-tests of the control and experimental group to determine whether the intervention programme was successful. In this study the Mann-Whitney test was used to test Hypotheses 1 and 2. The Mann-Whitney test is the non-parametrical counterpart of the *t*-test for independent groups. The test determines whether the null hypothesis – which states that the distributions of two non-related populations, including their central values, are identical in all respects – is true (Huysamen 1997:129). In contrast, the alternative hypothesis states that the two populations are not identical. The alternative hypothesis to Hypothesis 1 is directional; therefore a one-tailed test was used. In this case, the decision rule is as follows:

If $U \leq U_{0.05}$, for ^{n}EXPERIMENTAL and ^{n}CONTROL, reject H_0.

Pearson correlation analyses were used to investigate Hypothesis 2, namely the possible relationship that exists between the eight subtests of the ASB tests and Grade 1 learners' mathematical achievements (Ballard + and – tests). The correlation coefficient is a measure of the strength of the linear relationship between two quantitative variables (Maree & Pietersen 2007:234).

The Pearson correlation coefficient has the following properties:

- It is denoted by *r*

[2] The ASB test battery consists of the following eight subtests: Perception, Spatial, Reasoning, Numerical, Gestalt, Coordination, Memory and Verbal Comprehension.
[3] Mathematical abilities were measured by the Ballard + and – tests

- Its minimum is –1 and the maximum is +1
- Values close to +1 (positive) or –1 (negative) are indicative of a strong linear relationship
- Values close to 0 indicate a weak linear relationship

(Maree & Pietersen 2007:236).

The alternative hypothesis to Hypothesis 2 is non-directional; therefore a two-tailed test was used. In this case, the decision rule is as follows:

If /*tr*/> *t* (1- α/2) with N-2 degrees of freedom, reject H_0.

In order to investigate the results in the study the 5% level (α = 0.05) of significance was used.

6.6 Results and hypotheses testing

6.6.1 Pre-test results

A quasi-experimental design with matched groups was used in the study. In order to determine whether the experimental and control groups were reasonably similar with regard to the dependent variable before the beginning of the investigation, the two groups were compared with respect to the pre-scores obtained in the ASB and mathematics tests. Descriptive and inferential analyses were conducted using SPSS 16.1 (2000). The means and standard deviations for learners in the control and experimental groups of all the biographical and test variables are presented in Table 1. These include: learners' average age; gender; non-verbal IQ scores (Raven's); the scores of the ASB tests (Test 1, Visual Perception; Test 2, Spatial Orientation; Test 3, Reasoning; Test 4, Numerical; Test 5, Gestalt; Test 6, Eye-hand Coordination; Test 7, Visual memory and Test 8, Verbal Comprehension), as well as the scores for the two tests measuring mathematical abilities, namely the Ballard + and Ballard –.

From the pre-test scores it is evident that learners in the experimental and control groups were very similar prior to the intervention (see Table 1). In order to determine whether a significant difference existed between the average pre-test scores, the Mann-Whitney U-test was used (5% level of significance). For the given group sizes (experimental, *n* = 30; control, *n* = 29), the critical value in this case was 305. Mann Whitney U-tests revealed no statistically significant differences between the experimental and control groups before the experimental intervention: chronological age (*U* = 443; *df* = 57*; p* = 0.92), non-verbal IQ (Raven's) (*U* = 423.5; *df* = 57; *p* = 0.86); Ballard + (*U* = –0.12; *df* = 57*; p* = 0.96); Ballard – (*U* = –0.12; *df* = 57*; p* = 0.96); Visual Perception (*U* = –0.12; *df* = 57*; p* = 0.96); Spatial (*U* = –0.12; *df* = 57*; p* = 0.96); Reasoning (*U* = –0.12; *df* = 57*; p* = 0.96); Numerical (*U* = –0.12; *df* = 57*;*

p = 0.96); Gestalt (U = –0.12; df = 57; p = 0.96); Coordination (U = –0.12; df = 57; p = 0.96); Memory (U = –0.12; df = 57; p = 0.96) and Verbal Comprehension (U= –0.12; df = 57; p = 0.96).

Table 1: Mann Whitney U comparisons between the experimental and control groups for age and Raven's test scores (N = 59)

Groups	Age	Raven's
	M (*sd*)	*M* (*sd*)
Experimental group (n = 30)	6.6 (0.30)	15.5 (3.91)
Control group (n = 29)	6.6 (0.39)	15.68 (4.97)

*p = < 0.05

After the pre-testing learners were randomly assigned to the two groups (i.e. the experimental or control group). Learners in the experimental group were exposed to the mathematics intervention, which involved a combination of intervention techniques, such as explicit instruction and guided scaffolding to develop mathematical proficiency. These intervention strategies included concrete-representational-abstract (CRA) representations of new skills and concepts. The experimental group was withdrawn from two Grade 1 classes at the school while the control group followed the mathematics curriculum of the school. The same amount of time was spent on mathematics instruction compared to the experimental group. However, learners in the control group did not receive explicit instruction in mathematics proficiency.

6.6.2 Post-test results

From Table 1 it is evident that the average pre-test scores (all tests that were administered prior to the intervention) of both the experimental and the control group were very low and that learners experienced delays in both the Ballard + and – test, as well as in all eight subtests of the ASB. From the post-test scores in Table 2 it is evident that the mean scores of learners in the experimental group demonstrate a noteworthy improvement in all the tests mentioned above, for example: In the Ballard tests, which deal with basic arithmetic, i.e. direct fact retrieval, the post-test scores improved across the group in the experimental group. The average rate of correct responses for addition in the experimental group was 3.46 (SD = 1.67) and in the control group 3.44 (SD = 1.95). According to the comparative pre- and post-test addition scores of learners in the experimental group, the rate of correct responses showed a very significant improvement (post-test score 10.9), while in the control group they also showed a slight improvement (post-test score 4.6). Similar results were shown with regard to learners' subtraction skills (basic arithmetic). The pre-test scores of both groups were very low (experimental: 1.46; control: 1.48). However, the average post-test scores of the experimental group improved considerably (average score 9.1) compared to the average post-test score of the control group, which only showed a marginal improvement (average score 3.3).

From Table 2, it is evident that when comparing learners' pre- and post-intervention results for the ASB subtests, similar results were obtained with regard to all the test outcome variables of the ASB test, for example, the average score for Test 1 (Visual Perception) improved from 7.56 to 9.5 correct responses. Spatial Orientation improved from 5.56 to 8.03; Reasoning showed a slight improvement from 7.8 to 9.2, while Numerical Abilities demonstrated a very significant improvement from 3.6 to 7.2. On the Gestalt measure the score improved from 60.83 to a mean score of 85.76 after the intervention period. ASB test scores for coordination also showed similar growth and increased considerably from the pre-intervention to the post-intervention condition (i.e. from an average score of 11.90 to 20.1). Results for Visual Memory showed very good progress and increased from an average score of 5.9 to an average score of 8.93. Verbal Comprehension also improved significantly from a score of 9.03 to 13.28. From a comparison of the number of correct responses with those of learners in the control group, it is clear that the control learners demonstrated a slight and marginal improvement in most of the ASB subtests, for example in Visual Discrimination their scores increased from 7.6 to 7.8; in Spatial Orientation they improved from 5.6 to 6.6; in Numerical aspects from 3.5 to 4.3; in Visual Memory from 5.7 to 6.4 and in Verbal Comprehension a small improvement, from 9.1 to 10.51, but none of these were significant. With regard the other ASB subscales the control learners also demonstrated good progress, but compared to the improvement of learners in the experimental group, their average post-test scores with regard to Reasoning (7.9) and Gestalt (75.31) were still significantly weaker.

