MODELLING ECONOMIC- ENVIRONMENTAL
TRADE-OFFS OF MAINTAINING NITRATE POLLUTION
STANDARDS

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I, Nicolette Matthews, hereby declare that this thesis work submitted for the degree of Philosophiae Doctor in the Faculty of Natural and Agricultural Sciences, Department of Agricultural Economics at the University of the Free State, is my own independent work, and has not previously been submitted by me to any other university. I furthermore cede copyright of the thesis in favour of the University of the Free State.

____________________
Nicolette Matthews

Date

Bloemfontein

January 2014
“You must be the change you wish to see in the world”

Mahatma Gandhi

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Nicolette Matthews
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The main objective of this research was to develop the methods and procedures to more accurately quantify the trade-offs between improving production risk and environmental degradation using state-contingent theory to quantify economic and environmental risk with empirical distributions.

The first step in developing the economic-environmental trade-offs is to model the risk efficiency of fertiliser applications through the development of a utility maximisation programming model. Separate state-contingent nitrogen maize yield response functions estimated from simulated crop yields for each state of nature characterise production risk empirically. The unexplained variability not captured by the response function is taken into account by adding the residuals to the expected response to produce a stochastic response function. The same procedure quantified the environmental fate of fertiliser applications. An upper partial moment (UPM) ensured that the optimised farmers’ response complied with an environmental pollution goal of 28kg/ha. The upper frequency method (UFM) was developed to ensure a stricter probability bound which was used to determine the conservativeness of the UPM.

The results showed that the state-contingent representation of production risk were able to capture the changes in outcome variability without any distributional assumptions. More importantly, fertiliser can act as a risk-reducing input, risk-increasing input or both depending on soil choice while not considering the environment. The risk-reducing nature of fertiliser emphasises the importance of taking risk preferences into account when modelling economic-environmental trade-offs. The UPM results indicated that an environmental constraint hold substantial compliance costs for agricultural producers. To minimise compliance costs producers had to make extensive and intensive margin changes to ensure compliance. Soil choice is identified as being more important than fertiliser application method in reducing compliance costs. An interesting finding is that environmental compliance resulted in fertiliser being a risk-reducing input. Comparison of the modelling results of the UPM and UFM showed that the UPM is very conservative in estimating the economic-environmental trade-offs. The size of the conservativeness is very situation specific and is determined by the combination of fixed resources used, fertiliser application method, compliance probability and the conservativeness measure used.

The main conclusion is that state-contingent theory provides the opportunity to model the impact of management decisions on outcome variability due to the effect of the state of nature in which the production decision is made and not due to the input use decision. The state-contingent theory is therefore the more appropriate mechanism to model the influence of uncertainty on production risk and more importantly environmental risk. The application of the state-contingent
theory requires transformation functions, which captures the relationship between management
decisions and outcome variability due to the state of nature. Much more research is necessary
on the development of appropriate transformation functions.
CHAPTER 1

INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

The National Water Act (Act 36 of 1998) provides for proper water pricing of South Africa’s water resources as part of the water resource management strategy. Water use as defined in the National Water Act is not only the extraction and use of water from a water resource but also includes any action that may impact the water resource. Under the National Water Act waste discharge charges can be levied for the discharge of waste into water resources to internalise the externality created. A Waste Discharge Charge System (WDCS) document is being developed to create the necessary structures and guidelines to internalise the externality costs associated with the waste and pollution of water resources. Although the WDCS and the pricing strategy associated with the WDCS document is still being developed the charges will be used to recover the direct and indirect costs from the user and will address both point (PS) and non-point source (NPS) pollution. The National Water Act (Act 36 of 1998) therefore, provides the legislative means to target NPS pollution and allows for the development of source-specific procedures that address NPS pollution.

NPS pollution is seen as one of the remaining major sources of water quality problems due to nutrient and sediment losses (Cartwright et al., 1991; Shortle et al., 1998; Novothy, 1999; Peterson & Biosvert, 2001; Rossouw & Görgens, 2005, Görgens, 2012). Quibell (2000) argues that a lack of legislative and regulatory authority on the one hand and poorly defined linkages between implementable management actions and the processes that lead to NPS pollution on the other hand have hampered the management of NPS pollution in South Africa in the past. Through the development of the National Water Act the legislative means are set in place to ensure that NPS pollution is regulated. The remaining problem with controlling NPS pollution is with regard to the development of management actions and processes that can be implemented to regulate NPS pollution. Designing management actions and processes to control agricultural NPS pollution is difficult. In part, this is due to the complex relationship between agricultural production and damages from water pollution involving physical, biological and economic links. How well management actions and processes perform often depends on how well these links are understood (Ribaudo et al., 1999). As a result the demand for information about the economic and environmental properties of agricultural production systems have increased.
Trade-off analysis applies the principle of opportunity cost to derive information about the sustainability of agricultural production systems. During trade-off analysis the inter-relationships among sustainability indicators implied by the underlying biophysical processes and the economic behaviour of producers are quantified. Stoorvogel et al. (2004) stated that trade-off curves are two-dimensional graphs representing the trade-off between two sustainability indicators. The slope of a trade-off curve shows the opportunity cost of increasing agricultural production in terms of foregone environmental quality. The information generated with the trade-off analysis is critical for informed policy decision-making, as it allows policy makers and the public to assess whether a given improvement in environmental quality is worth the sacrifice in agricultural production.

Modelling economic-environmental trade-offs is typically achieved through the integration of modelling procedures to estimate producers’ response and procedures to quantify the environmental impact of responses. Fertiliser use is of special importance since nitrogen is one of the key factors that determine crop productivity (Sadras, 2004; Grandorfer et al., 2011) but is also identified as a major contributor to NPS pollution (Yadav et al., 1997; Tilman et al., 2002; Eickhout et al., 2006). Modelling producers’ fertiliser use decisions is complicated since they typically over-apply nitrogen because of uncertainty with regard to profit maximising fertiliser rates (Abedullah & Pandey, 2004) or the fact that producers try to minimise downside risk (Rajsic et al., 2009). Empirical evidence further shows that input use may affect the skewness of yield variability (Torriani et al., 2007; Rajsic & Weersink, 2008; Hennessy, 2009; Du et al., 2012). Antle (2010) argues that complex interactions between the biophysical and management processes jointly determine production variability resulting in asymmetrical changes in the positive and negative tails of distributions. The impact on variance and skewness is therefore not self-evident, which void the application of moment-based approaches to quantify the impact of input use on production variability. More flexible methodologies are required to study the impact of input use on production risk without any assumptions with respect to the way input use will change production risks (Antle, 2010).

Incorporating environmental compliance as one of the sustainability indicators into economic-environmental trade-off analyses is difficult. Nutrient losses that may potentially harm the environment depend on the amount of variable production inputs used, production practices, soil characteristics, topography and weather conditions. The indicator that is used to identify environmental sustainability is therefore stochastic and dependent on management practices which have a significant bearing on policy design (Shortle & Dunn, 1986; Segerson, 1988; Horan et al., 1998). The stochastic nature of environmental outcomes requires that pollution control strategies should be aimed at improving the distribution of outcomes rather than some scalar value. Most often researchers have used chance-constrained programming to incorporate the stochastic nature of environmental outcomes into trade-off analyses (Burn & McBean, 1985; Segarra et al., 1985; Zhu et al., 1994; Koo et al., 2000). Application of chance-constrained
programming to environmental outcomes poses problems since the distribution of the environmental indicator is determined endogenously during optimisation. Consequently, researchers are experiencing problems to apply chance-constrained programming since it is difficult to assign a distributional form for the endogenously determined distribution of the environmental indicator (Xu et al., 1996; Gren et al. 2012; Kataria et al., 2010). Furthermore, making distributional assumptions about the endogenously determined environmental outcomes can have a significant impact on the estimated trade-offs (Zhu et al., 1994; Qiu et al., 2001; Kampas & White, 2003) and may not hold for all situations due to the site-specific nature of agricultural NPS pollution (Qui et al., 2001). As an alternative to chance-constrained programming researchers have developed methods such as the Environmental Target-MOTAD (Teague et al., 1995) and the upper partial moment (UPM) (Qiu et al., 2001) to enforce environmental compliance through the use of empirical characterisations of the environmental risks (Teague et al., 1995; Qiu et al., 2001). The UPM is superior to the Environmental Target-MOTAD methods because it provides a method of enforcing a probabilistic constraint while characterising the environmental outcome with an empirical distribution. A potential problem with the application of the UPM is that it is conservative with respect to the enforcement of the probabilistic constraint (Qiu et al., 2001; Krokhmal et al., 2002; Kong, 2006). Although the researchers acknowledge the fact that the UPM is conservative, no research is conducted to quantify the impact of the conservativeness on the economic-environmental trade-offs. Conservative economic-environmental trade-offs may result in over-regulation.

Swinton and Clark (1994) motivate the inclusion of objective function risk into economic-environmental analyses. Applications of trade-off analyses that include both environmental and production risks are scarce. Within a South African context Aihoon et al. (1997) used a Target-MOTAD model with an environmental constraint to evaluate pollution insurance as an environmental policy. Although production risk was considered, the model did not allow for changes in production risk due to the changes in the fertiliser application rates included in the model. Grové (2004) evaluated economic-environmental trade-offs of nitrate pollution to determine cost-effective policy instruments to control nitrate pollution. The research by Grové (2004) did not consider the stochastic nature of pollution emissions neither is the impact of input use on production risk considered.

1.2 PROBLEM STATEMENT

Estimation of economic-environmental trade-offs require a clear understanding of the linkages between production risk and environmental risk. Currently the linkages between production decisions, production risk and environmental risk are not clearly understood. Appropriate modelling of production and environmental risk due to input decisions requires continuous relationships between input use and the resulting output variability and associated
environmental outcome variability. The continuous relationships are necessary to overcome the problem of input level diversification for the same technology set when discrete activities represent input use, the associated production and environmental outcome variability. A lack of procedures that relates input use on a continuous basis to flexible distributions production and environmental outcomes hampers the evaluation of an integrated analysis of the trade-off between production and environmental risk. Failure to correctly model these trade-offs will result in failure to develop applicable environmental policy that can be used to regulate agricultural NPS pollution.

The inherent output risk of production has been recognised and researched by many researchers (Quiggin & Chambers, 2006). Just and Pope (1979) developed an econometric two-stage procedure that is able to model production risk. The procedure fits a yield variance function to correct for yield variability in the mean yield function. The estimated mean yield and yield variance functions determined with the Just and Pope model have been incorporated into optimisation models to determine the optimal behaviour for decision makers (Lambert, 1990; Roberts et al., 2004). However, due to the use of a multiplicative error term in the Just and Pope model the model has difficulty estimating second and higher moments of output therefore heteroscedasticity is not addressed appropriately. Antle (1983) developed a moment-based approach that assumed that changes in the moments are symmetrical. However, the parametric Just and Pope model and the moments-based approach assume that the risky outcomes are normally distributed which is not always the case. More recently Antle (2010) developed a partial moment approach to show that changes in input use has an asymmetrical impact on moments. The abovementioned procedures use a distributional approach to model yield variability while ignoring the impact of input use on the environment.

Economic-environmental trade-offs are typically modelled with the use of chance constrained programming (Kampas & White, 2003), Target-MOTAD (Teague et al., 1995; Aihoon et al., 1997; Qiu et al., 1998; Umoh, 2008) or safety-first constraint models such as the Upper Partial Moment method (Qiu et al., 2001; Intarapapong et al., 2002). Chance-constrained programming evaluates right hand side risk by determining the constrained variables deterministically. Application of chance-constrained programming, therefore, requires that a functional form be specified to represent the distribution of the environmental variable (Qiu et al., 2001). To correctly capture the skewed distribution of the non-negative environmental variable skewed probability models are used. The skewed probability models that have been used to describe environmental data include the Poisson, negative binomial, Weibull, gamma, exponential and the lognormal, with the lognormal being the most used. However, choice of distributional assumption is not straightforward and an incorrect distributional choice can result in a significant impact on the estimated trade-offs and objective function as argued by Zhu et al. (1994), Qiu et al. (2001) and Kampas and White (2003). Qui et al. (2001) also argued that a distributional assumption may not hold due to the site-specific nature of agricultural NPS pollution. To
overcome the problems with making distributional assumptions the well-known Chebyshev inequality can be used as a distribution-free deterministic equivalent for a probabilistic constraint. The Chebyshev inequality assumes that the number of standard errors, $\phi^\rho$, is used to determine the standardised distribution of emissions where $\phi^\rho = (1 - \rho)^{-\frac{1}{2}}$, and $\rho$ is the chosen probability level (Gren et al., 2012). The Chebyshev’s non-linear mean-standard-error inequality usually generates conservative probability bounds (Atwood et al., 1988). To overcome the conservativeness of the Chebyshev inequality Atwood (1985) developed a more general lower partial moment (LPM) stochastic inequality to enforce the safety-first constraints. Atwood’s (1985) LPM approach requires that the random variable be finitely discrete and uses the empirical distribution of the random variables. Qiu et al. (2001) built on research done by Atwood (1985) to develop an UPM inequality approach to impose the safety first constraint in a Target-MOTAD framework that will ensure that the target pollution level will be met at a certain specified probability level. The UPM model treats the variables like an empirical distribution and determines the desirable or target pollution level endogenously. A potential problem with the UPM is that it is still conservative in the determination of the desirable pollution level, albeit not as conservative as the Chebyshev inequality (Qiu et al., 2001; Krokhmal et al., 2002; Kong, 2006). Many researchers acknowledge the conservativeness of the UPM, however, none of the studies investigated the size of the conservativeness of the UPM. Atwood et al. (1988) suggested that if conservativeness is of concern alternative nonlinear or exogenously constrained methods are needed to evaluate the conservativeness.

An alternative method to model production and environmental risk that is not well exploited in literature is the state-contingent theory. The state-contingent theory allows the researcher the opportunity to estimate empirical skewed production risk and environmental risk through the development of response functions for every state of nature (weather, year). Chambers and Quiggin (2000) were the first to address the fact that insight into production under uncertainty requires the researcher to acknowledge the effect of the state of nature. Production uncertainty is eliminated if the decision maker is aware of what the production function will be for a given state of nature (Rasmussen & Karantininis, 2005). Optimal production decision behaviour can be determined if all production functions (Rasmussen & Karantininis, 2005), producers’ risk preferences and expected outcomes (Asche & Tveteras, 1999) are known. Recently, application of state-contingent theory to model production risk has increased, however, applications with environmental risk were not found in the literature. State-contingent theory provides the ideal vehicle to portray the link between input use decisions and the environmental impact using empirically distributed risks.
1.3 OBJECTIVES

The main objective of this research is to develop methods and procedures to more accurately quantify the trade-off between improving production risk and environmental degradation using state-contingent theory to quantify economic and environmental risk with empirical distributions.

The main objective is achieved through the following sub-objectives:

**Sub-objective 1:** To model input use decision-making behaviour with skewed production risk through the development of a state-contingent direct expected utility model.

The first step in quantifying the economic-environmental trade-offs is the identification of the relationship between input use decisions and production risk. State-contingent theory provides the foundation for quantifying the production risk associated with nitrogen input use decisions under conditions of no water stress through the estimation of state-contingent response functions for every production year. Separate state-contingent response functions capture changes in water use between states of nature as a function of nitrogen applications within a specific state of nature. Combining both the state-contingent response functions allows for the estimation of a continuous function which relates nitrogen applications to gross margin variability without making any distributional assumptions or how production variability changes with input use. Gross margin variability is therefore the result of the production risk as a function of fertiliser applications and the changes in water use between different states of nature. The risk efficiency of alternative fertiliser application rates is determined through the use of a mathematical programming model that maximises the certainty equivalent.

**Sub-objective 2:** To model the economic-environmental trade-offs using a safety-first constraint while taking production and environmental risk into account.

The model developed in sub-objective 1 is extended to estimate the economic-environmental trade-offs for agricultural decision makers’ fertiliser use decision with an UPM. The trade-off model requires an empirical distribution of environmental risk associated with the input use to calculate the UPM which is used to model compliance with an environmental goal. State-contingent response functions quantified empirically distributed environmental risk as a function of fertiliser applications in every state of nature. The trade-off models allows the researcher to evaluate the impact of environmental compliance on the risk efficiency of fertiliser applications while considering both intensive and extensive margin changes when modelling economic-environmental trade-offs of increasing compliance probability.
**Sub-objective 3:** To develop an alternative nonlinear trade-off model that can be used to quantify the conservativeness of the UPM.

Two types of conservativeness are considered when estimating the conservativeness of the UPM. The exogenous conservativeness of the UPM is determined by comparing results of an UPM model at a specified compliance probability with the results of another UPM model that achieves the same level of compliance based on the exogenous calculation using the optimised distribution of the environmental outcome in the second optimisation. The endogenous conservativeness stems from the fact that the optimal response of producers to a conservative probability bound will be different from the response optimised as a result of a probability bound that is close to the actual compliance probability. As part of this research a new method is developed to enforce a probability bound that is close to the actual compliance probability which allows for the estimation of the endogenous conservativeness of the UPM. The newly developed method counts the number of deviations from the environmental goal in an effort to ensure that the number of deviations above the goal does not exceed the number of deviations that is used to specify the compliance probability.

The organisation of the thesis is discussed next.

### 1.4 ORGANISATION OF THE THESIS

The thesis consists of six chapters including the Introduction (Chapter 1) and the Conclusions and recommendations (Chapter 6).

Chapter 2 discusses the state-contingent approach as the theoretical framework for the study. The discussion on the theoretical framework starts by distinguishing between parametric distribution approaches and the state-contingent approach as techniques to characterise decision-making under uncertainty. The Chapter continues the discussions on the theoretical background of state-contingency in particular the theory and the optimality conditions for alternative input classifications.

Chapter 3 to Chapter 5 are structured in such a manner that every chapter addresses a sub-objective of the study. The state-contingent approach is used in Chapter 3 to fit nitrogen response functions for every state of nature. The state-contingent nitrogen response functions were used to model input use decisions with skewed production risk. The Chapter first provides the background to the problem statement that leads to objective 1 and a discussion of relevant literature. The data simulation process and the procedures follow the identification of the objective and sub-objectives. The results and conclusions bring the Chapter to an end.
Chapter 4 builds on the results of Chapter 3 by using the production response functions for every state of nature and newly developed state-contingent emission loss functions into a trade-off model. The economic-environmental trade-offs was then modelled using a safety-first constraint while taking the production and environmental risk into account. Chapter 4 follows the same layout as Chapter 3 with an introduction, literature review, procedures, results and conclusions.

Sub-objective 3 is addressed in Chapter 5 by investigating the conservativeness of the UPM in estimating the economic-environmental trade-offs. The layout of Chapter 5 is similar to Chapter 3 and 4.

The thesis concludes with conclusions from the research in Chapter 6. Finally some policy recommendations and suggestions for further research are made.
Chapter 2 consists of two sections. The first section discusses the use of the parameterised distribution approach and state-contingent approach as techniques to characterise decision-making under uncertainty. The second section continues the discussion on state-contingent approach focusing on the theory and the optimality criteria for input use.

2.1 CHARACTERISATION OF RISK DECISION-MAKING

2.1.1 PARAMETERISED DISTRIBUTION APPROACH

When a decision maker faces uncertain future consequences of a current choice the decision maker is said to face a risky choice. The principal theory underlying risk decision-making is expected utility theory developed by von Neumann and Morgenstern (1947), (Kaiser & Messer, 2010). Expected utility theory uses an ordinal utility function to rank risky alternatives through the maximisation of expected utility (Boisvert & McCarl, 1990).