In the following section the researcher will determine whether the improvement of scores with regard to the ASB tests and mathematical tests was statistically significant by testing the formulated Hypothesis 1 (see section 6.4).

Table 2. Pre- and post-test scores of the experimental and control groups (*N* = 59): Ballard + / – and ASB subtests

Tests	Groups			
	Experimental group		Control group	
	Pre	Post	Pre	Post
Ballard +	3.46	10.1*	3.44	4.65
Ballard –	1.46	9.16*	1.48	3.34
ASB: Test 1	7.56	9.5*	7.61	7.8
ASB: Test 2	5.5	8.03*	5.6	6.6
ASB: Test 3	7.8	9.20*	7.8	7.9
ASB: Test 4	3.6	7.20*	3.5	4.3
ASB: Test 5	60.83	85.76*	61.3	75.31
ASB: Test 6	11.90	20.91*	11.86	16.0
ASB: Test 7	5.90	8.93*	5.7	6.44
ASB: Test 8	9.03	13.28*	9.1	10.51

Note: The subtests of the ASB 1-8 are Visual Discrimination, Spatial, Reasoning, Numerical, Gestalt, Eye-hand Coordination, Visual Memory and Verbal Comprehension.
*p = < 0.05

6.6.3 Hypothesis testing

Hypothesis 1

In order to test Hypothesis 1 the Mann-Whitney U-test was used. Since the alternative hypothesis is non-directional, a two-tailed test was used. To investigate the results, the 5% level (α = 0.05) of significance was used. The average pre-scores of the two groups were compared, and the results are set out in Table 2.

From the calculated U-values for the different subtests it is evident that the U-values for all subtests were greater than the calculated critical value for the specific sample, for example: Ballard + (50.5 < 305); Ballard – (27 < 305); Visual Perception (29.5 < 305); Spatial (199 < 305); Reasoning (171 < 305); Numerical (102 < 305); Gestalt (275 < 305); Coordination (226 < 305); Memory (114.5 < 305) and Verbal Comprehension (74 < 305). Thus the null hypothesis was retained with regard to all the pre-test scores. In practice this means that there was no significant statistical difference between the average pre-test scores of the experimental and the control group prior to the intervention. Therefore the average post-test scores of the two groups were compared after the intervention. These results are presented in Table 2. They show a noticeable difference between the average post-test scores of the experimental and the control group with regard to all the tests that were administered. Further inspection revealed that the calculated U-values for each subtest were smaller than the critical value of 305. For example: Ballard + (50.5 < 305); Ballard – (27 < 305); Visual Perception (29.5 < 305); Spatial (199 < 305); Reasoning (171 < 305); Numerical (102 < 305); Gestalt (275 < 305); Coordination (226 < 305); Memory (114.5 < 305) and Verbal Comprehension (74 < 305). In practice this implies that the null hypothesis can be rejected in favour of the alternative hypothesis with regard to all the test measures applied in the study. This further demonstrated that statistically significant differences existed between the average post-test scores (all subtests) with regard to the experimental and the control group. Hence, it can be concluded that the mathematical performance of Grade 1 learners who were exposed to the response to intervention (RtI) mathematics programme was significantly better than that of Grade 1 learners who did not participate in the intervention programme.

Furthermore, as mentioned above, a statistically significant difference existed between the experimental and the control group with regard to all test measurements (post-tests) after the intervention of six months, hence the null hypothesis was rejected in favour of the alternative hypothesis with regard to hypothesis one. These include Ballard + (U = 10.40; df = 57; p < 0.000; r = 0.87); Ballard – (U = 10.40; df = 57; p < 0.000; r = 0.88); Visual Perception (U = 10.40; df = 57 ; p < 0.000; r = 0.66); Spatial (U = 10.40; df = 57; p < 0.000; r = 0.74); Reasoning (U = 10.40; df = 57; p < 0.000; r = 0.57); Numerical (U = 10.40; df = 57; p < 0.000; r = 0.86); Gestalt (U = 10.40; df = 57 ; p < 0.000; r = 0.82); Coordination (U = 10.40; df =57 ; p < 0.000; r = 0.64); Memory (U = 10.40; df = 57 ; p < 0.000; r = 0.83) and Verbal Comprehension (U = 10.40; df = 57; p < 0.000; r = 0.73).

According to Gay and Airasian (2003), practical significance refers to the educational value of the results obtained in the study, and an effect size is the measure of the practical significance. With regard to the Mann Whitney U-test, it is evident that the calculated r for the different test measures ranged from 0.57 to 0.88. This implies high practical significance for this study.

Hypothesis 2

With regard to Hypothesis 2, as stated in section 6.4, the following null hypothesis was tested: There is no significant positive correlation between the ASB subtests and the mathematical abilities of Grade 1 learners.

This hypothesis was stated for each of the eight ASB subtests (namely Visual Perception, Spatial Orientation, Reasoning, Numerical Abilities, Gestalt, Eye-Hand

Coordination and Verbal Comprehension) and the two mathematical tests, namely the Ballard + and the Ballard –.

In order to test the null hypothesis, the correlations between each of the eight subtests and the achievement in mathematical abilities (Ballard + and Ballard –) for Grade 1 were calculated. In testing the null hypothesis all 60 participants were used. The results are reflected in Table 3.

Table 3. Correlation coefficients between the eight ASB subtests and mathematical achievement in Grade 1 (Ballard + and –) (N = 59)

Maths Tests	Visual discrimi-nation	Spatial	Reasoning	Numerical	Gestalt	Co-ordination	Memory	Verbal compre-hension
Ballard +	0.61**	0.66**	0.36*	0.65**	0.61**	0.28*	0.62**	0.49*
Ballard –	0.55**	0.55**	0.29*	0.53**	0.50**	0.32*	0.66**	0.41*

* $p < 0.05$

** $p < 0.01$

Focusing on the results in Table 3 the researcher will firstly discuss the results pertaining to the possible relationships that exist between the eight ASB subtests and the Ballard + scores (i.e. basic addition skills). From the results in Table 3 it is evident that the null hypothesis for Grade 1 can be rejected for all the variables. The highest correlation was yielded between learners' spatial abilities and their math addition abilities (r = 0.66, p < 0.00). Further inspection revealed that significant moderate positive correlations were also demonstrated between Grade 1 learners' Visual Discrimination (r = 0.61, p < 0.00), Numeracy (r = 0.65, p < 0.00), Gestalt (r = 0.61, p < 0.00) and Memory (r = 0.62, p < 0.00) and their mathematical abilities (Ballard +). The other subtests, namely Reasoning (r = 0.36, p < 0.01), Coordination (r = 0.28, p < 0.01) and Verbal Comprehension (r = 0.49, p < 0.01), all yielded significantly low correlations with the Grade 1 learners' mathematical achievements.

It is also evident from Table 3 that moderate significant correlations were yielded for the majority of ASB subtests and Grade 1 learners' mathematical subtraction abilities. These include Visual Memory (r = 0.62, p < 0.00); Visual Discrimination (r = 0.55, p < 0.00); Spatial Orientation (r = 0.55, p < 0.00); Numeracy (r = 0.53, p < 0.00) and Gestalt (r = 0.50, p < 0.00). Significant but low correlations were yielded between Verbal Comprehension (r = 0.41, p < 0.00); Eye-hand Coordination (r = 0.32, p < 0.00); Reasoning (r = 0.29; p < 0.00) and Grade 1 learners' mathematical subtraction abilities. It is evident from these results that the null hypothesis can be rejected in favour of the alternative hypothesis, demonstrating moderate to low correlations between all the ASB subtests and Grade 1 learners' mathematical abilities.

From results like these researchers postulate that basic foundational visual perceptual skills assist learners to initially recognise letters and numbers (parts) and ultimately words and figures, thus facilitating the development of both mathematical and literacy concepts (Scheiman & Gallaway 2006:388). Furthermore, it has been suggested that visual perceptual skills such as visual discrimination, visual memory, visual spatial reasoning and gestalt are interrelated and that spatial competency plays a major role in learners' abilities to develop good visual perceptual skills (Groffman 2006; Scheiman & Gallaway 2006). Previous studies involving children with learning disabilities have also shown that visual-motor integration (i.e. Bender test scores) highly correlates with success in both mathematics and reading (r = 0.75) (Koppitz 1975), which is further corroborated by more recent empirical results (Balouti & Nazari 2012:36-37). The results from the present study also correspond to these previous findings, demonstrating significant moderate correlations between subtests measuring visual perception and visual motor-integration and Grade 1 learners' mathematical skills (Ballard +).

From the results it is evident that a learner's mathematics proficiency (skills and concepts) can be improved significantly by following intervention strategies that are evidence-based, explicitly taught and involve both lower and higher order math skills to improve automaticity, as well as by guiding learners through various scaffolding techniques to apply mathematics problem-solving strategies. In addition, the results demonstrated the interrelatedness of cognitive processes, perceptual development and mathematics proficiency. In practice this implies that underlying cognitive skills and mathematically related factors have specific implications for learners' math comprehension and therefore teachers should include practising these skills as part of their mathematics curriculum.

In the following section the author will present a detailed discussion of the results of the study, and integrate the findings with previous research that investigated the role and interrelatedness of visual-perceptual and developmental skills and mathematical abilities among early childhood learners.

6.7 General discussion

The study was preventive in nature and showed the advantages of effective instructional procedures focusing on the basic skills that underpin mathematics development, of immediate corrective feedback within a small group setting and of implementing the five anchors for successful intervention within a response to intervention (RtI) framework (see section 6.3.1). With reference to previous paragraphs, the purpose of preventive intervention as in this study was to decrease the probability of a specific problem developing into a mathematics learning disability (MLD) by addressing identified risk factors, e.g. poor counting skills. Learners who scored below the expected level on the pre-test variables were identified as being at risk for MLD and therefore qualified as candidates for the study. In the following paragraphs, the researcher will reflect on the findings during the six-month period and weigh them against existing empirical research from similar studies.

With regard to the first aim the results of the study demonstrate the benefits of the intervention programme for improving mathematics performance. The results indicated that both groups experienced gains in the course of the study, but that the participants showed larger gains compared to the control learners based on the post-test scores of the Ballard and Westwood Timed Test. This test gives an indication of a learner's knowledge and weaknesses regarding basic arithmetic facts. It also assesses progress and can indicate readiness for higher order mathematics skills such as multiple sign operations (8 – 2 + 3 = ?). A reason for these outcomes might be that specific skills were targeted in the intervention activities (counting, number knowledge and number magnitude reasoning). Mathematics proficiency is to a great extent influenced by the number of opportunities available for building on acquired skills and concepts (Van Nes & De Lange 2007:213). Moreover, the results from the study demonstrated that counting automaticity is an important prerequisite skill for success in mathematics. It serves as the basis for gaining arithmetic facts, and a certain amount of automaticity in counting will free up space in working memory, which directly influences the quality of learning experiences (Bull, Johnston & Roy 1999:422; Geary 2004:4-5; Ginsburg 1997:21). Dyson, Jordan and Glutting (2011:168) report that Grade 1 number sense (and counting skills) predicts computation fluency and problem-solving accuracy in Grade 2 and the presence of MLD in Grade 3. Results on the Ballard and Westwood Timed Test thus highlight the importance of core competencies in mathematics for later success.

The second aim related to the possible relationship that exists between perceptual skill development and mathematics performance. Positive correlations were found between Grade 1 learners' mathematics proficiency and their perceptual development based on the pre-test and post-test scores from the ASB subtests. In the following paragraphs the researcher will firstly discuss the perceptual and development skills that demonstrated the highest correlations with basic arithmetic, namely Spatial abilities, Visual discrimination, Numeracy, Gestalt and Memory. The possible value and interrelatedness of these skills will be presented as an integrated discussion, triangulating the present study results with existing literature that has demonstrated the importance of these skills in mathematics development.

6.7.1 Spatial abilities

The highest correlation was yielded between learners' spatial abilities and their math addition abilities. This is supported by findings by Bull, Davidson and Nordmann (2010:249) whereby better performance in spatial ability tasks correlated with higher scores in arithmetic. Bull et al. (2010:249) further support this finding by stressing young learners' dependence on visual-spatial representations to maintain information in working memory. Pienaar, Barhorst and Twisk (2013:375) and Mohammed (2013:62) found visual-spatial skills to be integral to higher order mathematics. This was confirmed by Gunderson, Ramirez, Beilock and Levine (2012:1238), who found that children's number line knowledge mediated the relation between spatial skill and later approximate symbolic calculation skill.

6.7.2 Visual discrimination

In the study a positive correlation was found to exist between Grade 1 learners' visual discrimination and mathematical abilities. Visual discrimination is used to distinguish between similarities and differences in pictures and symbols, for example 31 and 13. It has therefore a strong analytical component which is often necessary in mathematics (Olivier & Swart, 1974: Reprint 1995:3). Another example would be when learners use seriation and classification to sort a number of objects in sequence based on their similarities or differences in one or more dimensions, e.g. from smallest to largest (Desoete, Stock, Schepens, Boeyens & Roeyers 2009:253; Kivona & Bharagava 2002; Stock, Desoete & Roeyers 2010:251). A strong association was also found between visual discrimination and mathematics in the research conducted by Clutten (2009:38, 40) as well as by Assel and his colleagues (2003:28).