Hurley (2010) stated that there are three components to expected utility: the possible outcomes, the likelihood of the possible outcome (the probability of occurrence) and the utility of possible outcomes. The likelihood of outcomes can be presented by a probability distribution that is based on an individual's input choices or on the individual's perceptions of the likelihood that an outcome will occur. Chambers and Quiggin (2000) refer to the parameterised distribution approach as a distribution where the likelihood of outcome is based on the choice of input variables. Therefore, a random variable \( c \) is assumed to represent a continuous set of mutually exclusive outcomes that is bounded from above by \( \bar{c} \) and from below by \( \underline{c} \), where an individual's choices over alternative activities that affect the distribution of outcomes are represented by \( x \). Then a parameterised distribution is an individual's subjective perceptions about the likelihood of outcome \( c \) given the choice of \( x \). For a parameterised distribution expected utility is defined as (Hurley, 2010):
Theoretical Framework

\[ EU(x) = \int_{\underline{c}}^{\bar{c}} U(c) f(c|x) dc \]  

(2.1)

Where:

- \( c \) is a continuous mutually exclusive set of random outcomes bounded by \( \underline{c} \) and \( \bar{c} \)
- \( x \) is the individual’s decision variable (e.g. amount of fertiliser applied)
- \( U(c) \) is the utility of outcome \( c \)
- \( f(c|x) \) is the individual’s perceived likelihood of occurrence for outcome \( c \) given choice \( x \)

Many decision makers and researchers use parameterised distributions as the basis for optimising production under uncertainty (i.e. Lambert, 1990; Roberts et al., 2004). The most widely used technique to relate production risk to input decisions is the Just and Pope (1978) model. The procedure fits a yield variance function to correct for yield variability in the mean yield function. Given an estimate of the mean and the variance is provided by the Just and Pope model, researchers typically proceed by applying a Mean-Variance quadratic programming model to optimise the risk efficiency of input use. Several researchers have criticised the Mean-Variance approach. Abdullah and Pandey (2004) argue that the Just and Pope model has difficulty to estimate second and higher order moments of output because of the multiplicative error term. Therefore, heteroscedasticity is not addressed appropriately when estimating higher order moments of output. Antle (1983) recognised the restrictions of the Just and Pope model and proposed a moment-based approach. The moment-based approach regresses each moment of output on the inputs in a multi-stage approach. Therefore, changes in the moments are still symmetrical. To overcome the above-mentioned problem of symmetry Antle (2010) developed a partial moment regression system that allows the researcher the opportunity to model asymmetrical moments. The procedures of Antle (1983) and Antle (2010) have been used to determine the risk efficiency of input use decisions. Even though these procedures are able to model skew distributions of outcomes the likelihood of the outcome variable is still dependent on the level of input use which is a property of the parameterised distribution approach.

Rasmussen (2011) argues that the parameterised approach is not recommended, as it does not allow the researcher or decision maker the opportunity to exploit the possibility of actively responding to uncertainty, or to exploit the opportunities that uncertainty offers. As a result a need exists for the development of production economic models that can actively address uncertainty as uncertainty often plays an important role when making production economic decisions.
Chambers and Quiggin (2000) developed the state-contingent approach as an alternative to the parameterised distribution approach. The state-contingent approach is discussed next.

### 2.1.2 **State-contingent Approach**

Chambers and Quiggin (2000) used the work of Debreu (1952) and Arrow (1953) to extend utility theory to include state-contingent risk. State-contingent risk states that the outcome associated with input decisions are not primarily determined by the likelihood of occurrence of the resulting outcomes associated with an input decision but rather by the likelihood that a state of nature will occur. The state-contingent approach characterises individuals' perceptions based on the occurrence of the state of nature (level of rainfall) rather than the variation in the outcome variable. The state-contingent expected utility function is defined as follows (Hurley, 2010):

\[
EU(x) = \int \frac{U(c(x|s))f(s)ds}{s}
\]

Where:
- \(s\) is a continuous mutually exclusive set of state-contingent random outcomes bounded by \(\tilde{s}\) and \(\bar{s}\).
- \(U(c(x|s))\) is the utility from choice \(x\) given state of nature \(s\).
- \(f(s)\) is the decision makers subjective beliefs about the likelihood of state of nature \(s\).

The first term \(U(c(x|s))\) shows that the utility for outcome \(c\) is conditional on input choice \(x\) in a given state of nature \(s\). The implication is that when the state-contingent approach is used the producer is able to respond to differences in the states (e.g. weather) by changing input levels in every state. The producer can now respond actively to uncertainty or even exploit the opportunities offered by uncertainty. Rasmussen (2011) stated that it is not the product that provides utility, but rather the expectation of receiving certain quantities of the product conditional on the state that provides utility.

The second term \(f(s)ds\) estimates the likelihood of chance outcomes within a state-contingent world. While the parameterised approach assumes that decision makers' choices determine the likelihood of chance outcomes the assumption is that in a state-contingent world an individual's choices cannot affect the likelihood of chance outcomes. The state-contingent approach characterises the likelihood of outcomes based on the individual's perception of the likelihood of favourable weather conditions (e.g. state of nature). According to Hurley (2010) the basic idea is
that individual choices cannot affect the likelihood of chance outcomes in a state-contingent world, whereas individual choices do determine the likelihood of chance outcomes in the parameterised distribution approach.

### 2.1.3 Discussion

Researchers have acknowledged that production decisions are influenced by the uncertainty that a producer is exposed to. A major shortcoming of parameterised distribution approaches like the Just and Pope (1979) is that the inherent assumption is made that input use decisions remain the same between different states of nature and production uncertainty is a function of input use. These techniques therefore, do not allow a decision maker the opportunity to change input use between states of nature in response to production uncertainty which may result in the overestimation of production risk. In contrast the state-contingent approach allows the decision maker to react to changes in the state of nature by changing input use decisions. Production uncertainty is thus due to the state of nature and is not a function of input use. Critical to the application of the state-contingent approach is the availability of transformation functions. The transformation function shows the production possibility of a product given the available states of nature. Provided that the transformation functions are known the parameterised distribution approach and state-contingent approach are mathematically equivalent (Hurley, 2010).

Next the state-contingent theory and the optimality conditions for the alternative input classifications are discussed.

### 2.2 State-Contingent Theory

Rasmussen (2003) is of the opinion that most of the literature in economic decision-making under uncertainty discusses the sources of uncertainty and how to manage uncertainty through contracting, buying insurance, diversification, etc. These studies, however, neglect to discuss the criteria to be used when making basic input decisions such as the level of input to be used or how much product to produce. The reason is that the marginal principle that is used successfully for decision-making under certainty does not hold for decision-making under uncertainty. According to Rasmussen (2003) state-contingent theory provides the foundation for criteria that will hold for decision-making under uncertainty. State-contingent theory is rooted in the work of Arrow and Debreu (1954) who showed that production under uncertainty can be presented as a multi-output technology, if uncertainty is represented by a set of possible states of nature.
Next the optimality conditions for alternative input classifications are discussed.

### 2.2.1 Optimality Conditions for Alternative Input Classifications

The relationship between input and output can generally be described graphically through a production function (one input—one output), an isoquant (two inputs—one output) or a transformation function (production possibility curve) (one input—two outputs) (Rasmussen, 2011). Rasmussen (2011) stated that the transformation functions that are used to describe the relationship between different products can be used to describe production of a good between different states. The reason being that products produced in different states are classified as different products due to the state of nature effect. Furthermore the transformation functions could have different shapes. If suitable transformation functions for the random variables are available the state-contingent approach and the parametric approach are the same (Hurley, 2010). Thus, techniques like expected utility theory can be used to guide decision-making.

Deriving criteria for optimal input use is not easy when the exact form of the utility function is not known. For a risk-neutral decision maker with a linear utility function optimal input use criteria can be derived fairly easily. However, for a risk-averse decision maker it is not always easy to derive the criteria. Rasmussen (2003) derived optimal input use criteria based on the notion of whether a risk-averse decision maker is using more or less input than a risk-neutral decision maker. The development of these criteria depends on the notion of “good” and “bad” states of nature. The definition of a “good” or “bad” state of nature is subjective and depends on the decision maker’s risk preferences. Rasmussen (2011) uses a risk neutral decision maker as benchmark to compare the actions of a risk-averse decision maker.

Consider a risk-neutral decision maker who optimises production using \( x_n \). The state-contingent outcome \( y_s \) with subjective probability \( p_s \), yield a utility of \( EU(y_1, \ldots, y_s) = p_1y_1 + p_2y_2 + p_3y_3 \). Consider also a risk-averse decision maker with a general utility function \( EU(y_1, \ldots, y_s) \). As the scale of the utility function is arbitrary it may be rescaled so that:

\[
\sum_{s=1}^{S} EU_s \{y_1(x_n), \ldots, y_s(x_n)\} = 1 \quad (2.3)
\]

\( EU_s \) is the derivative of \( EU \) with respect to \( y_s \). In Equation 2.3 the sum of the derivatives of \( EU(y) \) with respect to \( y_s \) at the point \( y(x_n) \) is equal to one. Based on the scaling of the utility function
Rasmussen (2003) defined a good state of nature for a risk-averse decision maker at state $s$ as follows:

$$EU_s\left(y_1, \ldots, y_s\right) < p_s$$  \hspace{1cm} (2.4)

Where $p_s$ is the probability that nature will pick the $s^{th}$ state of nature. The marginal utility is measured on a utility function that is locally scaled so that the sum of marginal utilities over the $S$ states of nature is equal to one. Thus, a good state is defined as a state where the state-contingent net income of R1 gives a lower marginal utility than the probability of the state.

Rasmussen (2003) defined a bad state of nature as a state $s$ where:

$$EU_s\left(y_1, \ldots, y_s\right) > p_s$$  \hspace{1cm} (2.5)

Thus, in a bad state the state-contingent net income of R1 gives a higher marginal utility than the probability of the state. While for a risk-neutral decision maker Rasmussen’s (2003) neutral state of nature is defined as:

$$EU_s\left(y_1, \ldots, y_s\right) = p_s$$  \hspace{1cm} (2.6)

The states of nature may cause differences in the way different inputs are transformed into outputs. As a result it is necessary to make a distinction between different types of inputs when deriving optimality conditions. Based on the work of Chambers and Quiggen (2000) and Rasmussen (2003) optimality conditions are derived for state-general, state-specific and state-allocable inputs in the following sections.

2.2.1. State-general inputs

State-general inputs are inputs that influence production during one or more, possibly all states of nature. Chambers and Quiggen (2000) referred to these inputs as non-state specific inputs as the decision on input use is made prior to knowing the state of nature. State-general inputs are thus inputs that are applied with a view of overall increase in output, no matter what state of nature occurs. The formal definition of a state-general input $x_n$ is that:
Theoretical Framework

\[ \frac{\partial f_s(x)}{\partial x_n} \neq 0 \text{ for one or more states } (s \in \mathcal{S}) \text{ for some (relevant) level of } x_n \quad (2.7) \]

An example of a state-general input is the use of fertiliser in grain production. The assumption is made that maize is produced under two prevailing states of nature with \( y_1 \) representing a wet year and \( y_2 \) representing a dry year. The level of maize output when the year is wet is completely independent of yield when the year is dry, and depends only on the amount of fertiliser \( x_n \), applied. Two separate production functions for each of the state of nature can be used to determine the transformation function. Maize production functions for a wet and a dry year and the transformation function for maize production in the two states are shown in Figure 2.1.

![Figure 2.1: Production functions for state-general input use in a wet (y1) and dry (y2) state of nature and the resulting transformation function.](image)

Assuming that maize is produced in a wet year, the production function, \( y_1 = f(x) \) for that state is graphically illustrated by the function in Figure 2.1(a); while the production function for maize production in a dry year, \( y_2 = f(x) \) is represented by the function shown in Figure 2.1(b). The quantity of fertiliser applied to produce maize is the same between the two states of nature and is decided on before the state is known. Although the same amount of fertiliser is applied, maize yields realised will differ between the states due to the state of nature effect. As the amount of input is the same regardless of the state of nature, it is possible to derive the transformation function for the two products from the two production functions. The transformation function (Figure 2.1(c)) for different levels of fertiliser use is derived from the two production functions. If no fertiliser is applied, production is equal to point \( z_0 \) in Figure 2.1(c). By applying level \( x_1 \) of fertiliser
state-contingent outputs correspond to point $z1$, while production corresponding with $z2$ and $z3$ are achieved by applying $x_2$ and $x_3$ units of fertiliser. The transformation function for each input quantity is derived by calculating how much more of product $y_2$ can be produced if one produced slightly less of product $y_1$ (Rasmussen, 2011). Thus, the rate of substitution describes the slope of the transformation function. However, no such substitutions can be made. It is simply a situation of either/or. No matter what the decision maker does, production decisions cannot be adapted after the state of nature is known because the production decision was made in advance.

The following optimisation problem can be used to determine the optimality criteria for state-general inputs:

$$\max_x EU_s(y_1,\ldots,y_s)$$

(2.8)

The condition for optimal use of input $x_n$ is obtained by deriving the derivative of Equation 2.8 with respect to $x_n$ and equating it to zero:

$$\frac{\partial EU}{\partial x_n} = \sum_{s=1}^{S} EU_s(y) \left( Pr \frac{\partial f_s}{\partial x_n} - w_n \right) = 0 \quad (n = 1,\ldots,N)$$

(2.9)

Where $w_n$ is the price for input $n$, and the term $(Pr \frac{\partial f_s}{\partial x_n})$ is the value of the marginal product of input $x_n$ in state $s$. If the utility function $EU$ assumes risk-neutrality (linear) Equation 2.9 reduces to:

$$E \left( Pr \frac{\partial f}{\partial x_n} \right) = w_n \quad (n = 1,\ldots,N)$$

(2.10)

Where $Pr$ is the price of $y$, and $E$ is the expectation operator. Thus, a risk-neutral decision maker optimises the application of a state-general input $x_n$ by increasing the application as long as the expected value of the marginal product is larger than the input prices. As the production function $f_s(x)$ and output price $Pr$ vary over states of nature, the typical case is:

$$Pr_s \frac{\partial f_s(x)}{\partial x_n} \neq Pr_t \frac{\partial f_t(x)}{\partial x_n} \quad (s,t \in \Omega)$$

(2.11)
Thus, the marginal net return in state $s$ is different from that in state $t$. Meaning that for an optimal solution the marginal net return will be positive in some states and negative in other states. In some states $x_n$ will be either too high or too low amount of input, relative to the optimal allocation level given that the state is known in advance. This is true for both risk-neutral and risk-averse decision makers. The real question is whether the optimal application for a risk-averse decision maker who optimises production according to Equation 2.9 is higher or lower than that for a risk-neutral decision maker who optimises production according to Equation 2.10. It is not always possible to give a general answer, especially if all inputs are variable. However, if all inputs except $x_n$ are assumed to be fixed inputs, then a risk averse decision maker would use more of input $x_n$ than a risk-neutral decision maker would if the marginal utility is positive. A risk-averse decision maker would use more input than a risk-neutral decision maker would if the input improves the net return in the bad state of nature.

### 2.2.1.2. State-specific inputs

**State-specific inputs** are a special case of state-general inputs. A state specific input is applied with the view of increasing the output in one state of nature, or the input works in only one state. Thus, the formal definition of a state-specific input $x_n$ is (Rasmussen, 2003):

$$\frac{\partial f_s(x)}{\partial x_n} > 0 \quad \text{and} \quad \frac{\partial f_t(x)}{\partial x_n} = 0 \quad \text{for} \ s \neq t \ \text{for some (relevant) level of} \ x_n \quad (2.12)$$

Where $f_s(x)$ represents production in an alternative state. An example of a state-specific input is the use of a spray that protects against fungal infection during maize production. Again it is assumed that production takes place in either a wet year ($y_1$) or in a dry year ($y_2$). It is further assumed that fungicide applications are only effective in killing fungi in a wet state of nature. In a very dry year the use of fungicide will be ineffective. Cognisance should be taken of the fact that the spray is applied without knowing which state of nature will occur. The maize production functions associated with fungal spray use and the transformation function for a state-specific input is shown in Figure 2.2.
Theoretical Framework

**FIGURE 2.2:** Production functions for state-specific input use in a wet \( y_1 \) and dry \( y_2 \) state of nature and the resulting transformation function.

The production function for maize, \( y_1 = f(x) \) where maize production is a function of fungicide use during a wet year is shown graphically in Figure 2.2(a). In a wet year the increased use of fungal spray will increase maize production. Maize production as a function of fungicide use during a dry year \( y_2 = f(x) \) is shown in Figure 2.2(b). During a dry production year the spray will have no effect on production regardless of the quantity applied. The estimated production functions associated with fungicide use is combined to derive the transformation function shown in Figure 2.2(c). The transformation function illustrates that in a wet year the use of fungicide spray can increase production, where points \( z_0, z_1, z_2 \) and \( z_3 \) correspond to input quantities \( 0, x_1, x_2, \) and \( x_3 \). During a dry year the use of fungicide spray will have no impact on production. The transformation function again shows no substitution between \( y_1 \) and \( y_2 \) and now have the property that the horizontal parts of four transformation functions coincide (Rasmussen, 2011).

State-specific inputs are effective only in one state of nature, with the net returns estimated only for the state in question. The condition for optimal use of input \( n \) is then determined by estimating the state-specific utility function and setting the derivative with respect to \( x_n (n = 1, \ldots, N) \) equal to zero:

\[
EU_s Pr_t \frac{\partial f}{\partial x_n} = w_n \sum_{s=1}^{S} EU_s \quad (n = 1, \ldots, N)
\]  

(2.13)
Where \( w_n \) is the input price for input \( x_n \). If the decision maker is risk-neutral Equation 2.13 reduces to:

\[
p_t \left( P_t \frac{\partial f_t(x)}{\partial x_n} \right) = w_n \quad \left( n = 1, \ldots, N \right)
\]  

(2.14)

Thus, a risk-neutral decision maker should apply a state-specific input as long as the value of the marginal product \( P_t \frac{\partial f_t(x)}{\partial x_n} \) multiplied by the probability of being in that state \( (p_t) \) is larger than or equal to the input price \( w_n \). Again a risk-averse decision maker would use more of input \( x_n \) than a risk-neutral decision maker would if the marginal utility is positive as in Equation 2.15.

\[
\frac{\partial \text{EU}(y_1, \ldots, y_s)}{\partial x_n} \bigg|_{x=x^n} > 0
\]  

(2.15)

Performing the derivative in Equation 2.15 and using the value of \( P_t \frac{\partial f_t(x)}{\partial x_n} = \frac{w_n}{p_t} \) the condition in Equation 2.15 is equivalent to:

\[
\text{EU}_t > p_t \sum_{s=1}^{s} \text{EU}_s \bigg|_{x=x^n}
\]  

(2.16)

As the sum of the right-hand side is equal to one (see Eq 2.3) the condition in Equation 2.16 then indicates that state \( t \) is a bad state. Thus, it is concluded that if the state-specific input \( x_n \) is directed towards a bad state of nature, then a risk-averse decision maker will use more input than a risk-neutral decision maker will and vice versa.

2.2.1.3. State-allocable inputs

The free disposability that is assumed with state-contingent output can lead to inefficient production. To overcome this problem, inputs are allowed to be allocated to different actions. A state-allocable input is an input that may influence output in two or more states of nature and can be allocated to different states of nature. A state-allocable input can be considered as the sum of two (or more) state-specific inputs. The formal definition of a state-allocable input \( x_n \) is (Rasmussen, 2003):
Using Rasmussen's (2011) example, assume that a producer has labour available to either improve an irrigation system or to improve a drainage system that will result in improved production. During a wet year \( (y_1) \) an improved drainage system will increase yield because the producer will be able to drain excess water more effectively. During a dry year \( (y_2) \) the improved irrigation system would result in increased production. An improved irrigation system will have no effect during a wet year and a drainage system will have no effect during a dry year. The decision of what to improve is made before production is undertaken. Assuming that the producer can choose how to allocate the labour between improving the irrigation system and the drainage system the production functions and transformation function for a state-allocable input is given in Figure 2.3.

**Figure 2.3:** Production functions for state-allocable input use in a wet \( (y_1) \) and dry \( (y_2) \) state of nature and the resulting transformation function.
If the producer decides to allocate labour to upgrade the drainage system and a wet year \( (y_1) \) prevails the producer will achieve a higher yield because he will be able to drain more effectively as shown by the production function in Figure 2.3(a). During a dry year \( (y_2) \) upgrading the drainage system will have no effect on production as shown by the flat function in Figure 2.3(b). If the producer decides to allocate all the labour to improving the irrigation system and the subsequent production season is dry \( (y_2) \) a higher than normal yield will be realised, because the producer is able to irrigate more effectively. The production function used to illustrate production in a dry year \( (y_2) \) with an improved irrigation system is shown in Figure 2.3(d). If the subsequent season was a wet \( (y_1) \) the improved irrigation system would have no effect on production as shown by the flat function in Figure 2.3(c). The resulting transformation function is shown in Figure 2.3(e).