6.7.3 Numeracy

This subtest correlates positively with scores from the Ballard and Westwood Timed Test. In the manual (Olivier & Swart 1995:4) this subtest is described as indicative of a learner's ability to count and grasp quantities, proportions and numbers, all of which are necessary for arithmetic. Berch (2005:145) identified counting, number knowledge (number discrimination), number transformation (addition and subtraction), estimation and number patterns as components necessary for formal mathematics learning. This is parallel to findings in this study.

6.7.4 Gestalt

Van Nes and De Lange (2007:220) describe 'gestalt' or spatial structure as a spatial visualisation ability that enables a learner to mentally or physically reconstruct an object in space. A learner's ability to understand spatial structure is necessary for skills such as ordering, comparing, generalising and classifying. Olivier and Swart (1995:4) report that attentiveness, concentration and the absence of figure-ground confusion are important for success in this test. The study yielded positive results in relation to the Ballard and Westwood test scores.

6.7.5 Memory

Young learners (Grade 1) depend considerably on visual memory for acquiring knowledge in the classroom. Kulp, Earley, Mitchell, Timmerman, Frasco and Geier (2004:53) found that visual memory significantly relates to achievement in mathematics. Deficits in memory lead to difficulties in using concrete aids to count and working out multiple-step addition and subtraction sums (Mohammed 2013:62). Memory is also linked to executive functioning, number sense measures and addition fact retrieval in Grade 1 (Ebersöhn & Eloff 2004:122; Geary, Bailey, Littlefield et al. 2009:411; Kester, Lehnen, van Gerven &Kirschner 2006:94). The findings from this study agree with previous research.

The other subtests, namely Coordination, Reasoning and Verbal Comprehension, all yielded significantly low correlations with the Grade 1 learners' mathematical achievements.

6.7.6 Coordination

The aim of this subtest (Olivier & Swart 1995:5) is to evaluate a learner's ability to do pencil-and-paper activities, i.e. fine motor skills, visual-motor ability and eye-hand coordination. Visual-motor integration and fine motor skills were the best predictor of legible handwriting in studies from Kulp and Sortor (2003:313) and Pienaar (2013:376). Poor handwriting curbs cognitive functions and thus developing mathematic skills (Tükel 2013:5) as learners displayed problems with the size, alignment, shape and spacing of numbers. Handwriting forms part of the functional skills necessary to be successful in school. Poor handwriting constantly competes with learning processes and thus hinders learners' comprehension capacity (Ebersöhn & Eloff 2004:122; Kester et al. 2006:94). As a result, learning barriers develop as learners do not acquire any new knowledge, i.e. these learners do not store newly acquired arithmetic facts in long-term memory for later direct fact retrieval as the learning process is restricted.

6.7.7 Reasoning

Reasoning predicts comprehension and logical thinking (Olivier & Swart 1995:3-4), which are important aspects of learning. It forms part of executive functioning skills and plays a role in computation procedures, organisational skills, attention, mathematics fact retrieval, applying and switching between learned solution strategies and inhibiting irrelevant information from, for example, word sums (see Article 1, section 7).

6.7.8 Verbal comprehension

Comprehension of language is important for the development of mathematics understanding. A study by Gordon (2004:498) confirmed that numerical cognition is affected by the lack of language comprehension as words are used to indicate numerical quantities which form part of the basic vocabulary in a language such as English.

To comprehend and solve a mathematics problem while experiencing a delay in one or more of these skills may severely hinder effective computation and the acquisition of higher order skills (Van Nes & De Lange 2007:223).

Thirdly, the researcher investigated the RtI framework as a suitable approach for addressing and improving mathematics proficiency. The Grade 1 learners in the experimental group had significant delays in core mathematics and perceptual skills prior to intervention. After six months' training in math-related skills within an RtI framework, learners showed improved results. The value of the RtI framework lies in the fact that it provides early identification, intervention and progress monitoring, i.e. an improvement in mathematics proficiency was brought about before the onset of any serious deficits in most learners in the experimental group. Learners were thus guided to success through preventive intervention.

Finally a few comments need to be made on mathematics learning disability (MLD). Many researchers agree that learners with MLD use immature counting strategies, make frequent computational errors and have a delay in learning arithmetic facts (Beals 2012:32; Bryant & Bryant 2008:5; Geary, Bailey, Littlefield, Wood, Hoard & Nugent 2009:143; Geary 2011:255; Mazzocco & Myers 2003:143; Von Aster 2000:43-44). A learner would be identified as having a mathematical delay when improvement in his/her performance is evident after a period of intervention. Conversely, an MLD learner would experience persistent difficulty in mathematics problem-solving even after extended intervention has taken place. Collectively, the results from this study support the notion that learners benefit from intervention that is explicit, implements scaffolded techniques and fosters a solid conceptual foundation and effective procedural strategies. It is important to note that the learners who did not show adequate progress on the post-test scores in this study now receive individual (Tier 3) support at the school where the study was conducted. The intervention is aimed at minimising the effects of their slow progress in mathematics learning.

7. Conclusion

Results in this study demonstrated how fundamental areas of mathematics proficiency can be fostered in Grade 1 learners with an identified risk for mathematics difficulties or disabilities. Many of them started formal schooling without the learning experiences necessary to achieve success in mathematics. With regard to the aims of the study and by elaborating on other research it can be stated that the Grade 1 learners in the experimental group improved in the mathematical outcome variables included in the investigation and can now apply the knowledge gained during this intervention to the general classroom environment.

The researcher also examined the correlation between perceptual and mathematics development. The results have shown significant relations between the majority of ASB sub-skills. These outcomes add to the evidence (see section 8) that number sense, counting skills, number fact retrieval, procedural calculation and conceptual understanding can be explained by visual perception, motor skills, spatial skills, coordination and memory.

Spatial ability has been shown to bear the highest significance to mathematics in the study. This result proposes that when teachers attempt to expand learners' spatial thinking at a young age (Grade R and Grade 1), it will advance spatial reasoning and improve symbolic numerical representations, i.e. spatial skills are malleable and have the potential to cultivate more advanced mathematics skills (Dyson et al. 2011:178; Fuchs et al. 2012:266).

In summary, the results suggest that different combinations of cognitive processes and developmental skills sustain various forms of mathematical competence. It is also clear that learners in Grade 1 possess numerical skills that need to be cultivated in educational practices. The researcher gives a graphical representation (see Appendix 1) of the way all components of the study fit together. The outline includes the theoretical framework, response to intervention approach, intervention programme and activities. The researcher regards this type of intervention as a suitable approach and valuable resource to permit learners to experience long-term success in mathematics as it holds the potential for good educational service delivery in South African classrooms.