If the producer decides to allocate all of the available labour to improving the drainage system the state-contingent output achieved is shown by point A in Figure 2.3(e). If all of the labour is used to improve the irrigation system the state-contingent output is represented by point D. However, the producer can choose to allocate some of the labour force to improving the drainage system and the irrigation system. Assuming the possibility of other combinations, it is possible to connect the individual points to form a transformation function (Rasmussen, 2011). The state-allocable inputs thus allows for substitution between state-contingent outputs.

A state-allocable input allows the decision maker the possibility to allocate the input to whichever state is judged to be best. In principle the decision maker can combine a series of state-specific inputs in various combinations. Therefore, the optimality criteria for state-allocable input can be considered as a special case of state-specific input criteria (Rasmussen, 2011). The optimisation problem for the state-allocable input \( x_n \) is given by:

\[
\text{Max}_{x_{n1}, \ldots, x_{ns}} \text{EU}(y_1, \ldots, y_s) \quad (2.18)
\]

Assume that \( x_n \) is strictly state-allocable (each \( x_{nt} \) is state-specific). The optimal application of input \( x_{nt} \) is determined by the same conditions as for state-specific inputs:

\[
EU_t Pr_t \frac{\partial f_t(x)}{\partial x_{nt}} = w_n \sum_{s=1}^{S} EU_s \quad (t = 1, \ldots, S) \quad (2.19)
\]

Equation 2.19 reduces to Equation 2.20 because of the scaling in Equation 2.3. Therefore, the optimal allocation is determined for a risk-neutral decision maker by:
Theoretical Framework

\[ EU_t P_{t} \frac{\partial f_t(x)}{\partial x_{nt}} = w_n \quad (t = 1, \ldots, S) \]  

(2.20)

Because the expected utility \((EU_t)\) in state \(t\) is dependent on the likelihood that state \(t\) will occur \((EU_t = p_t)\), Equation 2.20 further reduces to:

\[ p_t \left[ P_{t} \frac{\partial f_t(x)}{\partial x_{nt}} \right] = w_n \quad (t = 1, \ldots, S) \]  

(2.21)

Thus, a risk-neutral decision maker should increase the application of the state-allocable input to state \(t\) as long as the marginal product multiplied by the probability of getting state \(t\) is larger than the input price \(w_n\).

To determine if a risk-averse decision maker uses more or less input than a risk-neutral decision maker, Equation 2.20 and Equation 2.21 should be compared. The result is similar as for a state-specific input. If a strictly state-allocable input is allocated to a bad state of nature, then the risk-averse decision maker will use more of the input in that specific state than a risk-neutral decision maker, and vice versa.

If only a limited amount of state-allocable input is available the condition in Equation 2.19 changes to:

\[ EU_t P_{t} \frac{\partial f_t(x)}{\partial x_{n}} = w_n \sum_{s=1}^{S} EU_s + \lambda \quad (t = 1, \ldots, S) \]  

(2.22)

Where \(\lambda\) is the Lagrange multiplier of restrictions on \(x_n\). For a risk-neutral decision maker the sum on the right-hand side is one and \(EU_t = p_t\), which means that the limited amount of input \(x_n\) should be allocated between state \(s\) and state \(t\) so that:

\[ p_t \frac{\partial f_t}{\partial x_{nt}} P_{t} = p_s \frac{\partial f_s}{\partial x_{ns}} P_{s} \quad (s, t \in \Omega) \]  

(2.23)
Thus, the marginal product of input $x_n$ in state $t$ multiplied by the probability of getting state $t$ should be equal to the marginal product of input $x_n$ in state $s$ multiplied by the probability of getting state $s$.

Although Rasmussen (2003) derived the criteria for optimal input decisions, it is concluded that it is not possible to make general statements concerning who will use more or less input. The answer will depend on specific preferences and partly on the decision makers view of what is a good or a bad state of nature.

2.3 CONCLUSIONS

Literature on decision-making under uncertainty has focussed primarily on the use of parameterised distributional approaches. However, the parameterised approach does not allow the decision maker the possibility to exploit the opportunities that uncertainty offers (Rasmussen, 2011). The state-contingent approach was developed as an alternative approach to evaluate decision-making under risk (Hurley, 2010; Rasmussen, 2011). The means of characterising the outcome distribution in the state-contingent approach differs significantly from that of the parameterised approach. While the distribution of the outcome variable in the parameterised approach is based on the choice of the input variable, the distribution of the outcome variable in the state-contingent approach is based on the state of nature. With the state-contingent approach the individuals' choice therefore cannot affect the likelihood of a chance outcome because the likelihood of a chance outcome is determined by the state of nature. The conclusion is that the state-contingent approach captures the uncertainty due to factors external to the decision makers decision-making process which is more realistic characterisation of risk compared to the parameterised approach.

The implication of the theoretical framework for this research is that state-contingent production functions should be estimated. The state-contingent production functions will enable the development of transformation functions that can be used to describe production of a good between different states. The transformation functions are essential to the application of the state-contingent approach because the existence of the transformation functions allows the decision maker the chance to exploit the opportunities that uncertainty offers. The estimation of the transformation functions is not easily attainable as the state-contingent production functions are difficult to estimate. The major concern with the estimation of state-contingent production functions is the characterisation of the states of nature which can include a large number of state-variables (Rasmussen, 2004; Rasmussen & Karantininis, 2005, O’Donnell et al., 2010). In real
life, however, the typical state-contingent variables for instance, sunshine, rainfall and temperature are not independent variables (Rasmussen, 2006). As a result it might be possible to combine state variables and assume that a production year is representative of a state of nature. Such an aggregation allows the estimation of state-contingent production functions.
CHAPTER 3

RISK EFFICIENT FERTILISER USE: A STATE-CONTINGENT APPROACH

3.1 INTRODUCTION

Research on producers’ input use decisions has created a rich body of literature. Agricultural producers’ fertilisation decisions are important for several reasons. Firstly, nitrogen is one of the key factors that determine crop productivity (Grandorfer et al., 2011) and therefore the potential to improve expected production income generation. Secondly, fertilisers are expensive and in many instances fertiliser costs represent the majority of the variable cost of production (Monjardino et al., 2013). Balancing the cost of production with potential gains stemming from applying fertiliser is not an easy task. The reason is that input use decisions have a significant bearing on the associated crop yield variability (Lambert, 1990; Sadras, 2002; Antle, 2001).

Most researchers (Lambert, 1990; Roosen & Hennessy, 2003; Abedullah & Pandey, 2004; Lobell, 2007; Rajsic & Weersink, 2008; Rajsic et al., 2009; Paulson & Babcock, 2010; Gandorfer et al., 2011; Picazo-Tadeo & Wall, 2011; Monjardino et al., 2013) have used the parametric distributional approach to study the impact of fertiliser use on crop yield variability using the Just and Pope procedure to estimate mean and variance functions (Just & Pope, 1978, 1979). Paulson and Babcock (2010) argue that producers do not equate yield variability as measured by variance to yield risk. Empirical evidence further shows that input use may affect the skewness of yield variability (Torriani et al., 2007; Rajsic & Weersink, 2008; Hennessy, 2009; Du et al., 2012). Therefore variance may be a poor proxy for risk if producers are concerned with downside risk. Methods are available to study the impact of input use on third and higher order moments of output. Finger (2013) used the moments based approach developed by Antle (1983) to show that water use may be underestimated if the impact of irrigation on skewness is ignored. The approach developed by Antle (1983) implicitly assumes that the changes in the moments are symmetrical. Recently Antle (2010) developed a partial moment approach to show that input use has an asymmetrical impact on moments. The asymmetrical impact on moments is the result of complex interactions between the biophysical and management processes that jointly determines production
variability. Since input use may change the positive and negative tails of distributions differently, the impact on variance and skewness is not self-evident. As a result Antle (2010) argues in favour of flexible methodologies to study the impact of input use on production risk.

An alternative procedure that is not well explored in literature is the state-contingent representation of input use decision-making under uncertainty. The research reported in this chapter contributes to the literature on modelling input use decision-making behaviour with skewed production risk through the development of a state-contingent direct expected utility maximising model that uses continuous response functions to quantify production risk empirically. Empirical production risk distributions were developed by viewing production response to fertiliser applications as state-contingent (Quiggin & Chambers, 2002). Consequently a nitrogen response function was estimated for each state of nature. Including all the different functions for each state of nature into the programming model made it possible to relate nitrogen fertiliser applications to an empirical distribution of production on a continuous basis. The benefit of using state-contingent response functions is that no distributional assumptions are necessary to model the impact of input use on changes in production risk. Since the model quantifies production risk empirically the procedure could be used with any of the decision models that use empirical representations of output distributions (e.g MOTAD (Hazell, 1971), Target-MOTAD (Tauer, 1983) and Direct Expected Utility Maximisation (Kaylen et al., 1987; Biosvert & McCarl, 1990)). Furthermore, the method overcomes the problem where the input use decision is represented by a combination of input levels for the same technology set if the stochastic production function is represented by different activities for discrete levels of input use (Grové, 2010).

In order to demonstrate the flexibility of the newly developed method to model production risk, the optimisation model was used to determine optimal fertilisation rates on two different soil types where nitrogen was applied in a single or split nitrogen fertiliser application under conditions of no water stress. The results showed that the state-contingent representation of production risk was able to capture the changes in outcome variable variability without making any distributional assumptions. Fertiliser use was classified as a risk-reducing input on a sandy clay soil while it could be risk-reducing or increasing on a sandy clay loam soil.

The chapter proceeds by discussing the data and procedures, followed by the results, discussion and conclusions.
3.2 EMPIRICAL APPLICATION

The estimation of state-contingent response functions requires state-contingent production data. A crop growth model was used to simulate the data necessary to estimate the response functions. The next section discusses the use of the crop growth simulation model, the optimisation model used to determine the risk efficiency of input decision, and how the response functions used in the optimisation model were estimated.

3.2.1 DATA SIMULATION

The Soil Water Balance (SWB) model is a daily time-step crop growth model developed for irrigation scheduling purposes (Annandale et al., 1999). Recently nutrient sub-routines were incorporated and validated by Van der Laan et al. (2009) to simulate nutrient loss. The newly developed model is referred to as SWB_Sci (Van der Laan et al., 2009). To simulate data for this research the validated SWB_Sci was set-up for late monoculture maize (planting date 15 December) under irrigation on a Sandy Clay Loam (SCL) and a Sandy Clay (SC) soil at Glen, South Africa. The two soils differ in structure which means it differs in water holding capacity. The water holding capacity for a SC soil is normally greater than for a SCL soil (Brouwer, 1985; FSSA, 2007) therefore the amount of irrigation water required on a SC soil is greater. Furthermore the SCL soil has a higher infiltration and drain rate compared to the SC soil.

Maize yield response to nitrogen fertiliser and the associated water use under conditions of no water stress were simulated for a period of 19 years while assuming 33kg of initial soil nitrogen. Nine levels of nitrogen fertiliser were applied either in a single application on the date of planting or in two separate applications. When using two separate applications, two thirds of the desired nitrogen level was applied on the day of planting while the remaining third of the nitrogen was applied seven weeks later. Only applications above 70 kg/ha was applied with a split application as it is typically too costly to apply lower levels of nitrogen in a split application. Conditions of no water stress was modelled by refilling the soil profile to field capacity once a soil water depletion level of 40% was reached. As a result the crop experienced no water stress.

3.2.2 DETERMINING OPTIMAL INPUT USE LEVELS

The risk efficiency of alternative input use decisions were evaluated through the optimisation of certainty equivalents. State-contingent response functions are used to quantify the transformation function that allows for a state-contingent representation of production risk within an expected utility framework (Hurley, 2010). The GAMS (Brooke et al., 2013) model specification is as follows:
Maximise  \[ CE = \frac{\ln[-\sum_s p_s(-e^{r_u(GM)}G M_s)]}{-r_u(GM)} \]  \hspace{1cm} (3.1)  

s.t.  \[ GM_s = Y_s(N)P_y - NP_N - W_s(N)P_w - B_a - B_y Y_s(N) \]  \hspace{1cm} (3.2)  \[ Y_s(N) = \beta_{1s} + \beta_{2s} N + \beta_{3s} N^2 + \varepsilon_s \]  \hspace{1cm} (3.3)  \[ W_s(N) = \omega_{1s} + \omega_{2s} N + \omega_{3s} N^2 + \mu_s \]  \hspace{1cm} (3.4)  \[ N \leq 220 \]  \hspace{1cm} (3.5)  

Where:  
- \( CE \) is the certainty equivalent that is determined with the optimisation model  
- \( p_s \) is the probability that state of nature \( s \) will occur  
- \( GM_s \) is the Gross Margin for state of nature \( s \) (R/ha)  
- \( r_u(GM) \) defines the coefficient of absolute risk aversion appropriately scaled to be relevant to the estimated gross margins  
- \( Y_s(N) \) is the crop yield as a function of nitrogen applications in state of nature \( s \) (ton/ha)  
- \( P_y \) is the price of maize (R/ton)  
- \( N \) is the level of nitrogen fertiliser applied (kg/ha)  
- \( P_N \) is the price of nitrogen fertiliser (R/kg)  
- \( W_s(N) \) is the applied irrigation water as a function of nitrogen applications in state of nature \( s \) (mm,ha\(^{-1}\))  
- \( P_w \) is the cost of applying irrigation water (R/mm,ha\(^{-1}\))  
- \( B_a \) is the area dependent cultivation cost (R/ha)  
- \( B_y \) is the yield dependent harvesting cost (R/ton)  
- \( \beta_{is} \) represents the \( i^{th} \) \( (i=1,2,3) \) estimated coefficient for the yield response function in state of nature \( s \)  
- \( \varepsilon_s \) is the estimated output residuals for every state of nature, \( s \)  
- \( \omega_{is} \) represents the \( i^{th} \) \( (i=1,2,3) \) estimated coefficient for the irrigation water response function in state of nature \( s \)  
- \( \mu_s \) is the estimated irrigation water residual for every state of nature, \( s \)  

A negative exponential utility function is used to estimate the certainty equivalents used to rank the alternatives in Equation 3.1. The state-contingent gross margin is estimated in Equation 3.2. The
variation of the gross margin is due to the effect of the state of nature on maize yield and irrigation water as estimated in Equation 3.3 and 3.4. The unexplained variability associated with maize production and water response in every state of nature is included as relative deviations from expected responses of the maize yield and irrigation water to nitrogen applications (Richardson et al., 2000). As a result each state-contingent response function includes a stochastic component that is modelled as a discrete empirical distribution. Consequently the nineteen estimated response functions plus their residuals characterise the production risk associated with nitrate applications. Therefore, there are 171 (19 production years multiplied by 9 estimation errors) possible random outcomes that are used to characterise risk.

The underlying assumption is that the decision maker decides on the level of fertiliser before the state of nature is known; therefore, nitrogen fertiliser is a state-general input. Within the model a state-general optimal fertiliser level is determined endogenously based on all possible outcomes in all possible states of nature and the probability of occurrence. Equation 3.3 shows that maize yield is only a function of fertiliser applications. Irrigation water was not included in the estimation of the maize production function because the crop never experienced water stress during the data generation process. Consequently no yield response to irrigation water use is expected in a specific state of nature. However, the level of irrigation water use is dependent on crop growth which is a function of fertiliser application in each state of nature. Therefore, irrigation water is determined endogenously in the model as a function of applied nitrogen.

The choice of the upper bound of the risk aversion coefficient used in the estimation of the certainty equivalent is based on the maximum risk aversion parameter of 2.5 usually reported in applied MOTAD studies. McCarl and Bessler (1989) derived a link between the MOTAD risk aversion parameter and the absolute risk aversion parameter used in mean-variance quadratic programming models. Grové (2007) used the link to demonstrate that the MOTAD risk aversion parameter is equivalent to the standard deviation of the outcome variable multiplied by the level of absolute risk aversion. The absolute risk aversion parameter was therefore chosen such that the upper bound of the risk aversion parameter corresponds to 2.5.

In order to run the optimisation model and determine optimal fertilisation levels state dependent gross margins, state-contingent yield response functions and irrigation water response functions is necessary. Next the estimation of the state-contingent gross margins, crop yield and water use is discussed in more detail.
3.2.2.1. Quantifying state dependent gross margin

Gross margin variability associated with different levels of input use is estimated using state general crop yield production functions and state allocable irrigation water functions. More specifically gross margin variability is estimated as follows:

\[
GM_s = Y_s(N)P_Y - NP_s(N)P_N - B_s - B_s Y_s(N)
\]  

(3.6)

Gross margin estimation is dependent on the income received by decision makers and the production cost. All production cost data and input prices used in the estimation of the gross margins were obtained from Griekwaland-Wes Cooperation (GWK Ltd), South Africa. The first term \( (Y_s(N)P_Y) \) in Equation 3.6 estimates the decision makers income. Income is determined by multiplying the output \( (Y_s(N)) \) with the product price \( (P_Y) \) realised by the decision makers. Decision makers' output is estimated as state-contingent yield response to fertiliser applied. Although the level of fertiliser remains constant between states, yield response does not, due to the effect of states of nature. The state of nature is representative of differences in production years (e.g. weather). The second term \( (NP_s(N)) \) estimates the cost of applying fertiliser to produce maize. The third term \( (W_s(N)P_w) \) is used to estimate the irrigation cost. The amount of irrigation water applied is determined by the state of nature that occurs and the level of fertiliser applied which influences plant growth and therefore irrigation requirements. Cultivation cost represented by \( B_a \), which is area dependent, and the decision maker faces this cost irrespective of which state of nature occurs. The last term \( (B_s Y_s(N)) \) represents the harvesting cost which is yield dependent and is thus determined by the state of nature. Harvesting cost is state dependent in the sense that the producer only faces harvesting cost if a crop is realised. The cost of harvesting increase or decrease as the crop yield produced increase or decrease in every state of nature.

The following sections provide details of the estimation of the state-contingent crop yield \( (Y_s(N)) \) and irrigation water requirement \( (W_s(N)) \).

3.2.2.2. State-contingent crop yield

The state-contingent approach suggests that an outcome obtained by a decision maker is dependent on the state of nature, therefore, a crop yield regression model was developed for every year in the research period. This research follows Quiggin and Chambers (2002) state-contingent approach to estimate empirical production functions. Literature (Abedullah & Pandey, 2004; Carew et al., 2009; Koundouri et al., 2009) indicated that the most commonly used fertiliser–crop yield
functions are Cobb Douglas functions and quadratic production functions. The quadratic production function was the better fit for the simulated data and was therefore used to relate nitrogen fertiliser use (kg/ha) to maize yield (ton/ha) as follows:

\[ Y_s(N) = \beta_{1s} N + \beta_{2s} N^2 + \varepsilon_s \]  

(3.7)

The estimated coefficients for every production year are different, therefore, a unique production function exists for every state of nature. The estimated coefficients for the state-contingent maize production functions for nitrogen are shown in Appendix A. The bulk of the estimations explain a great deal of the variation in the dependent variable with a \( R^2 \) well over 0.5. Only three functions explain very little of the variation in the dependent variable with a \( R^2 \) of 0.4 or less. The estimated constant term shows that even if no nitrogen fertiliser is applied a maize yield will be realised. The reason is that nitrogen applied is not the only source of fertiliser available to the crop. Residual nitrogen in the soil (about 33kg/ha), rainwater and irrigation water also contribute to nitrogen availability that influences crop growth.

3.2.2.3. State-contingent irrigation water

Irrigation water applied during the production process is a state-contingent input which is determined by the state of crop growth in each state of nature given the quantity of nitrogen applied. From the simulated SWB_Sci data a quadratic relationship between fertiliser applied and irrigation water applied was identifiable. The ability to correctly determine the amount of irrigation water applied in a state of nature allows for the calculation of the correct irrigation cost to be included in the gross margin estimation. The irrigation water–fertiliser response function is specified as follows:

\[ W_s(N) = \omega_{1s} N + \omega_{2s} N^2 + \mu_s \]  

(3.8)

The estimated irrigation water – fertiliser response functions show a different response function for every state of nature. The estimated coefficients for irrigation water – fertiliser response functions are shown in Appendix B. The bulk of the estimations explain a great deal of the variation in the simulated data with a good \( R^2 \). Only five of the estimations have a \( R^2 \) below 0.5 and do not explain a great deal of the variation in the simulated data.
3.3 RESULTS AND DISCUSSION

3.3.1 QUANTIFYING STATE-CONTINGENT RISK

Through the quantification of the state-contingent production risk the researcher can gain a better understanding of risk as a function of nitrogen fertiliser applied. The discussion of state-contingent risk starts by quantifying state-contingent production risk and irrigation water risk. The discussion concludes with the state-contingent gross margin risk. Gross margin risk was quantified with the use of the state-contingent production and irrigation water risk.