8. Pedagogical implications and recommendations

The researcher has identified the following as significant pedagogical implications with regard to appropriate methods of predicting MLD at an early stage and reducing subsequent complications.

- Many learners enter formal schooling without the necessary prerequisite foundation to learn mathematics successfully and with wide disparities in their mathematical knowledge (Francis, Rivera, Lesaux, Kieffer & Rivera 2006:34-36; Ginsburg 1997:21; Koshy, Ernest & Cassey 2009:215). Since early mathematical competence predicts favourable future scholastic performances it is essential to introduce preventive interventions for these learners before the onset of failure (Dowker 2005:328; Pienaar, Barhorst & Twisk 2013:376).
- The South African education system is facing a crisis concerning the poor quality of mathematics teaching (McCarthy & Oliphant 2013:3; Simkins 2013:5; Spaull 2013:10). The Centre for Development and Enterprise reports that teachers in elementary grades are not sufficiently prepared in their own knowledge of mathematics, which translates into ineffective instruction to their learners (Simkins 2013:6-7). Professional development reform rests on cultivating quality teacher knowledge, skills and practices (Ketterlin-Geller et al. 2008:34).
- The available research on mathematics instruction as an entity and on “what works” in intervention programmes is limited, particularly as it relates to learners with disabilities or barriers to learning. This may contribute to insufficient support being given to learners.

Regarding the obligation that teachers have in supporting children with barriers to learning an integrated system of early identification, the implementation of effective instruction and ongoing progress monitoring of learner growth is needed. The pedagogical value of this empirical investigation is apparent in the following:

- It significantly contributes to the outcomes of mathematical performance in early grades.
- It shows the value of early identification and the RtI approach in dealing with mathematical challenges.
- It provides a framework for the nature of intervention and a summary of the components, skills and concepts that are considered important.
- These findings might guide teachers to reflect critically on their teaching strategies and methods.

The researcher is confident that these findings will pave the way for future research in South Africa to validate the effectiveness of early mathematics intervention with special reference to a time-effective assessment tool that indicates delays in specific skill areas.

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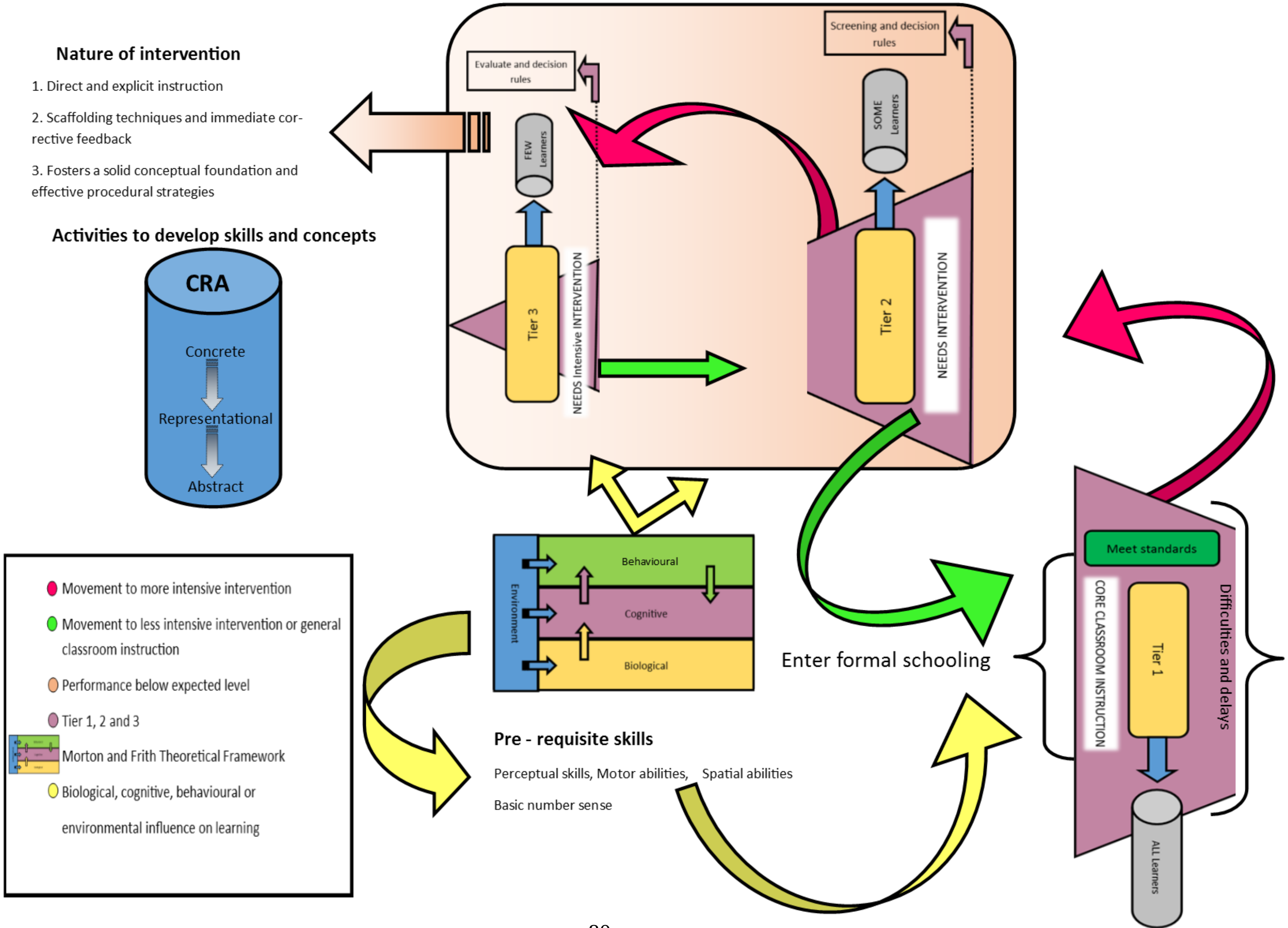
APPENDIX 1: GRAPHICAL REPRESENTATION OF THE STUDY



APPENDIX 2. COMPONENTS OF THE INTERVENTION PROGRAMME

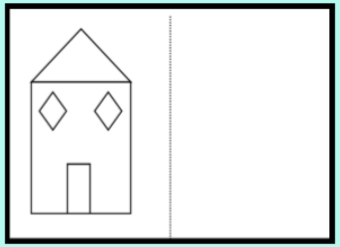
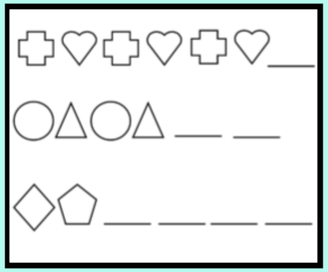
COGNITIVE: Working Memory

Central Executive	Phonological Loop	Visuo-spatial Sketchpad	Episodic Buffer
Domain general supervisory system	Domain specific component Retention of auditory-verbal information	Domain specific component Retention of visual-spatial information	Domain specific component Links information for sub-systems and long-term memory into multi-modal codes
Commands planning and inhibition Selecting and monitoring information Focus and switching attention	Counting Exact/mental arithmetic add and subtract) Acquisition of number facts Number ranking Single-digit + and – problems Multiplication accuracy	Supports approximation, counting procedures and exact computation/ calculation Quantity discrimination Number line Number writing and symbolic magnitude judgement	Problem- solving

PERCEPTUAL

Visual Perception

A mental process enabling us to perceive what we see with our eyes

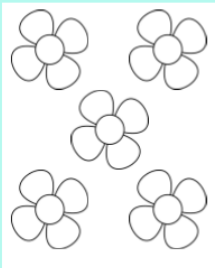
Visual memory	Visual discrimination	Visual sequencing	Visual form constancy
The ability to recall the visual appearance of an object, symbol or picture	The ability to take note of detail	The ability to see the sequence or pattern	The ability to observe forms irrespective of colour, size or the angle from which it is viewed
mechanical maths recall the appearance of numbers, operational signs etc.	to see the similarities or differences between symbols and objects	order of numbers, digits or objects	to recognise for example the number 3 as a three in different writing styles
			
Remember and draw	Match the pictures	Complete the patterns	Match animals with shadows

PERCEPTUAL: Continued

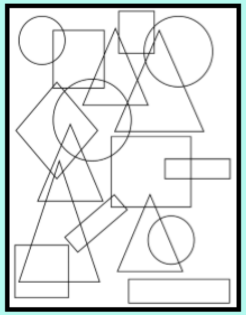
Visual Perception

A mental process enabling us to perceive what we see with our eyes

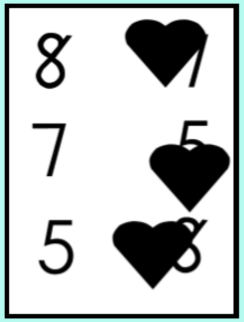
Visual position in space	Visual figure-ground	Visual closure
The ability to see that a symbol in a different position represents a different value or meaning	The ability to tell the difference between foreground and background	The ability to recognise the end of a sequence or a letter, number, picture etc. when part of it is missing
Position in space; for example 6/9	Isolate an object/symbol/shape in the foreground	To know for example the shape is a triangle when parts of it are missing



Make a square over the flower in the top left corner etc.



Colour all the triangles

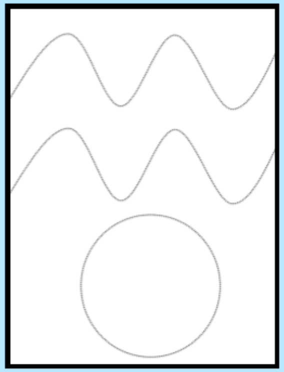


Match the numbers

PERCEPTUAL: Continued

Motor development

Infants actively explore their environment to construct knowledge. Motor ability and cognitive development are interrelated.



Cut/trace along the lines

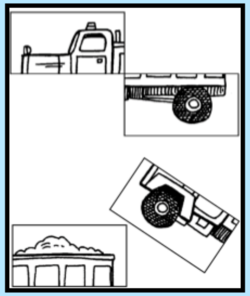
Fine motor skills and eye-hand coordination

Can be paired together as one notion, since fine motor skills by definition involve small muscle movements that depend upon close eye-hand coordination, which in turn is controlled by vision

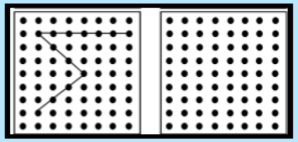
Form the basis for gaining important functional skills e.g. cutting and pasting, using manipulatives to do maths, holding and guiding pencils to produce legible handwriting, turning pages in a book and completing worksheets

Spatial ability

An over-arching concept that deals with the cognitive skills involved in understanding, manipulating, reorganising or interpreting relationships visually



Cut and complete the picture



Redraw the picture

Visual-spatial skills

Involve either rotation or transformation of visual material

Children depend on visual-spatial representations when they first learn to count because of the great emphasis put on concrete and pictorial representation

Significant when keeping track of number representation on a page and the order in which to write numbers

Spatial orientation or "closure"

Concerned with understanding the organisation of components within a visual representation

A spatial orientation task would for example be Gestalt completion; it requires an individual to interpret the individual parts of an object as a whole.

NUMERICAL

Basic number sense

Approximate magnitude representation

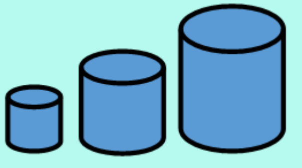
numerocity	Counting arrays by estimation
ordinality	Basic perception of more and less
simple arithmetic	Increase and decrease in quantity of small sets up to three

Basic Counting

Enumeration of sets up to three

Seriation	The logical ability to sort a number of objects in sequence based on their differences in one or more dimensions, e.g. from smallest to largest.
Classification	The ability to sort objects based on their similarities in one or more dimension.

Inclusion principle	Numbers are series containing each other
Conservation principle	The number of objects in a collection only changes when objects are added or removed

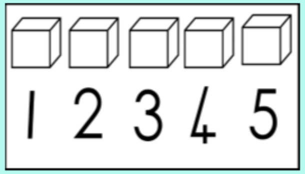


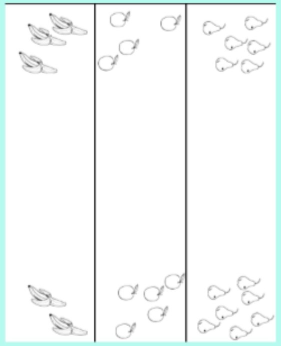
NUMERICAL: Continued

Higher order number sense

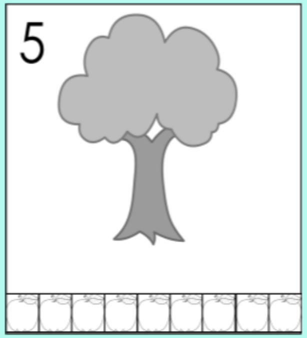
Number magnitude representation

Compare, classify and understand one-to-one correspondence	Number magnitude	Number knowledge	Number transformation	Number patterns and sequence