3.3.1.1 State-contingent production risk

The nitrogen fertiliser -maize yield response functions for all soil types for a single and split nitrogen fertiliser application are shown in Figure 3.1. The production functions are used to show the effect of states of nature on production variability.

The average yield depicted in Figure 3.1 shows that average maize yield is not very responsive to increased fertiliser use, as the average yield response appears flat. Average maize yield for nitrogen use in a single application on a SCL soil range from 8.2 ton/ha to 10.7 ton/ha, while average yields for a single application on a SC soil range from 8.8 ton/ha to 10.5 ton/ha. When nitrogen fertiliser is applied in a split application on a SCL soil, the average yields obtained range between 8.1 ton/ha and 10.6 ton/ha; while on a SC soil average yields range between 8.8 ton/ha to 10.4 ton/ha. To gain a better understanding of how the yield variability changes with changes in fertiliser applications the standard deviation and skewness are presented in Figure 3.2.
The standard deviation of maize yield as a function of nitrogen application follow the same general trend on both SCL and SC soils. Standard deviation decreases towards a minimum before showing a slight increase. Maize yield variation on a SC soil is lower than on a SCL soil, irrespective of the nitrogen application method if nitrogen application levels above 70kg/ha is considered. On a SCL soil maize yield variation reaches a minimum at a lower nitrogen use level than on a SC soil. Thus, the decision maker has greater potential to reduce yield variability on a SC soil by increasing nitrogen applications irrespective of nitrogen application method. The choice of single or split application adds a new dimension to the choice of soil type and level of application. At low levels of nitrogen fertiliser use on a SCL soil use of a split application of fertiliser will result in lower yield variability than a single application while at higher levels of fertiliser application (95kg/ha and more)
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A single application will result in lower yield variability. On a SC soil the opposite is true. At low levels of fertiliser use a single fertiliser application will result in lower yield variability while at higher levels of fertiliser application (70kg/ha and more) a split application will result in lower yield variability.

**FIGURE 3.2:** Standard deviation and skewness of maize yield for increased nitrogen fertiliser application (kg/ha) on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil in a single and split nitrogen application.

Skewness is a measure of symmetry, meaning the distributions of values are the same for the left as for the right. A negative skewness indicates that data is skewed left or left-tailed, while a positive skewness indicates that data is right-tailed. Another way of interpreting skewness is to view a negative skewness as an increased exposure to downside risk. Downside risk indicates that the individual is exposed to outcomes significantly lower than the average. For low levels of fertiliser use the decision maker will be more exposed to downside risk and the decision maker will be more exposed to upside potential at higher levels of fertiliser use. Production with a split nitrogen application at high fertiliser levels (greater than 145 kg/ha) shows a decrease in skewness (move towards zero). On a SC soil maize yield skewness shows movement towards negative skewness when applying nitrogen fertiliser in a split application. When applying nitrogen fertiliser in a single application even though maize yield skewness does show a slight decrease, skewness level-off and never moves towards negative skewness. Production on a SCL soil with a single fertiliser application remains the closest to zero skewness. The conclusion is that the state-contingent representation of production risk was able to capture the impact of nitrogen fertilisation rate on the variability and skewness of production without making any distributional assumptions. Risk decision-making is concerned with the trade-off between expected outcomes and risk. Due to differences in the expected outcomes and the variability of outcomes for the alternatives, it is expected that the risk efficiency of the alternatives will be different.
3.3.1.2. State-contingent irrigation water use

The estimated irrigation water use for increased nitrogen fertiliser on all soil types and all nitrogen fertiliser application methods is shown in Figure 3.3. Irrigation water is determined based on the state-general usage of nitrogen in the different states. Assuming production on a SCL soil with a single fertiliser application of 120kg/ha, the amount of irrigation water applied ranges from 278mm.ha⁻¹ to 843mm.ha⁻¹. The change in irrigation water applied is solely due to the state of nature that occurs during production and the level of nitrogen application.

**FIGURE 3.3:** Irrigation water use (mm.ha⁻¹) response functions for increased nitrogen fertiliser application (kg/ha) on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil for a single and a split nitrogen application.
The average irrigation water use shown in Figure 3.3 does not show huge increases for increased fertiliser use as the average irrigation water use appears fairly flat. On average the amount of irrigation water use on the SCL soil is lower than irrigation water use on a SC soil. Due to the lower infiltration rate of the SC soil, more of the water that is applied is lost on the SC soil due to run-off. To show the variability associated with fertiliser use on the two soil types with a single and split nitrogen application, the standard deviation and skewness associated with irrigation water use is shown in Figure 3.4.

![Graph showing standard deviation and skewness of irrigation water applied for increased nitrogen fertiliser application (kg/ha) on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil of a single and split nitrogen application.]

It is interesting to note the difference in the standard deviations of irrigation water use for the two different soils as fertiliser use increase. On a SCL soil the standard deviation associated with irrigation water use increases from 110 mm.ha⁻¹ to a maximum of almost 160 mm.ha⁻¹ as nitrogen fertiliser use increases before starting to decrease again. Although the standard deviation of irrigation water use on a SC soil also increases with increased nitrogen use, the increase in standard deviation is much smaller and the maximum is reached at higher nitrogen use levels than for a SCL soil. The conclusion is that a mean function is not representative of the distribution of outcomes between alternative production technologies, even though the estimated mean function seems similar and the distribution of outcomes around the mean function can be greatly different.

For all levels of fertiliser use irrigation water use is always positively skewed, meaning that there is a probability of realising irrigation levels that are significantly larger than the mean. Skewness on a SCL soil for single and split fertiliser use is the same, indicating that the distribution of irrigation amounts around the mean is the same for a single and split application of fertiliser on a SCL soil.
Skewness on a SC soil for both single and split fertiliser application decreases toward zero skewness with increased fertiliser use. Unlike on a SCL soil single and split application on a SC soil are not the same with a longer tail for a split application.

### 3.3.1.3. State-contingent gross margin

Figure 3.5 shows the gross margin as a function of nitrogen applications for each of the states of nature on all soil types and all nitrogen fertiliser application methods.

**FIGURE 3.5:** Nitrogen gross margin response functions on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil for a single and a split nitrogen application.
The average gross margin (R/ha) increases for increased fertiliser use. On average the gross margin obtained from production on a SCL soil reaches a higher maximum than on the SC soil. However, the minimum gross margin on a SCL soil is also lower than on a SC soil. It is thus expected that the variation in gross margin on a SCL soil will be greater than the variations in gross margins on the SC soil. Furthermore, the shape of the gross margin functions is very similar to that of the estimated state-contingent crop yield functions. To gain a better understanding of the distribution of the gross margin associated with fertiliser use on the two soil types with a single and split nitrogen application, the standard deviation and skewness is shown in Figure 3.6.

FIGURE 3.6: Standard deviation and skewness of gross margin for increased nitrogen fertiliser application (kg/ha) on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil of a single and split nitrogen application.

Generally the standard deviations of all the alternatives follow the same general trend as for maize yield variations. However, the differences between the two nitrogen application methods are not as profound. Standard deviation decreases towards a minimum before showing a slight increase if nitrogen application levels above 80kg/ha is considered. Gross margin variation on a SC soil is lower than on a SCL soil, notwithstanding the nitrogen application method. On a SCL soil gross margin variation reaches a minimum at a lower nitrogen use level than on a SC soil. A decision maker will therefore be able to reduce risk by applying more nitrogen on a SC soil when compared to a SCL soil.

In general the skewness of gross margin follows the same trend as maize production which shows the importance of maize production levels on gross margin estimation. Increasing fertiliser usage reduces downside risk to a point where the decision maker is exposed to a chance of realising gross margins that are significantly higher than the average.
3.3.2. **Optimal Nitrogen Input Levels under Uncertainty**

The optimisation model was used to maximise the certainty equivalents of gross margin through the use of the estimated stochastic state-contingent production functions. The optimisation model endogenously determines the level of fertiliser application that will result in the highest certainty equivalent as risk aversion increase. The optimised nitrogen application levels determined on a SCL and SC soil for single and split nitrogen application are shown in Figure 3.7.

![Figure 3.7: Optimal nitrogen fertiliser levels (kg/ha) for increased levels of risk aversion for single and split nitrogen application on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil.](image)

The most prominent difference in the estimated optimal fertilisation rates is the difference between fertiliser use on a SCL soil and a SC soil. Irrespective of the fertiliser application strategy, fertiliser use on a SCL soil is greater than on a SC soil. Except at extreme risk aversion (RAC) levels of 0.0024, where fertiliser use of a SC soil is greater than that on a SCL soil when applying fertiliser in a single application. Monjardio *et al.* (2013) also found that producers who produce on a sandier soil would do better if they used greater levels of fertiliser. Furthermore, when applying fertiliser in a split application more fertiliser is applied regardless of the soil used for production.

Risk preference has a significant impact on optimal fertiliser application levels. On a SCL soil fertiliser acts as a risk increasing and decreasing input. When applying fertiliser in a single application the decision maker would first increase and then decrease the amount of fertiliser...
applied with increased risk aversion. When applying fertiliser in a split application, fertiliser application decreases, increases and decreases again with increased risk aversion. Thus, nitrogen fertiliser acts as a risk increasing and decreasing input on a SCL soil, irrelevant of the fertiliser application method. From literature it was found that in some studies nitrogen acted as a risk decreasing input (Lambert, 1990 and Isik & Khanna, 2003) and in others nitrogen was a risk increasing input (Paulson & Babcock, 2010; Monjardino et al., 2013; and Gandorfer et al., 2011). Overall the difference between the upper and lower level of fertiliser applied with a single and a split application on a SCL soil, does not amount to more than 3kg/ha. So even though fertiliser acts as a risk increasing and decreasing input, the optimal nitrogen application rate stays fairly constant. Figure 3.7 clearly shows the risk decreasing effect of nitrogen applications on a SC soil with increasing levels of absolute risk aversion. The difference between the upper and lower level of fertiliser applied in a single and a split application is about 17kg/ha, which is much more when compared to a SCL soil. Although nitrogen application variability for production on a SC soil is higher, fertiliser acts as a risk decreasing input. Therefore, increased risk aversion result in increased use of fertiliser, irrelevant of the fertiliser application technique. In conclusion decision makers on a SC soil would preferably increase fertiliser use with increased risk aversion, as fertiliser is a risk decreasing input. It is more difficult to develop general guidelines on input use behaviour for decision makers who produce on a SCL soil as fertiliser acts as a risk increasing and decreasing input with increased risk aversion. Furthermore, the overall impact of risk aversion on optimal nitrogen levels is small. The conclusion is that soil-specific input recommendations are necessary for decision makers.

3.3.3. **Risk Efficiency of Fertiliser Use**

The optimised certainty equivalent (CE) for increased levels of risk aversion is shown in Figure 3.8.

A certainty equivalent indicates the minimum amount of money a decision maker requires to make him/her indifferent between a certain payoff and a risky alternative. Since the certainty equivalent is derived from the ordinal utility function (Hardaker et al., 2004), the certainty equivalent can be used to rank alternatives where the alternatives with the highest CE is preferred (Boisvert & McCarl, 1990). The optimised CE indicates that production with a single fertiliser application is preferred to a split fertiliser application irrespective of the soil that is used.
FIGURE 3.8: Certainty Equivalent (CE measured in R) for optimal fertiliser levels as determined in the optimisation model for increased levels of risk aversion (RAC).

The overall risk frontier consists of a combination of strategies. The frontier consists of production with a single fertiliser application on a SCL and SC soil. While decision makers with relatively low levels of risk aversion (RAC of 0.0013 or lower) would prefer production on a SCL soil, decision makers with a higher level of risk aversion (RAC greater than 0.0013) would prefer production on a SC soil. Production with a single fertiliser application is always preferred irrespective of the soil type, because the use of a single application always leads to a greater CE compared to a split fertiliser application. The only true trade-off effect is between the two soil types. The conclusion is that firstly, decision makers are willing pay a premium to use a single application of fertiliser during production, irrelevant of the soil type used for production. It should be noted that this result may hold for this study only. The study did not allow for changes in the timing of the fertiliser application and changes in fertiliser application timing can alter decision-making and the decision maker’s response under uncertainty. Secondly, the preferred choice of soil type used during production is dependent on the decision makers’ risk attitude. Therefore, information on the decision maker, specifically the decision makers’ risk attitude, must be known before advising the decision maker on the choice of soil type use.
3.4 CONCLUSIONS

The estimation of flexible state-contingent functions allows the researcher the opportunity to model skewed production risk without having to make distributional assumptions. A benefit of the state-contingent functions is that the functions are easily incorporated into Expected Utility models. During the estimation of the state-contingent functions a production year was used to characterise a state of nature. The results for the estimated functions indicate that the response functions do not capture all of the variation in the dependent variable. A possible explanation is that a true state of nature is characterised by a large number of state variables (Rasmussen, 2004; Rasmussen & Karantininis, 2005, O'Donnell et al., 2010) for instance sunshine, rainfall and temperature. During the reduction of these state-variables into one state-variable, some information that can explain the variance in the dependent variable could be lost. However, it is very difficult to capture all the state-variables that will explain the variance in the dependent variable. More research is necessary to correctly identify and capture the state-variables in the response functions.

Results for the optimisation model indicate that fertiliser acts as a risk decreasing input on a SC soil. This result is different from that found in recent studies (e.g. Finger, 2012 (Just & Pope model) and Monjardino et al., 2013 (Probability Density Function)) that found fertiliser to be risk-increasing. However, the authors of these papers attribute their results to the procedure use to capture production risk. What is also noteworthy from the results in this chapter is that nitrogen is risk decreasing on the sandy clay loam soil and act as a risk increasing and risk decreasing input on a sandy clay soil. The main conclusion is that generalisation of decision makers’ input decisions due to their risk attitude, is not possible because fertiliser can act as a risk increasing or decreasing input. As a result, soil-specific input strategies should be used to ensure optimal input use decisions for the soil type used during production. Monjardino et al. (2013) also recommended the use of soil-specific input strategies.

The results showed that increasing risk aversion increased the optimal fertiliser levels on a SC soil. Since fertiliser is acknowledged as a primary source of non-point source pollution (Sadras, 2002; Isik, 2002; Bonnem & Thomas, 2006; Gandorfer et al., 2011; Finger, 2012) the increased reliance on fertiliser during production could be detrimental to the environment. Therefore, further research is necessary to determine the impact of risk efficient input use decision on the environment and to what extent an environmental constraint can reduce the negative environmental effect of production.

The state-contingent framework allows for the determination of the value of information pertaining to which state of nature will occur. The value of information could be calculated by comparing the
results from this research with results where fertiliser applications are applied assuming complete knowledge of which state of nature will occur.

The fact that a single fertiliser application is better than a split application needs to be accepted with caution. No attempt was made in this research to optimise the timing of the split fertiliser applications. Changes in decision makers’ fertiliser timing strategy can potentially change the decision makers’ choice of production technology (e.g. soil type and single or split fertiliser application). Furthermore, during simulation of the production data no water stress was considered. Water limiting conditions could alter decision makers’ production behaviour under uncertainty, resulting in increased or decreased input use. Future research can therefore evaluate decision makers’ input use decisions under uncertainty when limited water is available.
4.1 INTRODUCTION

Weersink et al. (2002) argues that optimal resource allocation is important not only because of its effects on farm income but also due to its effect on environmental health. Non-point source (NPS) pollution stemming from agricultural practices is seen as a major cause of the remaining water quality problems in developed and developing countries (Shortle et al., 1998; Rossouw & Görgens, 2005). There is thus increased pressure on agriculture to use resources optimally to reduce agriculture’s negative environmental effect (Shortle & Horan, 2001). However, no market exists for environmental health; therefore, alternative methods are necessary to determine a value for environmental health. The relationship between production decisions and the effect of the environment is typically modelled with trade-off curves. Weersink et al. (2002) stated that trade-off curves show the reductions in residuals or the environmental variable and the associated reduction in farm income as abatement efforts require lower emissions. To estimate the trade-offs knowledge regarding farm income and the environmental variable is necessary.

Nitrogen fertiliser is one of the key factors that determine crop productivity (Sadras, 2004; Grandorfer et al., 2011) which is essential for improving expected production income. Because decision makers aim to improve production income, they tend to over-apply fertiliser because the response of yields to fertiliser is often overestimated (Sherriff, 2005; Rajsic & Weersink, 2008). Nitrogen fertiliser application decisions have been researched extensively, especially with regard to objective function risk (i.e., Abedullah & Pandey, 2004; Lobell, 2007; Rajsic & Weersink, 2008; Rajsic et al., 2009; Paulson & Babcock, 2010; Gandorfer et al., 2011; Picazo-Tadeo & Wall, 2011; Monjardino et al., 2013). Most nitrogen studies found nitrogen fertiliser to be a risk increasing input (i.e., Ramaswami, 1992; Abedullah & Pandey, 2004; Marenya & Barrett, 2009; Paulson & Babcock, 2010; Gandorfer et al., 2011); while other studies found fertiliser to be a risk decreasing input.
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input (Lambert, 1990; Babcock, 1992; Isik & Khanna, 2003). The risk reducing nature of fertiliser will lead to increased fertiliser use resulting in a greater potential to harm the environment. Since nitrogen is already considered to be one of the major agricultural NPS pollutants (Liu et al., 2005) it becomes increasingly necessary to evaluate the economic-environmental trade-offs for nitrogen fertiliser decisions.

Studies that model economic-environmental trade-offs typically use chance-constrained programming (Kampas & White, 2003), Target-MOTAD (Teague et al., 1995; Aihoon et al., 1997; Qiu et al., 1998; Umoh, 2008) or safety-first constraint models such as the upper partial moment method (Qiu et al., 2001; Intarapapong et al., 2002). The application of the chance-constrained programming requires the specification of a functional form for the distribution of the environmental variable (Qiu et al., 2001). Because environmental variables are non-negative, skewed distributions are used to show the distribution of the random variable (Kampas & White, 2003). The probability models that have been used most commonly to describe environmental data includes the Poisson, negative binomial, Weibull, gamma, exponential and the log normal, with the log normal being mainly used. Even though there is support for the use of skewed distributions such as the log normal, only independent variables can be examined since there are still some questions regarding the estimation of the distribution for the sum of the log normal variables. Furthermore, these distributional assumptions can have a significant impact on the estimated trade-offs (Zhu et al., 1994; Qiu et al., 2001; Kampas & White, 2003) and may not hold for all situations due to the site-specific nature of agricultural NPS pollution (Qiu et al., 2001). To overcome the problem techniques were developed to estimate economic-environmental trade-offs making use of empirical distributions. One such technique is the environmental Target-MOTAD, which models deviations from a specified environmental target (Teague et al., 1995; Aihoon et al., 1997; Qiu et al., 1998; Umoh, 2008). The environmental Target-MOTAD technique (or model) uses the sample pollution emissions as an empirical distribution while optimising over the column space of the sample. The model requires the specification of the risk level of the expected deviation from the environmental goal. A potential problem with applying the model is that the scientific bases for the selection of a reasonable environmental risk level are weak. The UPM provides a stronger scientific basis for modelling economic-environmental trade-offs since it provides a method to impose a probabilistic constraint using the Target-MOTAD framework with an empirical distribution of the outcome variable.

The review of literature has indicated that the impact of nitrogen fertiliser applications on production risk is well-researched and that procedures are available to evaluate the impact of production decisions on environmental outcomes. Integrated analyses of the impact of nitrogen fertiliser applications on production risk and the environment are, however, not well-documented
or failed to correctly characterise the impact of increasing fertiliser applications on production risk in programming models. Aihoon et al. (1997)\(^1\) for example, assumed that the distribution of production risk stays the same irrespective of fertiliser application rates. Of special concern is the fact that the results in Chapter 3 showed that fertiliser applications may be classified as a risk reducing input under certain circumstances. An inverse relationship therefore exists between applying fertiliser to reduce production risk and the possibility to harm the environment. Very little information is available in literature regarding the inverse relationship.