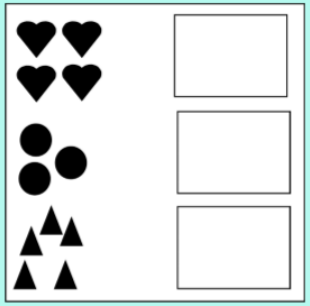




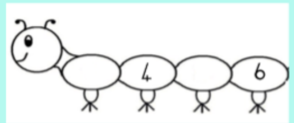
Circle the group with less



Paste 5 apples in the tree



Draw 2 more



Higher order counting

6 principles

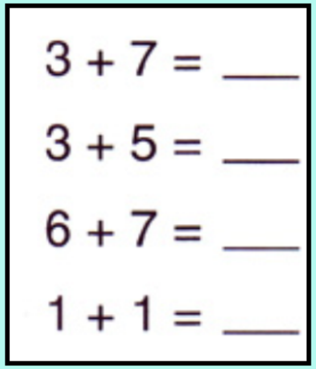
Order Irrelevance	One-to-one	Stable order	Cardinality	Abstraction
designating one counting word to each object counted	knowing that the list of number words is always in the same repeatable order	the number word assigned to the final object in a set represents the number of objects in that set	the order in which objects in a set are counted is irrelevant as long as every object is counted once and only once	all the above principles can be applied to any unique collection or set of objects

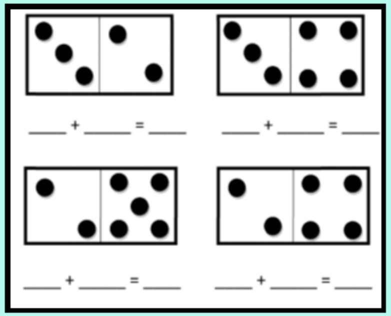
Arithmetic

Arithmetic

The continued use of counting results in long term memory-based representations for direct arithmetic fact retrieval and reconstruction of the answers based on retrieval of a partial sum.

Sum procedure	Min procedure
Adding by starting at one, e.g. 3+2=? “one, two, three, four, five”	Use the first number and adding the second addend, e.g. 3+2=? “three, four, five”





SUMMARY

Mathematics is a universal language and a pervasive component of everyday life that serves society at many different levels (Schoenfeld 2002:14; Tai, Qi Liu, Maltese Fan 2006:1143). It is essential for entering higher education and is a fundamental requirement for many modern professions (McCarthy & Oliphant 2013:3). Escalating evidence in two recent reports from the Centre for Development and Enterprise reveals that performance and teaching in mathematics are of poor quality in the majority of South African schools (Simkins 2013:4; Spaull 2013:4). Furthermore, the Foundation Phase CAPS for mathematics places emphasis on the importance of fundamental competence in mathematics skills, concepts and processes for later educational achievement, yet a disturbing number of learners enter formal schooling without the prerequisite skills to learn mathematics successfully. The wide disparities in these learners' mathematics knowledge make them vulnerable and leave them with the potential to develop mathematics learning disabilities (MLD) (Francis, Rivera, Lesaux, Kieffer & Rivera 2006:34-36; Koshy, Ernest & Cassey 2009:215). A learner's susceptibility to acquiring basic mathematics skills and concepts relies on various aspects, including the interplay of cognitive, perceptual and other developmental factors.

The research was presented as two articles, namely a theoretical and an empirical article.

The theoretical article aimed to investigate the early predictors and fundamental issues related to mathematics learning disability (MLD) and to analyse the core elements of mathematics proficiency at school-going age. Learners with MLD have difficulty with number sense, counting skills, number fact retrieval, procedural calculation and the conceptual understanding of mathematics (Ebersöhn & Eloff 2004:122; Geary, Bailey, Littlefield, Wood, Hoard, Nugent 2009:411; Kester, Lehnen, Van Gerven & Kirschner 2006:94).

The second article aimed to alleviate or possibly prevent MLD in later grades and examined the effects of a six-month intervention programme with a response to intervention (RtI) framework and focused specifically on Tier 2 intervention for learners that had been identified as at risk. The intervention programme purposefully targeted the skills identified as pivotal to early mathematics development by means of explicit instruction, scaffolding techniques and the use of concrete-representational-abstract (CRA) activities.

In comparing pre-test-post-test scores, results yielded positive correlations with outcome variables. It was evident that the average pre-test scores of both the experimental and

the control group were very low and that learners experienced delays in both the Ballard and Westwood + and – tests, as well as in all eight subtests of the Aptitude Test for School Beginners (ASB). From the post-test scores (Ballard and Westwood) it was evident that the mean scores of learners in the experimental group demonstrated a significant improvement, while those of the control group showed a slight improvement. Learners' pre- and post-intervention results on the ASB subtests indicated that similar results were achieved with regard to all the test outcome variables of the ASB. The highest correlation was yielded between learners' spatial abilities and their mathematics addition abilities. This was in accordance with by findings by Bull, Davidson and Nordmann (2010:249) whereby better performance in spatial ability tasks correlated with higher scores in arithmetic.

The researcher regards this type of intervention as a valuable resource for long-term success in mathematics. There is a need to validate the effectiveness of early mathematics intervention with special reference to a time-effective assessment tool that indicates delays in specific skill areas. The researcher is confident that these findings will pave the way for future research in South Africa as it holds the potential for good educational service delivery in the country's classrooms.

OPSOMMING

Wiskunde is ’n universele taal en ’n integrale komponent van die alledaagse lewe. Dit dien die samelewing op talle verskillende vlakke (Schoenfeld 2002:14; Tai, Qi Liu, Maltese Fan 2006:1143). Dit is noodsaaklik vir toelating tot hoër onderwys en is ’n fundamentele vereiste vir baie moderne professies (McCarthy & Oliphant 2013:3). Getuienis in twee onlangse verslae deur die Sentrum vir Ontwikkeling en Onderneming wys op die swak onderrig en prestasie in wiskunde in die meerderheid Suid-Afrikaanse skole (Simkins 2013:4; Spaull 2013:4). Voorts beklemtoon die KBAV Grondslagfase vir wiskunde die belangrikheid van fundamentele bekwaamheid in wiskundige vaardighede, konsepte en prosesse vir latere opvoedkundige prestasie; nogtans tree ’n ontstellende groot getal leerders tot formele skoolonderrig toe sonder die noodsaaklike vaardighede om sukses in die aanleer van wiskunde te behaal. Die wye dispariteite in hierdie leerders se wiskundige kennis stel hulle kwesbaar en oop vir die moontlikheid om wiskundige leergeremdheid te ontwikkel (Francis, Rivera, Lesaux, Kieffer & Rivera 2006:34-36; Koshy, Ernest & Cassey 2009:215). ’n Leerder se ontvanklikheid om basiese wiskundige vaardighede en konsepte te verwerf is van verskillende aspekte afhanklik, insluitend die wisselwerking tussen kognitiewe, perseptuele en ander ontwikkelingsfaktore.

Die navorsing word as twee artikels aangebied, te wete ’n teoretiese en ’n empiriese artikel.

Die teoretiese artikel ondersoek die vroeë voorspellers van fundamentele probleme rakende wiskundige leergeremdheid. Dit analiseer ook die kernelemente van die wiskundige bekwaamheid van leerders wat skoolgaande ouderdom bereik. Leerders met wiskundige leergeremdheid ondervind probleme met getalbegrip, telvaardighede, getalfeitonttrekking, prosedurele berekenings en die konseptuele begrip van wiskunde (Ebersöhn & Eloff 2004:122; Geary, Bailey, Littlefield, Wood, Hoard, Nugent 2009:411; Kester, Lehnen, Van Gerven & Kirschner 2006:94).