In this chapter a novel approach that allows integrated analyses of the impact of nitrogen fertiliser applications on production risk and environmental health, is presented. The novelty of the approach is that both production risk and environmental risk are characterised by state-contingent functions which allows for an empirical representation of these risks. Through the use of state-contingent production functions and environmental response functions in the UPM a clear link is created between the source of the pollutant, the production risk stemming from its use and the environmental outcome. The newly developed modelling framework is applied to achieve three sub-objectives. The first sub-objective is to determine how the presence of an environmental constraint will affect the risk efficiency of production. The sub-objective is achieved by evaluating changes in the risk efficiency frontiers modelled for no environmental constraint and the frontiers associated with environmental compliance. The second sub-objective is to estimate the cost of compliance faced by risk averse decision makers for increasing levels of assurance. While the last sub-objective of the chapter is to determine how decision makers will achieve compliance. To achieve compliance it is expected that decision makers would make production changes at either the extensive margin or the intensive margin.

The chapter proceeds by discussing the theoretical background of the upper partial moment, the data and procedures, followed by the results, discussion and conclusions.

### 4.2 ENFORCING CHANCE CONSTRAINTS WITH THE UPPER PARTIAL MOMENT

Qiu et al. (2001) built on research by Atwood (1985) to develop an UPM inequality approach to impose the safety first constraint in a Target-MOTAD framework that will ensure that the target pollution level will be met at a certain specified probability level.

\(^1\) The paper did not provide enough detail to justify the conclusion. The reader is therefore, referred to Aihoon (1994).
An UPM is defined as follows for a continuous case:

\[ \rho(\alpha, t) = \int_{-\infty}^{t_j} (x_j - t)^\alpha f(x_j) \quad x_j \geq t \]  

(4.1)

Where: \( \rho(\alpha, t) \) is the upper partial moment  
\( \alpha \) is a constant greater than zero  
\( t \) is the reference pollution level  
\( x_j \) is the pollution variable  
\( f(x_j) \) is the relative frequency distribution of the pollution variable \( x_j \)

In Equation 4.1 the upper partial moment \( (\rho(\alpha, t)) \) is the integral of the deviation from the target \( (x_j - t) \) multiplied by the relative frequency distribution of the pollution variable. Therefore, the upper partial moment is a function of the deviations from the target pollution level and the probability of such an occurrence. If the frequency distribution is discrete the appropriate summations apply rather than integrals (Atwood, 1985).

\( \alpha \) places an upper limit on the probability of \( x \) being more than \( p\theta(\alpha, t) \) units above \( t \). Setting \( \alpha = 1 \) expresses the inequality in terms of absolute deviations from \( t \). Fishburn (1977) proved that a model that examines the trade-offs between \( t \) and \( \rho(\alpha, t) \) would generate solutions that are a subset of the second-degree stochastic dominance efficiency set if \( \alpha \geq 1 \). If \( \alpha \geq 2 \) the solution will be a subset of the third-degree stochastic dominance efficiency set. The UPM is useful to model safety-first behaviour where safety-first behaviour is defined as behaviour in which the probability of failing to achieve a goal impacts and constraints the activities undertaken (Atwood, 1985).

Regardless of the safety-first behaviour, information on the probability of not achieving a goal is needed. The estimation of the probabilities is difficult and the information is not always available. Atwood (1985) suggested the following general inequality to estimate the lower limits on the probability of goal failure:

\[ \Pr(x \geq G) = \Pr(x \geq t + p\theta(\alpha, t)) \leq \left(1/p\right)^\alpha \]  

(4.2)

with \( \theta(\alpha, t) = [\rho(\alpha, t)]^{1/\alpha} \geq 0 \)  
(4.3)  
\[ p = (G - t)/\theta(\alpha, t) \]  
(4.4)
Where: 

\( Pr \) is the probability that an event will occur

\( G \) is the pollution standard set for pollution variable \( x \)

\( \theta(\alpha, t) \) is the positive \( \alpha^{th} \) root of Fishburn’s partial moment \( \rho(\alpha, t) \)

\( p \) is a constant greater than zero

Equation 4.2 places a lower limit on the probability of \( x \) increasing more than \( p\theta(\alpha, t) \) units above \( t \). In Equation 4.2 the choice of some level of \( t \) between \( G \) and the population mean will often result in probability limits close to the actual probability limits (See Atwood, 1985). However, since the mean does not enter the proof of Equation 4.2 the mean, variance or semi-variance for the distribution is not necessary. The proof only requires that \( \rho(\alpha, t) \) exists and that it’s bounded (Atwood, 1985). Furthermore \( p \) should be defined as the limit of Equation 4.4 when \( \theta(\alpha, t) \) is zero with \( t > G \). When \( t = G \) and \( \theta(\alpha, t) \) exceed zero, then \( p \) must be zero, and the probability limit is defined as \( 1/p^\alpha \). Furthermore the choice of \( \alpha \) can also affect the probability limits. The choice of \( \alpha = 1 \) will result in lower probability limits than setting \( \alpha = 2 \) because by setting \( \alpha = 1 \) deviations from \( t \) is expressed in absolute terms.

The choice of \( t \) affects the size of the probability limits and the selection of the appropriate level of \( t \) is difficult in applied research. Therefore, Atwood (1985) developed a constraint that allows a mathematical optimisation algorithm to endogenously select the appropriate and least constraining level of \( t \) given the data set.

Atwood (1985) showed that enforcing Equation 4.5:

\[
 t + q^*\theta(\alpha, t) \leq G
\]

is sufficient to guarantee:

\[
 Pr\left(x \geq G\right) \leq \left(\frac{1}{1/p}\right)^\alpha \leq \left(\frac{1/q^*}{q^*}\right)^\alpha
\]

Equation 4.6 allows the model to endogenously select \( t \). Since \( t \) is always non-negative, Equation 4.5 requires that \( t \leq G \). If Equation 4.5 is constraining the model will select a level of \( t \) that is least constraining but that will still satisfy Equation 4.2. Enforcing Equation 4.5 for all levels of \( a \geq 0 \) will not be easy. However, with \( a = 1 \) the linear Target-MOTAD model can be used to enforce Equation 4.5 (Atwood et al., 1988; Qiu et al., 2001). Based on the work by Atwood (1985) the safety-first constraint is imposed in the environmental Target-MOTAD with the following equation:
where $G$ is the environmental goal set by the environmental regulator and $e_{rj}$ represents the environmental variable that is controlled. Enforcing Equation 4.7 in the environmental Target-MOTAD ensures that the probability of achieving an environmental variable in excess of the specified goal ($\sum_r Pr(\sum_j e_{rj} \geq G)$) is less than a specified acceptable probability level ($1/L^*$.)

Application of the UPM requires knowledge about the environmental variable $e_{rj}$. The environmental variable is used to capture the relationship between agricultural input use and environmental health in every state of nature. The next section therefore focuses on quantifying the environmental variable used in the UPM.

4.3 QUANTIFYING ENVIRONMENTAL RISK

4.3.1 SIMULATING THE ENVIRONMENTAL INDICATOR

Information on the environmental indicator is necessary to estimate economic-environmental trade-offs of production decisions. Obtaining information on the environmental indicator is not always easy. However, if a well-designed, reliable biophysical simulation model is available then the model can be used to provide information on non-point source pollution (Xepapadeas, 1997; Kampas & White, 2004).

The Soil Water Balance (SWB) model is a mechanistic, generic crop model originally developed for irrigation scheduling (Annandale et al., 1999). Van der Laan et al. (2009) developed the SWB_Sci model through the addition of nitrogen and phosphorus simulation routines and algorithms to SWB that allows for salt and nutrient simulations. Algorithms incorporated into SWB_Sci allows the model to simulate above ground nitrogen mass, grain nitrogen mass and soil water content. More importantly the fate of nitrogen is also modelled creating an environment for the user to model an environmental indicator.

The simulation model was used to simulate production data and the environmental indicator. Production data and the corresponding environmental indicator were simulated for production of late monoculture maize under irrigation on two soil types at Glen, South Africa. Data on maize yield, irrigation water use and the environmental indicator were simulated for conditions of no
water stress for 19 different weather years while assuming an initial soil nitrogen level of 33kg. Nine levels of nitrogen fertiliser were applied, in either a single or a split application. When using a split application two thirds of the desired nitrogen level was applied on the day of planting while the remaining third was applied seven weeks later. Only applications above 70kg/ha was applied in a split application. The environmental indicator consists of the simulated amount of nitrate that was lost through leaching and run-off.

4.3.2. **Quantifying the State-Contingent Nitrate Losses**

A state-contingent approach (Chambers & Quiggin, 2000) is adopted to quantify the risk of nitrate losses on the two soils using a split and a single application method. Quantifying the risk of nitrate losses within a state-contingent approach requires a separate response function to relate fertiliser applications to nitrate losses in each state of nature where a state of nature is represented by a specific weather year. Nitrate losses were related to fertiliser applications on each soil and application method since Rudra *et al.* (2011) and Gowda *et al.* (2008) found that nitrogen fertiliser applications were the most important determinant of nitrate losses.

Some studies (IPCC, 1996; Nangia *et al.*, 2010) modelled nitrate losses assuming a linear relationship with nitrogen fertiliser application rates. Malone *et al.* (2007), however, found a quadratic relationship between nitrogen fertiliser application and nitrate losses. In this research a quadratic response function was estimated for each state of nature since the functional form reduces to a linear relationship if the coefficient of the squared term is not statistically significant. Thus, the possibility of a non-linear response in some states was not excluded.

The results for the quadratic response functions used to relate nitrogen fertiliser applied (kg/ha) to the level of nitrate loss (kg/ha) are given in Appendix C. The empirical distribution shows all possible nitrate loss levels associated with fertiliser application in every state of nature. The bulk of the estimations explains a great deal of the variation in the simulated data with a good $R^2$. However, not all of the estimations show a high $R^2$, indicating that not all of the variation in nitrate losses is due to the amount of nitrogen fertiliser applied.

The nitrate loss response functions for all soil types and for single or split nitrogen fertiliser applications are shown in Figure 4.1. The response functions show the effect of states of nature on nitrate loss variability as a function of fertiliser use.
FIGURE 4.1: Nitrate loss response functions (kg/ha) for increased nitrogen fertiliser application (kg/ha) on a Sandy Clay Loam (SCL) soil and a Sandy Clay (SC) soil for a single and a split nitrogen application.

The average nitrate losses in Figure 4.1 show that average nitrate loss is not very responsive to the amount of nitrogen fertiliser applied as average responses appears flat. Average nitrate loss for a single application on a SCL soil ranges from 24.3kg/ha to 30.8kg/ha, while a single application on a SC soil ranges from 28.7kg/ha to 30.1kg/ha. When nitrogen is applied in a split application on a SCL soil nitrate loss ranges from 24.2kg/ha to 30.9kg/ha. Average nitrate losses range from 29.3kg/ha to 30.1kg/ha on a SC soil using a split application.

The results show that the amount of nitrate lost in some states of nature is higher than the quantity of nitrogen fertiliser applied. The amount of nitrate available for losses is not only...
dependent on the amount of fertiliser applied but also the nitrogen levels in the soil prior to production and nitrogen available in irrigation and rainwater. Before production nitrogen fertiliser to the amount of 33 kg/ha is available in the soil, SWB_Sci assumes that irrigation water and rainwater contains fertiliser that would be available for production or could be lost during the production process. Furthermore, in a number of states of nature the nitrate loss functions appear to be linear, although in some states of nature the relationship between nitrogen fertiliser applied and nitrate loss is linear, there are instances when the quadratic term is significant but very small (see Appendix C). During one state of nature no function could be fitted through the data for fertiliser use on both soil types and fertiliser application methods as no losses were realised. The state of nature in question was a low rainfall state with less opportunity for losses because the irrigation water that was applied was used for crop growth with very little losses due to percolation.

The empirical distribution of the risk of nitrate losses is represented by the estimated nitrate losses for each state of nature given a specific level of fertiliser is applied. The nine residuals from the nitrate loss response function were added to the calculated expected losses to capture any unexplained variability in the environmental variable. Thus, each state-contingent production function was treated as a stochastic production function with an empirically distributed random component (Richardson et al., 2000). As a result 171 (19 production years multiplied by 9 estimation errors) nitrate loss quantities were estimated to represent the nitrate variability at each fertiliser use level.

4.4 MODELLING ECONOMIC-ENVIRONMENTAL TRADE-OFFS

Modelling the economic-environmental trade-offs with the UPM requires a baseline emission level that can be used as the environmental goal ($\mathcal{G}$). The first section will therefore discuss the identification of a baseline emission level to represent the environmental goal. The baseline emissions were then incorporated into the UPM as the environmental goal ($\mathcal{G}$) to model compliance using a trade-off model.

4.4.1 DETERMINING BASELINE EMISSIONS

Producers typically do not consider the environment when making fertiliser application decisions and therefore, nitrate pollution is considered an externality to the production decisions made by producers. The state-contingent production risk programming model that was developed in the previous chapter is used to determine risk efficient optimal fertiliser application rates assuming
nitrate pollution is an externality to production. The risk of nitrate losses associated with the risk efficient fertiliser applications is then calculated using the optimal fertiliser application rates.

The mathematical specification of the mathematical programming model is given below:

Maximise \[ CE = \frac{\ln[-\sum_s p_s (-e^{r_s(GM)}GM_s)]}{-r_s(GM)} \] (4.8)

s.t.

\[ GM_s = (Y_s(N)P_Y - NP_s - W_s(N)P_W - B_A - B_Y Y_s(N))HA \] (4.9)
\[ Y_s(N) = \beta_{s1} + \beta_{s2}N + \beta_{s3}N^2 + \varepsilon_s \] (4.10)
\[ W_s(N) = \omega_{s1} + \omega_{s2}N + \omega_{s3}N^2 + \mu_s \] (4.11)
\[ N \leq 220 \] (4.12)
\[ HA \leq 1 \] (4.13)

Where: \( p_s \) is the probability that state of nature \( s \) will occur. 
\( GM_s \) is the Gross Margin for state of nature \( s \) (R) 
\( r_s(GM) \) defines the coefficient of absolute risk aversion appropriately scaled to be relevant to the estimated gross margins 
\( Y_s(N) \) is the crop yield produced as a function of nitrogen applications in state of nature \( s \) (ton/ha) 
\( P_Y \) is the price for maize (R/ton) 
\( N \) is the level of nitrogen fertiliser applied (kg/ha) 
\( P_N \) is the price for nitrogen fertiliser (R/kg) 
\( W_s(N) \) is the applied irrigation water as a function of nitrogen applications in state of nature \( s \) (mm.ha⁻¹) 
\( P_W \) is the cost of applying irrigation water (R/mm.ha⁻¹) 
\( B_A \) is the area dependent cultivation cost (R/ha) 
\( B_Y \) is the yield dependent harvesting cost (R/ton) 
\( \beta_{si} \) represents the \( i^{th} \) (\( i=1,2,3 \)) estimated coefficient for the yield response function in state of nature \( s \) 
\( \varepsilon_s \) is the estimated output residuals for every state of nature, \( s \)
\[ \omega_{is} \] represents the \( i^{th} \) \((i=1,2,3)\) estimated coefficient for the irrigation water response function in state of nature \( s \)
\[ \mu_s \] is the estimated irrigation water residual for every state of nature, \( s \)
\[ HA \] is the area cultivated (ha)

The programming model maximises the utility a risk averse decision makers derive from applying nitrogen fertiliser. Utility is maximised by maximising the certainty equivalent (Hardaker et al., 2004). The production risk associated with different fertiliser applications is quantified through state-contingent production functions. State-contingent response functions are also used to capture changes in water applications in each state of nature as a function of fertiliser applications. Together the state-contingent production (Equation 4.10) and water applications (Equation 4.11) give rise to the enterprise gross margin variability (Equation 4.9) which is used to calculate the certainty equivalent in the objective function. The cultivation area \((H A)\) in Equation 4.9 is constrained to one hectare and can therefore be interpreted as the absolute area cultivated or as a proportion of the area available for cultivation. The detail of the estimation of the state-contingent nitrate response functions is provided in Chapter 3.

Table 4.1 provides an estimate of the optimal amount of fertiliser used for increasing levels of risk aversion for production on a SCL and SC soil using a single or split fertiliser application as well as the average amount of nitrate lost.

The average nitrate loss calculations showed that the amount of nitrate loss on a SC soil is greater than the losses on a SCL soil. However, the difference is negligible. Furthermore, nitrate losses were higher for a split application than for a single application, irrespective of the soil type. On investigation of the data it was found that shortly after fertiliser application it rained. Due to the combination of the soils lower infiltration rate and the increased opportunity of run-off losses occurring there were more nitrate losses when fertiliser were applied in a split application. Risk aversion has almost no impact on nitrate losses if a specific combination of soil and fertiliser application method is considered. The last mentioned is true in spite of the fact that fertiliser application levels are increasing with increasing levels of risk aversion.
TABLE 4.1: Baseline for maize production on a Sandy Clay Loam (SCL) and Sandy Clay (SC) soil for a single and split nitrogen application.

<table>
<thead>
<tr>
<th></th>
<th>Absolute Risk Aversion Coefficient</th>
<th>0</th>
<th>0.0001</th>
<th>0.0006</th>
<th>0.0011</th>
<th>0.0016</th>
<th>0.0021</th>
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<tr>
<td></td>
<td>Nitrogen used kg/ha</td>
<td>130</td>
<td>131</td>
<td>133</td>
<td>133</td>
<td>132</td>
<td>131</td>
<td>130</td>
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<tr>
<td></td>
<td>Nitrate lost kg/ha</td>
<td>27.17</td>
<td>27.18</td>
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<td>27.30</td>
<td>27.33</td>
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<tr>
<td></td>
<td>STD† kg/ha</td>
<td>29.24</td>
<td>29.26</td>
<td>29.33</td>
<td>29.35</td>
<td>29.33</td>
<td>29.28</td>
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<td>0.579</td>
<td>0.579</td>
<td>0.579</td>
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<tr>
<td>Sandy Clay Loam Single</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nitrogen used kg/ha</td>
<td>136</td>
<td>136</td>
<td>135</td>
<td>136</td>
<td>136</td>
<td>134</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>Nitrate lost kg/ha</td>
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<tr>
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<td>Nitrogen used kg/ha</td>
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<tr>
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<td>Nitrate lost kg/ha</td>
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<td>28.73</td>
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<tr>
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<td>Sandy Clay Loam Split</td>
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<tr>
<td></td>
<td>Nitrogen used kg/ha</td>
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<td>126</td>
<td>129</td>
<td>130</td>
<td>131</td>
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<td></td>
<td>Nitrate lost kg/ha</td>
<td>29.29</td>
<td>29.29</td>
<td>29.29</td>
<td>29.29</td>
<td>29.29</td>
<td>29.29</td>
<td>29.29</td>
</tr>
<tr>
<td></td>
<td>STD† kg/ha</td>
<td>43.35</td>
<td>43.47</td>
<td>43.78</td>
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<td>0.632</td>
<td>0.632</td>
<td>0.632</td>
<td>0.632</td>
<td>0.632</td>
<td>0.632</td>
</tr>
</tbody>
</table>

† STD is the nitrate lost standard deviation at the relevant level of fertiliser use.
* Compliance refers to the percentage of scenarios for which the environmental goal of 28kg/ha is maintained in the baseline model (0.579 indicate that in 11 of the 19 states the compliance target is achieved).

The environmental goal was established by assuming that the policy maker is concerned with reducing the probability that no more than the average amount of nitrate will be lost. As a result the nitrate losses of all four alternatives were averaged to determine a homogenous environmental goal of 28 kg/ha.