Die tweede artikel beoog om wiskundige leergeremdheid in later grade te verminder of moontlik te voorkom. Dit doen verslag oor ’n ondersoek omtrent die uitwerking van ’n intervensieprogram van ses maande met behulp van die respons-op-intervensie-raamwerk en fokus spesifiek op Vlak 2-intervensie met leerders wat vir potensiële wiskundige leergeremdheid geïdentifiseer is. Die intervensieprogram teiken doelgerig die vaardighede wat as sentraal vir vroeë wiskundige ontwikkeling geïdentifiseer is deur middel van

gedetailleerde onderrig, verminderdebystand-tegnieke en die gebruik van aktiwiteite gerig op die konkrete en abstrakte voorstelling van getalle.

Wanneer die tellings van die voor- en ná-toetse vergelyk word, toon die resultate positiewe korrelasies met die uitkomsveranderlikes. Uit die vooraf-toetse het dit duidelik geblyk dat die tellings van beide die eksperimentele en die kontrolegroep baie laag was en dat leerders agterstande getoon het in sowel die Ballard en Westwood + en – toetse as in al agt die subtoetse van die Aanlegtoets vir Skoolbeginners (ASB). Uit die tellings van die toetse wat ná die intervensie afgelê is (Ballard en Westwood) het dit geblyk dat die gemiddelde tellings van leerders in die eksperimentele groep ’n beduidende verbetering getoon het, terwyl die tellings van die kontrolegroep ’n geringe verbetering getoon het. Leerders se tellings in die BSB voor en ná intervensie het daarop gedui dat soortgelyke resultate behaal is met betrekking tot al die toetsuitskomsveranderlikes van die ASB. Die hoogste korrelasie was dié tussen leerders se ruimtelike vermoëns en hulle vermoë om optelsomme te doen. Hierdie bevinding is in ooreenstemming met navorsing deur Bull, Davidson en Nordmann (2010:249) dat beter prestasie in take wat ruimtelike vermoëns betrek, met hoër tellings in rekenkunde korreleer.

Die navorser beskou hierdie tipe intervensie as ’n waardevolle hulpmiddel vir langtermynsukses in wiskunde. Dit is nodig dat die effektiwiteit van vroeë wiskundige intervensie met spesifieke verwysing na ’n tyd-effektiewe assesseringsinstrument wat op agterstande in bepaalde vaardighede aandui, gevalideer word. Die navorser vertrou dat hierdie bevindings die weg sal baan vir toekomstige navorsing in Suid-Afrika aangesien dit oor die potensiaal beskik vir uitnemende dienslewering in die klaskamers van die land.

CLARIFICATION OF CONCEPTS

Mathematical learning disability

The DSM-V of the American Psychiatric Association defines mathematics learning disability (MLD) as a below-expected performance in mathematics, given an individual's age, intelligence and years of education, and which for adults significantly interferes with their daily activities. Learners with MLD have poor number sense, use immature counting strategies, make frequent computational errors and have a delay in learning arithmetic facts (Ebersöhn & Eloff 2004:122; Geary, Bailey, Littlefield, Wood, Hoard, Nugent 2009:411; Kester, Lehnen, Van Gerven & Kirschner 2006:94).

Number sense

Number sense is awareness of quantity. It is a skill which enables a learner to determine, compare and manipulate quantities. A good number sense helps learners to progress to higher-order mathematics skills and concepts with ease. As outlined by Berch (in Prinsloo 2001:344 and Van Nes & de Lange 2007:213), "processing number sense ostensibly permits one to achieve anything from understanding the meaning of numbers to developing strategies for solving complex math problems".

Early numeracy skills

Numerical ability comprises two essential competencies, namely number comprehension and counting efficiency. Skills include numerocity (counting arrays by estimation), ordinality (basic perception of more and less), counting (enumeration of sets up to three) and simple arithmetic (increase or decrease in quantity of small sets). Seriation, classification, counting knowledge (procedural and conceptual) and magnitude comparison are preliminary arithmetic abilities (Stock, Desoete & Roeyers 2010:253).

Basic number knowledge

Basic number knowledge includes number identification, number discrimination, counting, magnitude comprehension, estimation, number sequencing and number transformation (calculating simple addition and subtraction combinations). It serves as a measure of mathematics proficiency (Berch 2005:145).

Working memory

Working memory is part of a range of cognitive systems that contribute to learning across all academic domains. Working memory has limited retention and serves cognition as a temporary workspace where task-relevant information is being maintained, manipulated and processed. It has been called "the brain's Post-it note". It consists of a domain-general central executive system that regulates and coordinates three domain-specific subsystems: the phonological loop, the visuo-spatial sketchpad and the episodic buffer.

Preventive intervention

Preventive intervention is a proactive measure designed to keep a potential problem or condition from becoming a disability. It is aimed at reducing the incidence of a problem, decreasing the prevalence of identified problems or minimising the effect of a specific disability, i.e. intervention which invests in preventing the number of new candidates rather than waiting for a specific problem to occur and then following up with less than ideal solutions.

Response to intervention (RtI) framework

This approach is characterised by intervention in growing levels of intensity with quality general education as the starting point. It involves the early screening of all learners, intervention and progress monitoring. Most RtI approaches entail a three-tiered prevention system.

Visual-perceptual ability

Visual-perceptual ability is a mental process that enables us to perceive what we see with our eyes. Skills include visual memory, visual discrimination, visual sequencing, form constancy, visual position in space, visual figure-ground and visual closure.

Visual-motor skill

Visual-motor skill is the ability to integrate visual images with the appropriate motor response. It has an impact on functional skill areas and allows us to perform daily activities, for example cut, draw and write. It is influenced by factors such as fine motor skills and eye-hand coordination.

Spatial ability

Spatial ability is an overarching concept that deals with the cognitive skills involved in understanding, manipulating, reorganising or interpreting relationships visually. Spatial skills can be subdivided into visual-spatial skills and spatial orientation or “closure”.

Spatial orientation

Spatial orientation is concerned with understanding the organisation of components within a visual representation, i.e. objects in relation to other objects. It is often used in determining direction.

Visual imagery

Visual imagery involves the construction of internal (mental) and external (e.g. pencil and paper) images and then using those images for mathematical discovery and understanding.

Scaffolding techniques

Scaffolding is regarded as a fundamental component of effective teaching. It involves temporary support in subsequent levels to guide learners to more advanced comprehension levels and skill acquisition. It includes explanations, demonstrations, questioning etc.

Concrete-representational-abstract (CRA)

It is a three-stage instructional approach:

Concrete stage: physical manipulation of concrete objects

Semi-concrete stage: involves pictorial representation of concrete objects

Abstract (symbolic) stage: comprises numbers and symbols only