The distribution of losses determines the probability with which the environmental goal will be met. Compliance in Table 4.1 indicates how often the estimated nitrate losses are lower than the average nitrate losses of 28kg/ha. On a SCL soil 57.9% of the estimated losses are below the average nitrate losses estimated with the optimal fertiliser use levels irrelevant of the fertiliser application technique, while for production on a SC soil 63.2% of the outcomes in nitrate loss would be below the average.
4.4.2. **Modelling Environmental Compliance**

The baseline model that was used to determine the environmental goal ($\mathcal{G}$) does not consider the environment since nitrate loss was estimated exogenous to the optimisation model. The following equations were added to the baseline model in order to model compliance to the environmental goal ($\mathcal{G}$):

\[
E_s(N) = e_{1s} + e_{2s} N + e_{3s} N^2 + \tau_s
\]  
\[
t - (E_s(N))HA + d_s \geq 0
\]  
\[
\sum_s p_s d_s - \theta(t) = 0
\]  
\[
t + L^* \theta(t) \leq G
\]

Where:
- $E_s(N)$ is the level of nitrate that is lost in state $s$ as a function of application rates (kg/ha)
- $e_{is}$ represents the $i^{th}$ ($i=1,2,3$) estimated coefficients for the nitrate loss function
- $t$ is the endogenously determined reference level for the environmental variable
- $d_s$ is the deviation of pollution emissions above the pollution target in state of nature $s$
- $p_s$ is the probability that state of nature $s$ will occur
- $G$ is the environmental goal set by the environmental regulator
- $\theta(t)$ is the endogenously determined environmental risk level or the expected deviation above the reference level $t$
- $L^*$ the inverse of the acceptable probability of the environmental pollution being greater than the environmental goal $\mathcal{G}$.

In order to enforce environmental compliance the distribution of nitrate losses associated with different levels of fertiliser application rates have to be endogenously determined during the optimisation process. The estimated state-contingent nitrate loss response functions are represented by Equation 4.14 and are used to determine the empirical distribution of environmental risk associated with production. Compliance with the environmental goal is modelled using the safety-first constraint suggested by Atwood (1985) which is enforced in the UPM through Equations 4.15 to 4.17. The nitrate loss response function is incorporated into Equation 4.15 to estimate the expected deviation ($d_s$) above an endogenously determined target level ($t$) in every state of nature. Equation 4.16 is then used to estimate the expected pollution...
level ($\theta(t)$) by weighing the deviation ($d_s$) from the endogenous target by its probability of occurrence ($p_s$). Compliance with the environmental goal is ensured through Equation 4.17 which is used to specify the inverse of the acceptable probability of nitrate losses ($L^*$) being greater than the user-specified environmental goal ($G$).

A unique feature of the UPM specification is that it endogenously determines the reference level for the environmental variable ($t$) as well as the expected deviation above the reference level while satisfying the probabilistic constraint in Equation 4.17. Qiu et al. (2001) argue that the UPM overcomes the problem of specifying the level of compliance ($\lambda$) in the Environmental Target-MOTAD specification since their model only requires the specification of $L^*$ rather than $\lambda$. Based on the user-specified environmental goal ($G$), the user-specified compliance probability ($L^*$) and the frequency distribution of nitrate loss ($E_s(N|H|A)$) an endogenous target ($t$) is determined by the UPM. The endogenous target level is selected such that the expected deviation of pollution ($p_s d_s$) from the environmental target is as small as possible which ensures the least constraining solution is obtained given the model specification.

4.4.3. **Estimating Compliance Cost**

The cost of compliance is estimated as the difference between the certainty equivalent for a base scenario ($CE_b$) and the estimated certainty equivalent for an alternative scenario ($CE_a$). The cost of compliance is therefore the cost for the producer for him to switch between a preferred and a less preferred alternative (Hardaker et al., 2004).

Estimation of the cost of compliance requires knowledge of a base scenario and an alternative scenario. Because the chapter estimates the cost of compliance to an environmental constraint the optimisation results for production on a SCL and SC soil using a single or split fertiliser application when an environmental constraint is present, is $CE_a$. $CE_b$ is represented by production on a SCL and SC soil using a single or split fertiliser application without considering compliance to an environmental constraint. It is assumed when estimating cost of compliance that the decision maker will not change his/her production technology when faced with an environmental constraint. The estimation of cost of compliance is therefore specific for each soil type and fertiliser application method.
4.5 RESULTS AND DISCUSSION

The economic-environmental trade-offs model was used to answer the three objectives of the chapter. Firstly, the risk efficiency of fertiliser use under environmental compliance is discussed. The second section evaluates the cost for environmental compliance, while the last section shows how compliance will be achieved through changes in the intensive and extensive margin. Although extensive margin (area planted) results can be interpreted as the proportion of the area available for cultivation, all interpretations are made based on absolute changes in area cultivated.

4.5.1. RISK EFFICIENCY OF FERTILISER USE FOR ENVIRONMENTAL COMPLIANCE

The CE estimated for the baseline and the environmentally constrained model is shown in Figure 4.2. The constrained model is solved for a user-specified environmental goal of 28kg/ha at compliance probabilities of 0.6, 0.7 and 0.8.

The risk efficiency frontier consists of the optimal risk portfolio for the decision maker, based on the highest estimated CE. Assuming that the producer has access to both soil types the baseline the risk efficiency frontier with no environmental constraint consists of production on a SCL and SC soil with a single fertiliser application (Fig 4.2 unconstrained). Decision makers that show relatively low risk aversion (RAC of 0.0013 or lower) would prefer production on a SCL soil while decision makers with a relatively higher risk aversion (RAC higher than 0.0013) would prefer production on a SC soil. Farmers might, however, not have access to both soil types for production. Whenever producers are limited to one soil type, production with a single fertiliser application level will always result in the greatest CE when no environmental constraint is present irrespective of the soil type available and risk aversion level.

When an environmental constraint is introduced the risk efficiency frontier for production changed from that estimated for unconstrained production. For all levels of compliance the environmental risk efficiency frontier consists of production on a SCL soil using a single fertiliser application. The CE estimated on a SCL soil is always greater than the CE on a SC soil, irrespective of the fertiliser application technique. It was expected that the homogenous environmental goal ($G$) of 28kg/ha will be more constraining when producing on a SCL than for production on a SC soil because the goal is greater than the average losses on a SCL soil. However, this does not seem to be the case. On average the environmental goal ($G$) is more constraining to production on a SC soil. The last mentioned results emphasise the importance of considering the distribution of the environmental outcome. Based on the distribution of the environmental variable the UPM
Modelling Economic-Environmental Trade-offs using Safety-First Constraints

determines an endogenous target that it maintains at the specified compliance probability. The greater standard deviation for production on a SC soil results in a lower endogenous target, therefore, the effect of the environmental constraint is greater on the SC soil than on a SCL soil. Irrespective of the soil used for production the estimated CE when using a single application of fertiliser is always higher compared to the use of a split fertiliser application. Producers limited to a SCL or SC soil will therefore always apply fertiliser in a single application.

**FIGURE 4.2:** Estimated certainty equivalent (CE in R) for production on a Sandy Clay Loam (SCL) and Sandy Clay (SC) soil using a single and split fertiliser application across a range of risk preferences for compliance to an environmental goal of 28kg/ha for four compliance probabilities.
Unconstrained production results in a risk efficiency frontier that consists of production on a SCL and SC soil. When the environment is considered, the risk efficiency frontier consists of production on a SCL soil only irrespective of the compliance probability. The conclusion is that the presence of an environmental constraint has a significant impact on the risk efficiency frontier. Decision makers faced with an environmental constraint need to respond differently compared to decision makers who do not face an environmental constraint. The mismatch between production risk efficiency and environmental risk efficiency poses problems to risk decision support. Therefore, research in understanding decision-making when faced with an environmental constraint in the presence of uncertainty is necessary to understand and guide decision-making under uncertainty.

4.5.2. COST OF ENVIRONMENTAL COMPLIANCE

The cost of compliance to an environmental constraint was determined as the difference between maize production when an environmental constraint is not enforced (CE₀) and when an environmental constraint is enforced (CEₐ). The compliance cost for compliance to an environmental goal of 28kg/ha at compliance probabilities of 0.5, 0.6, 0.7 and 0.8 are shown in Figure 4.3. Results are shown for a RAC of 0.0001, 0.0006, 0.0011, 0.0016, 0.0021 and 0.0026.

When compliance to ₀ is at 0.5 the risk neutral decision makers compliance cost for production on a SC soil is R8 344 compared to the R7 053 on a SCL soil when using a single fertiliser application. With increased risk aversion the compliance cost for production on a SC and SCL soil decreased to R7 072 and R6 037, respectively. The difference in compliance cost due to a different technology (soil type) decreased from R1 291 (R8 344 compared to R7 053) to R1 035 (R7 072 compared to R6 037) with increased risk aversion. The result shows a similar trend for production with a split application. The choice of soil is therefore easy. Since a decision maker who wishes to minimise his/her cost of compliance would prefer production on a SCL soil, the choice of single or split fertiliser application on a specific soil type is not as straightforward. Decision makers using a SC soil will prefer production with a single application at all levels of risk aversion because the estimated compliance cost will be between R15 and R169 less than for a split application. A decision maker on a SCL soil would for the most part prefer production with a split fertiliser application. A risk neutral decision makers cost of compliance when using a split application is R6 950 which is R103 less than for a single application. With increased compliance the difference in compliance cost decrease to a RAC of 0.0011, where the compliance cost of production with a single application is R3 less than for production with a split application. With a continued increase in RAC the cost of compliance associated with a split application becomes
increasingly lower than the compliance cost for a split application with a difference of R22 at a RAC of 0.0026.

FIGURE 4.3: Estimated compliance cost (measured in Rand) for production on a Sandy Clay Loam (SCL) and Sandy Clay (SC) soil using a single and split fertiliser application for increased risk aversion (RAC) for compliance to an environmental goal of 28kg/ha for four compliance probabilities.

With increased compliance to 0.8 the cost of compliance for all technology sets continue to increase to between R10 000 and R11 900 for a SC soil and R8 900 and R10 600 for a SCL soil. Interesting to note is that although production on a SCL soil is still preferred due to the lower compliance cost the choice of fertiliser application has changed. Risk neutral decision makers on a SC soil would prefer using a split application. With increased risk aversion the difference in
compliance cost decrease before becoming more for a split application. As compliance increase from 0.6 to 0.8 the RAC at which decision makers change from preferring a split application to using a single application change from 0.0006 (compliance of 0.6) to 0.0011 for a compliance of 0.7 and 0.0021 for a compliance of 0.8. Decision makers on a SCL soil on the other hand, will always prefer production with a split application. When using a split application compliance cost will be between R141 and R15 less than for a single application at a compliance of 0.7. The difference in compliance cost due to production with a single and split application range between R182 and R32 when compliance increase to 0.8.

From the results it is concluded that decision makers would prefer producing on a SCL soil irrelevant of the fertiliser application technique, compliance level or risk aversion. Cost of compliance is always lower on a SCL soil than on a SC soil. The choice between a single and a split fertiliser application is not as straightforward as the compliance level, soil type and risk aversion influence the choice of application technique. Less risk averse decision makers tend to prefer a split application while more risk averse producers tend to move towards a single application. However, the difference in compliance cost due to choice of fertiliser application technique is relatively small. The greatest reduction in compliance cost is due to the choice of SCL or SC soil. The conclusion is that soil specific recommendations are necessary to ensure that decision makers react appropriately to environmental constraints given their fixed resources.

4.5.3. **Production Decisions to Ensure Compliance**

Compliance to the environmental constraint requires that decision makers change their production practices. The amount of fertiliser used (N) and the area cultivated (Ha) for optimal production when complying with the environmental goal ($\vec{g}$) is given in Table 4.2.

The discussion of Table 4.2 will start by comparing production on a SCL and SC soil for risk neutral decision makers at a compliance of 0.5 before discussing higher compliance levels. The discussion continues to evaluate the effect of risk aversion (RAC of 0.0001) on management decisions. Lastly the effect of increased risk aversion (RAC of 0.0026) is evaluated.
Modelling Economic-Environmental Trade-offs using Safety-First Constraints

TABLE 4.2: Nitrogen fertiliser use (N in kg) and area cultivated (Ha in hectare) estimated with the environmental compliance model for compliance of 0.5, 0.6, 0.7 and 0.8 on a Sandy Clay Loam (SCL) and Sandy Clay (SC) soil for a single and split nitrogen application.

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<th>N*</th>
<th>Ha**</th>
<th>N</th>
<th>Ha</th>
<th>N</th>
<th>Ha</th>
<th>N</th>
<th>Ha</th>
<th>N</th>
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<th>N</th>
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<th>N</th>
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</table>

A risk neutral decision maker on a SCL soil using a single fertiliser application will decrease fertiliser use form 82.0kg/ha to 72.6kg/ha when compliance increase from 0.5 to 0.8. Compared to

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* N indicates the optimal level of nitrogen fertiliser applied measured in kg/ha
** Ha indicates the optimal area produced to comply with the environmental constraint, measured in hectares

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the baseline application of 130kg/ha, the reduction in fertiliser use due to the presence of an environmental constraint is substantial at 48kg/ha (130 kg/ha – 82 kg/ha) to 57.4kg/ha (130 kg/ha – 72.6 kg/ha). The same decision makers would decrease the area cultivated to 0.618ha and 0.417ha respectively to ensure increased compliance of 0.5 and 0.8. Next the use of a split application strategy on a SCL soil is considered. A decision maker on a SCL soil who applies fertiliser in a split application would ensure compliance by applying less nitrogen and producing a larger area compared to decision makers using a single application. The maximum fertiliser use for a risk neutral decision maker is 70.8kg/ha for a compliance probability of 0.5 while the fertiliser application decreases by 12.2kg/ha for increased compliance (0.8). The area cultivated is also a maximum at 0.625ha for a compliance probability of 0.5 and decrease to 0.423ha as compliance increase to 0.8.

To ensure compliance fertiliser application on a SC soil is significantly higher than on a SCL soil while the area cultivated is less when using a SC soil irrespective of the fertiliser application strategy. A risk neutral decision maker would use 111kg/ha nitrogen fertiliser and cultivate 0.513ha to ensure a compliance probability of 0.5. For increased compliance (0.8) the amount of fertiliser used to ensure increased compliance is lower at 91.4kg/ha while the area cultivated decreases by 0.202ha. Fertiliser use to ensure compliance with an environmental goal is greater for split fertiliser applications than for a single application. Although the fertiliser use is higher at 111.7kg/ha, 112.6kg/ha, 104.6kg/ha and 101.1kg/ha for production with a split application, the area under cultivation is lower at 0.504ha, 0.424ha, 0.371ha and 0.309ha for compliance probabilities of 0.5, 0.6, 0.7 and 0.8 respectively.

With increased risk aversion (RAC of 0.0026) the decision maker’s decision on the intensive (fertiliser use) and extensive (area cultivated) margin changes with increased risk aversion. Decision makers who produce on a SCL soil will increase the amount of fertiliser used during production while reducing the area cultivated. For example, risk averse decision makers who produce on a SCL soil with a single fertiliser application will increase fertiliser use by 23.49kg/ha to 105.4kg/ha while reducing the area cultivated to 0.599ha when the compliance probability is 0.5. The same is true at compliance of 0.8 although the increase in fertiliser applied is greater at 24.2kg/ha and the reduction in area produced is less at 0.016ha. Similarly, increasing risk aversion on a SC soil would result in increased fertiliser use by decision makers and a reduction in the area cultivated irrespective of compliance probability. A decision maker using a single fertiliser application will increase fertiliser use to 121.0kg/ha while reducing area cultivated to 0.512ha when compliance is 0.5. At a compliance probability of 0.8 the decision maker will increase fertiliser use by 17.5kg/ha to 108.9kg/ha while decreasing area cultivated to 0.309ha. The reduction in area cultivated is marginal for both a single and split application on a SC soil and...
for most compliance probabilities with a maximum reduction of 0.0034ha for a split application and a compliance of 0.9. It seems as if the area cultivated remains constant at 0.432ha for a single fertiliser application and compliance probability of 0.6; there is however, a marginal reduction of 0.0002ha in area cultivated.

Fertiliser application decreases from the baseline levels once an environmental constraint is implemented. However, with increased risk aversion the amount of fertiliser applied increases, therefore, it can be concluded that fertiliser is a risk-decreasing input on all soil types and fertiliser application techniques. Area produced associated with the increase in fertiliser use shows a decrease. The implication of the results is that decision makers who are risk-averse will react differently to an environmental constraint compared to risk-neutral decision makers. More importantly a risk-averse decision maker will apply more fertiliser than a risk-neutral decision maker. However, the same level of compliance will be maintained due to a reduction in area cultivated. The development of incentives and standards to ensure compliance by decision makers need to take the risk-reducing behaviour of fertiliser and the behaviour of risk averse decision makers into account.

4.6 CONCLUSIONS

This chapter builds on Chapter 3 where state-contingent production functions were developed to capture skewed distributions. The state-contingent production function and environmental response functions creates a clear link between the source of the pollutant, production risk and the environmental outcome.

Results showed when a binding environmental constraint is introduced the risk efficiency frontier changes. The conclusion is that the presence of a binding environmental constraint has a significant impact on the risk efficiency frontier. The mismatch between production risk efficiency and environmental risk efficiency pose problems for environmental regulation. Producers typically do not consider the effect that production decisions will have on the environment. Because the environment risk efficiency of production decisions would not come naturally to the decision makers, it is necessary that the externality be internalised through the correct incentives or environmental standards. Results furthermore indicate that the decision maker's risk-behaviour affects the production decision made. Since fertiliser acts as a risk-decreasing input, more risk averse decision makers tend to apply more fertiliser than a risk neutral decision maker. To ensure compliance with the environmental constraint the more risk-averse decision maker reduces the size of the area cultivated. It is therefore, important that during the development of the incentives
Modelling Economic-Environmental Trade-offs using Safety-First Constraints

and standards that will ensure the internalisation of the externality that the risk behaviour of
decision makers be taken into account. Furthermore, policy makers should also take into account
the fact that decision makers respond with changes in the intensive and extensive margin
simultaneously to ensure compliance to the environmental constraint that is enforced.

The results further showed that the presence of an environmental constraint impact production
decisions significantly with substantial compliance costs, which are the direct results of changes in
the intensive and extensive margin responses of risk-averse decision makers. The compliance
cost results indicated that irrelevant of risk aversion, decision makers would prefer producing on a
SCL soil. The choice between a split and single application of fertiliser is not as straightforward as
the compliance level, soil type and risk aversion influence the choice of application technique.
However, changes in fertiliser application technique result in relatively small differences in
compliance cost. The greatest difference in compliance cost is due to changes in the use of fixed
resources. The conclusion is, however, that soil specific recommendations are necessary to
ensure that decision makers react appropriately to environmental constraints given their fixed
resources and risk behaviour.

The safety-first upper partial moment was successful in evaluating economic-environmental trade-
offs. Due to the estimation of an endogenous target the upper partial moment might be
conservative in its estimation of the economic-environmental trade-offs. Future research should
address the conservativeness of the upper partial moment and evaluate the effect of such
conservativeness. Furthermore, the study did not account for timing of fertiliser application when
applying fertiliser in a split application. It was assumed during the data simulation process that
fertiliser is applied using a fixed application schedule. However, changes in the timing of fertiliser
can change the environmental impact of the decision maker’s fertiliser application decision and
therefore the environmental risk efficiency frontier. Application of genetic algorithms could be
explored to optimise the timing of fertiliser applications. Another shortcoming of the analysis is
that the background pollution from uncultivated land is not considered. As a result the impact of
reducing irrigation area on nitrate loss reduction is overestimated. The estimated compliance cost
may therefore be conservative.
5.1 INTRODUCTION

Trade-off analysis applies the principle of opportunity cost to derive information about the sustainability of agricultural production systems. During trade-off analysis the inter-relationships among sustainability indicators implied by the underlying bio-physical processes and the economic behaviour of producers are quantified. Stoorvogel et al. (2004) stated that trade-off curves are two-dimensional graphs representing the trade-off between two sustainability indicators. Crissman et al. (1998) stated that trade-offs are an essential ingredient in setting research priorities and in designing and implementing the criteria of sustainable agriculture. The slope of a trade-off curve shows the opportunity cost of increasing agricultural production in terms of foregone environmental quality. The opportunity cost also represents the shadow prices for environmental quality and can be used as an economic incentive to achieve the environmental objective. The information generated with the trade-off analysis is critical for informed policy decision-making, as it allows policy makers and the public to assess whether a given improvement in environmental quality is worth the sacrifice in agricultural production (Stoorvogel et al., 2004). Therefore, it is of utmost importance that the trade-offs used to provide information for policy development are modelled correctly.

Trade-offs are typically modelled with the use of probabilistic programming due to the stochastic nature of the environmental outcomes. A potential problem with the application of probabilistic programming is that the available techniques such as the UPM (Qui et al., 2001) are conservative in the estimation of the compliance probability. The conservative estimation of the trade-offs can result in overregulation of agricultural production practices which will result in reduced agricultural production and losses in farm profit. Several researchers (Atwood et al., 1988; Qiu et al., 2001; Krokhmal et al., 2002; Kong, 2006) have raised their concern over the conservativeness of the UPM however, none of the researchers investigated the size of the conservativeness of the UPM. The conservativeness of the UPM is due to the use of the partial moment inequality that
generates a conservative probability limit. In other words, the UPM estimates the gross margin such that the environmental constraint is achieved more often than indicated by maintaining a higher compliance probability than that specified.

Atwood et al. (1988) indicated that the conservativeness of the UPM can be investigated using exogenously constrained or alternative nonlinear methods. An exogenous conservativeness is determined by comparing trade-off results of an UPM model at a specified compliance probability with the trade-off results of another UPM model that achieves the same level of compliance based on the exogenous calculation of compliance using the optimised distribution of the environmental outcome in the second optimisation. The estimated exogenous conservativeness might, however, not give an indication of the true conservativeness of the UPM model. The endogenously determined probability limits that ensure compliance in the UPM are determined by the specified compliance probability and the distribution of the environmental variable. Any change in the specified compliance probability or the distribution of the environmental variable will therefore result in probability limit changes that will influence how strict the environmental constraint is enforced. The estimation of the true conservativeness, the endogenous conservativeness, is based on the fact that optimised trade-offs are different when producers face a conservative probability bound compared to a probability bound closer to the actual compliance probability. The estimation of the endogenous conservativeness requires the use of a trade-off model that can model compliance without the conservative probability bounds of the UPM. Currently no technique is available to model the trade-off with a smaller probability bound than that used by the UPM.

The objective of the Chapter is to develop an alternative non-linear trade-off model that can be used to investigate the conservativeness of the UPM. The newly developed Upper Frequency Method (UFM) counts the number of deviations from the environmental goal in an effort to ensure that the deviations above the goal do not exceed the number of deviations allowed by the model.

5.2 CONSERVATIVENESS OF THE UPPER PARTIAL MOMENT

Safety-first rules are concerned with the probability of a variable falling above a critical or target level. Probabilistic safety-first constraints can be imposed using different approaches such as chance-constrained programming and the Chebyshev stochastic inequality. Imposing the probabilistic constraints through the use of the Chebyshev’s inequality generates highly conservative probability bounds (Atwood et al., 1988). Berck and Hihn (1982) introduced a semi-variance inequality that evaluates safety-first rules and is able to generate a smaller upper
Economic-Environmental Trade-offs and the Conservativeness of the Upper Partial Moment

probability limit than the Chebyshev. Atwood (1985) extended Berck and Hihn's (1982) semivariance inequality with a more general lower partial moment stochastic inequality to enforce such constraints. The LPM developed by Atwood (1985) requires that the random variable be finitely discrete and uses the empirical distribution of the random variables. Atwood (1985) demonstrated that the LPM generates a smaller upper probability limit than the Chebyshev.

Qui et al. (2001) stated that the Upper Partial Moment (UPM) is parallel to the LPM. Like the LPM approach the UPM requires a finite discrete sample and uses the empirical distribution of the random environmental variables.

The UPM is defined as:

\[ \rho(\alpha, t) = \sum_i (x_i - t)^\alpha f(x_i), \quad x_i \geq t \] (5.1)

for a discrete case and for a continuous case as:

\[ \rho(\alpha, t) = \int_{t}^{+\infty} (x - t)^\alpha f(x) dx \] (5.2)

Where: \( \alpha \) is a constant greater than zero
\( t \) is the reference pollution level
\( x_j \) is the pollution variable
\( f(x_j) \) is the relative frequency distribution of the pollution variable \( x_j \)
\( f(x) \) is the probability density function

The upper partial moment \( \rho(\alpha, t) \) is the integral of the deviation from the target \( (x_j - t) \) multiplied by the relative frequency distribution of the pollution variable. \( \alpha \) places an upper limit on the probability of \( x \) being more than \( p\theta(\alpha, t) \) units above \( t \). Setting \( \alpha = 1 \) express the inequality in terms of absolute deviations from \( t \). Fishburn (1977) proved that a model that examines the trade-offs between \( t \) and \( \rho(\alpha, t) \) would generate solutions that are a subset of the second-degree stochastically dominant set if \( \alpha \geq 1 \). If \( \alpha \geq 2 \) the solution will be a subset of the third degree stochastically dominant set. The use of absolute deviation can provide less conservative estimates for the probability (Kim et al., 1990).
Assume $\theta(\alpha, t) = [\rho(\alpha, t)]^{1/\alpha}$, which would be greater than or equal to zero since $\rho(\alpha, t) \geq 0$ and given a positive number $p$, then:

$$t + p\theta(\alpha, t) \geq t$$

The integral in Equation 5.2 can then be expressed as the sum of two integrals

$$\rho(\alpha, t) = \int_t^{+\infty} (x-t)^\alpha f(x)dx + \int_{t+p\theta(\alpha, t)}^{+\infty} f(x)dx$$

(5.4)

Since $\int_t^{+\infty} (x-t)^\alpha f(x)dx \geq 0$, then

$$\rho(\alpha, t) \geq \int_{t+p\theta(\alpha, t)}^{+\infty} (x-t)^\alpha f(x)dx$$

(5.5)

Over the interval $[t + p\theta(\alpha, t), +\infty]$, the expression $(x-t)^\alpha \rho(\alpha, t)$ holds since $\theta(\alpha, t) = [\rho(\alpha, t)]^{1/\alpha}$. Therefore, the term $p^\alpha \rho(\alpha, t)$ can replace $(x-t)^\alpha$ in Equation 5.5 with no loss of generality, which gives

$$\rho(\alpha, t) \geq \int_{t+p\theta(\alpha, t)}^{+\infty} p^\alpha \rho(\alpha, t)f(x)dx$$

(5.6)

$$= p^\alpha \rho(\alpha, t) \int_{t+p\theta(\alpha, t)}^{+\infty} f(x)dx$$

The integral $\int_{t+p\theta(\alpha, t)}^{+\infty} f(x)dx$ is the probability that $x$ is larger than $t + p\theta(\alpha, t)$.

Pr$[x \geq t + p\theta(\alpha, t)]$. Rearranging Equation 5.6 generates:

$$\Pr[x \geq t + p\theta(\alpha, t)] \leq (1 / \rho)^\alpha$$

(5.7)

Let $g$ be the standard that should be achieved for $x$ and $g = t + p\theta(\alpha, t)$ then Equation 5.7 holds.
\[
\Pr(x \geq g) = \Pr[x \geq t + p\theta(\alpha, t)] \leq (1 / \rho)^\alpha
\]  
(5.8)

where \( p = (g - t) / \theta(\alpha, t) \)

With \( \theta(\alpha, t) = \rho(\alpha, t) |^{1/\alpha} \geq 0 \) and \( p = (g - t) / \theta(\alpha, t) \), where \( g \) is the standard set for the pollution variable \( x \). Equation 5.8 places a lower limit on the probability of \( x \) increasing more than \( p\theta(\alpha, t) \) units above \( t \). In Equation 5.8 the choice of some level of \( t \) between \( g \) and the population mean will often result in probability limits close to the actual probability limits (See Atwood, 1985).

However, the distribution of the environmental variable determines the size of \( \theta(\alpha, t) \) which influences the choice of \( p \) and therefore the choice of \( t \). Since \( p\theta(\alpha, t) \) represents the allowable deviation from \( t \) an increase in the size of the allowable deviation will result in the choice of a lower environmental target, \( t \) which is maintained. As a result the distribution of the environmental variable may determine the magnitude of the conservativeness of the UPM.

A sufficient condition to guarantee that
\[
\Pr(x \geq g) = \Pr[x \geq t + p\theta(\alpha, t)] \leq (1 / \rho)^\alpha \leq (1 / q^*)^\alpha,
\]
is derived as follows. Note \( (1 / \rho)^\alpha \leq (1 / q^*)^\alpha \) requires \( \rho \geq q^* \). Since \( p = (g - t) / \theta(\alpha, t), p \geq q^* \) implies
\[
(g - t) / \theta(\alpha, t) \geq q^*
\]
(5.9)

Given that \( \theta(\alpha, t) \) is greater than zero, rearranging Equation 5.9 generates
\[
t + q^* \theta(\alpha, t) \leq g
\]
(5.10)

By enforcing Equation 5.10 the following constraint is possible.
\[
\Pr(x \geq g) \leq (1/p)^\alpha \leq (1/q^*)^\alpha
\]
(5.11)

Through the use of Equation 5.11 it is possible for the model to select a level of \( t \) endogenously. Since \( t \) is always non-negative, Equation 5.10 requires that \( t \leq g \). If Equation 5.10 is constraining the model will select a level of \( t \) that is least constraining but will still satisfy Equation 5.8. Enforcing Equation 5.10 for all levels of \( a > 0 \) will not be easy. However, with \( a = 1 \) the linear Target-MOTAD model can be used to enforce Equation 5.11 (Atwood et al., 1988; Qiu et al., 1985).
2001). Qiu et al. (2001) built on research by Atwood (1985) to develop an upper partial moment (UPM) inequality approach to impose the safety first constraint in a Target-MOTAD framework that will ensure that the target pollution level will be met at a certain specified probability level. Qui et al. (2001) imposed the following safety-first constraint in the UPM to ensure the target pollution level is maintained.

\[
\sum_r PR \left( \sum_j e_{rj} \ge G \right) \le \frac{1}{L^*}
\]  

(5.12)

Where \( G \) is the environmental goal set by the environmental regulator. By enforcing Equation 5.12 in the environmental Target-MOTAD ensures that the probability of achieving an environmental variable in excess of the specified goal \( \sum_r PR(\sum_j e_{rj} \ge G) \) is less than a specified acceptable probability level \( (1/L^*) \).

5.3 PROCEDURES

5.3.1 ECONOMIC-ENVIRONMENTAL TRADE-OFF MODELS

Chapter 3 and Chapter 4 estimated production and environmental risk using state-contingent production functions. In Chapter 4 the state-contingent production and environmental risk functions were used to estimate the economic-environmental trade-offs using an UPM. In this Chapter the state-contingent production and environmental risk functions are incorporated into a UPM and UFM model to estimate the conservativeness of the UPM. Both models consisted of generic constraints as indicated in the generic model and constraints specific to the compliance models. Next the generic model will be discussed followed by the UPM and the UFM models.

5.3.1.1. Generic model

The generic model was used to determine production decisions for risk neutral decision makers when no environmental constraint is enforced. The following equations were used to optimise fertiliser usage:
Maximise \[ GM = (Y(N)P_Y - NP_Y - W(N)P_W - B_Y - BY(N)) HA \] (5.13)

s.t.

\[ Y(N) = \sum_{s=1}^{S} p_s (\beta_{1s} + \beta_{2s} N + \beta_{3s} N^2 + \varepsilon_s) \] (5.14)

\[ W(N) = \sum_{s=1}^{S} p_s (\omega_{1s} + \omega_{2s} N + \omega_{3s} N^2 + \mu_s) \] (5.15)

\[ E_s(N) = e_{s1} + e_{s2} N + e_{s3} N^2 + \tau_s \] (5.16)

\[ N \leq 220 \] (5.17)

\[ HA \leq 1 \] (5.18)

Where:

- \( p_s \) is the probability that state of nature \( s \) will occur
- \( GM_s \) is the Gross Margin for state of nature \( s \) (R/ha)
- \( Y(N) \) is the average crop yield produced as a function of nitrogen applications (ton/ha)
- \( P_Y \) is the price for maize (R/ton)
- \( N \) is the level of nitrogen fertiliser applied (kg/ha)
- \( P_N \) is the price for nitrogen fertiliser (R/kg)
- \( W(N) \) is the average irrigation water applied as a function of nitrogen applications (mm.ha\(^{-1}\))
- \( P_W \) is the cost of applying irrigation water (R/mm.ha\(^{-1}\))
- \( B_A \) is the area dependent cultivation cost (R/ha)
- \( B_Y \) is the yield dependent harvesting cost (R/ton)
- \( \beta_{is} \) represents the \( i^{th} \) \((i=1,2,3)\) estimated coefficient for the yield response function in state of nature \( s \)
- \( \varepsilon_s \) is the estimated output residuals for every state of nature, \( s \)
- \( \omega_{is} \) represents the \( i^{th} \) \((i=1,2,3)\) estimated coefficient for the irrigation water response function in state of nature \( s \)
- \( \mu_s \) is the estimated irrigation water residual for every state of nature, \( s \)
- \( E_s(N) \) is the level of nitrate that is lost in state \( s \) as a function of application rates (kg/ha)
- \( e_{is} \) represents the \( i^{th} \) \((i=1,2,3)\) estimated coefficient for the nitrate loss function
- \( \tau_s \) is the estimated emission residual for every state of nature, \( s \)
- \( HA \) is the area cultivated (ha)
The generic model maximises the gross margin associated with alternative fertiliser application rates for a risk neutral decision maker. The average yield response due to fertiliser applications was estimated as the average response of all the state-contingent production function. The same is true for the estimation of water use as of a function of fertiliser applications. Fertiliser applications are limited to a maximum of 220kg/ha while the area planted are constrained to be no more than one hectare.

Equation 5.16 is included in the generic model to quantify the impact of the optimised production decisions on nitrate losses. A state-contingent approach is used to characterise the environmental variable. As a result Equation 5.16 represents nitrate losses as an empirical distribution which is continuously related to the amount of fertiliser applied. Equation 5.16 therefore, plays an important role in enforcing compliance with the environment goal of 28kg/ha since the equation determines the distribution of the environmental variable.

The following two sections describe how to enforce compliance with the UPM and UFM methods.

5.3.1.2. Environmental compliance with the Upper Partial Moment (UPM)

The compliance model requires additional equations to model compliance with the user-specified environmental goal of 28kg/ha. The additional equations allow the optimisation model to determine the economic-environmental trade-offs. The equations that were added to the generic model to complete the UPM model are given below:

\[ t - E_s(N) + d_s \geq 0 \]  
\[ \sum_s p_s d_s - \theta(t) = 0 \]  
\[ t + L^* \theta(t) \leq G \]

Where:  
\( t \) is the endogenously determined reference level for the environmental variable  
\( d_s \) is the deviation of pollution emissions above the pollution target in state of nature \( s \)  
\( p_s \) is the probability that state of nature \( s \) will occur  
\( G \) is the environmental target set by the environmental regulator  
\( \theta(t) = \theta(1,t) = \rho(1,t) \), endogenously determined environmental risk level or the expected deviation above the reference level \( t \)
The UPM uses the user-specified environmental goal \( G \), an acceptable probability level \( L^* \) and an endogenous environmental risk level \( \theta(t) \) to estimate the endogenous target, \( t \) which will be maintained in the UPM (Equation 5.21). The endogenous environmental risk level \( \theta(t) \) is estimated in Equation 5.19 based on the expected deviation \( p_s d_s \) of the decision makers nitrate loss from the endogenous target, \( t \). The deviation of pollution emissions \( d_s \) is estimated in Equation 5.19 as absolute deviation in nitrate loss \( E_s(N) \) from the endogenously determined target \( t \). While \( L^* \) can be interpreted as the inverse of the acceptable probability \( \varphi \) of environmental pollution being greater than the environmental goal \( G \) (Qiu et al., 2001).

The estimation of the endogenously determined target \( t \) relies heavily on the underlying distribution of the environmental variables (Qiu et al., 2001) and the level of compliance. Therefore, the results obtained with the use of the UPM are typically conservative.

### 5.3.1.3. Environmental compliance with the Upper Frequency Method (UFM)

The UFM of enforcing probabilistic environmental compliance is based on the premises that any compliance probability can be expressed for the discrete case as the frequency by which a target may be exceeded. Restricting the number of states in which the environmental target might be exceeded guarantees compliance. The modelling procedure utilises the Environmental Target-MOTAD model specification to identify states of nature in which the environmental target is exceeded and uses binary variables to restrict the number of times the target is exceeded. The following equations were used to ensure compliance:

\[
G - E_s(N) - d_s \geq 0 \tag{5.22}
\]

\[
-lB_s + d_s \leq 0 \tag{5.23}
\]

\[
\sum_s B_s \leq uf \tag{5.24}
\]

Where:

- \( B_s \) is a binary variable indicating whether the environmental target is exceeded in state of nature \( s \)
- \( uf \) is the upper frequency indicating the number of times a target might be exceeded to enforce compliance
is a large number which is used to give permission for a state of nature to exceed the target given $B_z$ has a value of one

Absolute deviations ($d_z$) are estimated in Equation 5.22 as the deviation in nitrate loss ($E_z(N)$) from the environmental goal ($G$). Equation 5.22 is the same as for the UPM (Equation 5.19) with the exception that the deviations are calculated from $G$ and not $t$ as in the UPM. The UFM therefore overcomes the conservativeness of the UPM in maintaining the true environmental goal and not an endogenously determined target that is dependent on the distribution of the environmental variable. Equation 5.23 uses a binary variable to identify whether a specific state of nature exceeds the environmental goal. Every time $E_z(N)$ exceeds $G$, $B_z$ takes a value of one. The $B_z$'s are counted to determine the frequency by which the environmental goal is exceeded. The probabilistic constraint is enforced by Equation 5.24 which restricts the number of times the environmental goal is exceeded to $uf$. The value of $uf$ is calculated as $(1 - \psi)S$ where $\psi$ specifies the compliance probability and $S$ the total number of states of nature. The choice of $uf$ is an integer value that corresponds with a value closest to the estimated discrete compliance probability without exceeding the compliance probability. The UFM can, therefore, also be conservative in the estimation of the trade-offs although the UFM will never be as conservative as the UPM.

5.3.2 Estimation of UPM Compliance Conservativeness

The conservativeness of the UPM was estimated with exogenously calculated actual compliance probabilities and the compliance probabilities that were achieved with the UFM.

The exogenous conservativeness is captured through the use of exogenously constrained methods. First the UPM was solved for a user-specified environmental goal, $G$, and a user-specified compliance level. However, the UPM maintains the environmental goal, $G$, at a compliance level greater than that specified. The expectation was that the exogenously estimated compliance level will be greater than that specified because the UPM determines an endogenous target, $t$, which is maintained at the specified compliance level. To estimate the conservativeness of the UPM a second UPM model was solved to achieve an actual compliance equal to the specified compliance in the first UPM through an iterative procedure. The exogenous conservativeness of the UPM was estimated as the difference in the gross margins determined with the first and second UPM.

The estimation of the endogenous conservativeness is based on the development of an alternative trade-off model that is not as conservative as the UPM. The specified compliance of
the first UPM was converted into integer values that indicate the number of times the environmental goal should be maintained \((u_f)\) in the UFM. The estimated \(u_f\) was then used to determine the economic-environmental trade-offs with the UFM model. The endogenous conservativeness of the UPM was estimated as the difference between the gross margin for the first UPM and UFM.

5.4 RESULTS

The results in Table 5.1 are divided into three sets of results. The first set of results (Upper Partial Moment Model 1) shows the optimisation results for the user specified compliance level. To determine the exogenous conservativeness of the UPM a second UPM (Upper Partial Moment Model 2) was solved to ensure an exogenously estimated actual compliance equal to the specified compliance in UPM model 1. The results for the second UPM optimisation are shown in the second set of Table 5.1 (Upper Partial Moment Model 2). The third set of results is the UFM results (Upper Frequency Method). The specified compliance of the UPM Model 1 was incorporated into the UFM to estimate the trade-offs with the UFM.

The first column in Table 5.1 shows the compliance probability \((\varphi)\) specified by the researcher. The second column \((GM)\) indicates the gross margin estimated with the trade-offs model under the environmental constraint and specified compliance. The third column \((\varepsilon)\) indicates the endogenous environmental target that is maintained in the UPM. The UPM maintains the environmental goal \((\vartheta)\) by maintaining the endogenous environmental target \((\varepsilon)\). Since the environmental target \((\varepsilon)\) is much stricter than the environmental goal \((\vartheta)\) the actual compliance level to the environmental goal is estimated exogenously using the optimised distribution of the environmental variable. The exogenously determined actual compliance level indicates the actual compliance to the user specified environmental goal \((\vartheta)\) of 28kg.

Next the exogenous and endogenous conservativeness of the UPM will be discussed based on the results shown in Table 5.1.

5.4.1. EXOGENOUS CONSERVATIVENESS

The exogenous conservativeness of the UPM is determined as the difference between the gross margin estimated in UPM model 1 and UPM model 2 as shown in Table 5.1.
TABLE 5.1: Estimated compliance to an environmental constraint using an Upper Partial Moment (UPM) and Upper Frequency Moment (UFM) results for a Sandy Clay Loam (SCL) and a Sandy Clay (SC) soil using a single and split fertiliser application (kg/ha).

<table>
<thead>
<tr>
<th>Sandy Clay Loam</th>
<th>Specified compliance</th>
<th>GM (R)</th>
<th>Target (t)</th>
<th>Actual compliance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper Partial Moment (UPM) Model 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single</td>
<td>0.596</td>
<td>9348</td>
<td>13.9</td>
<td>0.895</td>
</tr>
<tr>
<td>0.649</td>
<td>8757</td>
<td>16.0</td>
<td>0.895</td>
<td></td>
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The exogenous conservativeness of the UPM shows that even though a compliance level of 0.596 was specified, an actual compliance of 0.895 is maintained while a gross margin of R9 348 is realised for production on a SCL soil using a single fertiliser application. The actual compliance is greater than that specified because the UPM determines an endogenous target of 13.9kg that is maintained at the specified compliance level. The UPM chooses an endogenous environmental target ($r$) based on the distribution of the environmental variable. The endogenous environmental target is maintained at the specified compliance while the user-specified environmental goal is achieved at the actual compliance. Similarly, for a specified compliance level of 0.848 the environmental goal is achieved with an actual compliance of 0.947 while realising a gross margin of R6 209. However, the UPM solved the optimisation problem for an endogenous target of 15.9kg therefore $G$ was maintained at an exogenous compliance level of 0.947. The cost of conservativeness is estimated by comparing the estimated GM of R6 209 to the GM when the exogenously determined actual compliance in UPM model 2 is 0.848. Solving the UPM for the specified compliance level of 0.596 and 0.848, results in a gross margin of R17 551 and R10 883 respectively. The exogenous conservativeness of the UPM is therefore, R8 203 (R17 551-R9 348) for a specified compliance of 0.596 and R4 674 (R10 883-R6 209) for a specified compliance level of 0.848. With increased compliance to the environmental constraint the exogenous conservativeness decreases. The same is true for production on a SCL soil using a split fertiliser application.

For production on a SC soil when using a single fertiliser application the gross margin for a specified compliance level of 0.649 is R6 868 while the actual level of compliance to $G$ is 0.930. Optimising for the exogenously determined compliance of 0.649, results in a gross margin of R13 568. The exogenous conservativeness of the UPM at a compliance level of 0.649 is therefore R6 700 (R13 568-R6 868) while the gross margin for a specified compliance level of 0.895 is R3 737 with an exogenously estimated actual compliance of 0.953. Optimising for an exogenously estimated actual compliance of 0.895, results in a gross margin of R7 428. The exogenous conservativeness faced by the decision maker amounts to R3 691 (R7 428-R3 737). Similar to the results for production with a single fertiliser application the exogenous conservativeness will decrease with an increase in conservativeness. Although the exogenous conservativeness associated with production on a SCL soil and SC soil follow a similar trend the exogenous cost of conservativeness is greater for production on a SC soil compared to production on a SCL soil. It should also be noted that at relatively low levels of specified compliance the exogenous cost of compliance, although high for production on a SCL and SC soil is within the same ranges. When the specified compliance is very high (0.900) the exogenous cost of compliance is substantially greater on a SC soil than on a SCL soil.
Results showed that exogenous conservativeness decrease with increased compliance to the environmental goal, \( g \). The decision makers’ fixed resources also influences the size of the conservativeness with a higher conservativeness cost on the SC soil. The implication is that enforcing the environmental constraint in an incorrect manner will result in a significant conservativeness. Such conservativeness will put strain on the agricultural decision maker and agricultural production. The conclusion is that the choice of specified compliance should be carefully researched before policy makers take any decisions regarding the preferred compliance level. Furthermore, the decision makers’ fixed resource can contribute to the size of the conservativeness of the UPM, therefore, the decision makers’ fixed resources should be considered when evaluating alternative compliance levels. In essence, soil specific information is necessary before any decisions can be made.

### 5.4.2. ENDOWNEOUS CONSERVATIVENESS OF THE UPPER PARTIAL MOMENT

The endogenous conservativeness of the UPM is shown by comparing the results for the UPM model 1 with that of the UFM for the specified compliance. The results for compliance to an environmental constraint estimated with the UPM and the UFM for risk neutral decision makers are shown in Table 5.1.

Assuming a specified compliance of 0.596 on a SCL soil for a single fertiliser application, the GM for the UPM is R\( 9^{348} \) compared to the R\( 17^{556} \) realised with the UFM. The endogenous cost of conservativeness of the UPM is therefore R\( 8^{208} \). However, with increased compliance (0.895) the difference in the gross margin estimated with the UPM and UFM decreases to R\( 4^{227} \) (R\( 5^{274} \) for the UPM and R\( 9^{501} \) for the UFM). Similarly to the exogenous conservativeness the estimated endogenous conservativeness decreases with increased compliance. The same is true for production using a split application where the estimated endogenous conservativeness will also decrease with increased compliance. The estimated conservativeness cost will, however, be higher when applying fertiliser in a split application compared to a single application.

Decision makers, who produce on a SC soil with a single fertiliser application with a specific compliance of 0.649, will realise a gross margin of R\( 6^{868} \) using the UPM model compared to the R\( 14^{464} \) from the UFM optimisation. The endogenous conservativeness of the UPM is therefore R\( 7^{596} \) (R\( 14^{464}-R^{6^{868}} \)) and will decrease to an endogenous conservativeness of R\( 4^{158} \) (R\( 6^{737}-R^{2^{516}} \)) with an increase in compliance to 0.947. Decision makers who use a split fertiliser application on a SC soil will show a decrease in endogenous conservativeness due to increased actual compliance. An actual compliance probability of 0.649 will result in an endogenous
conservativeness of R7 330 (R13 965-R6 635). With increased compliance (0.947) the endogenous conservativeness will decrease to R4 051 (R6 567-R2 579).

Results show that the soil type used can influence the endogenous conservativeness of the UPM. For all compliance scenarios and fertiliser application techniques, production on SC soil resulted in a higher endogenous conservativeness compared to the SCL soil. The response to fertiliser application technique depends on the soil type used and the compliance level specified. The conclusion is therefore that the decision makers’ fixed resources and production decisions should be carefully considered when evaluating environmental constraints with the use of the UPM.

5.4.3. **Comparison of Exogenous and Endogenous Conservativeness**

Results showed that the conservativeness cost estimated with the endogenous and exogenous procedures both show a decline in conservativeness with increased compliance irrespective of soil choice or fertiliser application method. When a single fertiliser application is considered, the compliance cost is consistently higher on the SC soil irrespective of the conservativeness measure used. However, the results for the split application show mixed results when comparing the two different soils. Another noticeable result is that the exogenous conservativeness decreases significantly at high compliance probabilities on the SCL for both fertiliser application methods.

On a SCL soil the exogenous compliance cost conservativeness is almost the same for the two fertiliser application methods. When considering the endogenous estimate of conservativeness the split application tends to be higher than the single application method at high compliance probabilities. On a SC soil both application methods conservative measures closely follow each other irrespective of the conservativeness measure used. The exogenous measure tends to give more conservative estimates for the single fertiliser application method at lower compliance probabilities.

The conclusion is that it is difficult to clearly determine the impact of fertiliser application method on conservativeness while soils have a more profound impact.
5.5 CONCLUSIONS

The newly developed UFM is easy to use and requires no assumptions regarding the distribution of the environmental variable as the empirical data is used. The UFM behaved well during the optimisation process and is much less conservative in the estimation of the trade-offs due to the probability limit which is closer to the actual probability limit displayed by the data. Although the UFM provides a stricter probability bound than the UPM there are some concerns regarding the application of the UFM. The UFM ensures compliance by ensuring that the number of deviations above the goal does not exceed the number of deviations allowed, therefore, a fairly large number of observations is necessary to ensure probability limits close to the actual probability. The UFM may furthermore be subject to data-mining problems where statistical outliers are mined. Further research is necessary to determine the sensitivity of the UFM to sample size and mining of statistical outliers.

Results showed that the conservativeness cost is higher on SCL soil compared to a SC soil, irrespective of conservativeness method. The effect of fertiliser application method used affects conservativeness cost differently between the two soil types and conservativeness measures. Results also showed that the exogenous and endogenous conservativeness estimated with the UPM and UFM is very high. With increased compliance the exogenous and endogenous conservativeness decrease with the greatest reduction in conservativeness realised when estimating exogenous conservativeness on a SCL soil at high compliance probability levels. The estimated endogenous conservativeness is always greater than the exogenous conservativeness and more so when the conservativeness is very high, irrespective of soil type or fertiliser application method. The conclusion is that the conservativeness of the UPM as measured by the exogenous and endogenous conservativeness is very high. However, the size of the conservativeness is very situation-specific and varies due to differences in fixed resources, fertiliser application methods and conservativeness measure.

The conservativeness of the UPM will result in over-regulation since the shadow price for the environmental outcome is derived from conservative responses. Failure to consider the trade-offs generated with the UFM may result in misidentification of management options to control pollution.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

An estimation of economic-environmental trade-offs requires a clear understanding of the linkages between production risk and environmental risk. In this research it is argued that the linkages between production decisions, production risk and environmental risk are not clearly understood. The obscurity is the direct result of a general lack of procedures and methods to appropriately model the interactions between risk-averse decision makers’ input use decisions and the resulting environmental outcomes. A review of the literature showed that state-contingent production theory may provide the theoretical foundation to model these interactions more realistically through the use of an empirical representation of risk.

The main objective of this research is therefore to develop methods and procedures to more accurately quantify the trade-off between improving production risk and environmental degradation using state-contingent theory to quantify economic and environmental risk with empirical distributions. Although policy instruments to control NPS pollution was not evaluated the modelling framework provides a powerful basis to evaluate these instruments more realistically. The research thus focussed on modelling the complex relationship of agricultural input decision-making, production risk and environmental risk which is a prerequisite for policy design.

The main objective is achieved through the achievement of three sub-objectives. Next, the conclusion for each of the sub-objectives of the research is discussed followed by some recommendations.

6.2 MODELLING INPUT USE DECISIONS WITH SKEWED PRODUCTION RISK

Sub-objective 1: To model input use decision-making behaviour with skewed production risk through the development of a state-contingent direct expected utility model.
Conclusions and Recommendations

The results from the state-contingent approach that was used to model the risk efficiency of fertiliser input decisions clearly show the ability of the state-contingent approach to model skewed outcome distributions. The continuous relationship between input use and the empirically distributed production risk provide a flexible and empirically tractable means of modelling skewed outcome distributions which easily combines with utility maximisation to study risk decision-making. Antle (2010) argues that flexible procedures, such as the ones developed in this study, may advance the empirical understanding of how management decisions influence production risk.

Results also indicated that the risk preferences of the decision makers play a very important role in fertiliser application decisions. Optimised fertiliser application rates show that fertiliser act as a risk reducing input on sandy clay (SC) soils while application rates could be either risk increasing or risk reducing on a sandy clay loam (SCL) soil. The risk efficiency analyses indicated that a single fertiliser application rate is preferred irrespective of soil type while less risk-averse farmers will choose a SCL soil and more risk-averse farmers will prefer a SC soil. The main conclusion is that the generalisation in classifying fertiliser input as either risk decreasing or increasing is not possible since the results showed that fertiliser can act as a risk reducing, risk increasing or both. Fertiliser recommendations are therefore complicated. The fact that fertiliser use is risk reducing on a SC soil further emphasises the importance of taking risk preferences of decision makers into consideration when modelling economic environmental trade-offs.

6.3 MODELLING ECONOMIC-ENVIRONMENTAL TRADE-OFFS USING A SAFETY-FIRST CONSTRAINT

Sub-objective 2: To model the economic-environmental trade-offs using a safety-first constraint while taking production and environmental risk into account.

The results from the trade-off analysis confirmed that the state-contingent approach provides a theoretical framework to simultaneously model the impact of risk aversion while taking environmental pollution risk into account. The state-contingent production functions and environmental response functions create a clear link between the source of the pollutant, the production risk stemming from its use and the environmental outcome.

The results further showed that the presence of an environmental constraint impact production decisions significantly with substantial compliance costs, which are the direct results of changes in the intensive and extensive margin responses of risk-averse decision makers. The importance
of soil choice on minimising compliance costs is evident from the results since the risk efficiency frontier is associated with production on a SCL soil only. The choice between a single and a split fertiliser application is not as straightforward as the compliance level, soil type and the level of risk aversion influence the choice of application technique. Less risk-averse decision makers tend to prefer a split application, while more risk-averse farmers tend to move towards a single application. However, the difference in compliance cost due to choice of fertiliser application technique is relatively small. The conclusion is that soil choice is more important than application method when decision makers aim at reducing compliance costs. Results on intensive margin changes show that fertiliser application rates are risk decreasing, irrespective of application method or soil type and that extensive margin reductions in the area irrigated is necessary to ensure environmental compliance. Producers therefore, substitute land with increased levels of fertiliser use. The conclusion is that policy that aims at a reduction in input use per hectare may not yield the intended environmental consequences. The extensive margin responses of decision makers play an important role in reducing the magnitude of the environmental outcome. Effective policy should consider intensive and extensive margin responses simultaneously.

6.4 QUANTIFYING THE CONSERVATIVENESS OF THE UPPER PARTIAL MOMENT

Sub-objective 3: To develop an alternative non-linear trade-off model that can be used to quantify the conservativeness of the UPM.

The newly developed Upper Frequency Method (UFM) was able to estimate the economic-environmental trade-offs without the conservativeness associated with the UPM. The less conservative UFM ensures that the probability limits that are enforced are very close to the true probability limits. The probability bound is, however, a function of sample size and a large number of observations are necessary to ensure close probability bounds.

The results indicated that the cost of conservativeness on a SCL soil is higher compared to the cost on a SC soil, irrespective of conservativeness method. The choice of fertiliser application method affects compliance cost differently between the soil types and the choice of fertiliser application method is therefore not easy. The exogenous and endogenous conservativeness estimated was also very high with greater conservativeness estimated using the endogenous conservativeness measure. The conservativeness estimated decrease with increased compliance with the greatest reduction in conservativeness when estimated with the exogenous conservativeness measure on a SCL soil at high compliance probability levels. The conclusion is
that the conservativeness of the UPM is high. The size of the conservativeness is very situation specific and is determined by the combination of fixed resource used, fertiliser application measure, compliance probability and the conservativeness measure used. The conservativeness of the UPM can result in the over-regulation of agricultural production since the shadow prices used to ensure that the externality is internalised, are estimated based on a conservative response. Failure to consider the trade-offs generated with the UFM may result in misidentification of management options to control pollution.

6.5 RESEARCH RECOMMENDATIONS

Application of state-contingent theory provides a strong basis to enhance our understanding of how management actions influence risk which will lead to better risk decision-making models. The main reason for such a statement is that the application of state-contingent theory directs the researcher towards thinking about modelling the impact of management decisions within a specific state of nature on the outcome variable rather than determining the parameterised distribution of the outcome variable as a result of a specific decision. As a result the linkages between the management decisions and the outcome variables are better understood.

The following examples demonstrate how state-contingent theory may provide answers to research questions that are currently not answered satisfactorily.

- Grové (2007) developed procedures to incorporate the impact of risk into dynamic linear programming models to research irrigation technology investment decisions with the aim of conserving water. Long-run risk simulation procedures were used to parameterise the price and yield distributions to quantify the gross margin variability of activities. The DLP model maximises the long-run cashflow while considering the deviations in gross margins as the risk measure with MOTAD. Deviations in gross margins are not reconcilable with deviations in the cashflow of the farm. Within a state-contingent framework it is realised that changes in gross margin will impact differently on cashflows in each state of nature. A better representation of risk will result if separate cashflow calculations were done for each state of nature while maximising the utility derived from changes in the cashflows.

- Right hand side risk is typically modelled with chance constrained programming. Such an approach clearly fails to consider the impact of management decisions with limited resource availability on the objective function. Within irrigation agriculture a producer can change his risk exposure by changing the area irrigated. A state-contingent representation of the decision problem will require a relationship that quantifies the
Conclusions and Recommendations

impact of resource availability on area irrigated and crop yield within a specific state of nature. Given the availability of the transformation functions the risk of resource availability is transferred to the objective function where utility is maximised. The resulting area irrigated is thus a function of the utility derived from the decision and not a specified safety margin. Incorporating multiple sources of risk within a state-contingent framework are rather straight forward

Critical to the application of state-contingent theory is the availability of transformation functions which shows how management actions in a specific state of nature impact on the outcome variable. The following recommendations apply to the development of transformation functions.

- In this research a state of nature is captured by the aggregate influence of various state variables within a specific year. As a result the response functions were unable to explain all the variation in crop yield. A large number of state variables would characterise a state of nature best. More research is therefore necessary to determine state variables that characterise a state of nature best.

- Agricultural production is characterised by a dynamic environment in which producers need to manage input applications. An inherent requirement of a response function is that of technical efficiency. By implication the timing of input applications should result in the highest possible output. In this research a fixed timing schedule was assumed for fertiliser applications due to the amount of crop growth simulations necessary to determine the optimal timing of fertiliser applications. In this regard the use of genetic algorithms should be investigated as these algorithms are able to optimise complex simulation models. Use of genetic algorithms will allow the researcher to evaluate the impact of water limiting conditions in conjunction with fertiliser input use.

- An important note to crop growth model developers is that these biophysical models can only be used to develop response functions for management actions that are satisfactorily simulated with these models.

The methods and models developed in this thesis provide the basis for further analysis:

- A prerequisite for policy design is the availability of models and methods to model economics-environmental tradeoffs. The methods and procedures introduced in this research provides a framework for the identification of first-best solutions provided that the transformation functions, soil data and the decision makers risk behaviour is known. The modelling technique therefore allows the researcher the opportunity to evaluate
Conclusions and Recommendations

alternative taxes, standards and market incentives and the effect of policy regulation on
agricultural decision-making.

- The state-contingent framework allows for the determination of the value of information
  pertaining to which state of nature will occur. The value of information could be calculated
  by comparing the results from this research with results where fertiliser applications are
  applied assuming complete knowledge of which state of nature will occur.

- Significant differences in the production risk between different soils were identified in this
  research. The differences in production risk between soils will allow farm managers the
  opportunity to use diversification to manage production risk which should be further
  researched. Further analyses should also include price risk as a source of risk.
REFERENCES


References


References


TABLE A1: Coefficients estimated for maize crop yield (tonnes/ha) production function for nitrogen applications (kg/ha) in state of nature s on a SCL and SC soil with single and split nitrogen application

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<td>Intercept</td>
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<td>0.959</td>
<td>8.821***</td>
<td>0.008***</td>
<td>-2.59E-05***</td>
<td>0.959</td>
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*** significant at a 1% level  ** significant at a 5% level  * significant at a 10% level
ESTIMATED STATE-CONTINGENT IRRIGATION RESPONSE FUNCTIONS
### TABLE B1: Coefficients estimated for irrigation water requirement (mm ha⁻¹) for nitrogen applications (kg ha⁻¹) in state of nature s on a SCL and SC soil with single and split nitrogen application

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<td>Intercept</td>
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**significant at a 1% level  **significant at a 5% level  *significant at a 10% level
APPENDIX C

ESTIMATED STATE-CONTINGENT NITRATE LOSS RESPONSE FUNCTIONS
## TABLE C1: Coefficients estimated for nitrate response function for nitrogen applications (kg/ha) in state of nature s on a SCL and SC soil with single and split nitrogen application

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<th>NN</th>
<th>R²</th>
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*** significant at a 1% level  ** significant at a 5% level  * significant at a 10% level